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Implications of ballast breakage on ballasted railway track based on numerical modeling

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Abstract

Large and frequent cyclic train loading from heavy haul and passenger trains often leads to progressive track deterioration. The excessive deformation and degradation of ballast and unacceptable differential settlement of track and/or pumping of underlying soft subgrade soils necessitate frequent and costly track maintenance. A proper understanding of load transfer mechanisms and subsequent deformations in track layers is the key element for safe and economical track design and optimum maintenance procedures. Many simplified analytical and empirical design methods have been used to estimate the settlement and stress-transfer between the track layers. However, these design methods are based on the linear elastic approach, and often only give crude estimates. Given the complexities of the behaviour of the composite track system consisting of rail, sleeper, ballast, sub-ballast and subgrade subject to repeated traffic loads in a real track environment, the current track design techniques are overly simplified. The track design should also account for the deterioration of ballast due to breakage and subsequent implications on the track deformations. Considering this, an elasto-plastic constitutive model of a composite multi-layer track system is proposed. Constitutive models and material parameters adopted in this numerical model are discussed. A hardening soil model with a non-associative flow rule is introduced to accurately simulate the strain-hardening behaviour of ballast. The breakage of ballast observed in large scale triaxial tests is also simulated based on this model. In conjunction, numerical simulations are also performed using a two-dimensional plane-strain finite element analysis (PLAXIS) capturing the effects of ballast breakage and track confining pressure. The paper also demonstrates the advantages of the proposed elasto-plastic finite element simulations when compared to conventional analytical methods used by practitioners that are primarily based on a linear elastic approach.

1 INTRODUCTION

Australia relies heavily on rail for the transportation of bulk commodities and passenger services, and has introduced faster and heavier trains in recent years due to the growing demand. This has resulted in an increase in the rail track deformations, and consequently has increased the frequency and cost of maintenance. In order to compete with other modes of transportation, rail industries face challenges to minimise track maintenance cost, and to find alternative materials and better design and construction approaches to improve the performance of tracks. The track should be designed to withstand large cyclic train loadings to provide protection to subgrade soils against both progressive shear failure and excessive plastic deformation (Li & Selig 1998).

A proper understanding of load transfer mechanisms and subsequent deformations in track layers is the key element for safe and economical design. Several simplified analytical and empirical methods have been proposed in the past for the design of rail track including the method proposed by the American Railway Engineering Association (AREA 1996), European Union Rail (UIC 1994), British Rail (Heath et al. 1972) and the Japanese National Railways (Okabe 1961). However, these design methods are based on assumption of a homogeneous half-space for all track layers which neglect different properties of individual track layers. Several multi-layer track models have been developed for analysing stresses and deformations in all major components of track and subgrade, i.e., rails, fasteners, sleepers, ballast, sub-ballast, and subgrade. These methods include ILLITRACK (Robnett et al. 1975), GEOTRACK (Chang et al. 1980), KENTRACK (Huang et al. 1984) and FEARAT (Fateen 1972). However, these methods assume elastic behaviour of track layers, including ballast, which is a serious drawback.

Moreover, the breakage of ballast leads to significant compression of the ballast layer (Selig & Waters 1994, Indraratna & Salim 2005). The beneficial aspects of confining pressure on the track stability and reducing the maintenance cost is well established. The application of sufficient confinement to the ballast layer, leads to significant reduction in the vertical and lateral deformations, and assures more resilient long-term performance of the ballast layer.
(Lackenby et al. 2007, Indraratna & Salim 2003). In order to find out stress-strain behaviour of in-situ track layers and subsequent deformations in the rail track, a field trial was conducted on an instrumented track at Bulli, NSW Australia (Indraratna et al. 2009, 2010a). In this paper, the laboratory and field measurements were used for the calibration of the constitutive model and successive implementation in a finite element analysis capturing the elasto-plastic deformation characteristics of ballast. Further validation of the finite element model proposed herewith is conducted through comparison with field measurements.

2 FINITE ELEMENT ANALYSIS

2.1 Large Scale Triaxial Test Configuration

A series of isotropically consolidated drained triaxial tests were conducted on ballast using state-of-the-art large scale cylindrical triaxial equipment (Indraratna & Salim 2005). In the current study, an elasto-plastic constitutive model for ballast under triaxial loading is proposed and is implemented in PLAXIS (2006). PLAXIS has demonstrated its success in the limit analysis of geotechnical problems (de Borst & Vermeer 1984). Two dimensional axisymmetric finite element model is numerically simulated by the mesh discretisation shown in Figure 1. The 0.6 m high and 0.15 m wide finite element model is discretised to 1160 fifteen-node elements. The node at the left corner of the bottom boundary of the section is considered as pinned support, i.e., is restrained in both vertical and horizontal directions (i.e. standard fixity). The left (axis of symmetry) and bottom boundaries are restrained in horizontal and vertical directions respectively. The top and right boundaries are fully unrestrained. The effective confining stress is applied to the right boundary. The vertical monotonic deviator stress is applied at the top boundary.

2.2 Rail Track Model Configuration

An elasto-plastic constitutive model of a composite multi-layer track system including rail, sleeper, ballast, sub-ballast and subgrade is proposed. Numerical simulations are performed using a two-dimensional plane-strain finite element analysis PLAXIS (2006) to predict the track behaviour. A typical plane strain track model is numerically simulated in a Finite Element discretisation as shown in Figure 2.

The 3 m high and 6 m wide finite element model is discretised to 1464 fifteen-node elements, 37 five-node line elements and 74 five-node elements at the interface. The nodes along the bottom boundary of the section are considered as pinned supports. The left and right boundaries are restrained in the horizontal directions, representing smooth contact vertically. The vertical dynamic wheel load is simulated as a line load representing an axle train load of 25 tons with a dynamic impact factor of 1.4. The gauge length of the track is 1.68 m and the shoulder width of ballast is 0.35 m with the side slope of the rail embankment being 1:2.

2.3 Method of Analysis

Finite element modeling of rail track structure is essentially a three dimensional (3D) problem requiring huge computational power and resources. In engineering practices, it is prudent to simplify complex 3D problems into 2D so that extensive parametric study can be undertaken reasonably well to verify and optimize the rail track design concepts. This has many practical implications in routine engineering analysis and design. For this, the simplification process has to be appropriate and represent the ideal boundary conditions. In this paper, a higher order constitutive model and interface elements are used.
to capture adequately the real behaviour of the track. The geometry of the mesh for axisymmetric and plane strain condition is symmetrical about the centreline, therefore only one half of the cross section passing through the axis of symmetry is considered.

The ballast and other track layers are modeled using 15-node linear strain quadrilateral (LSQ) elements. In representing line elements, 5-node elements are used. Since it is also necessary to model the interaction between track layers, special interface 10-node elements are adapted. Figure 3 shows details of these elements used in finite element simulations. The 15-node isoparametric element provides a fourth order interpolation for displacements. The numerical integration by Gaussian integration scheme involves twelve Gauss points (stress points). This powerful 15-node element provides an accurate calculation of stresses and failure loads. It is postulated that the 15-noded, cubic strain triangle is theoretically capable of accurate computations in the fully plastic range for undrained situations which involve axial symmetry or plane strain (Sloan & Randolph 1982). The numerical integration of line and interface element is carried out by Newton-Cotes integration considering 4 sample points. The mesh generation of PLAXIS version 8.6 used here follows a robust triangulation procedure to form ‘unstructured meshes’, which are considered to be numerically efficient when compared to regular ‘structured meshes’.

2.4 Constitutive Models

Two different soil models have been adopted for the granular soil: the classical Mohr-Coulomb elastic-perfectly plastic model and the isotropic Hardening Soil Model (HSM), both available in the material models library of PLAXIS. The constitutive model parameters adopted in the investigation are based on the available data derived from laboratory test results (Indraratna & Salim 2005) as well as from the field investigation (Indraratna et al. 2010a, b).

2.4.1 Mohr-Coulomb Model

The M-C model is used to represent sub-ballast and subgrade soils. As a prototype of the classical approach to constitutive modeling of soil behaviour, the classical ‘Mohr-Coulomb’ (M-C) elastic-perfectly plastic model with non-associative flow rule is used. The high values of cone resistance \((q_u)\) and friction ratio \((R_f)\) obtained in EFCP tests revealed that the subgrade soil was a stiff overconsolidated silty clay (Indraratna et al. 2010a, 2011). The Cam Clay model is unsuitable for simulation of this heavily overconsolidated soil. An elastic, perfectly plastic, Mohr-Coulomb model with a constant value of Poisson’s ratio has been used to simulate the behaviour of the weathered silty clay. The Mohr-Coulomb yield criterion in terms of the principal stresses is given as:

\[
f = \frac{1}{2} \left| \sigma_1 - \sigma_3 \right| + \frac{1}{2} \left( \sigma_1 + \sigma_3 \right) \sin \phi' - c \cos \phi' = 0
\]  

(1)

where \(\sigma_1, \sigma_3\) and \(\sigma_2\) are major and minor principal stresses respectively; \(c, \phi, \phi'\) are cohesion and angle of internal friction, respectively.

In addition, the plastic potential function \(g\) is defined as below.

\[
g = \frac{1}{2} \left| \sigma_1 - \sigma_3 \right| + \frac{1}{2} \left( \sigma_1 + \sigma_3 \right) \sin \psi
\]  

(2)

where \(\psi\) is angle of dilation and is an additional plasticity parameter used to describe the plastic potential function. This parameter is required in modeling the positive plastic volumetric strain increments. The Mohr-Coulomb model involves five parameters, namely Young’s modulus, \(E\), Poisson’s ratio, \(\nu\), the cohesion, \(c\), the friction angle, \(\phi\), and the dilatancy angle, \(\psi\). For sub-ballast and subgrade soil, the dilatancy angle is considered as zero. The complete set of Mohr-Coulomb model parameters adopted in the numerical simulations is given in Table 1.

2.4.2 Hardening Soil (HS) Model

The HS model is used to represent the ballast layer. It is an isotropic hardening plasticity model intended to describe the mechanical behaviour of sand, gravel and stiff, heavily overconsolidated clays. In contrast to an elastic perfectly-plastic model, the yield surface of the HS model is not fixed in principal stress space, but will expand due to plastic straining. When subjected to primary deviator loading, the soil shows a decreasing stiffness and irreversible plastic strains develop. The HS Model is by far more superior than the hyperbolic model as the theory of plasticity is adopted including soil dilatancy and a yield cap. Its yield function is given by (Schanz et al. 1999):

\[
f = \frac{1}{E_{so}} \left( \frac{q}{q_f} \right) - \frac{2q}{E_{ur}} - \varepsilon_i
\]  

(3)

where \(E_{so}\) is the secant modulus at 50% failure load in drained triaxial compression, \(E_{ur}\) is the Young’s modulus describing the elastic response of the material, and \(\varepsilon_i\) is the plastic shear strain which is only (scalar) hardening parameter of the model defined.
as \((2\varepsilon_t^p - \varepsilon_t^p)\) where \(\varepsilon_t^p\) and \(\varepsilon_t^p\) are the major principal component and the volumetric component of plastic strain, respectively.

In Equation (3), the asymptotic value of the shear strength, \(q^\ast\), is given by:

\[
q^\ast = \frac{q_0}{R_i} = \frac{2 \sin \phi}{R_i (1 - \sin \phi')} \sigma_3'
\]

(4)

where \(q_0\) is the stress deviator at failure, provided by the Mohr-Coulomb criterion, \(\sigma_3'\) is the minor principal effective stress, and \(R_i\) is a material parameter, which should be smaller than 1.

Figure 4 exhibits the evolution of the hardening soil model parameters based on laboratory test data for clean ballast as reported by Indraratna & Salim (2005). As discussed previously by Indraratna et al. (1998), the effect of \(\sigma_3\) on the shear strength of clean ballast can be represented in terms of Mohr circles as shown in Figure 5. The stress-dependent secant stiffness modulus for primary loading, \(E_{50}\), is given as:

\[
E_{50} = E_0 \left(\frac{-\sigma_3' \sin \phi}{p_0 \sin \phi'}\right)^m
\]

(5)

where \(p_0\) is a reference pressure, and \(m\) a material parameter, typically in the range \(0.5 \leq m \leq 1.0\). For unloading and reloading stress paths, another stress-dependent stiffness modulus, \(E_{\delta_3}\), is defined as:

\[
E_{\delta_3} = E_{\delta_3}^0 \left(\frac{-\sigma_3' \sin \phi}{p_0 \sin \phi'}\right)^m
\]

(6)

where \(E_{\delta_3}^0\) is the reference Young’s modulus for unloading and reloading, corresponding to the reference pressure \(p_0\). In many practical cases, it is appropriate to set \(E_{\delta_3}^0\) equal to 3 \(E_{50}\).

The stress-dependent tangent stiffness modulus for primary loading, \(E_{\text{tangent}}\), is given by:

\[
E_{\text{tangent}} = E_0 \left(\frac{-\sigma_3' \sin \phi}{p_0 \sin \phi'}\right)^m
\]

(7)

where \(\sigma_3'\) is a tangent stiffness at a major principal stress of \(-\sigma_3 = p_0 \cos \phi\). The coefficient of earth pressure at rest for normal consolidation, \(k_0\), is expressed as \(1 - \sin \phi\) (Jaky 1944). The Poisson’s ratio for unloading/reloading conditions \(v_{\delta_3}\) is typically considered as 0.2. The flow rule adopted in HSM is characterized by a classical linear relation, with the mobilized dilatancy angle, \(\psi_m\), given by:

\[
\sin \psi_m = \frac{\sin \psi_m' - \sin \phi_m'}{1 - \sin \psi_m' \sin \phi_m'}
\]

(8)

where \(\psi_m'\) is a material constant (the friction angle at critical state).

\[
\sin \psi_m' = \frac{\sigma_3' - \sigma_3^p}{\sigma_3' + \sigma_3^p}
\]

(9)

The mobilised dilatancy angle, \(\psi_m\), depends on the values of friction, \(\phi\) and dilatancy angles at failure, \(\psi\) which control the quantity \(\phi_m'\). Note that equation (8) is comparable to the Rowe’s stress-dilatancy theory.
between increment of breakage index, (dB) representing as:

$$\frac{\sigma'_v}{\sigma'_s} = \left(1 - \frac{dE}{dV}\right) \tan\left\{45 + \frac{\phi'_v}{2}\right\} + \frac{dE_v}{\sigma'_v} \left(1 + \sin \phi'_v\right)$$

(10)

where, \(\phi'_v\) is the friction angle excluding the effect of dilation and particle breakage. The value of \(\phi'_v\) varies between \(\phi'_v\) (basic friction angle between particles) and \(\phi'_v\) depending on the sample density. The difference between \(\phi'_v\) and \(\phi'_v\) is attributed to the energy spent on the process of rearrangement of particles during shearing.

Recent studies (Indraratna & Salim 2002; Salim & Indraratna 2004) described the dependence of particle breakage and dilatancy on the friction angle of ballast. A modified flow rule considering the energy consumption due to particle breakage during shearing deformations is given by (Salim & Indraratna 2004):

$$\frac{dE_v}{dV} = \frac{9(M - \eta)}{9 + 3M - 2\eta M} + \frac{dE_v}{\rho dV} \left(9 - 3M\right) \left(6 + 4M\right) \left(\frac{6 + M}{6 + M}\right)$$

(11)

Indraratna & Salim (2002) proposed that the incremental energy consumption due to particle breakage per unit volume \((dE_v)\) is proportional to the increment of breakage index, \((dB)_v\), i.e. \(dE_v = \beta dB_v\) where \(\beta\) is a constant of proportionality and \(B_v\) is the breakage index proposed by Marsal (1973) for rock-fill materials. Therefore Equation (11) is further represented as:

$$\frac{dE_v}{dV} = \frac{9(M - \eta)}{9 + 3M - 2\eta M} + \frac{\beta dB_v}{\rho dV} \left(9 - 3M\right) \left(6 + 4M\right) \left(\frac{6 + M}{6 + M}\right)$$

(12)

In order to assess the breakage of ballast, a new parameter, Ballast Breakage Index (BBI), was proposed by Indraratna et al. (2005).

$$BBI = \frac{A}{A + B}$$

(13)

where, \(A\) is the shift in the PSD curve after the test, and \(B\) is the potential breakage or the area between the arbitrary boundary of maximum breakage and the final PSD. The method of determination of BBI is shown in Figure 6. In the current FE analysis, BBI proposed by Indraratna et al. (2005) is used. Incorporating BBI, Equation (12) can also be expressed as:

$$\frac{dE_v}{dV} = \frac{9(M - \eta)}{9 + 3M - 2\eta M} + \frac{\beta dB_v}{\rho dV} \left(9 - 3M\right) \left(6 + 4M\right) \left(\frac{6 + M}{6 + M}\right)$$

(14)

The experimental values of \(\eta\), \(\rho\), \(M\) and the computed values of \(\frac{dE_v}{dV}\) which are linearly related to the rate of particle breakage \(dB_v/dV\) can be readily used to predict the modified flow rule. Figure 7 shows method of determination of \(\phi'_v\) using non-linear relationship between of \(\phi'_v\) and the rate of particle breakage at failure, \((dB_v/dV)_f\). For latite ballast under triaxial testing, the value of \(\phi'_v\) was found to be approximately 44° (Indraratna and Salim 2002, Salim and Indraratna 2004).

The rate of energy consumption at failure \((dE_v/dV)_f\) can be calculated for given \(\phi'_v\) according to equation (10), using the values of effective stress ratio at failure \((\sigma'_v/\sigma'_f)\) and dilatancy factor at failure \((1 - dE_v/dV)_f\) obtained in triaxial tests.

The \((dE_v/dV)_f\) is related to the differential increment of ballast breakage index \((dB)_v\) corresponding to \((dE_v/dV)_f\) by a linear relationship defined as:

$$\frac{(dE_v/dV)_f}{(dB_v/dV)_f} = \kappa \left[\frac{dE_v}{dV}\right]$$

(15)

where, \(\kappa\) is a constant of proportionality (refer Fig. 8).
Thus the non-associated plastic flow rule incorporating the rate of particle breakage during shearing is represented by

$$\frac{d\varepsilon^p}{d\varepsilon^c} = \left[ 1 - \frac{\sigma'_s}{\sigma'_f} \right] \tan^{-1} \left( 45 - \frac{\phi'_m}{2} \right) + \kappa \left( \frac{dBBI}{\sigma'_s d\varepsilon^p} \right) (1 - \sin \phi'_m)$$

(16)

The effective stress ratio ($\sigma'_s/\sigma'_f$) and dilatancy factor ($1 - d\varepsilon^p/d\varepsilon^c$) can be expressed in terms of mobilised friction angle $\phi'_m$ and mobilised dilation angle $\psi_m$, respectively:

$$\frac{\sigma'_s}{\sigma'_f} = \frac{1 + \sin \phi'_m}{1 - \sin \phi'_m}$$

(17)

$$\frac{1 - d\varepsilon^p}{d\varepsilon^c} = \frac{1 + \sin \psi_m}{1 - \sin \psi_m}$$

(18)

The flow rule originally proposed in hardening soil model is further modified to incorporate ballast breakage. For plane strain condition (i.e. $\varepsilon_3 = 0$), the angle of dilatancy is obtained as (Bolton 1986):

$$\sin \psi_m = \frac{d\varepsilon^p}{d\varepsilon^c} = \frac{\left[ \frac{d\varepsilon^p}{d\varepsilon^c} \right]_1 + 1}{\left[ \frac{d\varepsilon^p}{d\varepsilon^c} \right]_1 - 1}$$

(19)

The Equation (19) is extended to include triaxial condition (i.e. $\varepsilon_3 = \varepsilon_1$),

$$\sin \psi_m = \frac{\left[ \frac{d\varepsilon^p}{d\varepsilon^c} \right]_2}{\left[ \frac{d\varepsilon^p}{d\varepsilon^c} \right]_2}$$

(20)

Analogous to the extension of the stress-dilatancy approach, the angle of dilatancy can be extended for ballast incorporating effect of particle breakage.

$$\sin \psi_m = \left[ 1 - \left( \frac{\sigma'_s}{\sigma'_f} \right) \left( \frac{1 - \sin \phi'_m}{1 + \sin \phi'_m} \right) + \kappa \left( \frac{dBBI}{\sigma'_s d\varepsilon^p} \right) \left( 1 - \sin \phi'_m \right) \right]$$

$$\left[ 1 + \left( \frac{\sigma'_s}{\sigma'_f} \right) \left( \frac{1 - \sin \phi'_m}{1 + \sin \phi'_m} \right) - \kappa \left( \frac{dBBI}{\sigma'_s d\varepsilon^p} \right) \left( 1 - \sin \phi'_m \right) \right]$$

(21)

It is interesting to know that the proposed modified stress-dilatancy relation reduces to Rowe's stress-dilatancy relation when particle breakage is ignored. The current formulation considers the contractive strains as positive and is consistent with geotechnical point of view. This is opposite of the PLAXIS code, where the contractive strain (& compressive stress) is negative and dilation (& tensile stress) is considered positive. Therefore, appropriate sign notations of current formulation are adopted during implementation in PLAXIS code. Further details of the HS material parameters and breakage parameters are given in Table 1. Rail and concrete sleepers are considered as linear elastic and their parameters can be found elsewhere (Indraratna et al. 2011). The roughness of the interaction is modeled by selecting an appropriate value for the strength reduction factor in the interface ($R_{int}$). This factor relates the interface strength to the soil strength.

The current formulation of finite element is incapable of conducting postpeak analysis into the strain-softening region however such large strains or large deformations are not permitted in the reality; hence the study is focused on the peak strength. The values of the friction and dilation angles in plane strain differ from those pertaining under triaxial conditions. The relationship between these two sets of angles depends on the assumed form of the yield function and plastic potential. The differences however are generally small (Collins et al. 1982). Also, as indicated by Schanz & Vermeer (1996), Equation (20) is applicable for both plane strain and triaxial strain. Thus no distinction is made between the plane strain and triaxial values of $\phi'_m$ and $\psi_m$ in the present study. The fasteners and rail pads have been excluded in this analysis and are outside the scope of this study. The two-dimensional (2D) finite element analysis does not take into account the spacing and width of sleepers.

3 RESULTS AND DISCUSSION

For investigating the performance of railway ballast under both triaxial monotonic loading and in-situ track situations, the distribution of vertical stresses

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and associated plastic deformations are considered. The results are summarized below.

### 3.1 Stress-Strain Response of Ballast Layer

Finite element (FE) simulations were employed to predict the stress-strain behaviour of railway ballast subjected to monotonic loading under different effective confining pressures as shown in Figure 9. It is evident that the $\sigma_3'$ has a significant effect on the stress-strain response of the ballast layer. The increase in $\sigma_3'$ causes increased interparticle contacts resulting in a more favourable redistribution of stresses, with enhanced degree of particle interlock or ballast friction angle (Indraratna et al. 2005). An increase in $\sigma_3'$ also leads to substantial reduction in the dilation of ballast. These results clearly show that increased lateral confinement results in decreased track settlement and greater track stability. It would therefore seem appropriate to maximize the lateral constraint on the in-situ ballast layer to improve the performance of the entire track system and thereby reduce the need for costly maintenance (Indraratna & Salim 2005, Lackenby et al. 2007).

The elasto-plastic constitutive model showed better agreement with the strain-hardening behaviour of ballast observed in the large scale triaxial tests, representing considerable ballast breakage. However, it could not accurately capture the post-peak behaviour of ballast. This is primarily attributed to the fact that stress-dependent stiffness moduli $E_{ref}^e$, $E_{ref}^d$ and $E_{ref}^u$ used in the current formulation are related to the peak strength of ballast. The current model is able to simulate well the volumetric strain response of ballast, but at large strains, the predicted volumetric compressive strains are lower than the test observations. This is because, a constant rate of particle breakage has been considered in the current analysis.

### 3.2 In-Situ Stress-Strain Response of Track Layers

In order to validate the findings of this finite element analysis, a comparison is made between the elasto-plastic analyses and the field data at the unreinforced section of track, as shown in Figure 10. It can be seen that the 2D elasto-plastic model predicts lower values of vertical stress along the depth than those obtained in the actual field measurements. One possible reason is that the real cyclic nature of wheel loading is not considered here and it is approximately represented by an equivalent dynamic plane strain analysis.

Furthermore, the values of vertical displacement predicted by the elasto-plastic analysis only shows a slight deviation from the field data. Considering the limitations of the elasticity based approaches, this prediction is still acceptable for preliminary design practices.

![Figure 9](image_url) Comparison of FE predictions with triaxial test results.

![Figure 10](image_url) (a) Comparison of vertical maximum cyclic stresses ($\sigma_v$) measured under the rail at Bulli with FE predictions. (b) Comparison of vertical displacement ($S_v$) measured under the rail at Bulli with FE predictions.

### 4 CONCLUSIONS

An elasto-plastic constitutive model for railway ballast both under triaxial loading and under in-situ track loading has been described through finite elements. The strain-hardening behaviour of ballast is accurately simulated by using a hardening soil model with a non-associative modified flow rule. Numerical simulations are performed using a two-dimensional
axisymmetric and plane-strain finite element analysis (PLAXIS) capturing the effects of ballast breakage and confining pressure. It is shown that the increased track confinement leads to significant reduction in vertical stresses and deformations. Provision of sufficient degree of lateral confining pressure improve the performance of the entire track system and reduce the need for costly maintenance. The main advantage of using a comprehensive elasto-plastic constitutive model for track layers as compared to conventional (simplified) analytical methods based on linear elasticity is elucidated.

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