2005

Response of Mains Connected Induction Motors to Low Frequency Voltage Fluctuations from a Flicker Perspective

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Publication Details

Tennakoon, S., Perera, S. & Robinson, D. A. (2005). Response of Mains Connected Induction Motors to Low Frequency Voltage Fluctuations from a Flicker Perspective. In M. Negnevitsky (Eds.), AUPEC 2005 Australasian universities Power Engineering Conference (pp. 1-5). Tasmania: The University of Tasmania.

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Keywords
flicker, fluctuations, frequency, low, response, motors, connected, mains, voltage, perspective, induction

Disciplines
Physical Sciences and Mathematics

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ABSTRACT

In radial power systems flicker transfer from a higher voltage level (upstream) to a lower voltage level (downstream) is seen to be significantly affected by the downstream load composition. Industrial load bases containing mains connected induction motors are known to be effective in the flicker attenuation process compared to residential load bases containing passive loads. For better understanding of the flicker attenuation influenced by induction motors their dynamic behaviour has to be closely investigated under fluctuating supply conditions. This paper presents the methodology and results of investigations undertaken to examine the response of an induction motor to small perturbations in the supply voltage using a transfer function approach (small signal modelling). The motor response established employing this approach is used to evaluate the effective impedance of the motor which in turn is used to explain the flicker attenuation at the point of common coupling (PCC) in a qualitative manner. Furthermore, the dependency of the effective impedance on the frequency of voltage fluctuations is also examined.

1. INTRODUCTION

Voltage fluctuations leading to lamp flicker are caused by chaotic loads such as electric arc furnaces. Flicker can penetrate into the power system by propagating though the transmission, sub transmission and distribution systems. Flicker propagation takes place with some level of attenuation depending on factors such as fault levels, loading levels at bus bars and compositions of loads. The degree of attenuation of flicker at one point (B) with respect to another (A) is described using the flicker transfer coefficient (T_{PstAB}), and is defined as the ratio between the P_{st} (short term flicker severity) values measured simultaneously at the two locations [1].

$$T_{PstAB} = \frac{P_{stB}}{P_{stA}} \tag{1}$$

Flicker propagation from a higher voltage level to a lower voltage level is significantly affected by the downstream load composition [1], [3]-[5]. Industrial load bases containing mains connected induction motors tend to assist flicker attenuation better compared to residential load bases containing passive loads. The better flicker attenuation caused by an induction motor can be crudely justified using the argument that dynamic impedance of an induction motor is relatively small compared to its steady state value. However, the validity of this argument has not been theoretically proven thus far. Further, the flicker produced by fluctuating loads such as electric arc furnaces contain numerous frequency components and whether all these components are equally attnuated or not is yet to be explored. Therefore, in order to understand how induction motors help attenuate flicker and to explore the dependency of the flicker attenuation on modulation frequency of the voltage fluctuations, the dynamic behaviour of induction motors has to be closely investigated.

The paper presents the methodology and the results of an investigation carried out with the objective of examining the dynamic behaviour of induction motors in relation to flicker attenuation. A transfer function has been formulated relating the small perturbations in the supply voltage and the resultant stator current. Transfer function is used to examine the response of the motor to small fluctuations in the supply. This enables the establishment of an effective impedance of the motor which in turn is used to predict the flicker attenuation due to induction motors at the (PCC) in a qualitative manner.

The paper is organised as follows: Section 2 describes the concept of the flicker transfer coefficient applied to a simple radial system where the crude argument behind flicker attenuation is illustrated. The key steps in the development of a small signal model suitable for the analysis of the induction motor behaviour in relation to flicker attenuation are described in Section 3 together with results in relation to a selected induction motor. Section 4 presents the variation of the effective impedance with injected frequencies for the motor obtained using the transfer function approach and Section 5 summarises the results and gives broad conclusions.

2. FLICKER TRANSFER COEFFICIENT FOR A SIMPLE RADIAL SYSTEM

For the simple radial system shown in Figure 1, assuming the flicker severity to be proportional to the relative changes in the voltage magnitudes (voltage fluctuation), upstream (A) to downstream (B) flicker transfer coefficient can be defined as [1]:

$$T_{PstAB} = \frac{|\frac{\Delta v_B}{v_B}|}{|\frac{\Delta v_A}{v_A}|} = \frac{|1 + \frac{Z_S}{Z_L}|}{|1 + \frac{Z'_S}{Z'_L}|} \tag{2}$$

where,
$\Delta v_A, \Delta v_B$ - fluctuations in the magnitudes of the voltages at A and B respectively,
v_A, v_B - magnitude of the steady state voltages at A and B respectively,
Z_L- steady state load impedance,
Z'_L - effective impedance of the load for small voltage perturbations,
Z_S - steady state impedance of the system that connects the upstream and the downstream (e.g. transformer impedance),
Z'_S - effective Z_S for small voltage perturbations,

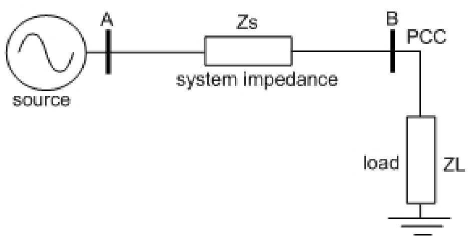

Figure 1: Simple radial system

(2) suggests that, the level of flicker attenuation at B with respect to A is mainly governed by the effective impedance of the load (Z'_L) as the effective impedance of the system under fluctuating conditions can be assumed to be equal the steady state system impedance (i.e. $Z_S = Z'_S$). For a passive load its steady state and effective impedances are approximately equal (i.e. $Z_L \approx Z'_L$) whereas for an induction motor its effective impedance is assumed to be approximately equal to its dynamic impedance which would be smaller than the steady state impedance (i.e. $Z'_L < Z_L$). This suggests that flicker transfer coefficient would be unity if the downstream load is passive whereas it would be less than unity if the downstream load is an induction motor. Further, (2) also suggests that the level of flicker attenuation depends on the magnitude of Z'_L.

3. SMALL SIGNAL MODELLING OF INDUCTION MOTORS

3.1. LINEARISED MACHINE EQUATIONS

The voltage (v) equations for an induction motor operating at steady state balanced condition can be expressed in the synchronously rotating reference frame considering flux linkage per second (ψ) as state variable as per (3) [6].

$$v = M\psi \tag{3}$$

where,

$$v = \begin{bmatrix} v_{qs} & v_{ds} & v'_{qr} & v'_{dr} \end{bmatrix}^T \tag{4}$$

$$M = \begin{bmatrix} \frac{r_s X'_{rr}}{D} + \frac{p}{\omega_b} & \frac{\omega_e}{\omega_b} & -\frac{r_s X_M}{D} & 0 \\ -\frac{\omega_e}{\omega_b} & \frac{r_s X'_{rr}}{D} + \frac{p}{\omega_b} & 0 & -\frac{r_s X_M}{D} \\ -\frac{r'_r X_M}{D} & 0 & \frac{r'_r X_{ss}}{D} + \frac{p}{\omega_b} & \frac{\omega_e - \omega_r}{\omega_b} \\ 0 & -\frac{r'_r X_M}{D} & -\frac{\omega_e - \omega_r}{\omega_b} & \frac{r'_r X_{ss}}{D} + \frac{p}{\omega_b} \end{bmatrix} \tag{5}$$

$$\psi = \begin{bmatrix} \psi_{qs} & \psi_{ds} & \psi'_{qr} & \psi'_{dr} \end{bmatrix}^T \tag{6}$$

where,
r_s - stator resistance [Ω],
r'_r - rotor resistance [Ω],
X_{ls} - stator leakage reactance [Ω],
X'_{rl} - rotor leakage reactance [Ω],
X_M - mutual reactance [Ω],
ω_b - base angular frequency [rad/s],
ω_e - synchronous speed [rad/s],
p - $\frac{d}{dt}$ operator,
subscripts / superscripts:
s - stator parameters,
r - rotor parameters,
$'$ - referred to the stator,

$$X_{ss} = X_{ls} + X_M \tag{7}$$

$$X'_{rr} = X'_{lr} + X_M \tag{8}$$

$$D = X_{ss}X'_{rr} - X_M^2 \tag{9}$$

The per unit electromagnetic torque (T_e), rotor angular speed (ω_r) and the load torque (T_L) are related according to:

$$T_e = \frac{X_M}{D}(\psi'_{dr}\psi_{qs} - \psi'_{qr}\psi_{ds}) \tag{10}$$

$$T_e = 2Hp\frac{\omega_r}{\omega_b} + T_L \tag{11}$$

where,
H - inertia constant.

Considering a load such as a pump with characteristics given by (12), load dynamics can be combined with (11) as per (13).

$$(T_L)_{pu} = k\frac{\omega_r^2}{\omega_b^2} \tag{12}$$

$$T_e = 2Hp\frac{\omega_r}{\omega_b} + k\frac{\omega_r^2}{\omega_b^2} \tag{13}$$

A linearised motor model from which transfer functions can be established by considering small variations about the steady state operating point of all control (input) and output variables of the motor [6]-[9]. Such a linearised model can be treated as a small signal model of the induction motor and represented in state space from as:

$$px = Ax + Bu \tag{14}$$

where,

$$u = \begin{bmatrix} \Delta v_{qs} & \Delta v_{ds} & \Delta v'_{qr} & \Delta v'_{dr} \end{bmatrix}^T \tag{15}$$

$$x = \begin{bmatrix} \Delta\psi_{qs} & \Delta\psi_{ds} & \Delta\psi'_{qr} & \Delta\psi'_{dr} & \frac{\Delta\omega_r}{\omega_b} \end{bmatrix}^T \tag{16}$$

A, B - constant matrices

3.2. Transfer function between small variations in motor supply voltage and stator current

The fluctuation in the normal supply voltage waveform could be a step change, a sinusoidal change or a square wave change. Sinusoidal or square wave changes are typical examples for voltage fluctuations considered in theoretical flicker studies. In the present study sinusoids superimposed on the fundamental voltage are used as the fluctuating components, thus forming the basis to examine the behaviour of an induction motor to any nominated supply voltage change of relevance to flicker.

With a sinusoidally modulated fluctuating voltage supply, the stator voltage is given by (17).

$$v(t) = V_p[1 + \frac{V_m}{V_p}\sin(2\pi f_m)t]\cos(2\pi f_b)t \qquad (17)$$

where,
f_m - modulating frequency
f_b - fundamental (base) frequency
V_m - amplitude of modulating waveform
V_p - amplitude of fundamental waveform
$\frac{V_m}{V_p}$ - modulation depth (m)

The per unit fluctuation in the amplitude of the supply voltage (Δv_s) can be expressed as:

$$\Delta v_s = \frac{V_m}{V_p}\sin(2\pi f_m)t \qquad (18)$$

In order to understand the motor dynamics under fluctuating supply conditions given by (17), it is necessary to examine the motor response to small voltage perturbations described by (18). This can be accomplished by formulating a transfer function between small perturbation in the amplitude of the stator voltage (Δv_s) and the resulting perturbation in the stator current (Δi_s) using the linearised machine equations given by (14).

For an induction motor operating with a balanced supply:

$$u = G\Delta v_s \qquad (19)$$

where,

$$G = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \end{array}\right]^T \qquad (20)$$

d-q axes small displacement stator currents (Δi_{ds} and Δi_{qs}) are related to the flux linkages as:

$$y = Cx \qquad (21)$$

where,

$$y = \left[\begin{array}{cc} \Delta i_{qs} & \Delta i_{ds} \end{array}\right]^T \qquad (22)$$

$$C = \frac{1}{D}\left[\begin{array}{ccccc} X'_{rr} & 0 & -X_M & 0 & 0 \\ 0 & X'_{rr} & 0 & -X_M & 0 \end{array}\right] \qquad (23)$$

Since the stator current is expressed in d-q axes components, there will be two transfer functions, $\frac{\Delta i_{qs}}{\Delta v_s}$ and $\frac{\Delta i_{ds}}{\Delta v_s}$ respectively. These can be established using (14) and (21) as per (24).

$$\left[\begin{array}{c} G_1(s) \\ G_2(s) \end{array}\right] = \left[\begin{array}{c} \frac{\Delta i_{qs}}{\Delta v_s} \\ \frac{\Delta i_{ds}}{\Delta v_s} \end{array}\right] = C(sI - A)^{-1}G \qquad (24)$$

$G_1(s)$ and $G_2(s)$ are fifth order transfer functions.

3.3. Evaluation of stator current response for an example motor

The transfer functions $G_1(s)$ and $G_2(s)$ have been evaluated for a three phase 2250hp, 4-pole, 60Hz squirrel cage induction motor that drives a pump load. Motor ratings and its parameters are as per Table 1 [6].

Table 1: 2250hp Induction motor data

Rated voltage$[V]$	2300
Rated current$[A]$	472
Rated power$[hp]$	2250
Rated speed $[rpm]$	1786
$r_s[\Omega]$	0.029
$X_{ls}[\Omega]$	0.226
$X_M[\Omega]$	13.04
$r'_r[\Omega]$	0.022
$X'_{lr}[\Omega]$	0.226
$J[kgm^2]$	63.87

Recalling (17), when the supply is amplitude modulated, small sinusoidal perturbation in the amplitude of the voltage (Δv_s) gives rise to two voltage perturbations (sidebands) $\Delta v_{LSB}(t)$ and $\Delta v_{USB}(t)$ at $f_b - f_m$ and $f_b + f_m$ respectively given by (25) and (26).

$$\Delta v_{LSB}(t) = \frac{V_m}{2V_p}\sin 2\pi(f_b - f_m)t \qquad (25)$$

$$\Delta v_{USB}(t) = \frac{V_m}{2V_p}\sin 2\pi(f_b + f_m)t \qquad (26)$$

These two voltage perturbations would cause a perturbation in the stator current ($\Delta i_s(t)$) which can be established using the response of $G_1(s)$ and $G_2(s)$. To establish the small variations in the stator currents $\Delta i_s(t)$, outputs of $G_1(s)$ and $G_2(s)$ have to be converted from d-q to a-b-c domain.

The frequency of the input signal to $G_1(s)$ and $G_2(s)$ was varied by varying modulation frequency, f_m, between 0.01Hz and 40Hz (usual range of frequencies of interest in flicker studies) while keeping the modulation depth constant. As expected, for each f_m, two side bands (Δi_{LSB}, Δi_{USB}) at $f_b \pm f_m$ appear in the resultant stator current perturbation. Figure 2 shows the resultant stator current perturbation and its frequency spectrum for f_m =10Hz for a modulation depth of 0.05 pu. Figure 3 illustrates the variation of the magnitudes of the two frequency components in the current perturbation (Δi_{LSB} and Δi_{USB}) with modulation frequency (f_m). Realising that there are two components in current having frequencies $f_b + f_m$ and $f_b - f_m$ respectively, the perturbations in the stator current of Figure 3 can be illustrated in an alternative form (Figure 4) where the injected frequency f is either $f_b - f_m$ or $f_b + f_m$. It is evident from Figure 3 that at low modulation frequencies, the magnitude of the stator current perturbations (Δi_{LSB} and Δi_{USB}) are relatively small (less than 20% of the steady state current for f_m <5Hz). However, as f_m increases Δi_{LSB} and Δi_{USB} tend to increase except Δi_{USB} tends stabilise

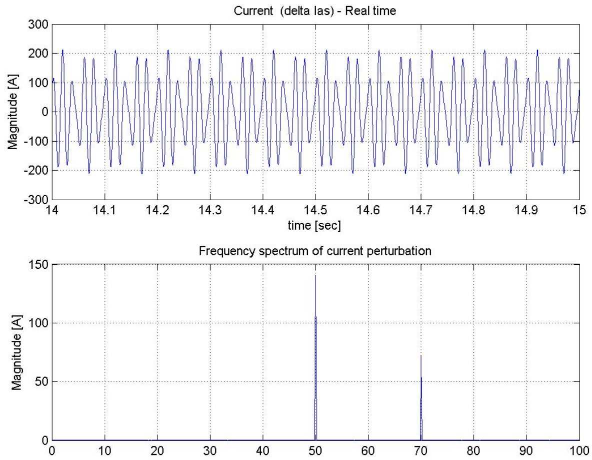

Figure 2: Stator current perturbation ($\Delta i_{as}(t)$) and its frequency spectrum for f_m =10Hz and m=0.05

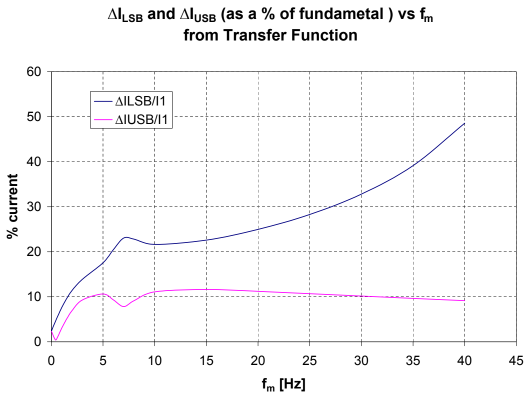

Figure 3: Variation of stator current perturbations (as a percentage of steady state current) with modulation frequency (f_m)

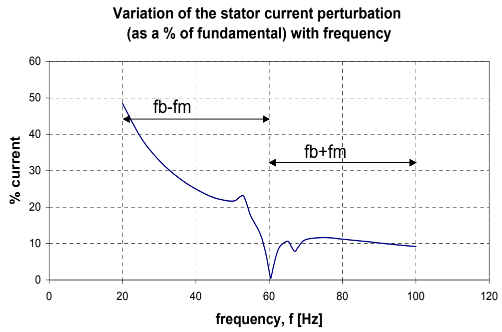

Figure 4: Variation of stator current perturbation (as a percentage of steady state current) (Δi_{as}) with frequency of the injected voltage perturbation (f); $f = f_b - f_m$ or $f = f_b + f_m$

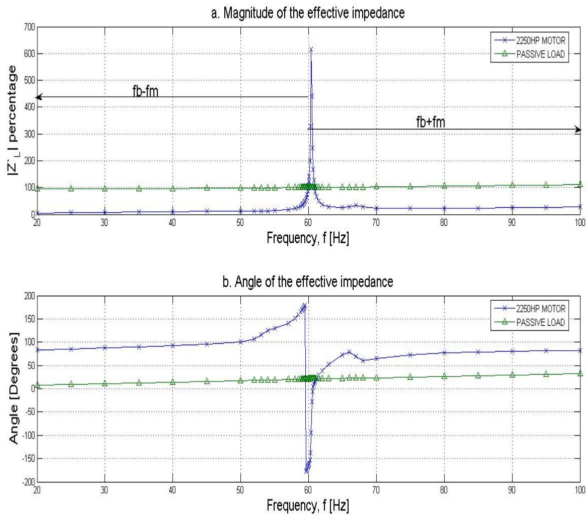

Figure 5: Variation of (a) the magnitude (b) angle of the effective impedance of the 2250hp motor and an equivalent passive load with frequency (f); $f = f_b - f_m$ or $f = f_b + f_m$

at a near constant level when f_m is greater than about 5Hz.

Although not reported explicitly the results presented in this section have been verified through time domain simulations of the example motor where there was very good agreement.

4. Effective impedance of the motor

In Section 3.3 the stator current responses were established for the motor operating with a fluctuating supply voltage which is amplitude modulated. These stator current responses can be used to determine the effective impedance of the motor (Z'_{motor}) for each injected voltage perturbation (Δv_i) as per (27).

$$Z'_{motor} = \frac{|\Delta_{vi}|}{|\Delta_i|} \angle \alpha_1 - \alpha_2 \tag{27}$$

where,
$|\Delta_{vi}|$- magnitude of the injected voltage perturbation ($\Delta v_{LSB}(t)$ or $\Delta v_{LSB}(t)$)
$|\Delta_i|$- magnitude of the current perturbation (Δi_{LSB} or Δi_{USB})
α_1- phase angle of injected voltage perturbation
α_2 - phase angle of current perturbation

In (27) subscript 'i' is used as a common notation to identify either sideband ($\Delta v_{LSB}(t)$ or $\Delta v_{USB}(t)$) as both are present at a single modulation frequency (f_m). Thus, Δv_i can be of any frequency between 20Hz and 100Hz. Figures 5(a) and 5(b) illustrate the variation of the magnitude (normalised using steady state impedance of the motor) and the phase angle of the effective impedance of the 2250hp motor and those of an equivalent passive load (i.e to the motor). It is seen from Figure 5(a) that the magnitude of the effective impedance of the motor ($|Z'_{motor}|$) for injected voltage perturbations varies between 5% and 30% of its steady state value ($|Z_{motor}|$) in the frequency range covered except for frequencies between 58Hz and 62Hz ($f \neq$60Hz). This implies that the motor under consideration would offer a relatively low impedance for the voltage fluctuations with modulation frequencies above 2Hz. Furthermore, $|Z'_{motor}|$ corresponding to the applied lower sideband component of voltage (i.e. perturbations with frequencies 20Hz-58Hz) is seen to be smaller than that for upper sideband components (i.e. perturbations with frequencies

62Hz-100Hz) At low modulation frequencies, that is as f approaches the fundamental frequency (60Hz), $|Z'_{motor}|$ tends to increase. According to Figure 5(b), between 59.5Hz$< f <$61Hz, effective impedance of the motor is greater than its steady state value ($|Z'_{motor}| > |Z_{motor}|$). In contrast to the varying effective impedance of the motor, the passive load exhibits a relatively constant impedance which is approximately equal to the steady state impedance ($|Z'_{passive}| = |Z_{passive}| = |Z_{motor}|$) at 60Hz. Thus, relatively small effective impedance of the motor compared to that of the passive load supports the crude argument used to justify flicker attenuation assisted by induction motors.

5. CONCLUSIONS

Investigations were carried out to examine the dynamic behaviour of induction motors in relation flicker attenuation. The response of a 2250hp induction motor operating under fluctuating supply conditions was examined using a transfer function approach. The stator current response to small perturbations in the supply voltage established from the transfer function was used to determine the effective impedance of the motor for voltage perturbations. It is seen that the stator current response for fluctuations in the supply depends on the frequency of the fluctuations (modulation frequency, f_m). The effective impedance ($|Z'_{motor}|$) offered by the motor for voltage perturbations by amplitude modulation is seen to be dependent on the frequency of voltage perturbation (f). $|Z'_{motor}|$ is relatively small compared to its steady state value for most of the frequencies and this small effective impedance would help attenuate voltage perturbations in the frequency range of 20Hz-58Hz and 62Hz-100Hz (covering most modulating frequencies up to 40Hz).

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