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New Results with Near-Yang Sequences
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Abstract. We construct new $TW$-sequences, weighing matrices and orthogonal
designs using near-Yang sequences. In particular we construct new $OD(60(2m + 1) +$
$4t; 13(2m + 1), 13(2m + 1), 13(2m + 1), 13(2m + 1))$ and new $W(60(2m + 1) +$
$4t; 13s(2m + 1))$ for all $t \geq 0$, $m \leq 30$, $s = 1, 2, 3, 4$.

1. Introduction
For definitions we refer the reader to [9, Introduction] and [11, Section 2]. We
give one new definition.

Definition 1. (near-Yang sequences) A triple $(F, G, H)$ of sequences is said
to be a set of near-Yang sequences for length $n$ (abbreviated as $NY(n)$) if the
following conditions are satisfied.

(i) $F = (f_r)$ is a $(0, 1, -1)$ sequence of length $n$.
(ii) $G = (g_k)$ and $H = (h_k)$ are sequences of length $n$ with entries
$0, 1, -1$, such that $G + H = (g_k + h_k)$ and $G - H = (g_k - h_k)$ are both
$(0, 1, -1)$ sequences of length $n$.

(iii)
\[
g_s + g_{n-s+1} \equiv 0 \pmod{2} \quad s = 1, \ldots, \left[\frac{n}{2}\right]
\]
\[
h_s + h_{n-s+1} \equiv 0 \pmod{2}
\]

(iv) $N_F(s) + N_G(s) + N_H(s) = 0$, $s = 1, \ldots, n - 1$.

where
\[
N_X(s) = \sum_{i=1}^{n-s} x_i x_{i+s}.
\]

2. Computational Results
In Koukouvinos, Kounias, Seberry, Yang and Yang [6] it is shown that if in (ii) of
the definition $G \pm H$ are both $(1, -1)$ sequences then conditions (i), (ii) and (iv)
imply condition (iii) but this is not true for near-Yang sequences. These sequences
are normal sequences $NS(\ell)$.

We searched for normal sequences $NS(\ell)$. $NS(\ell)$ do exist for the following
lengths $\ell \in \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 19, 20, 25, 26, 29,$
$32, \ldots\}$ and they do not exist for $\ell \in \{6, 14, 17, 21, 22, 23, 30, 46, 56, 62, 78,$
$\ldots\}$.
Table 1: Normal sequences via Simulated Annealing.

<table>
<thead>
<tr>
<th>Length $\ell$</th>
<th>Sequences</th>
<th>Weight</th>
</tr>
</thead>
</table>
| 20            | $F = -+--+-+-++--+-++--+$  
               | $G = 0+0+00+00+00+00+0+0$  
               | $H = -00++00+00+00+00+00+0$   | 14 |
| 25            | $F = -++--+-+-++--+-++--+$  
               | $G = 0--0+0+0+0+0+0+0+0+0$  
               | $H = -0-0-00+0+0+00+00+00+0$ | 13 |

94, ... } [3, 6, 16]. We note here that we found normal sequences of length 25 and 20 (Table 1) using simulated annealing; this is described in Gysin [3]. Sequences of length 25 can be obtained from Turyn sequences of lengths 13 and 12 and a complete search for these was carried out over 20 years ago. It is known that there are eight inequivalent sets of Turyn sequences of lengths 13 and 12 and hence by the construction discussed in [6] probably at least sixteen inequivalent sets of normal sequences of length 25. It would be interesting to know if there are $N\mathcal{S}(25)$ which cannot be made from Turyn sequences. There exist $N\mathcal{S}(20)$ which cannot be made from Turyn sequences, an example is given in Table 1. In those small cases where $N\mathcal{S}(\ell)$ do not exist we searched for $N\mathcal{Y}(n)$, which contain more zeros in appropriate positions. We obtained the following new results:

$N\mathcal{Y}(n)$ with weight $u = 12$ exist for the following lengths: $n \in \{7, 11, 13, 15\}$.

In Table 2 two conditions were imposed in counting the number of inequivalent triples of sequences: two triples of sequences were considered equivalent if one triple of sequences can be changed into the other triple of sequences by reversing and/or negating one or more sequences of the triple; if the three sequences $F$, $G$ and $H$ all started or ended with '0' they were considered to be of smaller length and not counted for this length.

This allows us to find new 4-complementary sequences of lengths $15(2m + 1)$, $23(2m + 1)$, $27(2m + 1)$, $31(2m + 1)$ and weights $13(2m + 1)$, $m \leq 30$.

3. Construction

Definition 2. (suitable sequences) [5, 8, 11] $A, B, C, D$ are suitable sequences $SS(m + p; m; w)$ with elements $0, 1, -1$ of lengths $m + p, m + p, m, m$ and total total weight $w$ if $A$ and $B$ are disjoint, $C$ and $D$ are disjoint and $A, B, C,$ and $D$ have zero non-periodic autocorrelation function.

We use a modified version of Yang’s [5, 8, 16] theorem
Table 2: New near-Yang sequences.

<table>
<thead>
<tr>
<th>Length n</th>
<th>No of Seq.</th>
<th>Sequence Examples</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2</td>
<td>$F = +++0--$</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$G = 0++0++0$</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$H = +00000+_+$</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F = +++0--$</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$G = 0+00000+0$</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$H = +00000+0+$</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>$F = -000+000+0$</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$G = 0+00000+0$</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$H = +0000000+$</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>24</td>
<td>$F = +000+000-0$</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$G = 000000000+0$</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$H = +00000000-$</td>
<td>4</td>
</tr>
<tr>
<td>15</td>
<td>26</td>
<td>$F = +0000000+0$</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$G = 00000000-0$</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$H = +000000000+$</td>
<td>4</td>
</tr>
</tbody>
</table>

Theorem 1. Let $A, B, C, D$ be $SS(m+p; m; w)$ and $F, G, H$ be $NY(n)$ with total weight $u$ and $0', 0$ be sequences of zeros of length $m+p$ and $m$ respectively and $X^*$ be the reverse sequence of $X$ then

\[ Q = \{A_{f_1}, C_{g_1}, -D_{h_1}, 0', 0; A_{f_{m-1}}, C_{g_{m-1}}, -D_{h_{m-1}}, 0', 0; \ldots; A_{f_1}, C_{g_1}, -D_{h_1}, 0', 0; B^*; 0 \} \]

\[ R = \{B_{f_1}, D_{g_1} + C_{h_1}, 0', 0; B_{f_{m-1}}, D_{g_{m-1}} + C_{h_{m-1}}, 0', 0; \ldots; B_{f_1}, D_{g_1} + C_{h_1}, 0', 0; -A^*; 0 \} \]

\[ S = \{0', 0; A_{g_1} + B_{h_1}, -C_{f_1}; 0', 0; A_{g_{m-1}} + B_{h_{m-1}}, -C_{f_{m-1}}; 0', 0; A_{g_1} + B_{h_1}, -C_{f_1}; 0', 0' \} \]

\[ T = \{0', 0; -B_{g_1} + A_{h_1}, D_{f_1}; 0', 0; -B_{g_{m-1}} + A_{h_{m-1}}, D_{f_{m-1}}; 0', 0; -B_{g_1} + A_{h_1}, D_{f_1}; 0', 0' \} \]

are $TW$-sequences of length $(2m+p)(2n+1)$ and total weight $(u+1)w$.

This gives many new $TW$-sequences, weighing matrices and orthogonal designs. Many other corollaries are also possible.

Example 1. Let $F = \{+++0--\}, G = \{00000+0\}, H = \{+0000+0\}$ and $A, B, C, D$ be suitable sequences of length $m+p$ and $m$ and total weight $w$. Then
with 0' and 0 zero vectors of length \(m + p\) and \(m\) respectively we have

\[
Q = \{-A,-D;0',0;A,C;0',0; -A,-D;0',0;0',0;0',0;A,C;0',0;A,-D;0',0;B^*,0\}
\]

\[
R = \{-B,C;0',0;B,D;0',0;-B,-C;0',0;0',0;0',0;0',0;0',0;B,C;0',0;B,D;0',0;0',0;0',0;0',0\}
\]

\[
S = \{0',0;B,-C;0',0;A,-C;0',0;B,C;0',0;A,-C;0',0;B,D;0',0;0',0\}
\]

\[
T = \{0',0;A,D;0',0;0',0;0',0;0',0;0',0;A,-D;0',0;0',0;0',0;0',0;\}
\]

are TW-sequences of of length 15(2 \(m + p\)) and total weight 13 \(w\).

**Corollary 1.** Suppose there are suitable sequences of length \(m + p\), \(m + p\), \(m\), \(m\) and total weight \(w\), \(SS(m + p, m; w)\) and near-Yang sequences of length \(n\) and total weight \(u\). Then there are TW-sequences of length (2 \(n + 1\))(2 \(m + p\)) and total weight \((u + 1)w\).

**Corollary 2.** Suppose there exist \(SS(m + p, m; w)\). Then since there are near-Yang sequences of length 7 and total weight 12 there are TW-sequences of length 15(2 \(m + p\)) and total weight 13 \(w\).

From [11] we see \(SS(m + 1, m; 2 m + 1)\) exist for all \(m \leq 30\) hence there exist TW-sequences of length 15(2 \(m + 1\)) and weight 13(2 \(m + 1\)) for all \(m \leq 30\). Using theorems 3.6, 3.7, 3.8 of [11] we have \(OD(60(2 m + 1) + 4 t; 13(2 m + 1), 13(2 m + 1), 13(2 m + 1), 13(2 m + 1))\) for all \(t \geq 0\) and \(m \leq 30\). Furthermore \(Q, R, S, T\) can be used in the Goethals-Seidel array to form \(OD(4 t; 13 w, 13 w, 13 w, 13 w)\) for every \(t > 15(2 m + 1)\).

Recalling that variables in an \(OD\) can be set equal or set zero to give weighing matrices, we obtain \(W(60(2 m + 1) + 4 t; 13 s(2 m + 1))\) and \(W(4 t; 13 s w)\), \(s = 1, 2, 3, 4\). Since \(NS(n)\) are \(NY(n)\) with total weight 2 \(n\).

**Corollary 3.** Suppose there exist \(SS(m + p, m; w)\). Then since there are normal sequences of length 25 and total weight 50 there are TW-sequences of length 51(2 \(m + p\)) and total weight 51 \(w\). If \(w = 2 m + p\) then we have \(T\)-sequences.

Again using theorems 3.6, 3.7, 3.8 of [11] we have \(OD(104(2 m + 1) + 4 t; 51(2 m + 1), 51(2 m + 1), 51(2 m + 1), 51(2 m + 1))\) for all \(t \geq 0\) and \(m \leq 30\). Furthermore \(Q, R, S, T\) can be used in the Goethals-Seidel array to form \(OD(4 t; 51 w, 51 w, 51 w, 51 w)\) for every \(t > (2 n + 1)(2 m + 1)\).

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References


