Laterally loaded rigid piles with rotational constraints

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Abstract
Elastic-plastic, closed-form solutions were developed recently by the author, to capture the nonlinear response of laterally loaded rigid piles. Presented in compact form, the solutions are convenient to use, and sufficiently accurate despite using only two input parameters of the net limiting force per unit length $pu$ along the pile, and a subgrade modulus $k$. Nevertheless, piles may be subjected to limited cap-restraints or loading below ground surface, which alter the response remarkably. This paper provides explicit expressions for estimating loading capacity of anchored piles and develops new solutions for lateral piles with cap-rotation by stipulating a constant $pu$ or a linear increasing $pu$ (Gibson $pu$) with depth. Lateral loading capacity $Ho$ (at the tip-yield state and yield at rotation point state) and maximum bending moment $Mm$ (at the tip-yield state) are presented against loading locations, and in form of the lateral capacity $Ho$. $Mm$ (applied moment) locus. The capacity is consistent with available solutions for anchored piles, and caissons with either $pu$ profile, allowing a united approach from lateral piles to anchored piles. The new solutions are also presented in charts to highlight the impact of rotational stiffness of pile-cap on nonlinear response, offering a united approach for free-head piles through fixed-head piles. Several advantages of the solutions are identified against the prevalent $p$-$y$ curve based approach. To estimate the key parameter $pu$, values of the resistance factor $Np$ (=ratio of pile-soil limiting resistance over the undrained shear strength $su$) are deduced using the current expressions against available normalised pile capacity involving the impact of gapping (between pile and soil), pile movement mode, pile slenderness ratio, inclined loading angle (anchored piles) and batter angles (lateral piles). The $Np$ is characterised by: (i) An increase from 5.6-8.6 to 10.14-11.6, as gapping is eliminated around lateral piles and caissons, and from 1.0-6.1 to 2.8-9.8, as translation is converted into rotation mode of footings. (ii) Similar variations with slenderness ratio between anchors and caissons (without gapping), and among anchors, caissons and pipelines (with gapping). And (iii) A reduction with loading angles (anchors) resembling that with batter angles (piles). © 2013 Elsevier Ltd.

Keywords
rotational, constraints, laterally, loaded, piles, rigid

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ABSTRACT

Elastic-plastic, closed-form solutions were developed recently by the author, to capture the nonlinear response of laterally loaded rigid piles. Presented in compact form, the solutions are convenient to use, and sufficiently accurate despite using only two input parameters of the net limiting force per unit length $p_u$ along the pile, and a subgrade modulus $k$. Nevertheless, piles may be subjected to limited cap-restraints or loading below ground surface, which alter the response remarkably.

This paper provides explicit expressions for estimating loading capacity of anchored piles and develops new solutions for lateral piles with cap-rotation by stipulating a constant $p_u$ or a linear increasing $p_u$ (Gibson $p_u$) with depth. Lateral loading capacity $H_o$ (at the tip-yield state and yield at rotation point state) and maximum bending moment $M_m$ (at the tip-yield state) are presented against loading locations, and in form of the lateral capacity $H_o$ - $M_o$ (applied moment) locus. The capacity is consistent with available solutions for anchored piles, and caissons with either $p_u$ profile, allowing a united approach from lateral piles to anchored piles. The new solutions are also presented in charts to highlight the impact of rotational stiffness of pile-cap on nonlinear response, offering a united approach for free-head piles through fixed-head piles.

Several advantages of the solutions are identified against the prevalent p-y curve based approach. To estimate the key parameter $p_u$, values of the resistance factor $N_p$ (= ratio of pile-soil limiting resistance over the undrained shear strength $s_u$) are deduced using the current expressions against available normalised pile capacity involving the impact of gapping (between pile and soil), pile movement mode, pile slenderness ratio, inclined loading angle (anchored piles) and batter angles (lateral piles). The $N_p$ is characterised by: (i) An increase from 5.6~8.6 to 10.14~11.6, as gapping is eliminated around lateral piles and caissons, and from 1.0~6.1 to 2.8~9.8, as translation is converted into rotation mode of footings. (ii) Similar variations with slenderness ratio between anchors and caissons (without gapping), and among anchors, caissons and pipelines (with gapping). And (iii) A reduction with loading angles (anchors) resembling that with batter angles (piles).

Keywords: soil/structure interaction, theoretical analysis, piles, plasticity, limit state design/analysis, bearing capacity, anchor piles
1. INTRODUCTION

Offshore exploration has propelled analytical, numerical and experimental investigation into bearing capacity of anchored piles [1-6]. This is seen in development of a complicated strength mobilization (SM) method [7], finite element method (FEM) [8] and plastic limit analysis (PLA) [9], among many others. The study provides the evolution law of the capacity with depth of loading attachment \( e^- \) for a constant \( p_u \) with depth (\( p_u = \) net force per unit length along the pile, \([\text{FL}^{-1}]\)) and a linearly increase \( p_u \) (Gibson \( p_u \)). [Note the symbol \( e \) is taken as negative (\( e^- \)) for depth of attachment to distinguish it from the positive (\( e^+ \)) loading above ground level]. The FEM and PLA analyses also reveal the variational law of the capacity with loading inclination angle (against horizon). To conduct practical design via these methods, one needs to determine the \( p_u \) that should incorporate the combined interaction among pile-movement (translation or rotation) mode, gapping and loading angle. This can be difficult and may be further complicated by ~ 4 times reduction in the \( p_u \) from free-head to fixed-head conditions [10-12]. A realistic \( p_u \) may be deduced by fitting available numerical and test results using closed-form solutions, as is evident in the deduced \( p_u \) profiles for 52 laterally loaded, flexible piles [13]. The corresponding closed-form solutions for lateral piles with rotating pile-cap, however, are not available. The impact of rotational constraints on the piles by the depth of attachment and/or cap-restraint remains to be determined.

The rigid piles refer to those with a pile-soil relative stiffness \( E_p/G_s \) being higher than 0.8322\((l/d)^4 \) [14] (Note: \( E_p \) is Young’s modulus of an equivalent solid pile \([\text{FL}^{-2}]\), \( G_s \) is soil shear modulus \([\text{FL}^{-2}]\), \( d \) is an outside diameter of a cylindrical pile \([\text{L}]\), and \( l \) is the pile embedded length \([\text{L}]\)). It should be cautioned that in the use of rigid-pile solutions to predict response of flexible piles [15], bending failure needs to be assessed against maximum bending moment in piles rather than against the applied bending moment.

The 52 \( p_u \) profiles deduced from test piles (\( e^+ \)) in clay, sand or multi-layered soil [13] allow the inadequacy of some prevalent \( p_u \) profiles to be revealed. To obtain \( p_u \) profile for an anchored pile or its like, pertinent literature for piles, caissons and footings in cohesive soil are reviewed herein. Murff and Hamilton [16] gained an elegant solution for estimating the \( p_u \) profile along rigid piles, but for the inability to incorporate the reverse resistance observed above pile-tip level. Aubeny et al [17] conducted the FEM and PLA
analyses on laterally loaded caissons with a slenderness ratio $l/d$ of 2~10. They demonstrated (i) a ~10% variation in the lateral capacity from anisotropic to isotropic strength profiles (smaller variation for $l/d > 6$, and no gapping between caisson and soil); (ii) a normalised rotation depth $z/r$ of 0.74~0.78 (FEM), or 0.70~0.76 (PLA) upon loading at mudline (without gapping between caisson and soil); (iii) a normalised capacity $H_0/(s_u dl)$ (ie. $N_p$) of 4.2~4.8 (without gapping) or 2.3~3.5 (with gapping), respectively for $l/d = 2~10$ and $e = 0$ ($e$ is a real or fictitious eccentricity of the load above ground level); $H_0$ is the lateral capacity; $s_u$ is undrained shear strength, which are rather close to those gained for anchor plates in clay [18]; and (iv) an $H_0/(s_u dl)$ of 10.2~11.9 (without gapping) or 5.2~8.6 (with gapping), respectively for loading at mid-depth ($e/l = - 0.5$). Yun and Bransby conducted FEM analysis on footing in cohesive soil [19], which resembles a short-rigid pile (with a full-length gap on one side). They also provided normalised capacity. These values of normalised capacity reviewed will be used to deduce the values of the factor $N_p$ for caissons, footings and anchored piles in cohesive soil.

The study to date has revealed that under a lateral load $H_l$ and moment loading $M_o (= H_l e)$, response of lateral piles [see Fig. 1a] is readily captured using a load transfer approach [14], underpinned by a series of springs (with a subgrade modulus $k$) distributed along the pile shaft (with limited impact of interaction among the springs). Each spring has a limiting force per unit length $p_u$ [varying with depth $z$, see Figs. 1b, 1c1 and 1c2]. With the ideal elastic-plastic $p-y(u)$ curves, Guo [20, 10] developed nonlinear closed-form solutions for free-head, rigid piles. The solutions reflect the consequence of mobilizing the on-pile force per unit length ($p$) along the limiting $p_u$ in the depth zones of $0 ~ z_o$ and $z_1 ~ l$, as indicated by the solid lines in Fig. 1c1 for a Gibson $p_u$ profile, and in Fig. 1c2 for a constant $p_u$ profile, respectively. The two $p_u$ profiles generally bracket real $p_u$ distributions with depth along piles [21, 22]. The associated solutions well capture nonlinear pile response against those based on real nonlinear p-y curves, despite use of only two input-parameters $k$ and $p_u$ [13]. Two critical states are defined: (i) Tip-yield state at which the on-pile force $p$ at the pile-tip just attains the limiting $p_u$ with $p = p_u$ at $z_1 = l$ [Fig. 1c1 or 1c2]; and (ii) the unachievable Yield at rotation point (YRP) state with the slip depths $z_0, z_1$ merging with the depth of rotation $z_r$ [i.e. $z_0 = z_1 = z_r$]. The values of lateral loads $H_l$ at the two states are taken as loading capacities $H_0$. 
This paper provides new expressions for estimating bearing capacity of anchored-piles, and develops new solutions for rigid piles with rotating cap for constant or Gibson $p_u$ profile. The expressions are compared with centrifuge tests, numerical (FEM, PLA) results, and complicated SM (rotation-dependent) solutions, and are subsequently used to examine the impact of $p_u$ profiles (with constant $k$), loading eccentricity, and yielding (at tip and rotation point) states on the capacity, displacement and rotation. The new solutions are used to examine the impact of rotational stiffness of pile-cap on nonlinear response, and to deduce the expressions for the capacity. Finally, the critical resistance factor $N_p$ for cohesive soil is deduced by matching the new expressions with the available normalised capacity involving the impact of gapping, translation or rotation, and loading angles. The $N_p$ is then synthesised into explicit expressions to facilitate estimating the $p_u$.

2. ELASTIC-PLASTIC SOLUTIONS

2.1 Model for Laterally Loaded Rigid Piles

In the framework of load transfer approach for lateral piles or anchored piles, the pile-soil interaction is characterised by the $p_u$ and the $k$ profiles.

In the elastic zone between depths $z_o$ and $z_1$ [Fig. 1c1 or 1c2], the on-pile force (per unit length), $p$ [FL$^{-1}$] ($< p_u$) at any depth is proportional to the local displacement, $u$ and the modulus of subgrade reaction, $kd$ [FL$^{-2}$] [see Fig. 1b]:

$$p = kdu$$  \hspace{1cm} \text{(Elastic state)} \hspace{1cm} (1)$$

where $k$ [FL$^{-3}$] is the gradient of the $p-u$ curve with $k = k_o z^n$, which is obtained using an average shear modulus $\tilde{G}$ over the pile embedment, and the expressions provided in Table 1. Later, a Gibson $k$ (n = 1) profile may be characterised by the gradient $k_o$ [FL$^{-3-n}$].

The slip (plastic) zone of $0 \sim z_o$ (prior to the tip-yield state) or both zones of $0 \sim z_o$ and $z_1 \sim l$ (after the tip-yield state) are developed once the $p$ attains the $p_u$:

$$p_u = A_r z^n d$$  \hspace{1cm} (2)$$

where $A_r$ [FL$^{-2-n}$] is the gradient of the limiting $p_u$ profile, with $A_r = N_p s_u$ and $n = 0$ for constant $p_u$, and $n = 1$ for Gibson $p_u$; $s_u$ is an average undrained shear strength over the pile embedment [FL$^{-2}$]. The $A_r$ for Gibson $p_u$ may be estimated, for instance, by employing Hansen’s plasticity solutions [23] with frictional angle and effective unit
weight of subsoil [20, 13]. The \( N_p \) is equal to 2.6~2.7 for footings with \( l/d \leq 1.0 \) [10] and 2~4 for slope stabilising rigid piles [24] with gapping between pile and soil. Further investigation into the \( N_p \) is conducted later.

At the transition (slip) depth \( z_0 \) from the plastic to the elastic state, the limiting force per unit length \( p \) of Eq. (1) is equal to the \( p_u \) of Eq. (2), at which the displacement threshold \( u^* \) (see Fig. 1) is deduced as

\[
u^* = \frac{N_p s_u}{k} \text{(Constant } p_u \text{ and } k) \tag{3a}
\]

\[
u^* = \frac{A_r}{k} \text{(Gibson } p_u \text{ and } k), \text{ or } u^* = \frac{A_r z_o}{k} \text{(Gibson } p_u \text{ and constant } k) \tag{3b}
\]

The pile-tip threshold displacement \(-u^*\) is shown in Figs. 1d1 and 1d2. Mathematically speaking, the on-pile force profile is a consequence of the \( p_u \) mobilised by a linear, rigid pile (with deflection \( u = \omega z + u_g \)) [20, 25] rotating about a depth \( z_r = -u_g/\omega \) to an angle \( \omega \) [note \( \omega \) is in radian, and \( u_g \) is a ground-level deflection, see Figs. 1d1 and 1d2]. More specifically speaking, the Gibson profile is characterised by \( p_u = A_r z_o d \) and \( u^* = \omega z_o \) (Gibson \( p_u \) and \( k \)), or Eq. (3b)) at the slip depth \( z_o \), and by \( p_u = -A_r z_1 d \) and \(-u^* = \omega z_1 + u_g \) at the slip depth \( z_1 \), respectively. The constant profile is described by \( p_u = N_p s_u d \) and \( u^* = N_p s_u /k \) at the depth \( z_o \), and by \( p_u = -N_p s_u d \) and \(-u^* = N_p s_u /k \) at the depth \( z_1 \). Both \( p_u \) and \( u^* \) profiles are schematically illustrated as the dashed lines in Figs. 1d1 and 1d2, respectively.

In the context of load transfer model, elastic-plastic solutions were developed to capture response of a rigid pile, concerning three typical pairs of the \( p_u \) and \( k \) profiles: (1) a constant \( p_u \) and a constant \( k \) [10]; (2) a Gibson \( p_u \) and a constant \( k \); and (3) a Gibson \( p_u \) and a Gibson \( k \) [20]. The response is presented in non-dimensional form such as normalised capacity \( \bar{H}_o = H_o/(A_r d l^{1+n}) \), normalised maximum bending moments \( \bar{M}_m = M_m/(A_r d l^{2+n}) \), and applied bending moment (at ground level) \( \bar{M}_o = M_o/(A_r d l^{2+n}) \), etc. The free-head solutions are simplified next to determine the capacity \( H_o \) of anchored-piles at both the tip-yield state and the YRP state. They are also used to examine the response of load, displacement, rotation and maximum bending moment of anchored piles. Afterwards, in the same framework, elastic-plastic solutions are developed to incorporate the impact of rotational stiffness of pile-cap on the pile response.
2.2 Ultimate Capacity

Assuming a constant $p_u$ (n = 0) and a constant $k$ with depth, the normalised capacity $\overline{H}_o$ at the tip-yield state or the YRP state [10] is given respectively by:

$$\overline{H}_o = \frac{H_o}{N_p s_o d l} = \frac{-3\overline{e} + (9\overline{e}^2 + 6\overline{e} + 3)^{0.5}}{3(1 + \overline{e}) + (9\overline{e}^2 + 6\overline{e} + 3)^{0.5}}$$  \hspace{1cm} (Tip-yield state) (4)

$$\overline{H}_o = \frac{H_o}{N_p s_o d l} = 2[-\overline{e} + (\overline{e}^2 + \overline{e} + 0.5)^{0.5}] - 1$$  \hspace{1cm} (YRP state) (5)

where $\overline{e} = e/l$, and $e = M_o/H_o$, or loading eccentricity. The normalised moment $\overline{M}_m$ at the tip-yield through the YRP state is given by

$$\overline{M}_m = \overline{M}_o + 0.5\overline{H}_o^2$$  \hspace{1cm} (6a)

where at tip-yield state, $\overline{M}_o = \overline{H}_o \overline{e}$ with $\overline{H}_o$ determined from Eq. (4); and at YRP state, the $\overline{M}_o$ is correlated with the $\overline{H}_o$ (depending on loading direction), e.g. for the fourth quadrant, by

$$\overline{M}_o = [-\frac{1}{4}(\overline{H}_o + 1)^2 - \frac{1}{2}]$$  \hspace{1cm} (6b)

Stipulating a Gibson $p_u$ (n = 1) and a constant $k$, the capacity $\overline{H}_o$ at the tip-yield state or the YRP state [20] is given respectively by

$$\overline{H}_o = \frac{H_o}{A_j d l^2} = \frac{0.5[-(3\overline{e} + 1) + (9\overline{e}^2 + 12\overline{e} + 5)^{0.5}]}{3(1 + \overline{e}) + (9\overline{e}^2 + 12\overline{e} + 5)^{0.5}}$$  \hspace{1cm} (Tip-yield state) (7)

$$\overline{H}_o = \frac{H_o}{A_j d l^2} = 0.25(\sqrt{A_0} + \sqrt{A_1} - \overline{e})^2 - 0.5$$  \hspace{1cm} (YRP state) (8)

where

$$A_j = (-\overline{e}^3 + 2 + 3\overline{e}) + (-1)^j[(2 + 3\overline{e})(-2\overline{e}^3 + 2 + 3\overline{e})]^{0.5}$$  \hspace{1cm} (j = 0, 1) (9)

The normalised moment $\overline{M}_m$ at the tip-yield though the YRP state is given by

$$\overline{M}_m = \overline{M}_o + \frac{1}{3}(2\overline{H}_o)^{1.5}$$  \hspace{1cm} (10a)
where at tip-yield state $\bar{M}_o = \bar{H}_o \bar{e}$ with $\bar{H}_o$ gained from Eq. (7), and at YRP state, the $\bar{M}_o$ is correlated with the $\bar{H}_o$, e.g. for the fourth quadrant, by

$$
\bar{M}_o = \left[ \frac{2}{3} \left( \bar{H}_o + 0.5 \right)^{1.5} - \frac{1}{3} \right]
$$

Eq. (4) or (7), and Eq. (5) or (8) allow the normalised capacity of lateral piles (with $\bar{e} \geq 0$) to be calculated for the tip-yield and the YRP states, respectively. The capacities for piles with an $l/d$ of 4-20 were obtained using the Broms’ solutions [21] underpinned by an ‘around-pile gap’ to a depth of $1.5d$, and were normalised by taking $N_g = 9$. All the normalised capacities are plotted in Fig. 2, which indicate that Broms’ solutions are well bracketed by the current solutions of constant $p_u$ ($n = 0$) and Gibson $p_u$ ($n = 1$). Note both solutions neglect the longitudinal resistance along the shaft and transverse shear resistance on the pile-tip.

### 2.3 Capacity of Anchored Piles and Loading Depth

Guo [10] demonstrates that upon approaching YRP state, lateral piles ($e^+$) may rotate about a depth of $(0.5-0.707)l$, and render a normalised capacity of $\bar{H}_o = \sim 0.414$ for constant $p_u$; or rotate about a depth of $(0.707-0.794)l$ with an $\bar{H}_o = \sim 0.113$ for Gibson $p_u$. A higher loading capacity is attained by shifting the depth of loading attachment (rotation) downwards (with $e < 0$ in anchored piles), as is demonstrated in Fig. 3 for three pairs of the $p_u$ and $k$ profiles, at the tip-yield and the YRP states:

(i) At the tip-yield state, the normalised capacity $\bar{H}_o$ for either $p_u$ profile (constant $k$), was calculated using Eqs. (4) and (7), respectively. It is plotted in Fig. 3 as the curves $a_b b_c$ and $b_c c_e$ (constant $p_u$), and as the dash line $a b$ (Gibson $p_u$). The $\bar{H}_o$ for a Gibson $p_u$ (Gibson $k$) was obtained using the expressions in [20] and is depicted as the dash line $a g b g$.

(ii) At the YRP state, the capacities for constant $p_u$ and Gibson $p_u$ (regardless of the $k$ profiles) were calculated using Eqs. (5) and (8) respectively, and are plotted as solid lines in Fig. 3. They slightly exceed the $\bar{H}_o$ at the tip-yield state at $\varepsilon = 0 \sim -0.4$ (constant $p_u$) or $0 \sim -0.667$ (Gibson $p_u$), but become indistinguishable from the latter at $\varepsilon = -0.5$ (constant $p_u$) or -0.667 (Gibson $p_u$).
Irrespective of the tip-yield or the YRP state, a maximum capacity occurs at translation mode (discussed later), with $\overline{H}_o = 1.0$ at $\bar{\varepsilon} = 0.5$ for constant $p_u$, and $\overline{H}_o = 0.5$ at $-\bar{\varepsilon} = 0.667$ (constant $k$) ~ 0.725 (Gibson $k$) for Gibson $p_u$. The capacity increases by $\sim 2.73$ (= $1/0.414$) times (constant $p_u$) and by $3.84$ (= $0.5/0.13$) times (Gibson $p_u$) as the attachment depth is shifted to $0.5l$ and $(0.667\sim 0.725)l$ below the ground level, respectively. This is physically associated with the shift of the on-pile resistance along two-sides of the pile (divided by rotation depth) to one-side (translation) for a specified $p_u$ of $N_ps_{u_d}$ or $A_rzd$.

The current capacity $\overline{H}_o$ (at the tip-yield state) is compared with published results in Figs. 4a and 4b for constant $p_u$, and Gibson $p_u$, respectively. The $\overline{H}_o$ shows

- A similar increase with the normalised depth to $-\bar{\varepsilon} = 0.5$ and decrease afterwards [see Fig. 4a] against the PLA analysis (with/without gapping between caisson and surrounding soil), and $\sim 15\%$ less than the SM prediction on caissons [7].
- A fair comparison, see Fig. 4a, with the measured capacities, at three typical $e/l$ ratios, of the anchored piles (tested in centrifuge) with flanges [26].
- An excellent agreement between the current solution (Gibson $p_u$ and Gibson $k$) and the SM prediction [7]; and between the current solution (Gibson $p_u$ and constant $k$) and the PLA prediction [9].

The ultimate lateral-moment loading capacity of anchored piles and lateral piles is governed by the same $p_u$ profile. Eqs. (5) and (6b) provide a concave portion of the $\overline{M}_o \sim \overline{H}_o$ curve in the fourth quadrant of Fig. 5a for a constant $p_u$, which overlaps the loading capacity locus for lateral piles [10]. Eqs. (8) and (10b) offer the same portion of locus in Fig. 5b for a Gibson $p_u$. The higher capacity $\overline{H}_o$ (e.g. $0.414\sim 1.0$ for constant $p_u$) of an anchored pile is owing to rotation about a depth $z_r$ either within its body at $-\bar{\varepsilon} = 0\sim 0.33$ [e.g. $z_r = z_d/l = 0.791\sim 0.833$ for $\bar{\varepsilon} = -0.25$, and $0.865\sim 1.055$ for $\bar{\varepsilon} = -0.35$], or outside its body as $-\bar{\varepsilon}$ approaches 0.5 [e.g. $z_r = 5.3\sim 83.7$ ($\bar{\varepsilon} = -0.499$), and $\propto (-0.5)$, respectively]. These diverse rotation depths resemble the mechanisms revealed in the SM method [7] that are characterised by four sets of complex expressions. The normalised capacity or moment as shown in Fig. 3 are independent of the slenderness ratio $l/d$, ...
gapping development, rotation-translation mode, and loading angle, but the normaliser \( N_p \) (or \( A_l \)) is, as discussed later.

The normalised moment \( \overline{M}_m \) was estimated, using Eqs. (6a) and (10a), respectively, for the tip-yield state [10], and is shown in Fig. 6. The figure indicates a limited effect of the \( p_u \) profiles on the \( \overline{M}_m \) in anchored piles at \(-\bar{\epsilon} = 0.25 \sim 0.49\) (and \( \overline{M}_m = 0 \) at \(-\bar{\epsilon} = 0.5\)(constant \( p_u \)) or \(0.667\) (Gibson \( p_u \))), compared to a remarkable effect on the \( \overline{M}_m \) in lateral piles with \( e^+ \).

Normalised responses of \( H_t, \overline{u}_g, \overline{\omega}, \) and \( \overline{M}_m \) were obtained using the free-head solutions [10] for a few typical \( e/l \) ratios in the range of \(3 \sim -0.5\) (\( n = 0 \)) or \(3 \sim -0.667\) (\( n = 1 \)), and are shown in Fig. 7. With anchored piles having a constant \( p_u \), Fig. 7 shows that a loading at middle depth of embedment (\( \bar{\epsilon} = -0.5 \)) causes pure translation of the pile (\( \omega = 0 \)), zero maximum bending moment (\( M_m = 0 \), see Fig. 6 as well), and a limited elastic displacement \( \overline{u}_g = 1 \) at \( H_t = 1 \) (not shown completely in the figure). A slip from the anchored pile-base may commence shortly after or even simultaneously with the incipient of the top-slip \( z_o \). This causes a limited impact of base-slip on the capacity, as is detected from the discrepancy between the tip-yield and the YRP states (see Fig. 3), and the small gap between the current and the previous solutions (see Fig. 4b). As for a Gibson \( p_u \), (1) similar features to constant \( p_u \) are noted, albeit that the translation occurs at \( \bar{\epsilon} = -0.667 \) (constant \( k \) \( \sim -0.725 \) (Gibson \( k \)); (2) the \( \overline{M}_m \) is doubled by using \( p_u = 2A_d dz \) (e.g. at tip-yield state and \( \bar{\epsilon} = 0.01 \), \( \overline{M}_m = 0.07728 \) in Fig. 7, compared to \( \overline{M}_m = 0.03864 \) in Fig. 6), and it occurs prior to reaching the tip-yield state. These conclusions for anchored piles, while expected, should be verified against displacement performance once available.

### 2.4 New Solutions for Lateral Piles with Rotating Cap

Piles are often installed in a group and cast into a pile-cap to restrain pile-head rotation. In reality, the pile-cap with a rotational stiffness \( k_r \) will rotate to an angle \( \omega \) [27]. With a head restraining bending moment \( M_o = k_r \omega \), new closed-form solutions are developed herein in the context of load transfer approach. The development employs the same technology as that for free-head piles [20], and it is thus not elaborated herein. For
instance, given a constant $k = p/(dz)$, from Eq. (1), and a constant $p_u = N_p s_u z$, Eq. (2),

Eq. (3a) and the expression of $u = \omega z + u_g$, allow the unknown rotation $\omega$ and groundline
displacement $u_g$ of a rigid pile to be determined, which in turn allow the pile response to
be related to the rotational stiffness, $k_r$ [FL/radian], in addition to the eccentricity, $e$ [L],
of the head load, and the slip depth, $z_o$ [see Fig. 1a], as outlined below

\[
H_i = 
\begin{align*}
\frac{0.5[(1+2\bar{z}_o)(\bar{z}_o-1)^2 + 12\bar{k}_r]}{(\bar{z}_o-1)^2(2+\bar{z}_o+3\bar{e}) + 6\bar{k}_r}
\end{align*}
\]  

(11)

\[
\bar{u}_g = 
\begin{align*}
\frac{2 + 3(1+\bar{z}_o^2)\bar{e} + \bar{z}_o^3 + 6\bar{k}_r}{(2+\bar{z}_o+3\bar{e})(1-\bar{z}_o)^2 + 6\bar{k}_r}
\end{align*}
\]  

(12)

\[
\bar{\sigma} = 
\begin{align*}
\frac{-3(1+2\bar{e})}{(2+\bar{z}_o+3\bar{e})(1-\bar{z}_o)^2 + 6\bar{k}_r}
\end{align*}
\]  

(13)

\[
\bar{z}_r = -\frac{\bar{u}_g}{\omega l}
\]  

(14)

\[
\begin{align*}
\bar{z}_m = H_i, 
\bar{M}_m = H_i(\bar{e} + 0.5\bar{z}_m) + \bar{k}_r\bar{\omega}
\end{align*}
\]  

(15)

\[
\bar{z}_s^3 + 3\bar{e}\bar{z}_s^2 - 1.5(1+2\bar{e})\bar{z}_s + 0.5(1+12\bar{k}_r) = 0
\]  

(16)

where $H_i = H/(N_p s_u d l)$, normalised pile-head load; $\bar{u}_g = u_g k/(N_p s_u)$, normalised groundline
displacement; $\bar{\omega} = \omega k l/(N_p s_u)$, normalised rotation angle; $\bar{M}_m [= M_m/(N_p s_u d l^2)]$ and $\bar{M}_o [= M_o/(N_p s_u d l^2)]$, normalised maximum bending moment, and moment at ground level;

$\bar{k}_r = k_r/[k l^3]$, normalised rotational stiffness of the pile-cap; $\bar{z}_o = z_0/l$, normalised slip
depth; $\bar{z}_r$ and $\bar{e}$ were defined earlier. The solutions for a Gibson $p_u (= A_r dz)$ with a
constant $k$ (i.e. $p = k d u$, thus $u^* = A_r z_o / k_o$), or with a Gibson $k$ (i.e. $p = k_o d z u$, thus $u^* =
A_r / k_o$) were also developed and are provided in Table 1.

Typical responses of $H_i \sim \bar{u}_g$, and $H_i \sim \bar{\omega}$ to the stiffness $\bar{k}_r (= 0, 0.05, 0.1, 0.2,
0.4, 0.8, 1.5, 3.0, and 10)$ were obtained for $n = 0$ and $n = 1$. They are plotted in Figs. 8a
and 8b, respectively, which exhibit the following salient features:

- A maximum lateral load for fixed-head ($\bar{k}_r > 10$) rigid piles is mobilised upon
reaching a unit normalised displacement ($\bar{u}_g = 1$, see Fig. 8a), as anticipated.

- Given a constant $p_u$ and a constant $k$, the rotation can be reduced by increasing the stiffness $k_r$ (see Eq. (13)), and at an infinitely large $k_r$, the fully fixed-head ($\omega = 0$)
occurs. The normalised load $H_i$ reaches a maximum of 1.0 (the normalised capacity)
as the slip depth extends from ground level to the pile tip ($\bar{z}_o \to 1$).
With a Gibson $p_u$ and a constant $k$, the rotation may also be reduced by increasing stiffness $k_r$. A fully fixed-head would theoretically occur at a normalised slip depth of $1.732\sim 2.0$ \[\text{using } \bar{z}_o = -1.5 \bar{\varepsilon} + 0.5(9 \bar{\varepsilon}^2 + 24 \bar{\varepsilon} + 12)^{0.5}\]. In other words, the fixed-head is practically unattainable given $\bar{z}_o \leq 1$, as is noted for Gibson $p_u$ and Gibson $k$ (see Table 2). The normalised load (or capacity) $\bar{H}_r$ reaches a maximum of 0.5 (the fixed-head capacity) as the slip depth extends to the entire pile length ($\bar{z}_o \rightarrow 1$).

The $z/l$ for the tip-yield state may be estimated using Eq. (16) and the expressions in Table 1. For instance, given $k_r/(k_l^3) = 0.02$ for a constant $p_u$ and a constant $k$, the tip-yield occurs at $z/l$ of 0.49 ($\bar{\varepsilon} = 0$), and 0.462 ($\bar{\varepsilon} = 0.03$), respectively. At a large $k_r$ value, the yield between pile and soil will occur almost simultaneously over the entire pile length. Theoretically speaking, fully fixed-head conditions will never occur in laterally loaded rigid piles with a constant $p_u$ through a Gibson $p_u$, rotation and/or cracking may be induced instead.

More importantly, the new solutions cover the free-head ($k_r = 0$) to fixed-head cases. Taking $k_r = 0$, for instance, Eqs. (11) and (16) reduce to

\begin{align}
\bar{H}_r &= \frac{0.5(1+2\bar{z}_o)}{2+\bar{z}_o+3\bar{\varepsilon}} \tag{11a} \\
\bar{z}_* &= -(1.5\bar{\varepsilon} + 0.5) + 0.5(3+6\bar{\varepsilon} + 9\bar{\varepsilon}^2)^{0.5} \tag{16a}
\end{align}

Substituting Eq. (16a) into Eq. (11a) allows Eq. (4) to be deduced. Likewise, with a Gibson $p_u$ and a constant $k$, the expressions in Table 1 offer

\begin{align}
\bar{H}_r &= \frac{0.5\bar{z}_o}{2+\bar{z}_o+3\bar{\varepsilon}} \tag{11b} \\
\bar{z}_* &= -(1.5\bar{\varepsilon} + 0.5) + 0.5(3+12\bar{\varepsilon} + 9\bar{\varepsilon}^2)^{0.5} \tag{16b}
\end{align}

Substituting Eq. (16b) into Eq. (11b) enables Eq. (7) to be gained. Furthermore, the solutions for a Gibson $p_u$ and a constant $k$ can be deduced using the expressions in Table 1 as well.

Conversely, the existing solutions for free-head piles with $M_o = H_t e$ (Guo 2012) are used to predict the response profiles of the current semi-fixed head piles (with $k_r > 0$) by replacing the $M_o$ with $M_o = H_t e + k_r \omega$. For instance, the $M(z)$ for a constant $p_u$ (constant $k$) at the pre-tip yield state is extended to

\begin{align}
\bar{M}_t(\bar{z}) &= \bar{H}_t(\bar{\varepsilon} + \bar{z}) + k_r \bar{\omega} - 0.5\bar{z}^2 \tag{17a} \\
\bar{H}_t(\bar{z}) &= \bar{H}_t - \bar{z} \quad (0 < z \leq z_o) \tag{17b}
\end{align}
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\[ M_2(z) = \bar{H}_f(\vec{c} + z) + k_c \vec{g} - (z_o z - 0.5z_o^2) \]
\[ - \frac{\vec{g}}{6}(\vec{z}^3 - 3\vec{z}_o^2 z + 2\vec{z}_o^3) - 0.5\bar{u}_g(\vec{z} - \vec{z}_o)^2 \] (18a)

\[ H_2(z) = \bar{H}_f - z_o - 0.5\vec{g}(z^2 - z_o^2) - \bar{u}_g(\vec{z} - \vec{z}_o) \] (z_o < z ≤ l) (18b)

Note the tip-yield state is excluded herein, as yielding may occur simultaneously over the pile length under working load. In light of Eqs. (17) and (18), typical shear force and bending moment profiles were obtained for \( z_o = 0.4 \) (corresponding to a factor of safety of 2.5 before reaching ultimate \( z_o = 1 \)) for nine typical values of normalised rotation stiffness. They are plotted in Figs. 9a1 and 9b1. The same profiles for Gibson \( p_u \) (constant \( k \)) were obtained using the expressions in Table 1, which are plotted in Figs. 9a2 and 9b2.

3. FACTORS \( N_p \) AND \( N_p\alpha \) FOR COHESIVE SOIL

We have established non-dimensional solutions for the capacity of anchored piles, and for lateral piles with rotating cap. The remaining question is how to estimate the normaliser \( N_p \) (e.g. in Eq. (4)) or \( A_t \) (see Table 1), as it varies from lateral piles to anchored piles, and with loading angle. A rotation (largely observed on lateral piles) may shift to translation (observed in anchored piles) under the combined lateral-moment loading, depending on pile-head and base restraints, depths of stiff layers, and loading positions. It is legitimate to integrate together the values of \( N_p \) or \( A_t \) across footing, lateral piles (caissons) and anchored piles, or even pipelines [10]. This is addressed next for cohesive soil. Note we defer the search for \( A_t \) expression, as there is scarcity of measured \( p_u \) profiles for anchored piles in drained or cohesionless soil, despite the extensive \( p_u \) profiles deduced for lateral piles [20]. To match a measured load-displacement curve of a semi-fixed head pile using free-head and fixed-head solutions, the associated \( N_p^{FreH} \) for free-head (\( \bar{k}_c = 0 \)) solutions may be reduced by 75\% (to 0.25\( N_p^{FreH} \)) when fixed-head (say, \( \bar{k}_c > 10 \)) solutions [12] are adopted. The real value of \( N_p \) for a typical \( \bar{k}_c \) may be interpolated from the two extreme values, which is referred herein to as capping effect. The study on the \( N_p \) is thus narrowed down to free-head cases.

With the normalised capacities for free-head cases (see Table 2), the \( N_p \) values for a uniform shear strength profile were deduced against solutions for a constant \( p_u \) (although the actual \( p_u \) may be somewhere between uniform \( p_u \) and Gibson \( p_u \) [13]), involving the impact of pile-movement mode, gapping, and slenderness ratios, as is elaborated below:
• With regard to $l/d = 2\sim 10$ and $e = 0$, the $N_p$ was deduced as $10.14\sim 11.6$ (no gapping) and $5.92\sim 8.33$ (with gapping), respectively. (i) The no-gapping $N_p$ was deduced using $H_o/(s_udl) = 0.414N_p = 4.2\sim 4.8$ (PLA on caissons), as is gained using Eq. (5) at YRP state. And (ii) As gapping (still translation movement) evolves, the $N_p$ observes $0.414N_p = 2.45\sim 3.5$ (PLA on caissons). The no-gapping $N_p$ of $10.14\sim 11.6$ (also see Table 2) is well bracketed by $9.14\sim 11.94$ of the upper bound solution [28] for translating piles. The $N_p$ is thus independent of the mobilisation of limiting force on either two-sides of a rotating pile (see Fig. 1c2, the current solutions) or one-side of a translating pile (the upper bound approach). The gapping $N_p$ of $5.92\sim 8.33$ agrees well with $5.6\sim 8.6$ [17] mentioned earlier, and is $40\sim 80\%$ the no-gapping $N_p$.

• As for short piles with $l/d = 0.08\sim 1$ or footings [19], the $N_p$ for $\overline{e} = -0.5$ (translational movement) was deduced as $1.0\sim 6.1$ using $N_p = H_o/(s_udl)$ at the YRP state; and the $N_p$ for $e = \infty$ (rotational movement) was gained as $2.8\sim 9.8$ on the base of $0.25N_p = 0.7\sim 2.45 = M_o/(s_udl^2)$ at YRP state, Guo 2012].

• The $N_p$ varies with pile-movement modes, and may be estimated using Eq. (19a) and Eqs. (19b and c) for pure rotation and rotation-translation (R-T), respectively.

$$N_p = 2 + 8.9\arctan(0.9l/d) \quad \text{(Pure rotation)} \quad (19a)$$

$$N_p = 1 + 7.5\arctan(0.8l/d) \quad \text{(R-T mode without gapping)} \quad (19b)$$

$$N_p = 1 + 5\arctan(0.7l/d), \quad \overline{e} = 0.7\sim 1.0 \quad \text{(R-T mode with gapping)} \quad (19c)$$

Eq. (19a) fits well with those induced from rotation of footing under moment loading and with $N_p = 15$ from cone penetration (CPT) tests. A higher $N_p$ than 15 [29] is likely seen for CPT tests that force rotational soil movement around the cone tip, as is evident from the analytical solutions for a cone angle of $30^\circ \sim 180^\circ$ [30]. Eq. (19b) matches closely with those deduced from footing and caissons (lateral loading) without gapping. Eq. (19c) is based on those for footing, caissons (lateral loading with gapping) and anchors. Eqs. (19a, b and c) with $\overline{e} = 0.7$ are plotted in Fig. 10.

As a comparison, the upper bound solutions [16] were obtained for a rough-surface pile in uniform and non-uniform shear strength profiles; and for a smooth surface pile in the same strength profiles. They are plotted in Fig. 10, and confirm the current variation law of the $N_p$ with $l/d$ ratio, despite the scatter for $l/d < 1$. The latter may be caused by replacing the embedment depth of anchors or pipelines with the pile-length $l$ in
determining the \( N_p \) values using empirical expressions deduced numerically for vertical anchors without gapping [31] and with gapping [32] deduced from centrifuge tests for pipelines with gapping [33].

Lateral loading is pragmatically applied at an angle \( \alpha ^+ (>0) \) (between the horizon and loading direction) on anchored piles or exerted horizontally on a group of rake piles (with a batter angle of \( \alpha \), see the insert of Fig. 11). Measured capacities from centrifuge tests on anchored piles [34] were normalised using that of \( \alpha = 0 \) case, as were the measured lateral capacities from laboratory 1-g model tests [35] on rake piles. The normalised capacities were taken as the ratios of \( N_{pa}/N_p \) \( [N_{pa} = p_u/(s_u d)] \), a new version of \( N_p \) involving an inclined loading or batter angles, and are plotted in Fig. 11. The figure demonstrates a starkly similar variation in the \( N_{pa}/N_p \) (thus \( p_u \)) with the angle \( \alpha \) between the anchored and battered piles, regardless of \( e^+ \) or \( e^- \). The impact of batter angle \( \alpha \) on lateral piles is well mimicked by factoring the \( p_u \) for ‘vertical piles’, as is revealed in analysing the centrifuge tests on pile groups [36], and in finite element modelling on single piles [37]. The new factor \( N_{pa} \) thus should be equally sufficiently accurate to anchored piles, as they are governed by the same limiting \( p_u \). The \( N_{pa} \) is well fitted (see Fig. 11) by

\[
N_{pa} = N_p \exp(-0.014\alpha) \quad (20a)
\]

Assuming an identical variation law to inclined anchor in clay [32], the \( N_{pa} \) may follow

\[
N_{pa} = N_{pa=90^\circ} + (N_{pa=0^\circ} - N_{pa=90^\circ})\left(\frac{90^\circ - \alpha}{90^\circ}\right)^2 \quad (20b)
\]

For convenience, we drops the subscript ‘\( \alpha \)’, and rewrite Eq. (19a), (19b) and (19c) as

\[
N_p = \left[2 + 8.9\arctan(0.9l/d)\right]\exp(-0.014\alpha) \quad (\text{Pure rotation}) \quad (21a)
\]

\[
N_p = \left[1 + 7.5\arctan(0.8l/d)\right]\exp(-0.014\alpha) \quad (\text{R-T mode without gapping}) \quad (21b)
\]

\[
N_p = \left[1 + 5\arctan(0.5/c)\right]\exp(-0.014\alpha), \quad \varepsilon = 0.7\text{--}1.0 \quad (\text{R-T mode with gapping}) \quad (21c)
\]

The \( N_p \) of Eq. (21a) \(~ (21c)\) may be used in Eqs. (4) \~ (6) to calculate response of anchored piles under inclined loading, as the impact of the loading position on the \( N_{pa} \) is limited [8]. It may be reduced further to cater for the shadowing and capping effect to model capped piles.
Plasticity limit analysis (PLA) and finite element analyses were conducted \cite{9} to gain lateral capacity of caissons (with length over diameter ratios of 2, 6 and 10) under various loading angle. Given $-\bar{e} = 0, 0.25, 0.5, 0.75$ and 1.0, the normalised capacities ($= N_{p\alpha}/N_p$ for a uniform shear strength profile) are plotted in Fig. 11. The ratio for each $\bar{e}$ hardly reduces at $\alpha = 0 \sim 15^\circ$, but drops quickly at $\alpha > 75^\circ$, which is not seen in the cited test data (further experiment is required). The faster reduction in $N_{p\alpha}$ with the $\alpha$ may be catered for by replacing the factor $-0.014\alpha$ in Eq. (20) with $-0.018\alpha$.

It is customary to estimate the $p_u$ utilising $N_p$ and undrained shear strength. As for Gibson $p_u$, the $p_u$ may be obtained using plasticity solutions \cite{23, 13} with cohesion and frictional angle of soil \cite{38} especially for piles in layered soil. The $p_u$ may be fitted to gain the gradient $A_r$, which allows the corresponding solutions to be adopted, as is noted for flexible piles in stiff clay \cite{20}. Further study on the factor $A_r$ for anchors, anchored piles in Gibson $p_u$ is warranted.

4. CONCLUSIONS

This paper provides explicit expressions for estimating loading capacity of anchored piles and develops new solutions for lateral piles with rotating cap by stipulating a constant $p_u$ or a linear increasing $p_u$ (Gibson $p_u$) with depth. Lateral loading capacity $H_o$ (at the tip-yield state and yield at rotation point state) and maximum bending moment $M_m$ (at the tip-yield state) are presented against loading locations, and in form of the lateral capacity $H_o$ - $M_o$ (applied moment) locus. The capacity is consistent with available solutions for anchored piles, and caissons with either $p_u$ profile, allowing a united approach from lateral piles to anchored piles. The new solutions are also presented in charts to highlight the impact of rotational stiffness of pile-cap on nonlinear response, offering a united approach for free-head piles through fixed-head piles.

The current solutions are underpinned by the mechanistic load-transfer model, which demonstrate the following advantages over the conventional p-y curve based approach:

- A displacement- based capacity is firmly defined as the load at the tip-yield state and/or at the yield at rotation point;

- The response of lateral piles and anchored piles is rigorously coupled by simply employing positive and negative loading eccentricity.
• The two input parameters $k$ and $p_u$ are well calculated using existing elastic and plastic solutions;

• In compact expressions, the solutions may be fitted to measured response of piles, allowing the parameters $k$ and $p_u$ to be deduced for piles in layered soil, and justified against available solutions; and finally

• A cap-stiffness is integrated into the compact expressions, allowing nonlinear response of free-head to fixed-head piles to be readily captured.

To facilitate estimating the key parameter $p_u$, values of the resistance factor $N_p$ (= ratio of pile-soil limiting resistance over the undrained shear strength $s_u$) are deduced using the current expressions against available normalised pile capacity involving the impact of gapping (between pile and soil), pile movement mode, pile slenderness ratio, inclined loading angle (anchored piles) and batter angles (lateral piles). The $N_p$ is characterised by:

(i) An increase from 5.6~8.6 to 10.14~11.6, as gapping is eliminated around lateral piles and caissons, and from 1.0~6.1 to 2.8~9.8, as translation is converted into rotation mode of footings. (ii) Similar variations with slenderness ratio between anchors and caissons (without gapping), and among anchors, caissons and pipelines (with gapping). And (iii) A reduction with loading angles (anchors) resembling that with batter angles (piles).

The use of rigid-pile solutions and a negative loading eccentricity to capture response of anchored piles is ‘rigorous’ and convenient, underpinned by the same parameters for lateral piles. The predicted capacity is on conservative side for $l/d < 3$ (owing to ignoring base resistance). The predicted displacement and determination of the value of $A_r$ (for Gibson $p_u$) are yet to be corroborated once test data become available. It may be extended to model piles in multilayered soil, as was done previously for flexible piles.
REFERENCES


NOMENCLATURE

\( A_r \) = gradient of the limiting \([FL^{-2-n}])

\( d(r_o) \) = outside diameter (radius) of a cylindrical pile (caisson) \([L]\)

\( E_p \) = Young’s modulus of an equivalent solid cylinder pile \([FL^{-2}]\)

\( e \) = eccentricity (free-length) \([L]\)

\( e^+ \) = eccentricity (free-length), i.e. the height from the loading location to the mudline; or \( e = M_o/H_t \) for lateral piles \([L]\)

\( e/l \) = normalised eccentricity

\( G_s, \tilde{G}_s \) = shear modulus of the soil, and average of the \( G_s \) \([FL^{-2}]\)

\( H_o \) = lateral capacity, which is the load \( H_t \) at tip-yield or YRP state \([F]\)

\( \bar{H}_o \) = \( H_o/(A_r d l^{1+n}) \), normalised capacity

\( H_t \) = lateral load applied at an eccentricity of ‘e’ above mudline \([F]\)

\( k \) = modulus of subgrade reaction \([FL^{-3}]\)

\( k_o \) = gradient of the modulus of subgrade reaction \([FL^{-3-n}]\), \( k_o = k \) at \( n = 0 \)

\( k_r \) = rotational stiffness of a pile-cap \([FL/rad]\)

\( l \) = embedded pile (caisson) length \([L]\)

LFP = net limiting force profile per unit length \([FL^{-1}]\)

\( M_m \) = \( M_{max} \), maximum bending moment within a pile \([FL]\)

\( \bar{M}_m \) = \( M_m/(A_r d l^{2+n}) \), normalised maximum bending moment

\( M_o \) = bending moment at the mudline level for free-head piles; or restraining bending moment for semi-fixed-head piles \([FL]\)

\( \bar{M}_o \) = \( M_o/(A_r d l^{2+n}) \), normalised applied bending moment (at ground level)

\( n \) = power to the depth \( z \), \( n = 0 \) for constant \( k \) or \( p_u \), \( n = 1 \) for Gibson \( k \) or \( p_u \)

\( N_p \) = a resistance factor for limiting force per unit length

\( N_{p_u} \) = a resistance factor for limiting force per unit length under an inclined loading at an angle of \( \alpha \) (anchored piles), or a batter angle \( \alpha \) (lateral piles)

\( p, p_u \) = force per unit length, and limiting value of the \( p \) \([FL^{-1}]\)

\( s_u \) = undrained shear strength \([FL^{-2}]\)

\( u \) = lateral displacement of a rigid pile \([L]\)

\( u^* \) = local threshold \( u^* \) above which pile-soil relative slip is initiated \([L]\)

\( u_g, u_t \) = lateral displacement at mudline level, and pile-head level, respectively \([L]\)

YRP = yield at rotation point

\( z \) = depth measured from the mudline \([L]\)

\( z_m \) = depth of maximum bending moment \([L]\)

\( z_s(z_i) \) = slip depth initiated from mudline (pile-base) \([L]\)

\( z_i \) = depth of rotation point \([L]\)

\( z_r \) = \( z_i/l \), normalised depth of rotation point

\( z^* \) = slip depth at tip-yield state \([L]\)

\( \omega \) = rotation angle (in radian) of the pile

Superscripts ‘+’ and ‘-’ indicate a positive and negative value, respectively;
Superscript ‘FreH’ indicates free-head piles;
Bar ‘-‘denotes normalised values (see Table 1)
**Figure Captions**

Fig. 1 Schematic analysis for a rigid pile [19]: (a) Pile - soil system, (b) Load transfer model, (c) $p_u$ (LFP) profiles, (d) pile displacement features ($i = 1$ and 2 for Gibson $p_u$ and constant $p_u$ respectively)

Fig. 2 Normalised capacity for lateral and anchored piles at YRP & tip yield states

Fig. 3 Normalised capacity $H_o$ versus normalised depth of loading attachment (- $e/l$)

Fig. 4 Comparison with other solutions and measured data ($a = 0$) (a) in clay, (b) in sand

Fig. 5 Comparison of normalised capacity $H_o$ and normalised bending moment loci between lateral piles and anchored piles (a) constant $p_u$, (b) Gibson $p_u$.

Fig. 6 Normalised maximum bending moment $M_m$ versus normalised eccentricity ($e/l$) or depth of loading attachment

Fig. 7 Normalised response for typical ratios of $e/l$. (a) Pile-head load $H_t$ and mudline displacement $u_o$. (b) $H_t$ and rotation $\theta$. (c) $H_t$ and maximum bending moment $M_{max}$

Fig. 8 Response of (a) pile-head load $H_t$ and groundline displacement $u_g$, (b) $H_t$ and rotation $\theta$, ($n = 1$ for Gibson $p_u$ & constant $k$, and $n = 0$ for constant $p_u$ and constant $k$)

Fig. 9 Normalised profiles of $H(z)$ and $M(z)$ for typical normalised $k_t$ at $z_o/l = 0.4$ (constant $k$): (a1)-(b1) constant $p_u$; (a2)-(b2) Gibson $p_u$

Fig. 10 Variation of $N_p$ with gapping and movement (rotation, translation) modes

Fig. 11 Reduction of $N_p$ with loading angle
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Table 1  Solutions for a rigid pile at pre-tip and tip-yield states  

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<table>
<thead>
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<tr>
<td>Gibson $p_u$ ($n=1$) and constant $k$ ($n=0$)</td>
<td>Gibson $p_u$ and Gibson $k$ ($n=1$)</td>
</tr>
<tr>
<td>$u = \alpha z + u_g$, $z/l = -u_g/(\alpha d)$,</td>
<td>$kd = \frac{3\pi G}{2} \left( 2\gamma_b K_i(\gamma_b) - \gamma_b \left[ \frac{K_i(\gamma_b)}{K_o(\gamma_b)} \right]^2 - 1 \right)$</td>
</tr>
<tr>
<td>$u = \alpha z + u_g$, $z/l = -u_g/(\alpha d)$,</td>
<td>$\gamma_b = k_i(v_o/l); \gamma_b = 2.14 + \bar{v}/(0.2 + 0.6\bar{v})$, increases from 2.14 at $e = 0$ to 3.8 at $e = \infty$.</td>
</tr>
<tr>
<td>$p = k d u$, $p_u = A_d z$, $u^* = A_r z_o / k$</td>
<td>$p = k_o d u$, $p_o = A_d z$, $u^* = A_r / k$</td>
</tr>
<tr>
<td>$H = \frac{0.5\pi o_o[(\pi_e - 1)^2 + 6K_r(2 - \pi_e)]}{(\pi_o - 1)^2(2 + \pi_o + 3\bar{v}) + 6K_r}$</td>
<td>$H = \frac{1}{6 \left( (2 + \pi_o)(2\bar{v} + \pi_o) + 3(\pi_o - 1)^2 + 12K_r \right)}$</td>
</tr>
<tr>
<td>$\bar{u}_g = \frac{[2 + 3\bar{v} + 6K_r]z_o}{(2 + \pi_o + 3\bar{v})(1 - \pi_o)^2 + 6K_r}$</td>
<td>$\bar{u}_g = \frac{3 + [2 + 3\bar{v}]\bar{v} + \pi_o^2 + 12K_r}{[(2 + \pi_o)(2\bar{v} + \pi_o) + 3(1 - \pi_o)^2 + 12K_r]}$</td>
</tr>
<tr>
<td>$\bar{\sigma} = \frac{z_o[3\bar{v} - 3 - 3(2 + \pi_o)\bar{v}]}{(2 + \pi_o + 3\pi_o)(1 - \pi_o)^2 + 6K_r}$</td>
<td>$\bar{\sigma} = \frac{-2(2 + 3\bar{v})}{[(2 + \pi_o)(2\bar{v} + \pi_o) + 3(1 - \pi_o)^2 + 12K_r]}$</td>
</tr>
<tr>
<td>$\pi_m = \sqrt{2H}$</td>
<td>$\pi_m = \sqrt{2H}$</td>
</tr>
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</table>

At tip-yield state

$\pi_e^3 + 3\bar{v}\pi_e^3 + [3K_r - (2 + 4.5\bar{v})]z_o = (\pi_e - 1)[3\pi_e^3 + (2 + 1)(\pi_e^2 + \pi_e) - (1 + \bar{v})]$

$+ (1 + 1.5\bar{v} + 3K_r) = 0$

$12K_r = 0$

$H_l = \frac{H_l}{A_d l^3}$, $\bar{u}_g = u_g k / A_r l$, $\bar{\sigma} = \sigma k / A_r l$, $\bar{\sigma} = \sigma k / A_r l$

$\bar{\sigma} = \sigma k / A_r l$, $M_m = M_m / (A_d l^3)$, $\bar{M}_m = M_m / (A_d l^3)$, $\bar{\sigma} = \sigma k / (k l^4)$

Note: $\pi_o = z_o / l$, $z_o = z/o$, $z_o = -\bar{u}_g / \bar{\sigma}$, $\pi_m = z_m / l$, and $\bar{\sigma} = \sigma / l$. The solutions for free-head piles (Guo 2012) are used to predict response profiles by replacing $M_o = H e$ with the counterparts of $M_o = H e + k_o \omega$, e.g., the $M(z)$ for Gibson $p_u$ and constant $k$ is as follows:

$\bar{M}_l(z) = \bar{H}_l(\bar{v} + \pi) + \bar{K}_r \bar{\sigma} - \frac{1}{6} z^3 \quad (0 < z \leq z_o)$

$\bar{M}_m(z) = \bar{H}_l(\bar{v} + \pi) + \bar{K}_r \bar{\sigma} - (0.5\pi \pi^2 - \frac{\pi^3}{3}) - \frac{\bar{\sigma}}{6} (\pi^3 - 3\pi^2 \pi_o + 2\pi_o) - 0.5 \bar{u}_g (\pi - \pi_o)^2$

$(z_o < z \leq l, z_l \approx l)$
<table>
<thead>
<tr>
<th>$l/d$</th>
<th>0.08</th>
<th>0.2</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
<th>6.0</th>
<th>8.0</th>
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</thead>
<tbody>
<tr>
<td>$H_o/(N_p s d l)$</td>
<td>1.0</td>
<td>2.0</td>
<td>3.6</td>
<td>6.1</td>
<td>4.2</td>
<td>4.5</td>
<td>4.75</td>
<td>4.8</td>
<td>4.8</td>
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<tr>
<td>$N_p$</td>
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<td>2.0</td>
<td>3.6</td>
<td>6.1</td>
<td>10.14</td>
<td>10.87</td>
<td>11.47</td>
<td>11.59</td>
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</tbody>
</table>

**References**
Yun and Bransby [19], translation
Aubeny et al [17], rotation no gapping

<table>
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<th>0.2</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
<th>6.0</th>
<th>8.0</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_o/(N_p s d l^2)$</td>
<td>$0.7^a$</td>
<td>$0.925^a$</td>
<td>$1.3^a$</td>
<td>$2.45^a$</td>
<td>$2.45^b$</td>
<td>$2.8^b$</td>
<td>$3.1^b$</td>
<td>$3.3^b$</td>
<td>$3.45^b$</td>
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<tr>
<td>$H_o/(N_p s d l)$</td>
<td>$2.8$</td>
<td>$3.7$</td>
<td>$5.2$</td>
<td>$9.8$</td>
<td>$5.92$</td>
<td>$6.76$</td>
<td>$7.49$</td>
<td>$7.97$</td>
<td>$8.33$</td>
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</tbody>
</table>

**References**
Yun and Bransby [19], rotation
Aubeny et al [17], rotation with gapping
Fig. 1 Schematic analysis for a rigid pile [19]: (a) Pile-soil system, (b) Load transfer model, (c) \( p_u \) (LFP) profiles, (d) pile displacement features (i = 1 and 2 for Gibson \( p_u \) and constant \( p_u \), respectively).

N.B. \( H_e \) = lateral load; \( e \) = free-length; \( l \) = embedded length; \( u_t \) = pile-head displacement; \( \omega \) = angle of rotation (in radian); \( z \) = depth from mudline; \( z_o \), and \( z_t \) = depths of slip, and rotation point.
Fig. 2 Normalised capacity for lateral and anchored piles at YRP & tip yield states
Laterally Loaded Rigid Piles With Rotational Constraints

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$z_r \cdot \{ p_u = N_p ds_u \} - e \cdot H_o$ 

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Fig. 3 Normalised capacity $H_o$ versus normalised depth of loading attachment (- e/l)
Fig. 4 Comparison with other solutions and measured data ($\alpha = 0$) (a) in clay, (b) in sand
Laterally Loaded Rigid Piles With Rotational Constraints

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Fig. 5 Comparison of normalised capacity $\bar{H}_o$ and normalised bending moment loci between lateral piles and anchored piles (a) constant $p_u$, (b) Gibson $p_u$
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Using \(-e/l\) and Constant \(p_u\) & Constant \(k\)

\((n = 0, A_r = N_p s_u), \text{tip-yield}\)

- Lateral piles
- Anchored piles

Gibson \(p_u (n = 1)\) & constant \(k\), tip-yield

Lateral piles
Anchored piles

\(M_m (d_l e^{-n})\)

Fig. 6 Normalised maximum bending moment \(\bar{M}_m\) versus normalised eccentricity \((e/l)\) or depth of loading attachment
Laterally Loaded Rigid Piles With Rotational Constraints

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Fig. 7 Normalised response for typical ratios of $e/l$. (a) Pile-head load $H_i$ and mudline displacement $u_g$. (b) $H_i$ and rotation $\omega$. (c) $H_i$ and maximum bending moment $M_{max}$. (Legend:
With Constant k & identical average $p_u$:
(1) Guo (2008):
  * Gibson $p_u$ (n = 1)
  & $p_u = 2A_d dz$
  * Tip-yield state
(2) Guo (2012)
  * Constant $p_u$ (n = 0)
  & $p_u = A_d$
  * Tip-yield state
(1) Guo (2012):
  * Gibson $p_u$ (n = 1)
  & $p_u = 2A_d dz$
  * Tip-yield state
(2) Guo (2012):
  * Constant $p_u$ (n = 0)
  & $p_u = A_d$
  * Tip-yield state

(-0.25) 0 2 4 6 8 10 12
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8

$H_i/(A_i d^{n+1})$

Legend
With Constant k & identical average $p_u$:
(1) Guo (2008):
  * Gibson $p_u$ (n = 1)
  & $p_u = 2A_d dz$
  * Tip-yield state
(2) Guo (2012)
  * Constant $p_u$ (n = 0)
  & $p_u = A_d$
  * Tip-yield state

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(-0.25) 0 2 4 6 8 10 12
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$M_{max} = 0$ for $e/l = (-0.666)$ or -0.5

Legend
With Constant k & identical average $p_u$:
(1) Guo (2008):
  * Gibson $p_u$ (n = 1)
  & $p_u = 2A_d dz$
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(-0.25) 0 2 4 6 8 10 12
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8

$H_i/(A_i d^{n+1})$
Fig. 8 Response of (a) pile-head load $\bar{H}_t$ and groundline displacement $\bar{u}_g$, (b) $\bar{H}_t$ and rotation $\bar{\omega}$, ($n = 1$ for Gibson $p_u$ & constant $k$, and $n = 0$ for constant $p_u$ and constant $k$)
Fig. 9 Normalised profiles of $\overline{H}(z)$ and $\overline{M}(z)$ for typical normalised $k_r$ at $z_o/l = 0.4$ (constant $k$): (a1)-(b1) constant $p_u$; (a2)-(b2) Gibson $p_u$
Laterally Loaded Rigid Piles With Rotational Constraints

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Fig. 10 Variation of $N_p$ with gapping and movement (rotation, translation) modes
Laterally Loaded Rigid Piles With Rotational Constraints

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Fig. 11 Variation of $N_{pa}/N_p$ with loading angle

\[ N_{pa} = N_{pa=90} + \left( N_{pa=0} - N_{pa=90} \right) \frac{\left( 90 - \alpha \right)}{\alpha} \]