1997

Minimal Critical Set of a Room Square of order 7

Ghulam R. Chaudhry

Jennifer Seberry
University of Wollongong, jennie@uow.edu.au

Publication Details
Minimal Critical Set of a Room Square of order 7

Abstract
A Room square $R$ of order $r$ is an $r \times r$ array each of whose cells may either be empty or contain an unordered pair of objects $0, 1, 2, \ldots, r$, subject to the following conditions:

(i) each of the objects $0, 1, 2, \ldots, r$ occurs precisely once in each row of $R$ and precisely once in each column of $R$, and

(ii) every possible unordered pair of objects occurs precisely once in the whole array.

Disciplines
Physical Sciences and Mathematics

Publication Details
Minimal Critical Set of a Room Square of order 7

Ghulam R Chaudhry and Jennifer Seberry
Department of Computer Science
University of Wollongong, AUSTRALIA

A Room square \( R \) of order \( r \) is an \( r \times r \) array each of whose cells may either be empty or contain an unordered pair of objects \( 0, 1, 2, \ldots, r \), subject to the following conditions:

(i) each of the objects \( 0, 1, 2, \ldots, r \) occurs precisely once in each row of \( R \) and precisely once in each column of \( R \); and

(ii) every possible unordered pair of objects occurs precisely once in the whole array.

A critical set \( Q = [Q_1, Q_2, Q_3, \ldots, Q_c] \), \( |Q| = c \), in a Room square \( R \) of order \( r \), is a set of quadruples \( Q_i = [i, j, k, l] \) such that if any \( Q_i \) is removed from the set, it can no longer be uniquely completed. In \( Q_i \), \( (i, j) \) shows the position of the pair \( (k, l) \) in the square. A minimal critical set (min. cs) of a Room square \( R \) of order \( r \) is a critical set of minimum cardinality.

Through computer search, we have found a critical set of size 10 for a Room square of order 7. We believe that critical set of size 10 given below is the minimal because critical set of size 9 could not be found. We also note that its size is less than \( r^2/4 \).

Example A minimal critical set in a Room square of order 7 and its completion.

<table>
<thead>
<tr>
<th>07</th>
<th>**</th>
<th>34</th>
<th>56</th>
<th>**</th>
<th>**</th>
<th>**</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>**</td>
<td>17</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>35</td>
<td>**</td>
</tr>
<tr>
<td>**</td>
<td>**</td>
<td>27</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>**</td>
<td>**</td>
<td>**</td>
<td>15</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>03</td>
<td>**</td>
<td>40</td>
<td>06</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
</tr>
</tbody>
</table>

07 34 56 ** 12
7 46 17 -- -- -- --
** 27 01 -- -- -- --
** 27 -- 37 15 -- 04
** 03 -- 24 06 15 --
** 03 -- -- -- 23 14 16

where "**" shows the unknown pair positions and "--" shows empty positions in the Room square.

References


Bulletin of the ICA, Volume 20 (1997) 90