Laboratory study of small-strain behavior of a compacted silty sand

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Abstract
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Keywords
small, study, laboratory, sand, silty, compacted, behavior, strain

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Laboratory study of small strain behavior of a compacted silty sand

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Abstract: Small strain behavior is a key indicator for assessing the performance of compacted fills. Compaction conditions i.e. initial moisture content and applied energy, govern compaction effectiveness and, thus, the structure and matric suction of compacted soil. This paper presents an experimental study of the small strain behavior of compacted silty sand prepared with different compaction conditions. Specimens with varying initial moisture contents and compaction energies were tested with Bender elements to determine the small strain shear modulus ($G_0$), while the post-compaction matric suction was measured using the filter paper method and tensiometer. The experimental data suggests a pronounced relationship between $G_0$ and the degree of saturation ($S_r$) of the as-compacted soil specimens. X-ray computed tomography (CT) scans were performed to examine structural changes of selected specimens upon compaction. The laboratory results are also examined in light of common end-product specifications, which show that it is beneficial to compact the soil slightly dry of optimum moisture content from the modulus point of view.

CE Database subject headings: small strain shear modulus, compaction energy, degree of saturation, matric suction.
Introduction

During construction, soil is usually compacted to meet a specified laboratory criterion based on the optimum values (typically, the moisture content and dry unit weight) for a given compaction energy (i.e. standard or modified compaction). However, the compaction energy applied in the field may not always be constant due to differences in hydration time, lift thickness and compaction machinery. While the stress-strain response depends on the compaction conditions (Turnbull and Foster, 1956; Seed and Chan, 1959), evaluating variations in small strain behavior caused by differences in compaction conditions becomes imperative in a practical sense (Gens et al., 1995).

The small strain behavior of soil, particularly when used as a construction material, is often regarded as an indicator of its geomechanical performance. Despite being widely used for predicting the performance of fills and for routine investigation of their dynamic properties, a limited number of relationships between the small strain shear modulus ($G_0$) and other compaction properties for unsaturated compacted soil have been established. Recent studies on the small strain behavior of unsaturated soils showed that an increase in matric suction contributes to enhanced stiffness of the soil skeleton which in turn leads to higher values of $G_0$ (Wu et al., 1984; Cho and Santamarina, 2001; Mancuso et al., 2002; Ng and Menzies, 2007, Ng and Yung, 2008; Sawangsuriya et al. 2009; Asslan and Wuttke (2012) and Biglari and Ashayeri, 2012). Furthermore, Ng, et al. (2009) and Ng and Xu (2012) studied the effect of post-compaction changes induced by wetting and drying, stress ratio and the recent suction history on the small strain modulus. However, there have been limited studies on the small strain modulus behavior of soils under different compaction history. Claria and Rinaldi (2007) investigated the shear wave velocity of compacted silt by changing the
initial moisture content. Sawangsuriya et al. (2008) studied the small strain modulus behavior of three different compacted subgrade soils by changing the initial water content and level of compaction energy. Furthermore, the study of the relationship between shear stiffness and different compaction conditions is also beneficial, in view of implementing modulus-based, intelligent compaction control (ICC) systems that consider the soil modulus as an indicator of compaction quality (Briaud et al., 2006; Mooney and Rinehart, 2007).

Differences in compaction history are thought to produce different soil properties. The change of soil microstructure caused by variations in the compaction induced states, due to changes in moisture content and compaction effort, often result in distinct mechanical and hydraulic behavior (Vanapalli et al., 1996; Wheeler and Sivakumar, 2000). Microstructural studies conducted in clays (Monroy et al, 2010 and Romero et al, 2011) and silts (Delage et al., 1996 and Koliji et al, 2010) have shown that the type of soil structure developed through compaction can be associated with a specific set of compaction properties. Soil compacted on the dry side of optimum moisture content (OMC) exhibits an open pore structure dominated by aggregations, whereas soil compacted at the wet side of OMC displays a constricted pore and matrix dominated structure with fewer aggregations (Delage et al., 1996). Gens et al. (1995), and more recently Wheeler and Sivakumar (2000) examined the effect of compaction conditions on the stress-strain response of soil compacted on the dry and wet side of OMC when tested under similar conditions. The results show that a structure induced by compaction has a significant effect on the strains exhibited by the specimens during shearing.

Vanapalli et al. (1999) compared the soil-water retention curves (SWRCs) of glacial till samples statically compacted on the dry and wet side of the OMC. The
samples were first saturated and subsequently dried using the axis translation technique. Although the difference in SWRCs obtained for samples compacted on the dry side and at OMC can be explained by invoking the void ratio dependency (Gallipoli et al., 2003), the same cannot be said for the wet side of the OMC that is affected by the change in structure. The same experimental evidence was given for dynamically compacted residual soil tested with different compaction efforts (Marinho and Stuermer, 2000). This highlights that differences in the degree of saturation and suction observed at different compaction states are likely to be independent of the different compaction procedures. Furthermore, evidence of the importance of the degree of saturation on the shear behavior of compacted soil is provided by Toll (2000) and Gallipoli et al. (2003).

The small strain behavior of compacted soil is also dependent on the soil structure. Wu et al. (1984) tested a fine-grained soil (Glacier Way silt) in a standard resonant column torsional shear (RCTS) device with no suction control. The material was prepared in a mould placed directly on the pedestal of the cell, and then statically compacted at variable water contents and applied stresses to obtain sets of specimens with the same initial density but different degrees of saturation. Mancuso et al. (2002) investigated the small strain behavior of Metrano silt using torsional shear tests, by varying the matric suction in a post-compacted state using the axis translation technique. The $G_0$ of the specimens compacted at the optimum and wet side of OMC showed marked inflexions in two main phase transitions of the SWRC, namely at the air entry suction range and moving towards the residual range.

Sawangsuriya et al. (2008) established a relationship for the soil modulus-moisture and suction of compacted soils. The variation of the modulus with “as-compacted” properties such as moisture content, suction and dry unit weight for
different soils was discussed. This current study is an attempt to investigate further the effect of compaction conditions on the $G_0$ of a silty sand soil at its initial compaction state. It also aims to provide a detailed analysis of the variation of the modulus across the compaction plane for the assessment of compaction quality (i.e. when following mechanical wetting paths). In particular, critical issues that govern the small strain modulus behavior are addressed and a novel empirical model capturing the soil structure associated with particular compaction conditions is proposed through laboratory testing.

**Materials and testing program**

**Soil type**

The soil selected for this study is silty sand used extensively as embankment fill at Penrith, Australia. The soil has a plasticity index of 10%, a liquid limit of 25.5% and a specific gravity of 2.7; thus, it can be classified as SP-SC (Unified Soil Classification System). The particle size distribution of the soil shown in Fig. 1 represents 89% sand, 7% silt and 4% clay size fraction.

**Testing program**

Before compaction, the soil sample was air dried prior to mixing with the required amount of water using a masonry trowel. Any moisture lumps were disaggregated before placing the mixture in a plastic bag, which was kept overnight under constant temperature and humidity for moisture equilibration. The compaction characteristics of the soil sample were established using a Standard Proctor compaction test (AS 1289.5.1.1 -2003). Four different levels of compaction energy $(E_1 = 154.5 \text{ kN.m/m}^3$, $E_2 = 242.7 \text{ kN.m/m}^3$, $E_3 = 529.5 \text{ kN.m/m}^3$, $E_4 = 838.4 \text{ kN.m/m}^3$ corresponding to 25%, 41%, 89% and 140% in relation to the standard Proctor
compaction energy level, respectively) were then utilized to mould the material into 50mm diameter by 100 mm high specimens, following the procedure described by Sridharan and Sivapullaiah (2005). These dimensions were chosen so that the specimen diameter and height would minimize the near-field effects (Leong et al., 2005).

**Small strain measurements**

To investigate the small strain behavior of the compacted specimens, a non-destructive technique using a pair of Bender elements was adopted in a standard Bishop-Wesley triaxial apparatus. This system was able to generate and detect shear waves, which enabled the $G_0$ to be evaluated as follows:

$$G_0 = \frac{\gamma}{g}V_s^2$$

where, $\gamma$ is the bulk unit weight (kN/m$^3$), $g$ is the gravity acceleration (m/s$^2$) and $V_s$ is the shear wave velocity (m/s).

The specimens were tested in a compacted state, but were enclosed in a silicone membrane beforehand to minimize moisture losses. Signal generation was controlled by GDSBES v2.0 software (GDS Instruments) while the data acquisition system had two input channels with 16-bit resolution each. A sampling rate of 300 kHz was used to ensure an adequate resolution of the time and voltage of input and output signals (Clayton, 2011). In order to minimize background noise and improve the signal to noise ratio (SNR), a series of twenty sampled signals were stacked.

One of the difficulties often involved with measuring shear wave velocity using Bender elements is to adequately select the testing variables, including the waveform and testing frequency (Viggiani and Atkinson, 1995, Jovicic et al., 1996; Leong et al., 2005). Fig. 2 shows the shear wave velocity traces obtained in a specimen compacted
on the dry side of OMC (energy, $E = 529.5 \text{kN.m/m}^3$) tested with different excitation frequencies. The excitation frequencies above 3 Hz yielded approximately the same travel time, whereas lower values of 1.5 kHz, 1 kHz, and 0.5 kHz led to much larger values and a consequent underestimation of the shear wave velocity. Note that as the waveform frequency increases beyond 3kHz, the strength of signal was significantly reduced. This is consistent with the findings of Arulnathan et al. (1998), which demonstrated that a decrease in wavelength to bender element length ratio contributes to the deterioration of the received signals. In this study, it was found that a testing frequency of 3 kHz having a ratio between wave path length and wavelength ($L_t/\lambda$) exceeding 2 (Brignoli et al., 1996, Arulnathan et al., 1998), was adequate to minimize the effect of the near-field component effect and warrant the strength of the received signal.

The shear wave velocity ($V_s$) was computed based on the wave path length ($L_t$) that corresponds to the tip-to-tip distance between the transmitter and receiver (Viggiani and Atkinson, 1995) and the travel time ($\Delta t$), as follows:

$$[2] \quad V_s = \frac{L_t}{\Delta t}$$

The tip-to-tip distance was determined using a digital caliper, while considering the protruding heights of the bender elements in the specimen. The travel time ($\Delta t$) was taken as the time interval to the first bump maximum, as described by Lee and Santamarina (2005) or to the first deflection if the first bump was not visible.

The shear wave velocity propagating vertically, with the soil particles vibrating in a horizontal polarization plane, was monitored. Ng and Menzies (2007) investigated anisotropy in triaxial specimens using vertical and horizontal wave propagation with different polarization planes. Although an inherent anisotropy was reported among the three different planes of polarization, the difference was non-
substantial, and was consistent for specimens compacted under different moisture contents. In this study, the influence of inherent anisotropy on $G_0$ was not addressed; because, every specimen was prepared using the same method, so any induced anisotropy is likely to affect all of them in a similar manner. Nevertheless, to ensure test reliability, every compaction point shown later in the plots represents the average of two specimens compacted under the same conditions and tested with Bender elements.

**Matric suction**

The filter paper method was used to determine suction because of its simplicity and ability to measure a wide range of suction although in specimens for whom suction did not exceed 90kPa the tensiometer technique (ASTM D3404-91, 2004) was also utilised. The filter paper technique was used in accordance with ASTM D5298 (2003) and using the 55 mm Whatman No.42 ashless filter paper. Matric and total suction can be measured using the contact and non-contact methods, respectively. Most specimens were tested using the contact method, except in the case of a few specimens compacted on the dry side of OMC for which the non-contact method was adopted, because of difficulty in trimming when preparing the test specimens. The influence of osmotic suction on total suction was considered negligible because the salt content in the tested soil was very small. Equilibration time was typically 7 to 10 days and the matric or total suction was computed using the bi-linear calibration curves given by Bulut and Wray (2005). In each compacted specimen, a minimum of two filter paper determinations were performed and the suction value was taken as the average of two tests.
Medical grade computed tomography-scanner testing

Computed tomography (CT) scanning systems use X-rays to visualize thin, cross sections of specimens. During CT scanning, high voltage X-rays generated from a source located at one side of the gantry, are attenuated through the test specimen and then registered by a series of detectors placed in the opposite direction. As the X-rays penetrate through the test specimen, some of them are absorbed. The different rates of absorption reflect changes in the specimen density (Alshibli et al., 2006). The tests were carried out using an X-ray CT scanner (Toshiba Asteion S4) on the compacted specimens. The reconstruction function adopted in this study enabled the correction of image artefacts that could result from the absence of lower energy X-rays. The X-ray tube voltage and current was 135 kV and 200 mA, respectively. The X-ray beam was 3 mm wide (i.e. slice thickness), the exposure time was 1 second, and the field of view (FOV) was 21 cm with a zooming factor of four. The CT-scan images were analyzed using medical radiology software DicomWorks v. 1.3.5. (Puech et al., 2007), including post-processing filtering (i.e. sharpness and inversion filters). The X-ray Ct-scan technique was adopted to minimize sample disturbance effects. While the images presented portray well the general arrangement of the soil structure, they have some limitations in terms of resolution that may hinder the measurement of very fine details.

Results and discussion
Compaction characteristics

The soil moisture-dry unit weight curves associated with the four different compaction energies are shown in Fig. 3. The OMC points of each energy level i.e. the line of optima, were approximated to a line of equal degree of saturation, $S_r =0.80$. 
The lines that represent full saturation \((S_r = 1)\) and \(S_r = 0.67\) are shown to delineate the ranges of aggregation likely to occur along the compaction curves.

For each compaction energy, the dry unit weight increases as the moisture content increase to the OMC. Beyond this point (i.e. wet side of the compaction plane), the dry unit weight decreases with increasing water content. This tendency can be explained from a microscopic perspective considering the interaction between water, air, and grains on the menisci. On the dry side of the compaction plane (points located below the line of optima), the suction that acts on the particle contacts to oppose slippage is high, and the compaction process yields low dry unit weights and an aggregation dominated structure (Delage et al., 1996). The progressive addition of moisture reduces suction and facilitates particle slippage. Thus, the soil experiences higher dry unit weights until it reaches its maximum at OMC represented by the line of optima in Fig. 3a. Beyond the OMC, the air phase becomes discontinuous i.e. air is occluded in bubbles (Barden and Pavlakis, 1971). In this condition, any applied external compaction energy is likely to be supported by the water phase in the soil, since compaction occurs over a relatively short period and the system is undrained. Fig. 3a also shows that the curves representing different energy levels converge to a common asymptote i.e. under a constant water content condition an increase in compaction energy does not yield a substantially different dry unit weight \((w = 12.5\%\) for energy levels of 529.5 kN.m/m\(^3\) and 838.4 kN.m/m\(^3\)). This is better illustrated in Fig. 3b. The equal moisture content line of \(w = 11.5\%\) reaches the optimum at a higher compaction energy \(E_3\) (point 3), after which it starts converging to a nearly constant dry unit weight (point 4). This indicates that any additional applied energy will cause shear at a constant volume and density, because the soil is incapable of mobilizing further strength at that moisture content (Olson and Langfelder, 1965).
When the soil is compacted at a higher initial moisture content of $w = 12.5\%$, the initial $S_r$ is greater and the line of optima is reached at a lower energy. This explains the progressive shift of OMC to the lower water content range observed in Fig. 3a i.e. 14.5% for $E_1$ to 10.5% for $E_4$ energy levels.

**Matric suction of compacted specimens**

Fig. 4 shows the water retention data for specimens compacted at different energy levels. The suction values varied from 616 kPa on the dry side of the OMC (highest compaction effort) to 5 kPa on the wet side of the OMC (lowest compaction effort). In these tests, the amount of suction developed by compaction was not very high because the fraction of clay in the tested soil was very small ($< 12\%)$. Overall, matric suction decreases with increasing moisture content. Although there is no apparent relationship between matric suction and compaction energy, all data points seem to converge to a logarithmic regression line given by Eq. [3] ($R^2 > 0.95$).

$$w = -1.56 \ln(s) + 18.50$$

This indicates that the hydraulic behavior of compacted soil may be independent of the compaction characteristics (i.e. change in the moisture content and compaction energy). While the use of the relationship expressed in Eq. [3] enables a relatively simple and straightforward appraisal of field matric suction for a range of moisture contents, some degree of caution must be exercised because the matric suction values may change with increasing energy (i.e. Region II in Fig. 4b). Here, the contour lines have a positive slope indicating that suction increases with $S_r$ at a constant water content. The existence of distinct regions of equal suction contours has been previously highlighted in the experimental evidence of statically compacted
specimens provided by Romero et al. (1999) (Region I) and by Tarantino and De Col (2009) (Region II).

**Small strain shear modulus of compacted specimens**

An example of the results of the shear wave time domain series is shown in Fig. 5 for specimens compacted with the same energy ($E_3 = 529.5$ kJ/m$^3$) but different moisture content ($w = 9.1, 10.5, 11.8, 12.5, 14.5$ and $16.1\%$). The $G_0$ was computed based on the wave velocity traces obtained using Eqs. [1] and [2]. Fig. 6 shows the relationship between $G_0$, matric suction and dry unit weight for the specimens compacted with $E_3 = 529.5$ kN/m$^3$. The most striking aspect is that $G_{\text{max}}$ did not correspond to the maximum dry unit weight. In fact, the values of $G_0$ in a similar order for $8\% < w < 10\%$ and decreased sharply when $w = 10.5\%$, just before the OMC was exceeded. This behavior can be attributed to the combined effect of interrelated changes in dry unit weight, suction and soil structure. As the dry unit weight progressively increases, the soil skeleton compresses and $G_0$ is expected to increase (Ng and Menzies, 2007). Concurrently, a decrease in suction weakens the contact stresses acting on the soil skeleton; thus, $G_0$ is expected to decrease (Cho and Santamarina, 2001). The balance of the changes in dry unit weight and matric suction on the dry side of the OMC causes the shear wave velocity and, hence, the $G_0$, to remain approximately constant. Furthermore, in this range the compacted soil microstructure is similar i.e. is composed mainly of aggregations with a large percentage of macropores (dry side) Delage et al. (1996). As moisture content increases, the matric suction stresses decrease even further, and after exceeding the OMC, so too does the dry unit weight, causing $G_0$ to abruptly decrease. Similar experimental observations were made by Sawangsuriya et al. (2008).
The $G_0$ values obtained for different compaction energies are depicted in Fig. 7a. The measured $G_0$ values ranged from 153 MPa on the dry side of the OMC ($E_4 = 838.4$ kN.m/m$^3$) to 10 MPa on the wet side of the OMC ($E_1 = 154.5$ kN.m/m$^3$). The $G_0$ -moisture content-suction relationships shown in Fig. 7a and Fig. 7b indicate that on the dry side of the OMC, $G_0$ is controlled by the compaction energy, since an increase in energy translates to an increase in $G_0$ at nearly constant suction. At higher moisture contents $w \geq 13\%$, the effect of suction on $G_0$ becomes increasingly pronounced, since an increase in compaction energy does not yield a higher shear modulus. It is interesting to note that on the wet side of the OMC, lower $G_0$ values are obtained under the highest compaction energies ($E = 838.4$ kN.m/m$^3$). This suggests that imparting higher compaction energy levels in the field or over-compacting does not necessarily yield superior shear stiffness. Similar findings were obtained by Turnbull and Foster (1956) in CBR field tests on lean clays.

Fig. 8 shows the relationship between $G_0$ and compaction energy with the constant moisture content lines represented. In the lower moisture content range ($w \leq 10\%$) $G_0$ exhibits an almost linear logarithmic increase with increasing compaction energy. As the moisture content increases, $G_0$ become constant ($w = 16\%$) or in some cases ($w = 13, 14, 15\%$) decreases with increasing compaction energy.

Fig. 9 shows the relationship between $G_0$ and $Sr$ for various compaction energies, defining three distinct regions based on their extent of aggregation (i.e. extensive, moderate and insignificant). It is noted that, the three different regions differentiation relates specifically to the type of fabric and associated macroporosity. At $Sr < 0.67$ (Region $\emptyset$: extensive aggregation), $G_0$ remains approximately constant for each energy level, although accompanied by a decrease in matric suction and an increase in dry unit weight as shown earlier (see Fig. 3a and Fig. 7b). The soil in this region is
fully aggregated and an increase in compaction energy causes $G_0$ to increase logarithmically (highlighted in Fig. 8).

The type of fabric is reflected in the CT-scan images shown in Fig. 10. Given the limitation of scanner resolution the soil structure change is evaluated only in terms of the general arrangement of macroporosity (inter-aggregate pores), by examining differences in the grey scale color of the images. White areas correspond to air filled and water filled pores whereas grey and dark areas correspond to aggregations and sand particles, respectively. Fig. 10a, which belongs to a specimen compacted in Region $\Omega$, shows that the macrostructure is dominated by the presence of aggregations (grey areas) with large inter-pores (white areas), while the sand particles (dark colored areas) are not easily distinguished.

With an increase in $S_r$ in Region $\odot$, $G_0$ starts to decrease, where the greater the compaction energy, the greater the rate of decrease in $G_0$. For example, for $E_4 = 838.4$ kN/m$^3$, $G_0$ decreases from 150 MPa to $\sim 60$ MPa, while for $E_3 = 529.50$ kN/m$^3$ the corresponding drop is from $\sim 100$ MPa to 60 MPa. With an increase in $S_r$, the aggregations become deformable and gradually tend to to a ‘matrix dominated’ macrostructure during compaction. Since the aggregations have come closer together and matric suction is considerably reduced, a decrease in $G_0$ is expected. This is supported by Fig. 10b, which shows a decrease in macroporosity, as evidenced by a decrease in white and grey colored areas.

At $S_r > 0.80$ (Region $\odot$: insignificant aggregation), $G_0$ converges rapidly to a minimum value of 10 MPa, regardless of the compaction energy imparted. At this point, the suction values are very low and the aggregations are almost likely all fused together and the fine fraction includes sand grains with a “matrix” dominated macrostructure. This transition to a “matrix” dominated type of macrostructure,
characteristic of OMC conditions and Region 3, is easily observed in Fig 10c. The sand grains (black) can be individualized from the matrix. These results are also consistent with SEM images and MIP distributions of silt compacted on the wet side of the OMC obtained by Delage et al. (1996) and Koligi et al. (2010).

It is interesting to note that macrostructure changes associated with the increase in compaction energy, as illustrated in Figs 10a-c under constant moisture content conditions ($w = 12.5\%$), is possibly the reason for the decrease in $G_0$, particularly at higher compaction energies (Fig. 7). This indicates that $G_0$ exhibits a strong dependence on the soil structure, as outlined in studies on structured sands by Cuccovillo and Coop (1997).

**Proposed empirical equation for the as-compacted small strain shear modulus**

To describe adequately the $G_0$, the void ratio dependency and current mean effective stress should be considered. A general relationship for $G_0$ in saturated soil proposed by Mitchell and Soga (2005) can be expressed by:

$$G_0 = A f(e) (p')^n$$

where, $A$ is a parameter associated with soil structure, $f(e)$ is a function of the void ratio, $p'$ is mean effective stress and $n$ is a fitting parameter associated with the state of stress. Although the fitting parameters $n$ and $A$ are not dimensionless in Eq. [4] and dependent on the units chosen, they can be made dimensionless by normalizing the mean effective stress term and the $G_0$ by a reference pressure $p_r$ (e.g. atmospheric pressure, $p_a = 100$ kPa) and by a reference saturated shear modulus ($G_{r,sat}$) of a specimen compacted at OMC with standard compaction effort, respectively.

Furthermore, under unsaturated conditions the effect of pore air ($u_a$) and water pressures ($u_w$) or matric suction ($u_a - u_w$) on the behavior of $G_0$ are of key importance.
The mean effective term in Eq. [4] can be modified to describe the state of stress on unsaturated soil following an expression proposed by Khalili et al. (2004) with the effective stress parameter $\chi$ form suggested by Vanapalli and Fredlund (2000) as $\chi = S_r^\kappa$. Thus, Eq. [4] can be normalized and extended as follows:

$$\frac{G_r}{G_{r,\text{sat}}} = A f(e) \left[ \frac{(p - u_o) + S_r^\kappa (u_o - u_w)}{p_r} \right]^n$$

where, $\kappa$ is a parameter related to the plasticity index and the void ratio function $f(e)$ follows Lo Presti (1995) as suggested for a wide range of soil types, $f(e) = e^{-x}$, where $x$ is the void ratio exponent parameter. Recently, Alonso et al (2010) have proposed a relationship for $G_0$ which adopts a similar functional form to that of Eq.[5].

The parameter $A$ (Eq.[5]) is associated with soil structure and usually taken as constant. However, in the compacted specimens the soil structure changes and, thus, $A$ should rather be defined as a function of an alternative variable related to the compacted condition. Among the variables involved in the compaction process (i.e. moisture content, matric suction, dry unit weight), the degree of saturation $S_r$ seems to be the most adequate as it exhibits a unique relationship with $G_0$, as highlighted in Fig. 9. Furthermore, Toll (2000) suggested that $S_r$ can be associated with the degree of aggregation. Therefore, $A$ can be defined as $A(S_r)$, where changes in soil structure are assumed to be related to changes in $S_r$. In practical applications, the definition of the $A(S_r)$ relationship is likely to be developed for only one energy level, i.e. typically the standard compaction energy. For this reason, the points belonging to the equivalent standard compaction energy are used to establish the $A(S_r)$ relationship, whereas the data from the additional energies are used to validate it.

The parameter $A$ is calculated using Eq.[5] and the parameters adopted are summarized in Table 1. Fig. 11a shows the computed $A$ with respect to the variation
of $S_r$, which can be approximated by Eq. [6] based on the maximum saturation value ($S_{\text{max}}$) defined as the degree of saturation line asymptote shown in Fig. 3.

$$[6] \quad A(S_r) = a \left( \frac{S_{\text{max}}}{S_r} - 1 \right)^b$$

where, $a$ and $b$ are empirical parameters that can be found using the least square method fitting procedure. Additionally, available data in the past literature is also included for validation purposes (Fig 11b). The summary of the empirical parameters is given in Table 1.

Once the $A(S_r)$ function is known, $G_0$ can be predicted for any point located in the compaction plane using Eq.[5], assuming the net-confining stress term ($p - u_a$) is null since the tests were conducted under unconfined conditions. The $G_0$ values are compared with the predicted values in Fig. 12, using both Eq. [5] and Sawangsuriya et al. (2008) model. Both prediction procedures may be considered satisfactory (standard error of 3%). Particularly Eq. [5] considering that the structure derived from different compaction conditions (i.e. moisture content and energy level) was not directly quantified in the proposed relationship. The results support the argument that $S_r$ can be used as an indicator of the compacted soil structure. However, in view of the fact that $S_r$ is described by the changes in macroporosity, it should not be used as an absolute method to quantify the compacted soil structure (Romero et al., 1999).

The constant moisture content contours representing different compaction histories were predicted using Eq.[5] and Eq.[3] and are shown in Fig. 13. These contours represent the mechanical wetting paths that the soil experiences when compaction energy is increased, whereby the changes in void ratio and dry unit weight are directly represented by the degree of saturation. In the lower range of saturation ($S_r < 0.67$), an increase in the dry unit weight is accompanied by an
increase in $G_0$ and a reduction in macroporosity of the soil structure. This is consistent with the CT-scan images of the specimens compacted with increasing compaction energy (Fig.10a and b). Furthermore, MIP studies performed by Tarantino and De Col (2009) in statically compacted kaolin specimens also confirm that while the microporosity range is relatively unaffected by the increase of compaction pressure, the change in dry unit weight is directly associated with the reduction in the macroporosity. Similar findings were also reported by Cuisinier and Lalouì (2004) for clay specimens subjected to mechanical loading.

A further reduction in macroporosity contributes to an additional increase in $G_0$ leading to a peak at $S_r = 0.77$. Thereafter, the soil structure would gradually change from an “aggregated” type to a “matrix” type. This lends support to the assumption that $G_0$ is sensitive to changes in particle aggregation, as indicated by the three distinct regions in Fig. 9.

**Effect of Small strain shear modulus on compaction end-product specifications**

Preceding the placement of compaction fills, it is common to stipulate an end-product specification, usually based on the standard Proctor compaction curve that aligns the strategic importance of the fill with the desired project objectives. An end-product specification commonly adopted at most construction works consists of: (a) a minimum of 95% of the maximum dry unit weight at the OMC (AS 3798 - 2007), (b) acceptable moisture deviation interval, typically 2% of the OMC or (c) less than the maximum acceptable value of air voids, e.g. 10% (Mokwa and Fridleifsson, 2007). Fig. 14 shows the compaction curves as per the above specifications. If the compacted soil performance is solely related to $G_0$, then it seems likely that compacting the soil between the $S_r$ of 0.67 and 0.80 (i.e. in Region©) may be
preferable. In this range, $G_0$ attains its maximum and changes in $S_r$ (or applied energy under constant moisture content) do not cause substantial variations in $G_0$. However, if a minimum shear modulus is required, i.e. $G_0$ at OMC, compacting the soil slightly on the dry side may prove advantageous. This is because matric suction values on the wet side decrease significantly and the desired shear modulus may never be reached regardless of the applied energy level. While compacting the soil on the dry side of OMC may be beneficial in terms of the magnitude of $G_0$, soil compacted under these conditions exhibits higher permeability and may become more susceptible to increased brittleness and long-term shrink/swell problems associated with moisture variations compared to soil compacted on the wet side of the OMC.

**Conclusions**

From a number of Bender elements tests performed in compacted silty sand specimens, it was noted that $G_0$ varied with different compaction conditions (moisture content and compaction energy). This study demonstrated that $G_0$ was influenced predominantly by the imparted compaction energy on the dry side of the compaction plane, where an increase in energy corresponded to an increase in $G_0$; and by the structure of the soil on the wet side of the compaction plane, where $G_0$ remained almost constant or decreased with the compaction energy. This clearly suggests that when compacting in the field, applying additional energy by increasing the number of compaction roller passes will probably have a marginal effect on the resulting shear stiffness. The laboratory data also showed that $G_0$ is closely related to the $S_r$ of the compacted specimens rather than the moisture content. This confirms that the $G_0$ of the soil is controlled largely by the hydraulic behavior that governs the unsaturated condition i.e. the availability of water in the pores (volume) rather than the quantity
of water (weight). The existence of three distinct regions defined by different ranges of $S_r$ was outlined based on the $G_0$ observations for each energy level. The small strain behavior in each region was associated with the macrostructure or extent of aggregation illustrated in the CT-scans images. The CT-scan images interpretation also revealed that the soil macrostructure changes with increasing compaction effort, which may explain why lower values of $G_0$ are obtained for the highest compaction energy level.

To predict $G_0$ of specimens compacted under different conditions, a simple relationship was proposed based on the hydraulic and mechanical behavior coupled with compacted soil structure associated with different degrees of saturation. A close form relationship to account for the variation of soil structure with degree of saturation was validated against the current experimental data and an additional data set found in the referenced literature. The proposed relationship correctly predicted the measured $G_0$ with a relative small margin of error.

The relationship was further utilized to investigate the change in $G_0$ following the mechanical wetting paths that the soil is subjected during compaction under constant water content conditions. It was found that it is beneficial to compact the soil to a dry unit weight/moisture content located in region 2 ($0.67 < S_r < 0.80$), when the soil modulus experiences the highest values and smaller variations.

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Penrith Lakes Development Corporation and Tristan McWilliam and Michael Hughes of Coffey Geotechnics is appreciated. The Authors appreciate the assistance of Dr Laura Banasiak (ARC Research Fellow) and Bill Clayton (ANSTO) during paper editing phase.

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<tr>
<th>Parameters</th>
<th>Silty sand soil</th>
<th>Sandy clay soil</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Current study)</td>
<td>(Sawangsuriya et al., 2008)</td>
</tr>
<tr>
<td>Stiffness coefficient $n$</td>
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<td>0.2</td>
</tr>
<tr>
<td>Void ratio exponent $x$</td>
<td>1.7</td>
<td>1.9</td>
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<td>$p_r$ (kPa)</td>
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<td>100</td>
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<tr>
<td>$G_{sat}$ (kPa)</td>
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<tr>
<td>$\kappa$ parameter</td>
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<td>Confinement (kPa)</td>
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<td>0</td>
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<tr>
<td>$S_{max}$</td>
<td>0.872</td>
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<tr>
<td>$a$</td>
<td>4.55</td>
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</tr>
<tr>
<td>$b$</td>
<td>0.25</td>
<td>0.143</td>
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Dry unit weight, $\gamma_d$: kN/m$^3$

Moisture content, $w$: %

Compaction energy, $E$: kN.m/m$^3$
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<table>
<thead>
<tr>
<th>Moisture content (%)</th>
<th>$G_0$: MPa</th>
<th>Matric suction, $s$: kPa</th>
<th>Dry unit weight, $\gamma_d$: kN/m$^3$</th>
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<tr>
<td>8</td>
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</table>

*Small strain shear modulus, $G_0$: MPa
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Current study

\[ A(S_r) = a(S_{max}/S_r - 1)^b \]

Sawangsuriya et al. (2008)

Degree of saturation, \( S_r \)

\( \varphi \): Extensive aggregation region
\( \varphi \): Moderate aggregation region
\( \varphi \): Insignificant aggregation region

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