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An Investigation into Temporal Gamma Luminance for Digital Fringe Fourier Transform Profilometers

J. M. Baker  
*University of Wollongong*

Joe F. Chicharo  
*University of Wollongong, chicharo@uow.edu.au*

Jiangtao Xi  
*University of Wollongong, jiangtao@uow.edu.au*

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An Investigation into Temporal Gamma Luminance for Digital Fringe Fourier Transform Profilometers

M. J. Baker, J. F. Chicharo and J. Xi
School of Electrical, Computer and Telecommunications Engineering,
University of Wollongong, NSW 2522, Australia
Tel: +61 2 42 21 3244, Fax: +61 2 42 27 3277
E-mail: mjb06@uow.edu.au

Abstract – Analysis of the impact of temporal gamma luminance on Fourier Transform Profilometry (FTP) Digital Video Projection (DVP) based structured light profilmeters is undertaken. We investigate the spectral harmonic structure for typical DVP fringe images linking projector γ and 2nd order fringe harmonics. The validity of the presented study is verified through simulation, and subsequently we conclude that for typical projector γ, the 2nd order harmonic is the single most significant contribution to reconstruction error for the phase measuring technique. The impact of our analysis is further gauged by empirical measurement of the temporal variation of γ of a DVP device.

Keywords – Gamma, Harmonics, Non-linear Luminance, Digital, Fringe, Profilometry, FTP

I. INTRODUCTION

Structured light techniques for non-contact, dynamic and accurate profile measurement of diffuse surfaces have been widely studied due to their potential industrial applications. The objective of our research is investigation into the application of structured light sensing technology for the potential application of 3D feature extraction of the human face. More specifically we are interested in developing an economical yet sophisticated sensor in which could be utilised to replace avatars in network games with actual player features.

The most exploited techniques often utilise a projected periodic fringe pattern composed of parallel lines. The observed pattern is modulated by the diffuse surface in such a way that the modulation contains information about the height of the object perpendicular to the plane of observation. The modulated structured light pattern is recorded, commonly by a CCD camera and then processed by a fringe processing algorithm such as the popular Fourier Transform Profilometry (FTP) [1] technique to extract the spatial phase modulation. Figure 1 depicts the typical Crossed Optical Axes geometry utilised by many traditional structured light profilimeters. Noting that triangle ACE and triangle ACD are similar, the spatial phase modulation is thereby used to recreate the surface of interest in three-dimensional space.

$$\frac{AC}{-h(x, y)} = \frac{d_0}{l_0 - h(x, y)}$$

$$h(x, y) = \frac{l_0 AC}{AC - d_0}, \quad (1)$$

Traditionally, structured light fringe patterns are often generated using interferometric methods via a laser source. An alternative to conventional laser projection is Digital Video
Projection (DVP). DVP is a technology which has been actively pursued in this particular field of research as it can provide a number of key advantages. For instance, typical digital video projectors are capable of projecting standard 24 bit bitmap computer generated images and hence, when interfaced to a personal computer make a very affordable, flexible and robust projection source. However, for well exploited fringe processing algorithms such as FTP, nonlinear luminance effects commonplace with DVP significantly hinder the estimation of spatial displacement AC and hence system accuracy. Commonly, the nonlinear luminance associated with DVP is referred to as Gamma distortion. Gamma distortion is typical in visual display systems to enhance human perception of the sensation of lightness, which can be regarded as a power function of intensity [2]. Gamma correction for DVP based structured light profilometers was first identified by Guo et al. in [3] whereby the iterative statistical analysis of digital fringe patterns was undertaken to correct the gamma distortion in digitally projected images. Through successful identification of projector gamma distortion Guo et al. were able to considerably reduce reconstruction errors. Alternate solutions to counter gamma non-linearities typically involve photometric fringe calibration, whereby multiple intensity distributions varying over the full range of luminance values are recorded and a camera / projector luminance curve is fitted [4], [5], however, some solutions calibrate projected fringes via other means [6], [7].

All of the proposed solutions attempt to linearise the projector / camera luminance by treating the gamma luminance non-linearity as a static phenomenon. In fact, practically, gamma luminance presents temporal aspects, varying from fringe image to image and resulting in further reconstruction error. In this paper we analytically investigate the implications of temporal gamma and its impact on system accuracy for the well known FTP fringe processing algorithm. We subsequently verify our findings via simulation, showing that the influence of only the 2nd order harmonic component needs to be considered for the γ modeled fringe. To further gauge the impact of the temporal gamma variation, we empirically measure the variation for a typical digital video projector, and reconstruct a known surface.

II. Principle FTP Fringe Profilometry

The FTP fringe processing technique, obtains an estimate of AC by processing the spatial displacement as a spatial phase displacement. The approach typically requires the projection of a sinusoidal intensity distribution (although a Ronchi grating can also be used) with spatial frequency f0, therefore Equation (1) becomes

\[ h(x, y) \approx \frac{l_0}{2\pi f_0 d_0} \Delta \phi(x, y), \]

where

\[ \Delta \phi(x, y) = \phi(x, y) - \phi_0(x, y), \]

and where \( \phi(x, y) \) and \( \phi_0(x, y) \) are the phase modulations relating to the diffuse surface to be profiled and the reference plane \( R \) respectively, for the case where \( l_0 \gg h(x) \). For the sinusoidal case the following Equations represent the ideal captured fringe patterns.

\[ g(x, y) = a(x, y) + b(x, y) \cos[2\pi f_0 x + \phi(x, y)] \] (4)

and

\[ g_0(x, y) = a(x, y) + b(x, y) \cos[2\pi f_0 x + \phi_0(x, y)] \] (5)

where \( a(x, y) \) and \( b(x, y) \) are functions representing the captured fringe offset and contrast.

By the principle FTP method an estimation of phase modulation \( \Delta \phi(x, y) \) can be obtained by

\[ \Delta \phi(x, y) = \text{Im} \left[ \log(g(x, y)g_0^*(x, y)) \right] \]

where \( \text{log}(.) \) denotes natural logarithm function, \( \text{Im}(\cdot) \) is the operation to acquire the imaginary part of a complex number, \( g_0^*(x, y) \) is the complex conjugate of \( g_0(x, y) \) the appropriately filtered reference fringe image and \( \hat{g}(x, y) \) represents the appropriately filtered deformed fringe image.

Observing Equation (6) it is clear that the mathematics governing the evaluation of spatial displacement AC requires the adequate generation, capture and processing of pure sinusoidal fringe images. For DVP based structured light profilometers the geometric precision of projected fringe images is often compromised due to non-linear luminance effects known as Gamma distortion. Furthermore, since the FTP method is reliant on digital filtering, inadequate filtering can result in phase residual from higher order harmonics and hence, when combining such technology it is important to understand the implications Gamma non-linearities impose.

III. Modeling a γ Distorted Fringe

In general, the gamma distortion of a digital display can be modeled using the simple power function seen in Equation (7)

\[ u(x, y) = \alpha(x, y)^\gamma, \]

for \( \alpha \in [0, 1] \)

where \( u(x, y) \) is the normalised image delivered to the display device, \( u(x, y) \) is the actual normalised image output intensity distribution and \( \gamma \) is typically a fractional value \( 1 < \gamma < 3 \) specific to the display device. Considering the FTP scenario where we typically have the projection of a sinusoidal intensity distribution, Equation (7) becomes,

\[ \bar{u}(x, y) = [a + b \cos(2\pi f_0 x)]^\gamma, \]

where \( f_0 \) is the intended spatial carrier frequency of the projected fringe, and \( a \) and \( b \) are constants referring to the fringe offset and contrast respectively. It is clear from observation of Equation (8) the projection of a sinusoid will result in harmonics and thus we can represent a projected fringe as a Fourier Series with
infinite $m$th order harmonic components. Considering only the contribution of the 2nd order harmonic we can define

$$
p = \left[ a^2 \int_0^T \left[ 1 + \frac{b}{a} \cos(2\pi f_0 x) \right] \cos(4\pi f_0 x) \, dx \right]^2,
$$

the power ratio of the 2nd order harmonic to the fundamental, which is a function of the ratio of fringe contrast and offset parameters $\frac{b}{a}$ and $\gamma$, where $T$ corresponds to the spatial period of the fringe image. Now that we have gauged the harmonic distortion of a gamma distorted sinusoid we can use Equation (9) to further evaluate the impact of the parameters on system performance.

IV. STATIC $\gamma$ / HARMONIC ERROR ANALYSIS

Assuming that $\gamma$ can be considered static, based on a previous derivation [8], we denote $\Lambda(\theta)$ as the phase measuring error associated with the 2nd order harmonic letting $\theta$ denote $2\pi f_0 x + \phi(x, y)$ where $\phi(x, y)$ represent $\phi(x, y)$ and $\phi_0(x, y)$ for the deformed and reference fringe respectively, and show that

$$
\Lambda(\theta) = \left( \frac{\sqrt{\sin(\theta(m-1))}}{1 + \sqrt{\cos(\theta(m-1))}} \right)
$$

where clearly $m = 2$. It can be further shown that the maximum absolute phase measuring error can be given as

$$
\varepsilon_{max} = 2\arctan \left( \left( \frac{p}{1 - p} \right) \right)
$$

and hence using Equation (2), the maximum absolute height measurement error $\beta$ can be given as

$$
\beta = \frac{l_0}{\pi f_0 d_0} \cdot \arctan \left( \left( \frac{p}{1 - p} \right) \right)
$$

If we now assume system parameters $l_0, d_0$ and $f_0$ are 5m, 2m and 10m respectively, we can plot a curve describing the relationship, as depicted in Figure (2). It can be seen that for larger values of $\gamma$ i.e., stronger harmonics, the greater the profile measurement error. It is important to note that this derivation does not take filtering into consideration and hence, provides insight into the worst case scenario only. Nevertheless, the influence of filtering can simply be likened to reducing the magnitude of the $\gamma$ term.

V. TEMPORAL $\gamma$ / HARMONIC ERROR ANALYSIS

The above expressions describe the errors associated with $\gamma$ when it can be considered static. In practice it has been observed that the harmonic spectral distribution presents temporal aspects and hence further consideration of this analysis needs to be undertaken to measure the influence of this phenomenon.

If we consider $\gamma$ and $\gamma_0$, the gamma values for both the deformed and reference images respectively, we can define the captured $\gamma$ distorted fringe images as

$$
g(x, y) = a(x, y) + b(x, y) \cos(2\pi f_0 x + \phi(x, y))
$$

and

$$
g_0(x, y) = a(x, y) + b(x, y) \cos(2\pi f_0 x + \phi_0(x, y))
$$

where $c_m(x, y)$ and $c_{0m}(x, y)$ are the functions representing the $m$th order harmonic component for the deformed and reference fringe respectively.

Using the FTP method, we can obtain $\Delta \phi(x, y)$ an estimation of the phase map $\Delta \phi(x, y)$ by

$$
\Delta \phi(x, y) = l_0 \left( \log(g(x, y)) - \log(g_0(x, y)) \right)
$$

Discarding the dc terms and substituting Equations (13) and (14) it can be seen that

$$
\Delta \phi(x, y) = l_0 \left( \log(b \cos(2\pi f_0 x + \phi_0)) + \log(b \cos(2\pi f_0 x + \phi_0)) \right)
$$

and

$$
\Delta \phi(x, y) = l_0 \left( \log(b \cos(2\pi f_0 x + \phi_0)) + \log(b \cos(2\pi f_0 x + \phi_0)) \right)
$$

where $c_m(x, y)$ and $c_{0m}(x, y)$ are the functions representing the $m$th order harmonic component for the deformed and reference fringe respectively.
\[
\begin{align*}
\phi_b + \Delta \phi = & \quad \Delta \phi + I_m \left( \log(1 + \sqrt{p} \cdot e^{-i(2\pi f_0 x + \phi)(m-1)}) ight) \\
+ & \quad \log(1 + \sqrt{p} \cdot e^{-i(2\pi f_0 x + \phi)(m-1)}) - \Delta \phi
\end{align*}
\]

where \( p = \frac{c^2}{b^2} \) and \( p_0 = \frac{c^2}{b^0} \) represent the power ratio of the \( m \)-th harmonic to the fundamental, for the deformed and reference fringe images.

Hence, the estimation error of the phase can be expressed as:
\[
\varepsilon = \Delta \phi - \Delta \phi
\]
\[
= \Delta \phi + I_m \left( \log(1 + \sqrt{p} \cdot e^{-i(2\pi f_0 x + \phi)(m-1)}) ight) \\
+ \log(1 + \sqrt{p} \cdot e^{-i(2\pi f_0 x + \phi)(m-1)}) - \Delta \phi
\]
\[
= I_m \left( \log(1 + \sqrt{p} \cdot e^{i(2\pi f_0 x + \phi)(m-1)}) ight) \\
+ \log(1 + \sqrt{p} \cdot e^{-i(2\pi f_0 x + \phi)(m-1)})
\]

(17)

Now defining \( \Lambda(\cdot) \), the function to extract the phase angle of the complex number, we can now show
\[
\Lambda(\theta, \rho) = I_m \left( \log(1 + \sqrt{p} \cdot e^{i(m-1)}) \right) \\
+ \arctan( \frac{\sqrt{p} \sin(\theta(m-1))}{1 + \sqrt{p} \cos(\theta(m-1))})
\]

(18)

where we set \( m = 2 \) for the second order contribution and \( \rho \) represents the respective fringe power ratios.

Now rewriting Equation (17), the phase measuring error can be given as
\[
\varepsilon = \Lambda(2\pi f_0 x + \phi, p) + \Lambda(-2\pi f_0 x + \phi, p_0)
\]
\[
= \Lambda(2\pi f_0 x + \phi, p) - \Lambda(2\pi f_0 x + \phi, p_0)
\]

(19)

And furthermore, we can define a maximum measurement error
\[
\varepsilon_{\text{max}} = \Lambda_{\text{max}}(2\pi f_0 x + \phi, p) - \Lambda_{\text{min}}(2\pi f_0 x + \phi, p_0)
\]
\[
= \Lambda_{\text{max}}(\theta, p) - \Lambda_{\text{min}}(\theta, p_0)
\]

(20)

By letting \( \frac{d\Lambda}{d\theta} = 0 \), we can derive the maximum and minimum value of \( \Lambda \) respectively. That is
\[
\Lambda_{\text{max}} = \arctan \left( \frac{\sqrt{p}}{1 - \rho} \right)
\]

(21)

and
\[
\Lambda_{\text{min}} = -\arctan \left( \frac{\sqrt{p_0}}{1 - \rho_0} \right)
\]

Equation(20) can then be expressed as
\[
\varepsilon_{\text{max}} = \arctan \left( \sqrt{\frac{p}{1 - \rho}} \right) + \arctan \left( \sqrt{\frac{p_0}{1 - \rho_0}} \right)
\]

and hence substituting Equation (23) into Equation (2), the maximum absolute height measurement error \( \beta \) is given as
\[
\beta = \frac{l_0}{2\pi f_0 d_0} \cdot \arctan \left( \sqrt{\frac{p}{1 - \rho}} \right) + \arctan \left( \sqrt{\frac{p_0}{1 - \rho_0}} \right)
\]

(24)

A. Gamma Compensated Analysis

Since we now have an expression to describe the measurement error, the obvious question is what impact does the temporal variation of \( \gamma \) have on a \( \gamma \) compensated system. Clearly, this would be a factor dependent on the way in which \( \gamma \) was compensated and also the amount of temporal variation. For simplicity if we assume that just one of the fringe images had been appropriately compensated we can gauge the phase measuring error as
\[
\varepsilon = \Lambda(2\pi f_0 x + \phi, \rho)
\]

(25)

and hence the maximum measurement error as
\[
\beta = \frac{l_0}{2\pi f_0 d_0} \cdot \arctan \left( \sqrt{\frac{\rho}{1 - \rho}} \right)
\]

(26)

where \( \rho \) represents the temporal difference between the two \( \gamma \) compensated fringe images.

VI. SIMULATION

In order to verify the validity of the analytical study we simulated a range of digitally projected fringe images with various values of \( \gamma \) and \( \gamma_0 \) and measured the corresponding system accuracy for the FTP reconstruction of a simulated surface. System parameters \( l_0, d_0 \) and \( f_0 \) were chosen to be 5m, 2m and 10/m respectively, corresponding to a spatial period of 100mm if we assume an image spatial resolution of 1 pixel/mm. The simulated fringe images are given as
\[
g(x, y) = [0.5 + 0.5 \cos(2\pi f_0 x + \phi(x, y))],^{\gamma_0}
\]

(27)

and
\[
g_0(x, y) = [0.5 + 0.5 \cos(2\pi f_0 x + \phi(x, y))],^{\gamma_0}
\]

(28)

where \( \phi(x, y) \) corresponds to the hemispherical dome shape seen in Figure 4(a). \( \gamma_0 \) was varied by \( \pm 5\% \) of \( \gamma \) and based on the system parameters we plot the expected maximum absolute error \( \beta \), according to the analytical study and also measure the reconstruction error resulting from the simulated gamma distorted fringes. We reframed from filtering in our FTP simulation apart from the necessary dc and negative frequency components to reveal the full influence of higher order harmonics. The results are displayed in Figure (3). The
analytical error curves are represented with the solid line, with the simulation results indicated with the dotted lines.

As can be seen the analytical derivation accurately models the maximum absolute errors resulting from fringe temporal gamma distortion. In fact, it can be seen even as $\gamma \rightarrow 3$ and the magnitude of the higher order harmonics increase, there is little if any deviation between the analytical and FTP simulated curves. The only major discrepancy is seen for the very low values of gamma. This discrepancy is directly a result of the inherent accuracy limitations of the fft operation involved in FTP reconstruction.

Based on these findings we have verified the analytical study describing the relationship between temporal projector gamma and reconstruction error for the FTP fringe processing approach. Furthermore, we conclude that for typical projector $\gamma$ fringe modeling, the 2nd order harmonic component is the single most significant contribution to phase measuring error for the FTP technique.

### A. Example FTP Reconstruction

To further gauge the significance of this study we measure the variance in $\gamma$ for a typical DLP projector. Since DLP projectors temporally project RGB components, the temporal effects of $\gamma$ are easily observed, as each colour channel typically yields an individual $\gamma$ characteristic. We project a fringe image using an InFocus LP530 DLP projector onto a reference plane from a distance of approximately 1500mm. We capture the projected fringe image using a DuncanTech MS-3100 3-CCD camera. The integration time of the camera was adjusted to be approximately 20ms and a number of images of the projected fringe were obtained and the magnitude of $p$ was measured. The variation in $p$ is shown in Figure 5.

Based on the data obtained and displayed in Figure 5, using Equation 9 we measured a maximum $\gamma$ value of 2.47 and a minimum of 2.13. Using these two values we simulated the reconstruction of the diffuse surface seen in Figure 4(a) with system parameters $l_0$, $d_0$ and $f_0$ of 1500mm, 400mm and 10/m respectively, where 1pixel = 1mm. The simulated projected fringes can be seen as Figures 4(b) and (c) respectively, with the reconstructed surface shown as Figure 4(d). As can be seen the harmonic distortion significantly reduces the accuracy of the FTP method. The mean measurement error and standard deviation was found to be 8.207mm and 5.191mm respectively. The temporal variation of $\gamma$ is distinctly observable in the rippled reconstruction of the reference plane surface around the
VIII. CONCLUSION

In this paper we have investigated the influence of temporal $\gamma$ luminance for DVP based FTP's. The harmonic structure of a digitally projected fringe was investigated and consequently we derived an expression to link temporal harmonic distortion and phase measuring error for the FTP method. The validity of the analytical derivation was confirmed and it was concluded that for typical projector $\gamma$, the 2nd order harmonic is the single most significant contribution to reconstruction error for the phase measuring technique. With more and more industrial applications utilising digital technology in profilometric sensing, derivations such as that presented in this paper are essential to improve the understanding of the limitations and system accuracy of the FTP phase measuring algorithm.

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