A Follow up to Study the Behaviour of Cable Bolts in Shear: Experimental Study and Mathematical Modelling

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A FOLLOW UP TO STUDY THE BEHAVIOUR OF CABLE BOLTS IN SHEAR: EXPERIMENTAL STUDY AND MATHEMATICAL MODELLING

Naj Aziz, Ali Mirza, Jan Nemcik, Haleh Rasekh and Xuwei Li

ABSTRACT: A mathematical model which is able to determine the pure shear strength of cable bolt was introduced and calibrated for various conditions. The proposed equation is developed based on the Fourier series concept and a linear relationship between shear and normal forces generated during cable bolt shearing. The conventional double shearing instrument was up to date to evaluate the pure shear strength of cable bolt by removing the contribution of concrete block frictional force. The experimental data obtained from the revised double shear instrument was in good agreement with the prediction of the proposed mode.

INTRODUCTION

British Standard (BS 7861-2: 1996) since its development has been widely incorporated to determine shear strength properties of various tendons used in mining and civil industries. This standard indeed replicates guillotine box shearing whereby the bolt is cut across its cross section without being encapsulated in the host strata. As shown in Figure 1, the bolt in the passive mode (i.e. no pretension) was positioned inside the apparatus and loaded vertically until the pronounced failure was observed.

Despite of the fact that the British Standard can be carried out easily and without cumbersome initial sample preparation work, it possesses some conspicuous drawbacks as:

- The shear strength is underestimated as the bolt is not pretensioned prior to shearing,
- The testing assembly inevitably will lead to a metal to metal shearing (Figure 2),
- The bolt is not encapsulated inside the host strata, resulting in different confinement stiffness as compared to the field conditions.

Figure 1: Sectional diagram of double embedment shear frame with the unit being tested (BS 7861-2: 1996)
Aziz et al., (2003) incorporated the concept of double shearing to investigate the shear strength of three common types of bolt used in Australian mining industries. The instrument that was developed in this testing program is shown in Figure 3. Unlike the single shear test of British Standards (BS 7861-2: 1996), the bolt was encapsulated inside the concrete blocks, representing the host strata and pretensioned prior to shearing.

Aziz et al., (2005) continued the previous study to investigate the effect of resin thickness in shear for bolt-grout-concrete interaction by double shear testing. ANSYS commercial software was applied to simulate numerically the experimental results obtained from this study.

Craig and Aziz (2010) studied shear behavior of cable bolts using a large scale double shear instrument as shown in Figure 4. It consisted of three concrete blocks with outer two cube blocks of 300 x300 and 300 mm sides and the middle block of 300 x 300 x 450 mm sides. The concrete blocks were cast in the steel frame of the double shear apparatus, and before the concrete could be cast, a 20 mm diameter conduit pipe was placed through the pre-cut holes in the centre of the wooden ends and galvanized steel separators of the mould. The cable bolt was eventually encapsulated in the concrete blocks using an appropriate grout.
An experimental study on the shear performance of plain and spiral cable bolts was carried out by Aziz et al., (2014) using double shear apparatus. Other studies on shear behavior of cable bolts incorporating the same experimental instrument, were reported by Aziz et al., (2015a), Rasekh et al., (2015) and Li et al., (2015a and b).

An original mathematical model for the shear behavior of cable bolt was introduced by Aziz et al., (2015b). The model was associated with a set of systematic experimental study. Experimental study was performed and results were compared with the proposed mathematical equation.

The above mentioned mathematical equation designates the combination of shear strength of cable bolt and frictional force generated due to concrete block sliding. Since then the model has been further developed which calculates only the pure shear strength of cable bolt. This aspect of the study together with modified double shear equipment is the subject of this paper.

**MATHEMATICAL MODELING**

The mathematical model is based on the assumption of a linear relationship between the shear and normal loads as:

\[ S - N \tan(\phi) - c = 0 \]  

(1)

where, \( S \) is the shear load, \( N \) is the normal load, \( \phi \) is the friction angle and \( c \) is the cohesion.

The Fourier series concept is applied to replicate the variation of the normal load against shear displacement. Fourier series is a mathematical technique incorporated to solve a large variety of engineering problems mainly adopting the principle of superposition:

\[
N = \frac{a_n}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos \left( \frac{2n\pi u}{T} \right) + b_n \sin \left( \frac{2n\pi u}{T} \right) \right]
\]  

(2a)

\[
a_n = \frac{2}{T} \int_0^T \sigma_n \cos \left( \frac{2n\pi u}{T} \right) du
\]  

(2b)
\[ b_n = \frac{2}{T} \int_0^T \sigma_n \cos\left(\frac{2n \pi u}{T}\right) du \]  

(2c)

where, \( a_n \) and \( b_n \) are Fourier coefficients, \( n \) is the number of Fourier coefficient, \( u \) is the shear displacement and \( T \) is the shearing length.

Introducing Equations (2a, b, and c) in equation (1) by considering \( a_0 \) to \( a_3 \), the shear strength is obtained as:

\[ S = \left(\frac{a_0}{2} + \sum_{n=1}^3 a_n \cos\left(\frac{2n \pi u}{T}\right)\right) \tan(\phi) + c \]  

(3)

The shear displacement at peak shear strength is determined by taking derivation of the above relationship respect to the shear displacement and equating to zero as:

\[ \frac{d}{du} \left[ \left(\frac{a_0}{2} + \sum_{n=1}^3 a_n \cos\left(\frac{2n \pi u}{T}\right)\right) \tan(\phi) + c \right] = 0 \]  

(4)

Thus, the peak shear displacement at peak shear strength \( (u_p) \) is obtained as:

\[ u_p = \frac{T}{2\pi} \cos^{-1}\left[ \frac{-4a_2 + \sqrt{16a_2^2 - 48a_4a_3 + 144a_3^2}}{24a_3} \right] \]  

(5)

Introducing equation (5) in equation (3), the peak shear strength \( (S_p) \) is proposed as:

\[ S_p = \left(\frac{a_0}{2} + \sum_{n=1}^3 a_n \cos\left(\frac{2n \pi u_p}{T}\right)\right) \tan(\phi) + c \]  

(6)

The model coefficients including Fourier coefficients \( (a_n) \), cohesion \( (C) \) and angle of friction \( (\phi) \) are determined according to the measured data for various test conditions such as the cable type and pre-tension. Generally, the values of Fourier coefficients showed a decreasing trend with increasing the number of Fourier coefficients.

Equation 6 determines the total shear strength of reinforced concrete blocks. This consists of cable bolt shear strength and the additional shear force generated by the concrete surface friction. In order to obtain the pure shear strength of the cable bolt, the frictional term should be quantified and subsequently deducted from the total shear strength as indicated by equation 6.

The frictional force generated in the process of shearing follows the Coulomb tribological equation as:

\[ S = N \tan(\phi_f) \]  

(7)
where, $\varphi_b$ is the concrete surface basic friction angle determined by tilt testing.

Deducting equation 7 from equation 6, the pure shear strength of cable bolt ($S_p^b$) is obtained as:

$$S_p^b = \left(\frac{a_0}{2} + \sum_{n=1}^{3} a_n \cos\left(\frac{2n\pi T}{2\pi}\right) \cos^{-1}\left[\frac{-4a_2 + \sqrt{16a_2^2 - 48a_1a_3 + 144a_3^2}}{24a_3}\right] \right) \left[(\tan(\varphi) - \tan(\varphi_b)) + c\right]$$

Concrete surface basic friction angle

Double shearing test (Aziz et al., 2015b), without cable bolt as the reinforcing element, was carried out to determine the concrete surface basic friction angle. The normal load subjected to concrete blocks started with 50 kN and increased incrementally every 20 mm, reaching to 250 kN at the end of the test. The value of shear load against shear displacement was measured and subsequently incorporated to calculate the concrete surface basic friction angle as shown in Figure 5. The basic friction angle was indicated as 26.94°.

![Figure 5: test results of the concrete blocks sliding test](image)

By introducing the value of basic friction angle in Equation 8, the pure shear strength of cable bolt is obtained as:

$$S_p^b = \left(\frac{a_0}{2} + \sum_{n=1}^{3} a_n \cos\left(\frac{2n\pi T}{2\pi}\right) \cos^{-1}\left[\frac{-4a_2 + \sqrt{16a_2^2 - 48a_1a_3 + 144a_3^2}}{24a_3}\right] \right) \left[(\tan(\varphi) - \tan(26.94^\circ)) + c\right]$$

(9)
Model calibration

The coefficients in equation 9 were calibrated for various conditions of cable type, pre-tension value and bonding agent incorporating the experimental data reported by Aziz et al., (2015b) and listed in Table 1.

MODEL VERIFICATION

To verify the proposed equation for the pure shear strength of cable bolts, two new double shear tests were undertaken. In one test the concrete surfaces were maintained in contact with each other and in the other without, that is no frictional resistance. In both tests various parameters, such as pretension load, grout type and concrete strength were kept constant. To achieve cable bolt shearing without contact between concrete blocks, the double shear apparatus was modified by installing two lateral braces on each side of the assembly as shown in Figure 6a to impede subjecting normal load on concrete blocks during shearing. To further assure no friction between concrete blocks, a pair of Teflon sheets with negligible fiction coefficient was introduced between concrete joints as illustrated in Figure 6b. Table 2 compares the values of the pure shear strength of cable bolts obtained from the proposed equation and experiments. It is clear that the experimental test result is reasonably close with the mathematical equation modelling.

Figure 6: Up to date double shear instrument (a) the whole assembly inside compression machine (b) Teflon sheet layers between concrete blocks
## Table 1a: list of tested cables and the test environment Aziz et al., (2015b)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Product name</th>
<th>Cable Ø (mm)</th>
<th>Wire geometry</th>
<th>Cable cross-section</th>
<th>Cable geometry</th>
<th>Drill bit (mm)</th>
<th>Bonding agent</th>
<th>Pre-tension load (kN)/Peak axial load</th>
<th>Peak shear load (kN) [½ double shear]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Superstrand</td>
<td>21.8</td>
<td>Spiral</td>
<td>19 wire, PC strand</td>
<td>Non-birdcaged</td>
<td>28</td>
<td>Oil based resin</td>
<td>250/558</td>
<td>558</td>
</tr>
<tr>
<td>2</td>
<td>Superstrand</td>
<td>21.8</td>
<td>Plain</td>
<td>19 wire, PC strand</td>
<td>Non-birdcaged</td>
<td>28</td>
<td>Oil based resin</td>
<td>250/628</td>
<td>628</td>
</tr>
<tr>
<td>3</td>
<td>TG</td>
<td>28</td>
<td>Spiral</td>
<td>9 wires, hollow centre</td>
<td>Non-birdcaged</td>
<td>42</td>
<td>TD80 Grout</td>
<td>250/604</td>
<td>604</td>
</tr>
<tr>
<td>4</td>
<td>SUMO</td>
<td>28</td>
<td>Spiral</td>
<td>9 wires, hollow centre</td>
<td>35mm birdcage</td>
<td>42</td>
<td>TD80 Grout</td>
<td>250/414</td>
<td>414</td>
</tr>
<tr>
<td>5</td>
<td>SUMO</td>
<td>28</td>
<td>Spiral</td>
<td>9 wires, hollow centre</td>
<td>35mm birdcage</td>
<td>42</td>
<td>TD80 Grout</td>
<td>100/488</td>
<td>488</td>
</tr>
<tr>
<td>6</td>
<td>Plain SUMO</td>
<td>28</td>
<td>Plain</td>
<td>9 wires, hollow centre</td>
<td>35mm birdcage</td>
<td>42</td>
<td>TD80 Grout</td>
<td>250/711</td>
<td>711</td>
</tr>
<tr>
<td>7</td>
<td>Plain SUMO</td>
<td>28</td>
<td>Plain</td>
<td>9 wires, hollow centre</td>
<td>35mm birdcage</td>
<td>42</td>
<td>TD80 Grout</td>
<td>100/659</td>
<td>659</td>
</tr>
<tr>
<td>8</td>
<td>Gardford</td>
<td>15.2</td>
<td>Plain</td>
<td>2 x 7 wire, PC strand</td>
<td>25mm Bulbs</td>
<td>55</td>
<td>BU100 Grout</td>
<td>0/501</td>
<td>501</td>
</tr>
</tbody>
</table>

## Table 1b: Model coefficients for different types

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$\phi$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>624.59</td>
<td>-53.97</td>
<td>-28.72</td>
<td>25.73</td>
<td>52.13</td>
<td>8.82</td>
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<tr>
<td>2</td>
<td>619.70</td>
<td>-7.87</td>
<td>-77.06</td>
<td>65.73</td>
<td>51.41</td>
<td>0.27</td>
</tr>
<tr>
<td>3</td>
<td>636.82</td>
<td>-67.87</td>
<td>-14.88</td>
<td>18.57</td>
<td>37.53</td>
<td>0.16</td>
</tr>
<tr>
<td>4</td>
<td>387.59</td>
<td>83.53</td>
<td>-36.68</td>
<td>-5.05</td>
<td>51.23</td>
<td>67.8</td>
</tr>
<tr>
<td>5</td>
<td>335.31</td>
<td>-27.32</td>
<td>-62.84</td>
<td>40.02</td>
<td>61.7</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>534.76</td>
<td>3.47</td>
<td>-75.64</td>
<td>55.49</td>
<td>59.56</td>
<td>12.66</td>
</tr>
<tr>
<td>7</td>
<td>449.78</td>
<td>-136.34</td>
<td>16.39</td>
<td>-3.91</td>
<td>61.33</td>
<td>0.44</td>
</tr>
<tr>
<td>8</td>
<td>235.38</td>
<td>-157.50</td>
<td>42.83</td>
<td>-4.21</td>
<td>47.61</td>
<td>137.89</td>
</tr>
</tbody>
</table>

## Table 2a: Comparison between the proposed model for the pure shear strength of cable blots and experimental data

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Product type</th>
<th>Bonding agent</th>
<th>Pre-tension load (kN)</th>
<th>Peak shear load per face (kN)</th>
<th>Friction between surfaces</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>Plain</td>
<td>Strata binder HS</td>
<td>5</td>
<td>645.64</td>
<td>with</td>
</tr>
<tr>
<td>10</td>
<td>Plain</td>
<td>Strata binder HS</td>
<td>5</td>
<td>442.16</td>
<td>without</td>
</tr>
</tbody>
</table>

## Table 2b: Determination of shear load by the model

<table>
<thead>
<tr>
<th>Test</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>Model normal load (kN)</th>
<th>$\tan \phi$</th>
<th>$\tan 26.94^\circ$</th>
<th>$c$ (kN)</th>
<th>Measured peak shear load per face (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain superstrand</td>
<td>324.77</td>
<td>182.37</td>
<td>18.65</td>
<td>3.04</td>
<td>366.316</td>
<td>1.47</td>
<td>0.508</td>
<td>88.61</td>
<td>441.006</td>
</tr>
<tr>
<td>with 5 kN pre-tension</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CONCLUSIONS

- A new mathematical model is developed to calculate pure shear strength of cable bolts. The model was tested against the experimental data. The calculated shear failure load was in close agreement with the experimental test results. The model can also be used for other types of tendons used for ground reinforcement.
- The modified double shear apparatus that is capable of determining pure shear strength of the tendons alone was successfully tested. The initial test was undertaken using a cable bolt and further experimental studies are planned for testing of different marked cables used in Australia for both civil and mining engineering constructions.

REFERENCES


