Preservice teachers' understanding and representation of equality of fractions in a JavaBars environment

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Abstract
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Keywords
javabars, fractions, equality, environment, representation, understanding, teachers, preservice

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Preservice Teachers' Understanding and Representation of Fractions in a JavaBars Environment

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In recent years, considerable research effort has been invested in identifying the nature of the knowledge that drives mathematics teachers' actions in the classroom. While this investigation has generated a useful body of information, there has been little information about changes in the character of this knowledge when teaching involves the use of technology. In this paper, I address this issue by examining a group of preservice primary mathematics teachers' understanding of fractions. The participants were required to order fractions within software called JavaBars. The results suggest that, while the preservice teachers had built up robust knowledge about fractions, they experienced difficulty in translating this knowledge in the JavaBars environment.

Children encounter fractions and fraction-related concepts both in real-life and classroom situations. A sound understanding of what fractions are would help children make sense of a multitude of other ideas in their daily life. Regardless of the context in which children engage fractions, it is generally agreed that fractions provide teachers with insight into developments in children's understanding of numbers and relations among numbers. These understandings are built on children's personal experiences, intuitions, and formal knowledge gained in the classroom. Fractions are complex in character and provide important prerequisite conceptual foundations for the growth and understanding of other number types and algebraic operations in later years of their school experience. Despite the critical conceptual link between mathematics strands such as space and measurement provided by fractions, they continue to present difficulties for children in primary schools (Pitkethly & Hunting, 1996).

Mathematics educators and teachers have invested considerable effort in exploring the instructional value of computers in helping children develop a better grasp of mathematical concepts including fractions. The significant role that computers could play in enriching the classroom experiences teachers provide has received further attention in major curricular documents such as the Curriculum and Evaluation Standards (National Council of Teachers of Mathematics, 1989). Indeed, it is now generally agreed that the appropriate use of computers in the instructional process could promote the construction of deeper levels of conceptual understanding of fractions.

Concurrent developments in cognitive psychology and domain expertise have had significant effects on our understanding of the processing of mathematical information by the teacher. However, little effort has gone into utilising this knowledge about teachers' conceptual understanding of fractions in the examination of how computers could be used as effective instructional aids. Specifically, there is little information on how teachers utilise softwares that allow for visual representations of fractions. Work on this issue would not only inform us
Preservice Teachers' Understanding and Representation of Fractions in a JavaBars Environment

235

about computer use but also help us understand the link between teachers' understanding of fractions and their ability to represent fractions in different modes (Norman, 1993).

This general concern with teachers' knowledge of fractions could be addressed by describing the state of their knowledge at different stages in their career, including the early phase of their professional training. A survey of the literature shows that there is little data on the question of the quality of preservice teachers' knowledge of fractions, and the teaching of fractions within a technologically rich environment. The elucidation of the relationship between preservice teachers' understanding of fractions and their use of computers to foster the development of understanding of fractions among children is important. Data about this relationship could help us improve this interface and develop appropriate teaching activities.

In this paper, I report findings from a study that examined the above issue. The purpose of the study was to document preservice teachers' knowledge about fractions, their understandings about children's difficulties in the area of fractions, and how they would utilise software called JavaBars to help children. Accordingly, the study addressed the following three research questions.

What is the nature of knowledge that preservice teachers access in relation to:
- the concept of fractions,
- children's difficulties with fractions,
- the possible use of JavaBars to teach fractions.

Teachers' Knowledge Base for Fractions

Recent research, particularly in the area of teacher expertise, indicates that there are three major components which could be related to the knowledge base of teachers which permit them to perform their role effectively: teachers' mathematical content knowledge, the organisation of this knowledge, and the blend of content and pedagogical knowledge. Mathematical content knowledge includes information such as mathematical concepts, rules, and associated procedures for problem solving. The organisation of content knowledge refers to the links that teachers construct between the various components of content knowledge. The blend of content and pedagogical knowledge includes understandings about why some children experience difficulties when learning a particular concept while others find them easy to assimilate, knowledge about useful ways to conceptualise and represent a chosen concept (Feiman-Nemser, 1990), the quality of explanations that teachers generate prior to and during instruction (Leinhardt, 1987) and the characteristics of the learner. This latter knowledge has also been labelled as pedagogical content knowledge (Shulman, 1986). In recent years, researchers interested in improving children's mathematical performance have argued that the quality of teachers' own knowledge would have a strong influence on how that knowledge is accessed and exploited during planning for a lesson and during instruction (Clark & Peterson, 1986; Lawson & Chinnappan, 1994; Ma, 2000; Schoenfeld, 1992).

The above model could be used to characterise the quality of teachers' knowledge at different stages of their career including those who are in the early
phases of their professional training. In this study, I apply the above components in order to analyse preservice teachers' knowledge underlying the teaching of fractions. An important consideration here is the interaction between their knowledge and the use of technology, namely JavaBars, during the teaching process. While there may be other aspects of the teachers' knowledge base which are relevant for teaching mathematics, I regard the above three knowledge components to be essential if a teacher is to succeed in helping his or her children take an active part in the learning and appreciation of fractions.

The difficulty in learning fractions is reflected in studies that have examined different aspects of the concept. For instance, Mack (1990) investigated the role of informal understanding in the learning of fractions. She argued that there was a need to develop instructional strategies that draw on the prior knowledge of children in the area of fractions. A review of research in the area of fractions led Pitkethly and Hunting (1996) to the conclusion that instructional approaches need to consider children's own constructions about fractions. Their view, again, draws attention to the importance of the relationship between teachers' pedagogical knowledge and the learning of fractions.

Although some progress has been made in ways to teach fractions and related concepts, there is less information about how the use of technology will facilitate children's understanding of fractions. Investigations of teachers' knowledge of fractions and the teaching of fractions have given little consideration to the identification and analysis of the nature of knowledge that teachers access when computer aids are used in the teaching-learning process. Specifically, studies of teacher expertise have not examined the nature of teachers' knowledge that is relevant to (a) using computer software effectively in helping children come to terms with the concept and (b) the identification of types of fractions that are encountered in the classroom and other contexts. One way to generate data relevant to the issue of teacher knowledge in the context of utilising computer technology is to analyse the type of knowledge that preservice teachers activate in such situations, and trace the use of this knowledge in helping children grasp the concepts.

Growth of Fraction Knowledge Among Young Children

Young children are exposed to fractions at an early age as these numbers are used in a variety of real-life situations such as measuring and dividing continuous quantities. Prior experiences play a key role on the meanings that children develop about fractions. These experiences allow young children to develop a personal knowledge of fractions which matures through learning situations they encounter in the classroom. Thus, the growth of understanding of fractions could be characterised as involving a progressive change in the mixture of intuitive and formal knowledge.

Children develop an understanding of fractions in their preschool years, and experiences provided by the teacher ought to build on these understandings. Thus, any attempt to characterise the growth of fractions needs to show the links between the informal and formal knowledge that can be associated with children's understanding. In their attempts to address this issue, researchers have focused on
the identification of relations among the subconcepts underlying fractions and the development of these subconcepts among young children. In recent years, a significant body of research has focused attention on the development of fraction knowledge from children's understanding of whole numbers. Their relations to whole numbers, decimals, percentages, and ratio have been the subjects of a number of studies (Lo & Watanabe, 1997; Post, Cramer, Behr, Lesh, & Harel, 1993). These studies provide an important point of departure for the current investigation in which I examine teachers' own understanding of how fraction knowledge develops.

The multifaceted nature of fractions has made the task of describing its growth difficult. Several attempts have been made to capture the complexity of fractions and children's construction of these numbers. The most detailed analysis of fractions was undertaken by Kieren (1988). His analysis showed that fraction knowledge consists of many interwoven strands. He identified eight levels in his description of fraction thinking. This was a hierarchical model in which the higher levels of thinking were based on developments at lower levels. An important outcome of this model was the specification of cognitive structures that provided the basis for the maturing of fraction knowledge among young children. The structures which appear at levels three and four consist of what he referred to as subconstructs. Six subconstructs were identified in this model: partitioning, unit forming, quotients, measures, ratio, and operations.

Research by Behr, Lesh, Post, and Silver (1983) has also supported the arguments about the six subconstructs that are required for developments of fraction knowledge. While Kieren's model captured understandings shown by young children and advanced mathematics learners, the levels which are most relevant to children in early and middle grades lack detail. For instance, we do not have sufficient information about how, in the early grades of schooling, children access their personal knowledge and build any one of the six subconstructs. Neither is there an analysis of conceptual shift among these subconstructs.

Lesh, Landau, and Hamilton (1983) provided an important theoretical construct to capture the adaptive structure of mathematical learning and problem solving which they applied to the analysis of understanding of fraction. Central to their framework was the notion of a conceptual model, defined as an "adaptive structure consisting of (a) a within-concept network of relations and operations, (b) between-concept systems that link and/or combine within-concept networks, (c) systems of representations, and (d) systems of modelling processes" (p. 264). Lesh et al.'s work is important, as it is an improvement on the earlier models, providing information on the type of connections children could make as their understanding of fractions matures. However, the model does not specify potentially useful links within and between constructs that children in early grades might construct.

The model of teachers' knowledge of fractions proposed in Figure 1 not only disentangles fraction subconstructs but also draws out potential links among the constructs. These links are important for the analysis of the organisational quality of the knowledge of fractions. In the past, discussions on fractions were limited to common fractions. In the proposed model, I have included decimal fractions within the fraction schema. A further advancement of the model is that it identifies the array of prior knowledge that young children could bring to formal representations.
Computers and the Development of Fraction Concepts

A recurring theme in the research literature is that children experience difficulties in understanding how whole numbers combine to form fractions. The implication is that, for some children, the transition from whole number to fraction is not straightforward. This difficulty has been given importance in the design of instructional experiences involving computers. The dynamic learning environment provided by computers can be expected to assist children visualise their own understanding of whole numbers better and to transfer this knowledge to the learning of fraction concepts. This point is consistent with the view that “computers as cognitive tools actively engage learners in creating knowledge that reflects their comprehension and conceptualisation of information” (Jonassen & Reeves, 1996, p. 697).

Hunting, Davis, and Pearn (1996) conducted a teaching experiment with 8- and 9-year old children with a software called Copycat. They reported that the environment was suitable for externalising children’s prior understanding of whole numbers and their integration in forming fractions. Children were able to reason
that fractions are made up of two components, parts and wholes.

More recently, a team of researchers from the University of Georgia has been investigating children’s understanding of fractions with the aid of computer software known as JavaBars (Olive, 2000). This software has been primarily designed and developed to examine the type of representations of fractions constructed by children and to encourage students to develop conceptual understanding of fractions. The developers argue that JavaBars is an environment where students make sense of what they are doing. Figure 2 shows a JavaBars screen. The screen is divided into two parts: the control area (the shaded top section) and the working area (the blank bottom section). The Parts region is for creating and manipulating equal parts. The Pieces section of the control area can be used for creating and manipulating pieces in a bar.

![Figure 2. The JavaBars screen.](image)

The buttons in the control area allow students to construct bars of different shapes in the working area. These buttons provide students with options to draw bars of different shapes which can be modified in a number of ways by clicking the Break, Erase, Join, and Pullout buttons. For instance, a given bar can be divided into equal or unequal parts which in turn could either be filled with different colours or isolated from the parent bar. This facility is analogous to children’s manipulation of counters in learning about parts and wholes of fractions. Children are, therefore, able to transfer their skills with concrete objects into the computer environment. Since its development, a number of studies have used the software in
order to examine potential effects on children’s learning of fractions (Tirosh, 2000; Tzur, 1999). It was thus decided that JavaBars would be a suitable medium in which to examine preservice teachers’ knowledge underlying fractions.

Method

Participants

Participants in the present study were a group of eight volunteer preservice teachers who were enrolled in the first year of their Bachelor of Education (Primary) programme. Before this study, the teachers had three weeks of practicum experience. None of the participants had any prior experience with JavaBars. However, all the participants agreed that they had a good knowledge of computer use and Information Technology. All the preservice teachers had completed the New South Wales Higher School Certificate 2-unit mathematics in Year 12. This subject includes a strand on number types and their relations. Thus, it was expected that the participants would have developed a good understanding of fractions.

Materials and Procedure

The author, who was the investigator, met each preservice teacher individually for about two hours. The interview session consisted of three parts.

During the first part of the meeting, the participants were trained in the use of JavaBars installed on a Power Macintosh G3 computer. The investigator introduced the software and showed the various parts of the menu. Each participant was encouraged to raise questions about its various features and the function of the buttons on the screen. This training session was generally completed within 60-80 minutes, and all the participants reported that they were comfortable with the software.

1. Which is the bigger of the two fractions below?
   \[
   \frac{4}{7}, \frac{2}{3}
   \]

2. Order the following fractions from the smallest to the largest.
   \[
   \frac{3}{5}, 0.06, 0.30
   \]

Figure 3. Interview tasks.

In the second part of the interview session, the preservice teachers were given two problems to solve. Both problems required that a given set of fractions be ordered (see Figure 3). In developing these problems, the researcher was interested
to see (a) how the participants would activate and use their basic knowledge of fractions and (b) how they would represent their understandings in the computer environment. The fractions had different denominators and place values, making it difficult to make direct comparisons. Upon completing both problems, participants were asked to suggest a second solution method. This was done to ensure that participants searched a wider knowledge base for fractions. At this point, they were also asked to talk about potential areas of difficulty which children in Grades 5-6 might face in tackling the given problems.

During the final part of the interview session, the preservice teachers were asked to solve the problems again, but this time using JavaBars. They were also encouraged to explain or illustrate in the software context the solutions they had generated in the second part of the interview. At the end of this activity, participants were required to reflect on children’s possible difficulties and errors in using JavaBars, and how they could be helped. Throughout the interview, the investigator probed responses with questions relevant to the aims of the study.

The interview sessions were audiotaped and later transcribed. The transcripts were then analysed for evidence of four groups of knowledge: content knowledge about fractions, organisation of this content knowledge, pedagogical content knowledge, and the construction of links between these three knowledge components and the use of JavaBars. A unit of content knowledge refers to any identifiable concept in the area of fractions. A unit of content knowledge was deemed to be organised when there was an indication of links being made with another concept, procedure, principle, or rule.

Results

Content Knowledge

The participants’ content knowledge of fractions was analysed in two ways. Firstly, their solutions to the problems were scored using the following scheme: 2-correct solution, 1-partly correct solution, 0-incorrect solution. Because each participant had solved each problem twice, the range of possible scores for each problem was 0-4.

Table 1 shows results of the above analysis. In order to preserve the anonymity of the participants, the preservice teachers are labelled as PST A, PST B, and so on. All the participants except PST D were able to solve both problems. Three of the
preservice teachers solved both problems correctly using two different methods. Results also indicated that four of the preservice teachers had difficulties solving the problems the second time by a different approach. There was evidence that all the preservice teachers activated and used the three major schemes for fractions: equivalence, part-whole, and decimals. I give two examples.

During her solution attempt for Problem 1, PST E reasoned that the fractions needed to be expressed with the same denominator. She multiplied 3 by 7 and obtained 21 as the common denominator. This number was used in turn to express \( \frac{4}{7} \) as \( \frac{12}{21} \) and \( \frac{2}{3} \) as \( \frac{14}{21} \). She compared the numerators of the resulting fractions and decided that \( \frac{2}{3} \) is bigger than \( \frac{4}{7} \). In doing so, PST E showed an understanding of equivalence as well as the part-whole relation in a fraction. When asked to provide a second solution to Problem 1, she could not think of another way to sequence the fractions. In her first attempt at the second problem, she correctly expressed \( \frac{3}{5} \) as \( \frac{60}{100} \) and converted the decimal fractions to \( \frac{6}{100} \) and \( \frac{30}{100} \), respectively. At this point, she gave up. In her second attempt at Problem 2, she suggested that \( \frac{3}{5} \) should be changed to a decimal but did not explore this idea any further.

PST B used the same approaches to Problem 2 as PST E. But, in the first solution, she was able to use the size of the numerators to judge that \( \frac{3}{5} \) was the largest number, followed by 0.30 and 0.06. Also, in her second solution, she succeeded in translating \( \frac{3}{5} \) to a decimal fraction (0.6) and then applied her understanding of place value to decide that 0.6 > 0.30 > 0.06.

A number of preservice teachers followed similar procedures. These actions suggested that most of the preservice teachers had a good understanding of part-whole relations in fractions and of the transformation of fractions to equivalent common or decimal fractions. However, the analysis did not provide insight into other related concepts stored in their long-term memory. A more complete picture about the knowledge base of the preservice teachers was obtained by identifying fraction-related knowledge that was activated during the solution process and comments made in relation to other questions raised in the interviews. The data were analysed for their structure in two ways.

Firstly, I generated concept maps which showed the identification and use of the three components that were regarded as necessary for the solution of problems used in the present study: equivalence, part-whole, and decimals. As an example, Figure 4 shows the concept map for PST E. She had activated the three fraction-related subconcepts: equivalence (E), part-whole (PW), and decimals (D). The specific information that was activated in relation to these subconcepts is shown in parentheses. PST E had also shown an understanding of links among the three subconcepts. The concept map contains some information that is not significantly different from that activated during the solution process, but there are two new features: the language used to express \( \frac{3}{5} \) (PW3) and a comparison of two decimals.
One could not detect links to other related concepts, such as percentages or applications of fractions. There was also a lack of articulation of two-way relations among the three subconcepts, such as the translation of a common fraction to its decimal equivalent. These results showed that PST E’s knowledge of fraction was limited.

Secondly, the organisational quality of the participants’ knowledge was analysed by considering the connections that participants made with related concepts. The analysis was based on a modified form of the scoring system developed by Chinnappan, Lawson, and Nason (2000) in their investigation of teachers’ mathematical knowledge structure. This system, it was argued, provided a good estimate of links amongst the network of strands of knowledge built up by teachers. The connections were analysed in terms of the distance of links from the three core concepts found in the concept maps. A one-level link was defined as a relation made with another concept. If there was evidence of establishment of further connections with the new concept, such instances were regarded as higher-level links. According to the system, a knowledge base that is qualitatively superior would have more instances of higher-level links.

Accordingly, transcripts were searched to determine the frequency of links between the three core concepts and other concepts. For example, it can be seen from Figure 4 that PST E made one one-level link involving the equivalence (E1), two one-level links involving the part-whole subconcept (PW1 and PW2), and two one-level links involving decimals (D1 and D2). This participant also made one higher-level link, shown by PW2 and PW3. This was considered a higher-level link because PST E commented that the fraction \( \frac{3}{5} \) was three parts out of five equal parts and subsequently described the number as “three fifths”.
Table 2 shows the results of this analysis. The table shows that all participants activated each of the three fraction schemas at least once during the solution process. But while there was evidence of links made with other concepts, the extent of these connections was limited. In particular, participants tended to make fewer two-level links than one-level links, suggesting that preservice teachers' fraction schemas were not well developed.

<table>
<thead>
<tr>
<th>PST</th>
<th>Concept</th>
<th>Identification</th>
<th>One-level link</th>
<th>Higher-level link</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Equivalence</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Part-whole</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Decimals</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>Equivalence</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Part-whole</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Decimals</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>Equivalence</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Part-whole</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Decimals</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>Equivalence</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Part-whole</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Decimals</td>
<td>1</td>
<td>2</td>
<td>0</td>
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<tr>
<td>E</td>
<td>Equivalence</td>
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<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Part-whole</td>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Decimals</td>
<td>3</td>
<td>2</td>
<td>1</td>
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<tr>
<td>F</td>
<td>Equivalence</td>
<td>3</td>
<td>1</td>
<td>0</td>
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<tr>
<td></td>
<td>Part-whole</td>
<td>4</td>
<td>1</td>
<td>0</td>
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<tr>
<td></td>
<td>Decimals</td>
<td>3</td>
<td>1</td>
<td>0</td>
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<td>G</td>
<td>Equivalence</td>
<td>1</td>
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<td></td>
<td>Part-whole</td>
<td>1</td>
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<td></td>
<td>Decimals</td>
<td>2</td>
<td>0</td>
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</tbody>
</table>

**Pedagogical Content Knowledge**

While the above results show that preservice teachers had built up concepts and procedures to solve the given problems, the data did not reveal their knowledge about the difficulties that young children might encounter in tackling such problems. In order to generate data relevant to this question, I analysed the transcripts for instances where the preservice teachers talked about potential
difficulties that children might face during their attempts to solve the problems and how they might help them. The results are shown in Table 3.

Table 3
Frequency of Preservice Teachers’ Comments on Children’s Difficulties in Solving Fraction Problems and Proposed Help

<table>
<thead>
<tr>
<th>PST</th>
<th>Anticipation of children’s difficulties or misconceptions</th>
<th>Suggestions for helping children</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>1</td>
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<tr>
<td>B</td>
<td>5</td>
<td>3</td>
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<tr>
<td>C</td>
<td>2</td>
<td>1</td>
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<td>D</td>
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<td>G</td>
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<tr>
<td>H</td>
<td>4</td>
<td>1</td>
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</tbody>
</table>

Table 3 shows that almost all the participants tended to have a view on the learning and teaching of fractions. However, most of them provided no or few suggestions about how to assist children. Most of their comments tended to focus on what they would do to address their own weaknesses—with little regard for children’s own dispositions and misconceptions. Most of their responses suggested a lack of concern about prior knowledge of children and how that might impact on their understanding of fractions.

The following excerpts exemplify some of the concerns expressed by the preservice teachers.

PST A: They don’t understand that $\frac{4}{7}$ is part of a whole (in reference to Problem 1)

PST B: 7 is bigger than 3, so is the top two numbers. So students might say that $\frac{4}{7}$ is bigger than $\frac{2}{3}$ (in reference to Problem 1)

PST F: Children may have difficulty transferring fractions to decimals (in reference to Problem 2)

While these are legitimate concerns, participants were not able to advance possible reasons for these potential problem areas. Again, we have evidence of limited understanding of children’s learning difficulties.

Use of JavaBars

Even though responses from the participants seem to reflect a reasonable understanding of fractions and some of the problems young children might face, it was necessary to examine how preservice teachers would transfer this knowledge when working within JavaBars. That is, I wanted to investigate the manifestation of
this knowledge in the JavaBars environment. The expectation was that participants would draw on the interactivity of JavaBars in order to explain the concepts better to children. Participants' responses to the use of the software to explain the two target problems to children were analysed for the same two categories of pedagogical content knowledge as in Table 3. The results are shown in Table 4.

Table 4
Frequency of Preservice Teachers’ Comments on Children’s Difficulties in Solving Fraction Problems Using JavaBars and Proposed Help

<table>
<thead>
<tr>
<th>PST</th>
<th>Anticipation of children’s difficulties or misconceptions</th>
<th>Suggestions for helping children</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>3</td>
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<tr>
<td>C</td>
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<td>D</td>
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<td>H</td>
<td>1</td>
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</tbody>
</table>

The results here indicate that all the participants were able to identify at least one problem area for the children. But although the preservice teachers expected JavaBars to present difficulties for children, this concern was not supported by appropriate suggestions to help children. One of the common areas that was identified concerned children’s difficulty in transforming the decimals into fractions before children could construct the appropriate bars. For instance, PST H drew attention to the difficulty children would face in converting $\frac{3}{5}$ to a fraction out of a hundred and constructing a bar to represent this when the maximum number of parts allowed by the software was 99.

The following excerpts exemplify some of the concerns expressed by the preservice teachers in relation to use of the software:

PST C: Students may not have identical bars for the fractions. Some students may think that the parts have to be the same size but not the whole [bar].

PST G: Some students may not divide the bars into equal parts

In general, their own weakness in working within this environment was also reflected in comments about how young children might work with JavaBars.

Transfer of Fraction Knowledge into the JavaBars Environment

All the participants learnt to work within JavaBars in the short time they were given, and thought that children would have little difficulty in learning and using the various menus and drawing bars. This view was reflected in the ease with
which participants were able to draw bars, fill the bars with different colours, and break the bars into parts. Six of the preservice teachers considered in this study were able to show the solution to the first problem by first constructing two bars and dividing one into seven and the other into three equal parts. The fractions were then ordered by comparing four of the one-sevenths with two of the one-thirds. Thus, they were able to map the part-wholes of the fractions with the bars and their parts. There was agreement that the bars would assist in visualising the relative size of fractions. Five of the participants tended to overlook the need to ensure that the bars were identical, and three of them failed to explain the conceptual significance of this condition.

JavaBars could have been used to show the relationship between finding a common denominator for a given set of fractions and generating equivalent fractions. For example, before using JavaBars, all the preservice teachers solved Problem 1 by finding a common denominator for $\frac{4}{7}$ and $\frac{2}{3}$ (21) and expressing each of the fractions out of 21. However, while working in JavaBars, they tended to illustrate the solution by partitioning two bars into seven and three parts respectively. While this action was correct, they could also have used the software to show graphically why $\frac{2}{3}$ is the same as $\frac{14}{21}$. None of the preservice teachers articulated the links between these representations.

In order to examine this area of their understanding further, I analysed in detail participants' solutions for Problem 2. To illustrate the results, I give three examples.

![Figure 5. PST E's Solution for Problem 2.](image)
Figure 5 shows the solution attempt of PST E. She constructed the top bar, partitioned it into five equal segments, and then coloured three of them to show the fraction $\frac{3}{5}$. She then constructed the bottom bar, partitioned it into 10 segments, and coloured three of these to indicate the decimal fraction 0.30. She commented that she could not represent 0.06. She had understood the part-whole relationship in a fraction and was able to map it onto the bars. The second bar also shows that she was able to translate a decimal fraction into a common fraction. However, she had apparently overlooked the fact that the two bars had to be of equal length in order to compare the two fractions.

![Figure 6. PST D's solution for Problem 2.](image-url)

PST D’s solution (Figure 6) indicates that she was able to translate both the decimals to common fractions and to represent them in JavaBars. She correctly constructed three identical bars and aligned them vertically in order to show their equality. She first divided the bottom bar into five equal parts and then removed two of these parts, leaving three parts representing $\frac{3}{5}$. She then tried to represent 0.06 in the middle bar but realised that she could not divide this bar into 100 segments. She overcame this problem by converting 0.06 to three parts out of fifty, thus showing an understanding of equivalent fractions. She then divided the middle bar into fifty segments and removed three of them. Finally, she used the top bar to represent 0.30. She reasoned that 0.30 was equal to $\frac{30}{100}$, and in turn
translated this to \( \frac{15}{50} \). So she broke the bar into 50 parts and removed 15 of them. In order to work out the relative size of the three given fractions, she compared the fifteen parts from the top bar with the three from the middle and the three from the bottom bar. Although her method was correct in principle, it failed in practice because she appears to have overlooked the importance of aligning the three groups of bars to allow an accurate comparison.

![Figure 7. PST F’s solution for problem 2.](image)

Figure 7 shows PST F’s interesting method of segmenting the bars to get around the limitations of the software. PST F made good use of the software in two ways. Firstly, he constructed one large bar and divided it into 100 parts by activating the Up/down and Left/right options in the Parts section of the software. Secondly, he differentiated the three fractions by colouring them differently (not shown well in Figure 7). PST F’s analysis of Problem 2 shows that he had a good understanding of part-whole relationships and decimals fractions. His use of a single bar might, however, confuse children in that they would have to visualise the three numbers in one bar.

**Discussion**

The purpose of this study was to generate data about preservice teachers’ knowledge in three areas: knowledge about fractions, knowledge about the
teaching and learning of fractions, and the use of JavaBars to exhibit their understanding of fractions. These issues concerning preservice teachers' knowledge base involving fractions and the teaching of fractions were addressed by three research questions.

In order to address these questions, this study used a framework for teacher knowledge representation which included mathematical content knowledge, organisation of mathematical content knowledge, and pedagogical content knowledge. Analysis of the knowledge base of the eight preservice teachers' suggests that all them had built up a minimum level of content knowledge of fractions. This was evidenced by the fact that they activated and applied schemas involving equivalence, part-whole, and decimals. This set of their knowledge was relevant for the representation of fractions and understanding the relative size of the given set of fractions.

In the second area of interest, concerning the structure of their knowledge of fractions, one could detect a number of gaps in the knowledge base of the participants. For instance, while almost all of the participants could find a fraction that was equivalent to the given fraction, they were unable to generate more equivalent fractions. This was the case with common as well as decimal fractions. There was also limited evidence of understanding of relationships between the parts and wholes of a fraction. However, all the participants were skilful in using procedures for finding common denominators. This gap in their knowledge is particularly significant given that the software could have easily been utilised for this purpose. Prawat (1989) argued that a well-organised knowledge structure aids in the accessing and use of that knowledge during performance. It seems that preservice teachers' knowledge in this area is not sufficiently structured, suggesting that they could experience problems in constructing alternative representation of the concepts and in activating available content knowledge during the teaching process.

In their theories about the development of children's understanding of fractions, Kieren (1988) and Mack (1990) argued that children's learning of fractions needs to be supported by a rich store of prerequisite knowledge which includes their informal understandings. A well-developed prior knowledge network involving both formal and informal knowledge would not only help children assimilate new concepts but also facilitate the use of newly acquired concepts in understanding and solving problems. In the present study, the knowledge activated by the pre-service-teachers both in the problem-solving and JavaBars contexts tended to represent more of the formal aspects of fractions such as numerical symbols and procedures for finding common denominators. Data analysis suggests that the preservice teachers in the present study were concerned more with how they would approach fractions and less about difficulties children might face in understanding and solving problems involving fractions. There were few instances where the teachers referred to children's prior knowledge, their attitudes to, and beliefs about the topic, and the influence of these factors on using JavaBars. Knowledge about how children come to understand mathematics and how the child would process the given topic knowledge constitutes a critical factor in the acquisition and further development of the content knowledge (Peterson, 1988). On the basis of what the preservice teachers
said during the interviews, I am led to the conclusion that there is a general lack of awareness of the importance of understanding the learner in the teaching-learning situation. This could perhaps have been expected, as the participants were in the early stages of their career.

From the analysis which focused on how and to what extent the preservice teachers related their knowledge about the software to their knowledge of fractions, there is some evidence that they were able to perform most of the routine functions such as constructing the bar for a fraction and partitioning it. A large proportion of the preservice teachers commented about the advantages conferred by the visual features of JavaBars. However, this point was not supported by justifications about how and why such features would help young children understand fractions better. In their modelling of fractions, English and Halford (1995) argued that teaching needed to show the links between symbols and objects that are used to represent them. In the present study, this mapping process between the symbols and parts of the bar was not clearly articulated by the teachers. It seems that limited content knowledge of the preservice teachers could have influenced not only their pedagogical content knowledge but also their flexibility in the use of the software.

The results of the study indicate that the preservice teachers did not exhibit skills at using the software to provide different but pedagogically powerful solutions to the given problem within JavaBars. For example, despite being alerted to the possibility of grouping and ungrouping parts of a bar, none of the participants made use of this information to illustrate equivalence. This feature could be used to challenge children to make connections between two representations of equivalent fractions, which could provide insight into the procedures involved in finding common denominators. The ability to move flexibly between these two representations is considered indicative of deeper understandings of mathematics and the teaching of mathematics (Kaput, 1992).

Furthermore, JavaBars has the potential to be used as a tool for testing conjectures about fractions. While this was not necessary for solving the problems given in the present study, participants could have alluded to its value during the discussions about estimating the relative size of the given fractions. Participants' failure to examine or comment on this possibility is another indication of their limited knowledge about fractions and the teaching of fractions.

In generating the data shown in Table 2, I was mainly interested in determining instances of the three knowledge components relevant to the fraction problems of the study and the two levels at which that knowledge could be organised. Future research should examine these connections further in order to provide a more complete picture about the state of preservice teachers' knowledge underlying fractions.

One limitation of the present study is that, due to constraints of time and the techniques used during the interviews, participants did not always reveal all their understandings. A larger sample size and more training with JavaBars are needed in order to better answer the questions raised in the present study—but the data analysed here has provided information that could be used in a much larger study. Despite the limitations, the analyses here also provide us with ideas about how preservice teachers could be trained to use technology to teach fractions.
Implications

The use of computers is increasingly being accepted as a viable alternative to the traditional paper and blackboard approaches to teaching fraction concepts. While this view has its merits, it is based on the assumption that teachers will be working from a sufficiently developed and organised knowledge of (a) the content area of fractions and (b) the power of the software to help children access and use prior knowledge in understanding new concepts involving fractions. Results from this study suggest that while the participants showed acceptable levels of knowledge of fractions, they may not have this knowledge integrated sufficiently with their knowledge about the computer software. While it is too early to generalise on the basis of this study, the results do seem to suggest that teacher education programs need to analyse the mathematical content and software interface carefully. Such an approach should aim to generate learning activities in which preservice teachers could explore the interrelations between their own mathematical knowledge and how that knowledge could be transformed within the computer environment. In so doing, we can expect a better understanding of children's own learning difficulties by teachers of the future.

References


Preservice Teachers' Understanding and Representation of Fractions in a JavaBars Environment


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