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School geometry: Focus on knowledge organisation

Mohan Chinnappan
University of Wollongong, Mohan.Chinnappan@unisa.edu.au

Michaell Lawson
Flinders University

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Abstract
Given that geometry is an area of mathematics that has a firm and obvious basis in the real environment, senior secondary students have surprising difficulties in geometric problem-solving. One distinct difficulty appears to be in activating the particular concepts among those previously acquired that are applicable to the problem at hand. A model is presented for analysis of student understanding, based on five levels of geometric knowledge.

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Many students appear to have difficulty with geometry because they are unable to activate and use their geometric knowledge.

MOHAN CHINNAPPAN of the Queensland University of Technology and MICHAEL LAWSON of the Flinders University of South Australia

investigate this puzzling problem.

SCHOOL GEOMETRY: *Focus on*

There are some puzzling things about geometry. From the teacher's perspective it is an area of mathematics in which students can both explore their understanding of the environment through the investigation and manipulation of shapes and figures, and develop their understanding of forms of reasoning. Researchers have also found geometry attractive as a rich domain in which to observe students' methods of reasoning and problem solving. Yet at the senior level of high school, examination of student performance indicates that students often do not handle geometry concepts well compared to other areas of mathematics, perhaps seeing geometry as one of the less interesting areas (Bloom, 1986). Students continue to perform poorly in problems that require the application of geometric knowledge (see, for example, Senior Secondary Assessment Board of South Australia, 1988).

Why is it that students who can show high levels of skill in other areas of mathematics continue to have difficulty with geometry? One difficulty experienced by students in geometry classes that has been given recent attention concerns *knowledge access* – the activation and use of knowledge. The point of interest here is not with what students do not know, since it can be shown in most cases that students 'have' this knowledge in memory in one form or another. Rather, the problem is one of production of that knowledge at the appropriate point in the problem attempt. Whitehead (1929) referred to this as a situation in which the students' knowledge was 'inert'.

Accessing knowledge

In a recent study of geometry problem-solving, we examined the accessing of knowledge by high and low-achieving students (Lawson & Chinnappan, 1994). The number of problem-relevant geometry theorems and formulae that students could retrieve with and without any assistance was compared. Results indicated that on a set of four problems typical of those included in a Year 10 syllabus, the low-achieving group could, by themselves, access only about 50 per cent of the knowledge required for the development of solutions. However, with the assistance of prompts, this group could access a further 30% of the relevant geometry information. The corresponding figures for the high-achieving group were 80% without assistance and an extra 15% with assistance. These findings suggested that for some students who are not highly successful problem solvers, the problem is not simply one of not having the knowledge available for use on the problem, but that they experience difficulty in activation of that knowledge during memory search.

In searching for explanations for this problem of access we identified three broad sets of influences. The first factor is the state of organization of knowledge. It seems likely that the difficulty in activation is influenced by the way in which knowledge components are connected and grouped in memory. A second influence on accessing of knowledge is also related to memory. Many students do not seem to have developed effective strategies for memory search so that knowledge which is available cannot easily be activated and considered. In such cases the students, when working on their own, cannot retrieve information relevant to the problem. The final factor suggested to influence access is more affective in nature, being related to the students' views of themselves as problem solvers. A history of difficulty in searching for problem-relevant knowledge is likely to lower the student's level of persistence in the search process (Prawat, 1989). In this article we concentrate on a discussion of the first of these factors, namely the organization of knowledge.

knowledge organization

Knowledge organization

If the difficulties shown in the performance of these students derive in part from the state of their knowledge organization, how can we talk about that state of organization? Many teachers will be familiar with the van Hiele model of geometric thinking (Pegg, 1985). The van Hiele model describes geometric thinking in terms of five levels of reasoning. At each of the proposed levels, the model examines the complexity of reasoning skills associated with the manipulation of geometrical objects, symbols and concepts. It provides a useful framework within which to explore students' logical arguments and deductions. The van Hiele model, however, does not address the issue of knowledge organization. Rather, it is focused on differences in procedures used to manipulate that knowledge.

We have been investigating an alternative framework for thinking about knowledge organization, one that focuses on the degree of connectedness of geometry knowledge. Believing that memory has some form of network structure we assume that the quality of the links or connections among components of knowledge has a major influence on the likelihood of knowledge access. If this is the case we might expect students with better quality connections to display access to a wider set of knowledge, especially when they are working on their own without assistance from a teacher. Our recent research indicates that this does seem to be the case. Before we outline our model of geometry knowledge structure, let us explore major components of school geometry and some of the relations that can be constructed in this system.

Basic components of geometry knowledge structures

Several components of geometry knowledge are needed for a student to function effectively in situations that call for the use of this knowledge. First, during the initial exposure, students identify and learn about some *basic elements*, or *basic features*, of geometry knowledge. These are parts of the figures that are most obvious to the students and can be seen as constituting the foundation elements for further knowledge construction. For instance, the circle, curves, straight lines, angles and measures such as degrees might be such features. The second major component of geometry knowledge concerns relationships, or propositions, that students construct within and between these basic elements. These sets of propositions are generally known as *rules* or *theorems*. For example, one could establish that a straight angle is 180°, or a right angle is 90°. The third component of geometry knowledge concerns the types of knowledge structures or *schemas* that students could construct by using the previous two components as building blocks. These structures can be seen as larger, more integrated clusters of knowledge that have been developed as a result of experience in investigating and elaborating the relationships between sets of features and theorems. High-achieving students can be expected to show a more extensive network of the above classes of geometry knowledge compared to the low-achieving students.

The levels of connectedness framework

Based upon this analysis we propose that geometry content knowledge can be characterized as being in five states or levels of connectedness: the *Basic* level, the *Form* level, the *Rule* level, the *Application* level and the *Elaboration* level. We use the term 'levels of connectedness' to refer to states of *use* of the knowledge that is available to the student. In referring to these as states of use of knowledge we recognize that we can only make inferences upon the state of knowledge organization from the performance of the student, that is, from the way in which the student uses available knowledge. The assumption we make here is that the more the student can *extend* the particular set of knowledge the better the quality of organization and so the more likely it is that the knowledge will be activated during a problem attempt. The five levels are as follows:

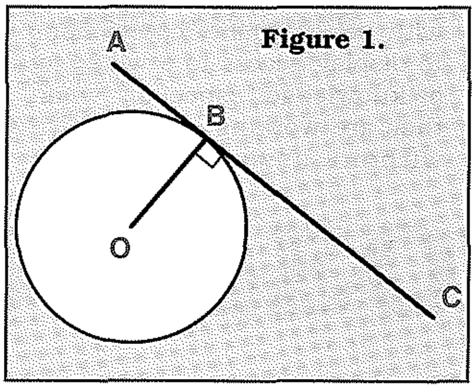
Basic level: At this level discrete features, constituting the building blocks of geometrical figures such as points, lines and curves, are established. These are described as discrete in the sense that they are not differentiated into sets of features among which relationships have been established.

Form level: Geometric forms are established in shapes and figures that reflect the linking of Basic level features or components to form more complex geometrical forms such as a radius or tangent.

Rule level: This level incorporates formal postulates, or propositions regarding forms, such as 'sum of the interior angles in a triangle is 180 degrees'. Knowledge at this level is generally represented as theorems or formulae. Each rule formalizes a particular result that is precipitated by the assembling of knowledge components that had their source in the Basic and/or Form levels. Any information that is connected at this level is basically *declarative* in nature (Anderson, 1990).

Application level: This level requires the use of contents of the Rule level to attempt a problem whose solution can be generated using rules. This level incorporates knowledge that can be seen as being more *procedural* and *conditional*, that is, knowledge of both how a certain sequence of analysis is to be carried out and when use of that sequence is likely to be effective. In a geometry problem this knowledge is likely

Figure 1.



to be cued by the details provided in a diagram. These details act to direct and constrain the student's search of available knowledge, suggesting perhaps that specific angle measures be calculated. This activity could be seen as *local* knowledge use, local in the sense that the knowledge used to explore the diagram is confined to a single rule. For instance, given two angles in a triangle, the magnitude of the third angle could be generated using the rule relating to the sum of angles in a triangle.

Elaboration level: This level contains information about ways of making geometrically consistent associations between clusters of knowledge forms, or schemas, at any of the above three levels. In this level knowledge use is not constrained by a single rule. The student may generate appropriate multiple representations, which will usually be diagrammatic, might extend a diagram to construct a new geometric form, and might then explore relationships between these newly generated components. In this sense, activity at this level is more *global*. For instance, the establishment of new connections between rules would constitute an elaborative activity in the sense that the network of meanings attached to the rules is expanded.

Central to this description of the state of organization of knowledge is the notion of connectedness. Growth in knowledge is viewed as involving not only addition of new features, forms or rules, but is also characterized by the development of increasingly rich connections both within these components and between them. We picture this knowledge growth as involving both an increasing degree of integration of related components or connections at the local level, and the extension or spread of connections across these locally integrated structures or schemas. Experience with a wider range of problems can therefore be expected to be associated with the 'packaging' of knowledge components into units which have an increasing degree of integration, or increasing coherence, or density. We see evidence of both these attributes of increasing experience in studies of expert performance (e.g. Bedard & Chi, 1993). Many of the moves made by experts during the development of a problem solution seem to involve large leaps in reasoning because the clusters of knowledge accessed by the expert are more densely integrated than those of novice problem solvers.

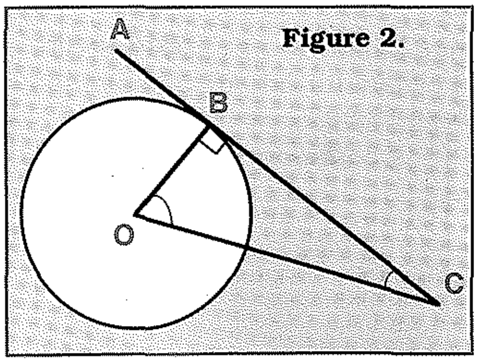
An illustration of the model

Let us first look at how we can apply this framework to a meaningful interpretation of the theorem that states that, 'A tangent to a circle is perpendicular to the radius drawn to its point of contact' by analyzing the various knowledge components embedded in this theorem.

Figure 1 shows a diagrammatic representation of this theorem. At the Basic level of knowledge connectedness, students need to recognize features such as points, curves, straight lines and angles. Figure 1 shows one instance of the linkages between these concepts. Firstly, there are four points in this figure: *A*, *B*, *C* and *O*. *AC* and *BO* are straight lines. At the point of intersection of *OB* and *AC* two angles are created: angle *OBA* and angle *OBC*.

Assuming that a student was able to recognize the above elements of knowledge at the Basic level, let us consider how this knowledge develops into structures at the Form level. At the Form level students are required to make the link between *OB* as straight line and *OB* as a radius. Likewise

Figure 2.



the straight line *AC* must now be recognized as a tangent, and finally, the point *O* identified as the centre of circle.

Knowledge structures linked at the Basic and Form levels are subsequently employed in developing further relations at the Rule level. For instance, when students are attempting to establish meaningful relationships between a radius, a tangent and the size of the angle created at the point of contact, that is, a right angle, then they are operating at the Rule level. The theorem is built from a set of features and forms, the meaningful integration of which is crucial to its understanding and further use.

We can now look at possible extensions a student could make at the Application level. For instance, given the appropriate diagram, the rule expressing knowledge of the angle sum in a triangle could be used to find a third angle of the right-angled triangle. For example, in Figure 2, a student could determine angle *BOC* if angle *BCO* is given. Here the student is establishing new connections within a specific rule.

Knowledge at the Elaboration level can be used to explore further parts of diagrams or to explore further connections between the theorem illustrated by Figure 1 and other theorems. For instance, a student could construct an elaboration of Figure 2 which involves the integration of the Pythagorean theorem with that of the tangent theorem indicated by Figure 1. This elaboration is shown in Figure 3. In this case the student is manipulating knowledge beyond a specific theorem so that the network of connections between rules is expanded.

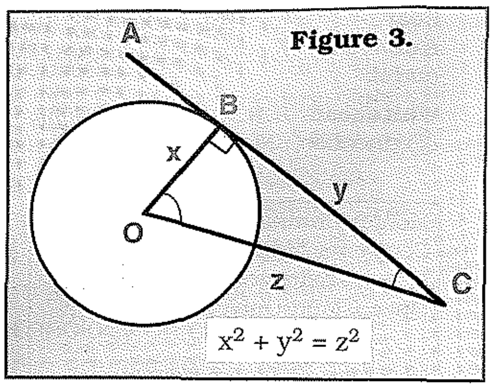
Use of the model

This model has assisted us in analysis of student understanding of geometry knowledge and level of performance in geometry tasks. The levels framework suggests a way of classifying students' understandings and their difficulties, and provides a scale for use in monitoring the growth of that understanding. Specification of these levels as levels of connectedness implies that students' knowledge can be seen as spreading within and between parts of an overall network. The more extensive the spread both within and between these parts, the more powerful will be the individual student's knowledge of geometry. Other things being equal in the student world, the student with the more powerful, more extensively elaborated network, will be able to solve more complex problems.

This framework also has implications for sequencing of material presented to students and for the assessment of problem-solving performance. The levels of connectedness framework suggests a basis for ordering a sequence of teaching, moving from the establishment of features to the interrelating of schemas associated with rules. The same logic can be applied to the assessment of geometry knowledge since the levels represent points on a scale of increasing complexity. The more extensive a student's elaborative activity, the more valued the performance.

The emphasis given to the notion of connectedness in this framework parallels the importance placed by James Hiebert (1984) on establishing links during teaching, links between the student's existing, everyday understanding and the forms of mathematical language and procedure. Hiebert's concern was with the importance of establishment of links at very specific levels, so that misinterpretations and gaps in understanding can be avoided. Hiebert emphasised the significance of specific teaching actions in the linking process. While our framework has been derived from detailed analysis of students' problem-solving behaviour, it is highly compat-

Figure 3.



ible with Hiebert's analysis of teachers' actions. In both sets of work a principal aim of mathematics lessons is seen as the fostering of well-connected knowledge clusters, so that emphasis is placed on activity that is designed to establish and extend links between knowledge components. In our case we see that such structures provide a powerful base from which to understand and solve problems in junior and senior high school geometry.

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Mohan Chinnapan
Centre for Mathematics and Science Education
Queensland University of Technology
GPO Box 2434 Brisbane
Queensland 4001

Michael Lawson
School of Education
The Flinders University of South Australia
GPO Box 2100 Adelaide
South Australia 5001

From the Editor

I have just received my bound copies of the Proceedings and Selected Lectures from the 1992 ICME-7 conference. What a good conference that was! I had some initial doubts about attending – 3500 participants was rather daunting. But it turned out to be very worthwhile.

I wonder why it is that relatively few mathematics teachers attend conferences? There are of course a number of negatives. One is the matter of cost, especially for an overseas conference. Another is the matter of time: there are often more pressing matters at hand, and even if the conference occurs at a relatively slack time, we may well think of a more attractive option. Then there is the thought of arriving on your own, and possibly not knowing anyone ... .

But there are many positive aspects too. Conferences are an important way of keeping up with what is happening, of learning about what is new on the teaching scene, of gaining new ideas. Every conference has an extensive display of the latest in books and teaching aids for perusal and purchase. Mixing with other teachers and sharing classroom experiences can give insights into the handling of student behavioural problems, and the teaching of 'difficult' parts of the course. Conferences provide an opportunity for hands-on experience with new software and the latest in calculators. And for the regular conference goer, there is the enjoyment of catching up with old friends.

I find that conferences are more significant for me when I prepare something to share with others. Such preparation of a talk or a workshop is a good discipline in itself, and can result in a publication – always good for the CV! I also find that attending a conference with a special aim in mind is helpful. Thus, as editor of *AMT* I am always on the lookout for good material; in the case of ICME-7 this resulted in the series of 'International' pages. Again, I found the ICME-7 conference particularly useful because I went with the express purpose of learning more about computer-aided teaching of university calculus. Having such an aim in mind, and making a report for one's school or institution focuses the attention and gives meaning to what might otherwise be a disparate set of talks and discussions.

Make a point of attending your next State conference!

Paul Scott

A conference encounter

During an intermission of the NCTM meeting held in Corpus Christi a couple of years ago, I found myself at a little table enjoying a Coke with Father Bezuska. The conversation turned to geometry and the old master, Euclid. I finally said, "I owe an immeasurable debt to Euclid. Reading the first six books of his Elements in school marked an important turning point in my life, for it decided that I would go into mathematics as my life's work." And I concluded by musing, "Isn't it remarkable that what a man did some 2000 years ago should so effect one's life?" And then I suddenly realized that perhaps there was nothing so remarkable about it after all, for there sitting across the table from me was Father Bezuska in clerical dress, and he similarly had had his life markedly effected by the doings of a man who lived about 2000 years ago.

– as told by Howard Eves in his book *Mathematical Circles Revisited.*