Deformation and degradation aspects of ballast and constitutive modelling under cyclic loading

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by

MD WADUD SALIM
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CERTIFICATION

I, Md Wadud Salim, declare that this thesis, submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Civil, Mining and Environmental Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

The following publications are fully or partially related to the research work conducted in this PhD program:


Md Wadud Salim

8 November, 2004
ABSTRACT

This thesis contains laboratory experimental results of strength, deformation and particle breakage of fresh and recycled ballast under monotonic and cyclic loadings, experimental evaluation of effectiveness of various geosynthetics in stabilising recycled ballast, and a new stress-strain constitutive model for ballast incorporating particle breakage. Ballast degrades progressively under heavy cyclic rail loadings, leading to deterioration of track substructure, rail alignment and train safety. Severely fouled ballast is often removed from the track and replaced by freshly quarried ballast, causing track maintenance very costly.

Discarded ballast can be cleaned, sieved and recycled to track foundation. In this study, the shear strength and stiffness of both fresh and recycled ballast were investigated in a series of consolidated drained shearing tests using a large-scale triaxial apparatus. The degree of particle breakage was quantified by sieving the ballast specimens before and after each test and recording the changes in ballast gradation. The stress-strain, shear strength, stiffness and particle breakage results of recycled ballast were compared with fresh ballast. The crushing strength characteristics of fresh and recycled ballast grains were investigated in a series of single grain crushing tests on various particle sizes.

A small track section comprising rail, sleeper, ballast, capping and subgrade was simulated in a large prismatic triaxial apparatus in the laboratory. The settlement, lateral deformation and particle breakage behaviour of fresh and recycled ballast under field-simulated loading and boundary conditions were studied in a series of cyclic loading tests using the prismatic triaxial rig. Three types of geosynthetics (geogrid,
woven-geotextile and geocomposite) were used in this study to stabilise recycled ballast. The cyclic test results of recycled ballast stabilised with geosynthetics were compared with the fresh and recycled ballast without geosynthetics. In order to study the effect of saturation, the cyclic tests were conducted in both dry and wet conditions.

Currently, there is a lack of appropriate stress-strain constitutive models for coarse aggregates like ballast, especially under cyclic loading incorporating particle breakage. The new constitutive model developed in this study incorporates the energy consumption due to particle breakage during shearing. A single non-linear function has been formulated to model particle breakage, and incorporated in the plastic flow rule assuming non-associated flow. The model has been developed based on the critical state framework and the concept of bounding surface plasticity. It captures the strain-hardening, post-peak strain-softening, dilatancy and cyclic hardening features of ballast behaviour accurately. The model has been examined and verified against the experimental results. Finite element analyses using ABAQUS were also conducted to compare with the analytical model. This study clearly shows that the new constitutive model predicts the stress-strain, volume change and particle breakage of ballast to an acceptable accuracy for both monotonic and cyclic loadings.
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<tr>
<td>$a$</td>
<td>empirical constant</td>
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<tr>
<td>$a_c$</td>
<td>semidiameter of the consolidation surface</td>
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<tr>
<td>$a_0$</td>
<td>semidiameter of the yield surface</td>
</tr>
<tr>
<td>$a'_0$</td>
<td>slope of the semi-logarithmic relation</td>
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<tr>
<td>$a_{l_1}$</td>
<td>semidiameter of the first loading surface</td>
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<tr>
<td>$a_r$</td>
<td>settlement at the reference tonnage</td>
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<tr>
<td>$b$</td>
<td>empirical constant</td>
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<td>$B$</td>
<td>model constant</td>
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<td>$B_g$</td>
<td>particle breakage index</td>
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<td>$c$</td>
<td>superelevation</td>
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<td>$C$</td>
<td>a coefficient for computing equivalent dynamic wheel load</td>
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<td>$C_1$</td>
<td>fitting parameter</td>
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<tr>
<td>$C_2$</td>
<td>fitting parameter</td>
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<tr>
<td>$d$</td>
<td>particle diameter</td>
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<tr>
<td>$dE_B$</td>
<td>infinitesimal increment of energy consumption due to particle breakage per unit volume</td>
</tr>
<tr>
<td>$dN$</td>
<td>infinitesimal increment of load cycle</td>
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<tr>
<td>$dS$</td>
<td>increment in surface area</td>
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<tr>
<td>$dS_v$</td>
<td>infinitesimal increase in specific surface area per unit volume</td>
</tr>
<tr>
<td>$dx_i$</td>
<td>infinitesimal increment horizontal displacement at contact $i$</td>
</tr>
<tr>
<td>$dy_i$</td>
<td>infinitesimal increment corresponding to $\delta y_i$</td>
</tr>
<tr>
<td>$d\varepsilon_1$</td>
<td>infinitesimal increment of major principal strain</td>
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<td>$d\varepsilon_{ij}$</td>
<td>infinitesimal increment of strain tensor</td>
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</table>
\(d\varepsilon_{ij}\)

- Infinitesimal increment of elastic strain tensor

\(d\varepsilon_{ij}^p\)

- Infinitesimal increment of plastic strain tensor

\(d\varepsilon_{ij}^{pc}\)

- Infinitesimal increment of plastic collapse strain tensor

\(d\varepsilon_{ij}^{pe}\)

- Infinitesimal increment of plastic expansive strain tensor

\(d\varepsilon_{vol,c}\)

- Plastic volumetric strain increment due to cyclic compaction

\(d\varepsilon_s\)

- Infinitesimal increment of distortional strain

\(d\varepsilon_s^e\)

- Infinitesimal increment of elastic distortional strain

\(d\varepsilon_s^p\)

- Infinitesimal increment of plastic distortional strain

\(d\varepsilon_v\)

- Infinitesimal increment of volumetric strain

\(d\varepsilon_v^e\)

- Infinitesimal increment of elastic volumetric strain

\(d\varepsilon_v^p\)

- Infinitesimal increment of plastic volumetric strain

\(d\kappa^p\)

- Infinitesimal increment of plastic distortional strain

\(D_w\)

- Diameter of the wheel

\(e\)

- Void ratio

\(e_i\)

- Initial void ratio

\(e_{ij}\)

- Strain deviator tensor

\(e_1\)

- Strain at load cycle 1

\(e_N\)

- Strain at load cycle \(N\)

\(E_{ur}\)

- Unloading-reloading elastic modulus

\(f\)

- Yield function

\(f_c\)

- Yield surface for plastic collapse

\(f_p\)

- Yield surface for plastic expansion

\(F\)

- Compressive force on a particle

\(F_{11}\)

- Force acting between two particles at contact \(i\) along major principal stress direction
\( F_2 \) a factor depending on the sleeper type and track maintenance

\( F_{3i} \) force acting between two particles at contact \( i \) along \( \sigma' \)

\( F_f \) failure value of force \( F \)

\( F_I \) Fouling Index

\( g \) plastic potential

\( g_c \) plastic potential for plastic collapse

\( g_p \) plastic potential for plastic expansion

\( G \) elastic shear modulus

\( h \) hardening function

\( h_i \) initial hardening parameter at the start of shear deformation in the first cycle of loading

\( h_{int} \) hardening function at the interior of bounding surface

\( h_{int(i)} \) initial value of \( h_{int} \) during reloading

\( h_{bound} \) hardening function at the bounding surface

\( H \) number of half cycles

\( H_s \) force required to initiate lateral displacement

\( H_w \) cross wind force

\( I_1 \) first invariant of stresses

\( I_2 \) second invariant of stresses

\( I_3 \) third invariant of stresses

\( k \) proportionality constant

\( K' \) a constant modelling dilatancy

\( K \) a scalar hardening modulus

\( K_{ur} \) dimensionless modulus number

\( K_y \) initial value of \( K \) on yield surface
$K_B$ bounding value of $K$ on consolidation surface

$K_1$ an empirical constant

$K_2$ an empirical constant

$l$ total length of sleeper

$L$ effective length of sleeper supporting the load $q_r$

$m$ a model constant

$m_{ij}^r$ flow direction for volumetric compaction

$m_{ij}^f$ flow direction for frictional sliding

$M$ critical state friction ratio (constant)

$n$ a model constant

$N$ number of load cycles

$N_i$ normal force between two particles at contact $i$ acting normal to the slip direction

$n_1$ number of contacts in aggregate per unit length in the direction of $\sigma_1$

$n_2$ number of contacts in aggregate per unit length in the direction of $\sigma_2$

$n_3$ number of contacts in aggregate per unit length in the direction of $\sigma_3$

$n_f$ unit vector normal to the yield surface

$n_g$ unit vector normal to the plastic potential surface

$p, p'$ mean effective normal stress

$p_a$ atmospheric pressure

$p_c$ distance between center of rails and center of gravity of vehicle

$p_{cs}$ value of $p$ on the critical state line for the current void ratio

$p_o$ value of $p$ at the intersection of the initial stress ratio line with an imaginary undrained stress path, which passes through the current stress ($p, q$) point and current ($p_{cs}, M_{p_{cs}}$) point corresponding to the current void ratio
\( p_w \)  vertical distance of resultant wind force from center of rails
\( P \)  Axle load
\( P_4 \)  percentage passing 4.75 mm (No. 4) sieve
\( P_{200} \)  percentage passing 0.075 mm (No. 200) sieve
\( P_a \)  average contact pressure
\( P_d \)  design wheel load
\( P_{di} \)  design wheel load
\( P_H \)  lateral force at curved track
\( P_{sl} \)  static wheel load
\( P_s \)  static wheel load
\( P'_w \)  equivalent dynamic wheel load for design
\( P_w \)  static wheel load
\( q \)  distortional (or deviator) stress
\( q_p \)  change in \( q \) in the previous half cycle
\( q_r \)  maximum rail seat load
\( Q_{\text{static}} \)  static wheel load
\( Q_{\text{centrifugal}} \)  increase in wheel load on the outer rail in curves due to non-compensated centrifugal force
\( Q_{\text{wind}} \)  increase in wheel load due to wind
\( Q_{\text{dynamic}} \)  dynamic wheel load component resulting from sprung mass, unsprung mass, corrugations, welds, wheel flats etc.
\( r_n \)  ratio of \( n_3 \) and \( n_1 \)
\( R \)  radius of a curved track
\( s \)  track width
\( s_{ij} \)  stress deviator tensor
$S$ ballast settlement

$S_1$ settlement after first load cycle

$S_e(t)$ mean settlement over unit length at tonnage $t$

$S_i$ shear resistance between two particles at contact $i$

$t$ total tonnage

$t_r$ reference tonnage

$V$ velocity of the train

$V_s$ volume of solids

$W$ width of sleeper

$W_{ki}$ percentage retained by weight in each grain size fraction before test

$W_{kf}$ percentage retained by weight in each grain size fraction after test

$W_p$ plastic work done

$\alpha$ dimensionless model constant

$\dot{\alpha}$ soil parameter for cyclic hardening

$\alpha'$ dimensionless speed coefficient for the impact factor $\phi$

$\beta$ proportionality constant between the rate of energy consumption due to particle breakage per unit volume and rate of particle breakage

$\dot{\beta}$ soil parameter for cyclic hardening

$\beta_c$ value of $\beta_i$ for minimum energy ratio

$\beta_i$ angle of slip plane with the direction of major principal stress $\sigma'_1$ at contact $i$

$\beta'$ dimensionless speed coefficient for the impact factor $\phi$

$\chi$ model parameter related to rate of particle breakage

$\delta_{ij}$ Kronecker delta

$\delta e_1$ finite increment of major principal strain

$\delta e_{pq}$ increment of plastic shear strain
\[\delta \varepsilon^p_v\] increment of plastic volumetric strain

\[\delta \mathcal{E}_{bi}\] incremental energy consumed due to particle breakage during sliding between two particles at contact \(i\)

\[\delta \mathcal{E}_B\] incremental energy consumed due to particle breakage during shearing in a unit volume of aggregate

\[\delta \mathbf{u}_i\] incremental displacement between two particles along slip plane at contact \(i\)

\[\delta x_i\] horizontal component of \(\delta \mathbf{u}_i\)

\[\delta y_i\] vertical component of \(\delta \mathbf{u}_i\)

\[\Delta W_k\] difference between \(W_{ki}\) and \(W_{kf}\)

\[\varepsilon_{ij}\] strain tensor

\[\varepsilon_1\] strain after first load cycle

\[\varepsilon_\gamma\] shear strain

\[\varepsilon_{\text{pref}}\] reference shear strain

\[\varepsilon_N\] permanent strain after \(N\) load cycles

\[\mathbf{\varepsilon}\] plastic strain increment vector

\[\varepsilon_s\] distortional strain

\[\varepsilon_s^p\] plastic distortional strain

\[\varepsilon_s^e\] elastic distortional strain

\[\varepsilon_v\] volumetric strain

\[\varepsilon_v^e\] elastic volumetric strain

\[\varepsilon_v^p\] plastic volumetric strain

\[\phi\] dimensionless impact factor

\[\phi_h\] interparticle friction angle between two particles

\[\phi_f\] basic friction angle of an aggregate excluding particle breakage and
dilatancy effects

\( \phi_{fb} \) apparent friction angle excluding dilatancy effect but including particle breakage effect

\( \phi_{cv} \) friction angle of an aggregate at constant volume

\( \phi_{cs, \text{crit}} \) friction angle of an aggregate at critical state

\( \phi_{mob} \) mobilised friction angle

\( \phi_{p, \text{max}} \) friction angle of aggregates at peak or maximum deviator stress

\( \gamma \) dimensionless model parameter for cyclic loading

\( \gamma' \) dimensionless speed coefficient for the impact factor \( \phi \)

\( \Gamma \) void ratio of the critical state line at \( p = 1 \)

\( \Gamma_s \) surface free-energy

\( \eta \) stress ratio \( (q/p) \)

\( \eta* \) modified stress ratio \( = \eta(p/p_{cs}) \)

\( \eta_2 \) a constant

\( \eta_{bound} \) stress ratio at the bounding surface

\( \eta_i \) initial stress ratio

\( \eta_j \) a specific value of stress ratio

\( \eta_o \) initial stress ratio

\( \kappa \) swelling/recompression constant in \( e-\ln p \) plane

\( \lambda_{cs} \) slope of the critical state line in \( e-\ln p \) plane

\( \mu \) model parameter related to rate of particle breakage

\( \nu \) model parameter related to particle breakage

\( \theta \) model parameter related to particle breakage

\( \sigma_{ij} \) stress tensor
\( \boldsymbol{\sigma} \) stress increment vector

\( \sigma'_1 \) major effective principal stress

\( \sigma'_2 \) intermediate effective principal stress

\( \sigma'_3 \) minor effective principal stress

\( \tau \) shear stress

\( \tau_{\text{max}} \) maximum shear stress

\( \xi \) cyclic hardening index

\( \xi_1 \) dimensionless model parameter

\( \xi_2 \) dimensionless model parameter

\( \xi_3 \) dimensionless model parameter
CHAPTER 1
INTRODUCTION

1.1 GENERAL BACKGROUND

Rail network forms an important part of the transport system in Australia and many other countries of the world. Railways in Australia play a vital role in its economy through transporting freight and bulk commodities between major cities and ports, and carrying passengers, particularly in urban areas. The rail has carried around one third of all domestic freight over the past 25 years, and represents around 0.5 percent of gross domestic product (Productivity Commission, 1999). Millions of passengers travel in trains each year, especially in the state of New South Wales (NSW). Sydney CityRail (Figure 1.1) for example, carried over 300 million passengers in the financial year 2000-2001 (SRA, 2002), and is the largest urban rail network in Australia.

There are more than 43,000 km of broad, narrow, standard, and dual gauge rail tracks in Australia (Figure 1.2). The cost of maintaining this mammoth rail infrastructure (rails, sleepers, ballast, signaling, communications, etc.) on the mainline network alone is estimated to $220 million per year (Bureau of Transport and Communications Economics, 1995). In terms of speed, carrying capacity and cost, there is continual competition with road, air and water transport with a resulting demand for faster, larger and heavier rail traffic, and improved safety and passenger comfort. On one hand this means continuous upgrading of track, and on the other, pressure to adopt innovative technology to minimise construction and maintenance cost.
Figure 1.1. Sydney City Rail network (courtesy: CityRail)
1.2 STATEMENT OF THE PROBLEM

In a ballasted rail track (which is the most common type worldwide), a large portion of the track maintenance budget is spent on ballast related problems (Indraratna et al., 2002a). Although ballast is usually comprised of hard and strong angular particles, which are derived from high strength unweathered rocks, it also undergoes gradual and continuing degradation under cyclic rail loadings (Figure 1.3). The sharp edges and corners are broken due to high stress concentrations at the contact points between adjacent particles. The reduction in angularity decreases its angle of internal friction (i.e. shear strength), which in turn increases plastic settlement of the track.
In low-lying coastal areas where the subgrades are generally saturated, the fines (clays and silt-size particles) of subgrade can be pumped up into the ballast layer as a slurry under cyclic rail loading, if a proper subbase or filter layer is absent (Indraratna et al., 2002b; Selig and Waters, 1994). The pumping of subgrade clay is a major cause of ballast fouling (Figure 1.4). The fine particles either from clay pumping or ballast degradation, form a thin layer surrounding larger grains; hence, increase the compressibility. The fine particles also fill the void spaces between larger aggregates and reduce the drainage characteristics of the ballast bed. The fouling of ballast usually increases track settlement and may cause differential track settlement (Figure 1.5). Where there is saturation and poor drainage, any contamination of ballast may also cause localised undrained failure (Indraratna et al., 2003a). In severe cases, fouled ballast needs to be cleaned or replaced to keep the track up to its desired stiffness (resiliency), bearing capacity, alignment and level of safety.
In conventional track design, ballast degradation and associated plastic deformation are generally ignored. This problem stems from a lack of understanding of complex particle breakage mechanism and the absence of a realistic stress-strain constitutive model including plastic deformation and particle breakage of ballast under a large number of load cycles, typically millions. The consequence of this limited understanding is an oversimplified empirical design and construction of track substructure, which usually requires frequent remedial action and costly track maintenance.
Over the past couple of years, routine replacement of fouled ballast during track maintenance has resulted in large stockpiles of waste ballast, for which there are limited and costly disposal options available, especially with the current strict regulations required by the Environment Protection Authority (EPA). Continuous replacement with fresh ballast demands additional quarrying, which degrades the environment and depletes limited resources. Therefore, to preserve the environment and minimise the track maintenance, discarded waste ballast may be cleaned, sieved, and recycled. However, due to its reduced angularity, it is anticipated that the settlement and lateral deformation of recycled ballast will be higher compared to those of fresh ballast. So, before using recycled ballast, its mechanical response compared to fresh aggregates must be investigated under field-loading and boundary conditions. With the advent of geosynthetic technology, the mechanical behaviour of used ballast can be improved. However, the degree of improvement with the inclusion of geosynthetics must also be studied before placing the used ballast in a live rail track.

1.3 OBJECTIVES AND SCOPE OF RESEARCH

The primary objective of this research study is to investigate the strength, deformation and degradation behaviour of ballast under both monotonic and cyclic loadings, and to advance the current state of theoretical and analytical methods of track analysis, particularly with regard to particle breakage and plastic deformation of ballast. The secondary objective is to study the role of various geosynthetics in stabilising recycled ballast. Within the scope of this research, only the mechanical degradation of ballast under monotonic and cyclic loadings has been studied. However, the natural degradation of ballast in the form of physical, chemical or bio-chemical weathering, and
the associated strength reduction and plastic deformation are beyond the scope of this research.

The specific objectives of this research study are summarised below:

(a) Investigation of stress-strain and degradation response of fresh and recycled ballast under monotonic (static) loading at various confining pressures. The results of this study will provide a direct comparison between the strength, deformation and degradation characteristics of fresh and recycled ballast.

(b) Study the deformation and degradation behaviour of fresh and recycled ballast under field-simulated cyclic loading and boundary conditions. The cyclic test results will reveal the performance of recycled ballast compared to fresh aggregates under realistic track loading.

(c) Examination of the role of geosynthetics in track and evaluation of the effectiveness of various geosynthetics in stabilising recycled ballast under cyclic loading. A series of cyclic loading tests on recycled ballast with and without geosynthetic inclusion will provide direct comparison and assessment of the benefits of using geosynthetics in track stabilisation.

(d) Development of a realistic stress-strain constitutive model for ballast incorporating particle breakage. The constitutive model will enable the railway engineers to compute and predict the deformation and degree of particle breakage for a given load magnitude and number of cycles, so that an appropriate and economic maintenance scheme could be undertaken.
(e) Evaluation of model parameters and verification of the constitutive model by comparing the model predictions with the experimental data.

(f) Prediction of ballast behaviour using other existing models, and compare and discuss those predictions with the new model and the experimental data.

1.4 OUTLINE OF THE THESIS

This thesis is divided into 8 chapters (including the introduction), which are organised as follows:

In Chapter 2, the current state of research on the behaviour of ballast, including the use of geosynthetics in track substructure, has been critically reviewed, after giving a brief overview of different components of track structure and loadings.

Chapter 3 presents the theoretical background of ballast modelling, and describes the widely used constitutive models for ballast and other granular aggregates under both monotonic and cyclic loadings, followed by a discussion on the available particle breakage models.

Chapter 4 describes the details of various laboratory investigations conducted in this study to evaluate the strength, deformation and degradation behaviour of fresh and recycled ballast under both monotonic and cyclic loadings, including the tests on recycled ballast stabilised with various geosynthetics.
Chapter 5 presents the experimental findings and discussions on the current test results, including the variations of principal stress ratio, friction angle, modulus of elasticity and particle breakage with increasing confining pressures. The results also include the performance of recycled ballast under cyclic loading compared to fresh ballast and the degree of stabilisation of recycled ballast using various types of geosynthetics.

In Chapter 6, a new stress-strain and particle breakage constitutive model for ballast is presented. It incorporates the energy consumption due to particle breakage during load changes in conjunction with conventional energy dissipation during frictional (plastic) deformation. The constitutive model includes shearing under monotonic loading from both isotropic and anisotropic initial stress states, and also for a more complex cyclic loading.

In Chapter 7, numerical implementation of the constitutive model including evaluation of the model parameters is presented. The model predictions were compared with the experimental data and also with the results of numerical analysis using a finite element code ABAQUS. Cyclic model predictions were also compared with the test results and 2 existing cyclic models.

Conclusions of the current study are given in Chapter 8. It summarises the findings of this research study, which include the strength, stiffness, deformation, and particle breakage of fresh and recycled ballast under monotonic and cyclic loadings, and the degree of stabilisation of recycled ballast using various geosynthetics. Finally, several recommendations have been made for further research. A list of References and Appendices follow Chapter 8.
CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

Rail tracks are conventionally founded on ballast for several reasons, including economy and ease of maintenance. ‘Ballast’, which means large size aggregates having a uniform grading with little or no fines, is a unique type of aggregates compared to other typical geomaterials. The behaviour of ballast is comparable to other coarse-grained aggregates (e.g. gravels, sands), except that the size and geometry of ballast grains undergo gradual changes over its service life. Most researchers concentrated their study on the strength and deformation behaviour of ballast under cyclic loading but neglected particle degradation. The current state of research on the behaviour of ballast is reviewed in this chapter, where the relevant components of railway track structure and various types of loadings imposed on the ballast bed are briefly discussed.

2.2 COMPONENTS OF RAIL TRACK

Railway track structure is designed and constructed to provide a safe and economical guideway for passenger and freight rail traffic. This requires a track stable enough in both vertical and horizontal alignments under the various speeds and axle loadings of different trains. A track system usually consists of several components, each of which must perform its specific functions to maintain the rail system safely and satisfactorily.
A ballasted track system consists of the following components: (a) rail, (b) fastening system, (c) sleepers or ties, (d) ballast, (e) subballast and (f) subgrade. Figure 2.1 shows a typical track section and its different components. The track components may be grouped into two main categories: (a) superstructure, and (b) substructure. Track superstructure consists of rails, fastening system and sleepers. The substructure comprises ballast, subballast and subgrade. The superstructure is separated from the substructure by the sleeper-ballast interface.

Figure 2.1. Typical section of a ballasted rail track (modified after Selig and Waters, 1994)

2.2.1 Rails

Rails are longitudinal steel members that guide and support the train wheels, and transfer concentrated wheel loads to the supporting sleepers, which are spaced evenly
along its length. The rails must be stiff enough to carry out the above functions without excessive deflection between the sleepers. The rails may also serve as electric signal conductors and as the ground line for electric power trains.

The vertical and lateral profiles of the rails and the wheel profile govern the smoothness of traffic movement as the wheels roll over the track. Consequently, any appreciable defect on the rail or wheel surface can cause a significant magnitude of dynamic load on the track structure when the trains are running fast. Excessive dynamic loads caused by rail or wheel surface imperfections are detrimental to other components of the track structure.

Rail sections may be connected by bolted joints or welding. In bolted joints, the rails are connected with drilled plates known as ‘fishplates’. However, the discontinuity resulting from this joint causes vibration and extra dynamic load, which lowers passenger comfort and causes accelerated failure around the joint. The combination of impact load and reduced rail stiffness at the joints produces exceptionally high stress on ballast and subgrade, and this increases the rate of ballast degradation, fouling, and settlement. Most track problems are found at bolted rail joints where frequent maintenance is required. Therefore, in most of the important passenger and heavily used freight lines, bolted joints have now been replaced by continuously welded rail (CWR). CWR has several advantages, including substantial savings in maintenance due to the elimination of joint wear and batter, improved riding quality, reduced wear and tear on rolling stock, and reduction in substructure damage (Selig and Waters, 1994).
2.2.2 Fastening System

Steel fasteners are used to hold the rails on the sleepers to ensure they do not move vertically, longitudinally, or laterally (Selig and Waters, 1994). Various types of fastening systems are used by railway organisations throughout the world, depending on the type of sleeper and geometry of the rail section.

The major components of a fastening system include coach screws to hold the baseplate to the sleeper, clip bolts, rigid sleeper clips, spring washers and nuts to attach the sleeper to the rail (Esveld, 2001). Rail pads are sometimes used on top of the sleepers to dampen the dynamic forces generated by high-speed traffic movements. Fastening systems are categorised into two groups: direct fastening and indirect fastening. In direct fastening, the rail and baseplate are connected to the sleeper using the same fastener. In contrast, in an indirect fastening system, the rail is connected to the baseplate with one fastener, while the baseplate is attached to the sleeper by a different fastener (Esveld, 2001). The indirect fastening system allows a rail to be removed from the track without removing the baseplate from the sleeper, and allows the baseplate to be attached to the sleeper before being placed on the track.

2.2.3 Sleeper

Sleepers (or ties) provide a solid, even and flat platform for the rails, and form the basis of a rail fastening system. They hold the rails in position and maintain the designed rail gauge. Sleepers are laid on top of compacted ballast layer, perpendicular to the rails, and at a specific distance apart. Mechanically, sleepers receive concentrated vertical, lateral and longitudinal forces from the wheels and rails, and distribute them over a wider ballast area to decrease the stress to an acceptable level.
Sleepers can be made of wood, concrete or steel. However, timber sleepers are predominantly used worldwide. In recent times, prestressed concrete sleepers (Figure 2.2b) have become popular and economic in several countries. Steel sleepers are expensive and are used only in special situations. Timber sleepers are cheaper and available in most parts of the world. Prestressed concrete sleepers are potentially more durable, stronger, heavier, and more rigid than timber sleepers. Moreover, the geometry of the concrete sleepers can be easily modified to extend the support area beneath the rails (see Figure 2.2b). The extended support area decreases ballast/sleeper contact stress, which minimises track settlement and particle breakage. Concrete sleepers provide an overall stiffer track, which also increases fuel consumption benefits. Some researchers however, indicate that timber sleepers are more resilient and less abraded by the surrounding ballast compared to concrete sleepers (Key, 1998).

2.2.4 Ballast

The term ‘ballast’ used in railway engineering means granular coarse aggregates placed above subballast or subgrade to act as a firm platform with high bearing capacity, and
support the track superstructure (sleepers, rails etc.). The sleepers (or ties) are embedded into the ballast layer. Ballast is usually composed of crushed rocks originating from high quality igneous or well-cemented sedimentary rock. Traditionally, crushed angular hard stones and rocks having uniform gradation, free of dust, and not prone to cementing action have been considered good quality ballast materials (Selig and Waters, 1994).

The source of ballast (parent rock) varies from country to country depending on the quality and availability of the rock, and economy. No universal specification of ballast for its index characteristics such as size, shape, hardness, abrasion resistance and mineral composition that will provide the best track performance under all types of loadings, subsoil and environments, has yet been established. Because there are no such universal specifications, a wide variety of materials (e.g. basalt, limestone, granite, dolomite, rheolite, gneiss, slag, gravel etc.) are used as ballast throughout the world.

2.2.4.1 Functions of ballast

Ideally, ballast should perform the following functions (Jeffs, 1989):

- Provide a firm and stable platform, and support the sleeper uniformly with high bearing capacity
- Transmit the high imposed pressure at the sleeper/ballast interface to the subgrade layer at a reduced and acceptable stress level
- Provide adequate stability to the sleepers against vertical, longitudinal and lateral forces generated by train movements within designed speed limits
- Provide dynamic resiliency
- Provide adequate resistance against crushing, attrition, bio-chemical and
mechanical degradation and weathering

- Provide minimal plastic deformation to the track structure
- Provide adequate hydraulic conductivity for drainage purposes
- Facilitate maintenance operations
- Inhibit weed growth
- Absorb noise, and
- Provide adequate electrical resistance.

2.2.4.2 Properties of ballast

In order to fulfil the above functions satisfactorily, ballast must have certain characteristics such as particle size, shape, gradation, surface roughness, particle density, bulk density, strength, durability, hardness, toughness, resistance to attrition and weathering, as discussed below.

Various standards and specifications are made by railway organisations throughout the world to meet their ballast requirements. In general, ballast particles must be angular, uniformly graded, strong, tough and durable under anticipated traffic loads and environmental changes. In Australia, Australian Standard AS 2758.7 (1996) specifies the general requirements, and the specification TS 3402 (2001) details the specifications and required properties of ballast for Rail Infrastructure Corporation (RIC) of NSW. The size and gradation of ballast as specified by both AS 2758.7 (1996) and TS 3402 (2001) are given in Table 2.1.
Table 2.1: Ballast size and gradation (AS 2758.7 and TS 3402)

<table>
<thead>
<tr>
<th>Sieve size (mm)</th>
<th>% passing by weight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>63.0</td>
<td>100</td>
</tr>
<tr>
<td>53.0</td>
<td>85-100</td>
</tr>
<tr>
<td>37.5</td>
<td>20-65</td>
</tr>
<tr>
<td>26.5</td>
<td>0-20</td>
</tr>
<tr>
<td>19.0</td>
<td>0-5</td>
</tr>
<tr>
<td>13.2</td>
<td>0-2</td>
</tr>
<tr>
<td>9.50</td>
<td>-</td>
</tr>
<tr>
<td>4.75</td>
<td>0-1</td>
</tr>
<tr>
<td>1.18</td>
<td>-</td>
</tr>
<tr>
<td>0.075</td>
<td>0-1</td>
</tr>
</tbody>
</table>

The Australian Standard AS 2758.7 (1996) also specifies the minimum wet strength and the wet/dry strength variation of the ballast particles when determined in accordance with AS 1141.22 for the fraction of aggregates passing 26.5 mm sieve and retained on 19.0 mm sieve, as shown in Table 2.2.

Table 2.2: Minimum ballast strength and maximum strength variation (AS 2758.7)

<table>
<thead>
<tr>
<th>Minimum wet strength (kN)</th>
<th>Wet/dry strength variation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>175</td>
<td>≤ 25</td>
</tr>
</tbody>
</table>

The durability of ballast is usually assessed by conducting several standard tests such as Los Angeles Abrasion (LAA) test (AS 1141.23), Aggregate Crushing test (AS 1141.21), Wet Attrition test (AS 1141.27) etc. Indraratna et al. (2002c) gives a comparison between the specifications of ballast used in Australia (AS 2758.7), USA (Gaskin and Raymond, 1976) and Canada (Gaskin and Raymond, 1976; Raymond, 1985), as given in Table 2.3.
Table 2.3: Ballast Specifications in Australia, USA and Canada (after Indraratna et al., 2002c)

<table>
<thead>
<tr>
<th>Ballast property</th>
<th>Australia</th>
<th>USA</th>
<th>Canada</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Crushing Value</td>
<td>&lt; 25%</td>
<td>&lt; 25%</td>
<td>&lt; 40%</td>
</tr>
<tr>
<td>LAA</td>
<td>&lt; 25%</td>
<td>&lt; 40%</td>
<td>&lt; 20%</td>
</tr>
<tr>
<td>Flakiness Index</td>
<td>&lt; 30%</td>
<td>&lt; 30%</td>
<td>&lt; 40%</td>
</tr>
<tr>
<td>Misshapen Particles</td>
<td>&lt; 30%</td>
<td>&lt; 10%</td>
<td>&lt; 25%</td>
</tr>
<tr>
<td>Sodium Sulphate Soundness</td>
<td></td>
<td>&lt; 5%</td>
<td>&lt; 10%</td>
</tr>
<tr>
<td>Magnesium Sulphate Soundness</td>
<td></td>
<td></td>
<td>&lt; 10%</td>
</tr>
<tr>
<td>Soft and Friable Pieces</td>
<td>&lt; 5%</td>
<td>&lt; 5%</td>
<td>&lt; 5%</td>
</tr>
<tr>
<td>Fines (&lt; No. 200 sieve)</td>
<td>&lt; 1%</td>
<td>&lt; 1%</td>
<td>&lt; 1%</td>
</tr>
<tr>
<td>Clay Lumps</td>
<td>&lt; 0.5%</td>
<td>&lt; 0.5%</td>
<td>&lt; 0.5%</td>
</tr>
<tr>
<td>Bulk Unit Weight</td>
<td>&gt; 1200 kg/m³</td>
<td>&gt; 1120 kg/m³</td>
<td>&gt; 1200 kg/m³</td>
</tr>
<tr>
<td>Particle Specific Gravity</td>
<td>&gt; 2.5</td>
<td></td>
<td>&gt; 2.6</td>
</tr>
</tbody>
</table>

In order to design the track substructure adequately and efficiently, it is essential to know the magnitude of sleeper/ballast contact stress and the distribution of stresses through the ballast, sub-ballast and subgrade layers. The depth of ballast layer required for a track structure depends on the maximum stress intensity at the sleeper/ballast interface, acceptable bearing pressure of the underlying layer (subballast or subgrade) and the distribution of vertical stress through the ballast layer. Several methods, including simplified theoretical models, semi-empirical and empirical solutions are used in practice to determine the distribution of vertical stress through the ballast layer (Doyle, 1980). These methods are based on calculating stress under a uniformly loaded strip of infinite length and circular loaded area.

Under cyclic loading from repeatedly passing wheel loads, ballast undergoes irrecoverable plastic deformation and particle degradation, in addition to recoverable elastic strains. The accumulated plastic deformation may become excessively high after
millions of load cycles. The continuous degradation process makes the originally sharp angular particles into relatively less angular and semi-rounded grains, thereby reducing inter-particle frictional interlock. Thus, the frictional resistance is decreased, which leads to a further increase in plastic strains. Ballast degradation and associated plastic deformation have been ignored in conventional design and analysis of track substructure. Traditionally, when the plastic deformation exceeds a tolerance level, or ballast becomes excessively fouled by mechanical degradation or other processes, these shortcomings in design and analysis are covered by frequent costly maintenance operations, which also disrupt traffics. Where timely maintenance is not carried out, the track section may not evenly support the running train, which may lead to a devastating accident with loss of lives and properties.

In order to design a more efficient track structure and minimise maintenance cost, ballast degradation and plastic track deformation must be examined and studied in detail. Moreover, the effects of particle breakage must be included in the constitutive stress-strain formulation so that a more appropriate and rationalised analysis and design method can be employed. With the advent of geosynthetic technology, the degradation and deformation of ballast may be minimised. These innovative ideas and techniques will be discussed in detail in the following Sections and Chapters.

2.2.5 Subballast

Subballast is a layer of aggregates placed between ballast layer and the subgrade, and usually comprised of well-graded crushed rock or sand/gravel mixtures. It prevents penetration of coarse ballast grains into the subgrade, and also prevents upward migration of subgrade particles (fines) into the ballast layer. Subballast therefore, acts as
a filter and separating layer in the track substructure, transmits and distributes stress from the ballast layer down to the subgrade over a wider area, and also acts as a drainage medium, to a limited extent.

When designing the subballast layer, attention is given to its filtering function. Therefore, it is usually made of widely graded materials where the empirical filter design methods govern its particle size distribution. Where there is no subballast or where poorly designed subballast is used, subgrade clay and silt size particles become mixed with ground/infiltrated water to form a slurry, which is pumped up to the ballast layer under cyclic loading (clay pumping). In low lying coastal areas of Australia and many other parts of the world, ballast fouling by clay pumping is commonly observed during and after heavy rainfall. Use of geosynthetics in track substructure may prevent or minimise ballast fouling, and this aspect will be discussed further in the following Sections and Chapters.

2.2.6 Subgrade

Subgrade is the ground where rail track structure is constructed. It may be naturally deposited soil or specially placed fill material, e.g. rail embankment. The subgrade must be stiff and have sufficient bearing capacity to resist traffic induced stresses at the subballast/subgrade interface. Instability or failure of subgrade will result in an unacceptable distortion of track geometry and alignment, even with excellent ballast and subballast layers. If a track is to be constructed on soft soil, the subgrade needs to be stabilised by one of the several ground improvement techniques, e.g. installing prefabricated vertical drains (PVD), lime-cement columns, deep cement/lime grouting etc.
2.3 TRACK FORCES

In order to analyse and design an adequate rail track substructure, the type and magnitude of loads that may be imposed on the ballast bed during its lifetime must be known. These loads are exerted by the sleepers on the ballast bed due to standing or running trains through wheel-rail-sleeper interactions, and are a combination of static load and a dynamic component superimposed on the static load.

2.3.1 Vertical Forces

The total vertical wheel load on a rail may be classified into two groups: quasi-static load and dynamic load (Esveld, 2001). The quasi-static load is composed of three components, as given below:

\[ Q_{\text{total}} = Q_{\text{quasi-static}} + Q_{\text{dynamic}} \]  

\[ Q_{\text{quasi-static}} = Q_{\text{static}} + Q_{\text{centrifugal}} + Q_{\text{wind}} \]

where, 

\( Q_{\text{static}} = \) static wheel load = half of static axle load,

\( Q_{\text{centrifugal}} = \) increase in wheel load on the outer rail in curves due to non-compensated centrifugal force,

\( Q_{\text{wind}} = \) increase in wheel load due to wind,

\( Q_{\text{dynamic}} = \) dynamic wheel load component resulting from sprung mass, unsprung mass, corrugations, welds, wheel flats etc.
Considering the equilibrium of forces acting on a vehicle, as shown in Figure 2.3, Esveld (2001) proposed the following expressions for the centrifugal and wind forces:

\[ Q_{\text{centrifugal}} + Q_{\text{wind}} = G \frac{p_c h_d}{s^2} + H_w \frac{p_w}{s} \]  \hspace{1cm} (2.2)

\[ h_d = \frac{sV^2}{gR} - h \]  \hspace{1cm} (2.3)

where, 
- \( G \) = weight of vehicle per axle,
- \( H_w \) = cross wind force,
- \( s \) = track width,
- \( V \) = speed,
- \( g \) = acceleration due to gravity,
- \( R \) = radius of curved track,
- \( h \) = cant (or superelevation),
- \( p_c \) = distance between center of rails and center of gravity of vehicle, and
- \( p_w \) = vertical distance of resultant wind force from center of rails.
The most uncertain part of the wheel load is the dynamic component, \( Q_{\text{dynamic}} \). In order to get a rough estimate of \( Q_{\text{dynamic}} \), the static wheel load may be multiplied by a dynamic amplification factor (Esveld, 2001). The major factors affecting the magnitude of dynamic load component are (Jeffs and Tew, 1991):

- Speed of train
- Static wheel load
- Wheel diameter
- Vehicle unsprung mass
- Track condition (including track joints, track geometry and track modulus)
- Track construction
- Vehicle condition

In general, the design vertical wheel load is expressed empirically as a function of the static wheel load, as given below (Jeffs and Tew, 1991):

\[
P_d = \phi P_s
\]

where, \( P_d \) = design wheel load (kN), incorporating dynamic effects,
\( P_s \) = static wheel load (kN), and
\( \phi \) = dimensionless impact factor ( >1.0).

Several empirical formulae for the impact factor \( \phi \), are used by different railway organisations. For the purpose of track design, Li and Selig (1998) proposed the following simple expression for the computation of design wheel load based on the recommendation by American Railway Engineering Association (AREA):

\[
P_{d_i} = \left(1 + \frac{0.0052V}{D_W} \right) P_{s_i}
\]

(2.5)
where, $P_{di}$ is the design wheel load,

$P_{si}$ is the static wheel load,

$D_w$ is the diameter of the wheel (m), and

$V$ is the velocity of the train (km/hour).

The most comprehensive method of determining the impact factor is that developed by the Office of Research and Experiments (ORE) of the International Union of Railways (Jeffs and Tew, 1991). In this method, the impact factor is made entirely based on the measured track forces of locomotives (ORE, 1965). This impact factor is defined in terms of dimensionless speed coefficients $\alpha'$, $\beta'$ and $\gamma'$, as given by:

$$\phi = 1 + \alpha' + \beta' + \gamma'$$

where, $\alpha'$ and $\beta'$ are related to the mean value of the impact factor, and $\gamma'$ is related to the standard deviation of the impact factor.

The coefficient $\alpha'$ depends on track irregularities, vehicle suspension and vehicle speed. Although, it is difficult to correlate $\alpha'$ with track irregularities, it was found that for the poorest case, $\alpha'$ increases with the cubic function of speed, and was empirically expressed by:

$$\alpha' = 0.04 \left( \frac{V}{100} \right)^3$$

where, $V = \text{vehicle speed (km/hour)}$.

The coefficient $\beta'$ is the contribution to the impact factor due to the wheel load shift in curves, and may be expressed by either Equation 2.8a or 2.8b:
\[ \beta' = \frac{2d.h}{G^2} \] (2.8a)

\[ \beta' = \frac{V^2(2h + c)}{127Rg} - \frac{2c.h}{G^2} \] (2.8b)

where, 

- \( G \) = horizontal distance between rail centerlines (m),
- \( h \) = vertical distance from rail top to vehicle center of mass (m),
- \( d \) = superelevation deficiency (m),
- \( c \) = superelevation (m),
- \( g \) = acceleration due to gravity (m/sec^2),
- \( R \) = radius of curve (m), and
- \( V \) = vehicle speed (km/hour).

The last coefficient \( \gamma' \), depends on the vehicle speed, track condition (e.g. age, hanging sleepers etc.), vehicle design, and maintenance condition of the locomotives. It was found that the coefficient \( \gamma' \), increases with the speed, and can be approximated by the following expression:

\[ \gamma' = 0.10 + 0.017 \left( \frac{V}{100} \right)^3 \] (2.9)

The ORE impact factor (\( \phi \)) for different train speeds and various standards of tangent track has been plotted graphically, as shown in Figure 2.4.
In Japan, the equivalent dynamic wheel load is computed using a simple equation, as given by (Atalar et al., 2001):

\[ P'_w = P_w \left( 1 + \frac{V}{100} \right)(1 + C) \]  

(2.10)

where, \( P'_w \) = equivalent dynamic wheel load for design,

\( P_w \) = static wheel load,

\( V \) = maximum velocity (km/hour), and

\( C \) = a coefficient \( \approx 0.3 \).

Typical distribution of wheel load to the rails, sleepers, ballast, subballast and subgrade, is shown in Figure 2.5.
Shenton (1975) studied the distribution of sleeper/ballast contact pressure in real tracks. He indicated that as the typical ballast size is in the range of 25-50 mm and typical width of a sleeper is about 250 mm, the number of particles involved in directly supporting the sleeper is relatively small. He estimated that a sleeper, which has been placed in a track for some time, may only be supported by 100-200 contact points, which means measuring the actual sleeper/ballast contact stress becomes excessively difficult. However, British Railways attempted to measure the sleeper/ballast contact pressure in track, and Shenton (1975) reported those measurements, as shown in Figure 2.6. The distribution of contact pressure is extremely erratic and highly variable from test to test. However, these field measurements (Figure 2.6) give a sound indication regarding the maximum pressure exerted by the sleeper to the underlying ballast layer for a known axle load.
For the purpose of design, the contact pressure between the sleeper and ballast is generally assumed to be uniform and simplified to the following expression (Jeffs and Tew, 1991):

\[ P_a = \left( \frac{q_r}{W \cdot L} \right) F_2 \]  

(2.11)

where, \( P_a \) = average contact pressure,

\( q_r \) = maximum rail seat load,

\( W \) = width of sleeper,

\( L \) = effective length of sleeper supporting the load \( q_r \), and

\( F_2 \) = a factor depending on the sleeper type and track maintenance.

Assuming one third of the total sleeper length as the effective length, Equation 2.11 becomes:

\[ P_a = \left( \frac{3q_r}{W \cdot l} \right) F_2 \]  

(2.12)

where, \( l \) = total length of sleeper.
The sleeper/ballast contact pressure following Equation (2.12) is represented in Figure 2.7.

In the Japanese Standard, a similar distribution of sleeper/ballast contact pressure is assumed, with only a different effective sleeper length, as shown in Figure 2.8.

Atalar et al. (2001) estimated the maximum sleeper/ballast contact stress for a train speed of 385 km/hour to about 479 kPa. Esveld (2001) stated the maximum permissible sleeper/ballast contact stress to be 500 kPa.

2.3.2 Lateral Forces

Lateral loads in tracks are far more complex than vertical loads and less understood (Key, 1998). Selig and Waters (1994) indicated that there are two principal sources of
lateral loads: (a) lateral wheel force, and (b) buckling reaction force. Lateral wheel forces are generated by the lateral component of frictional forces between wheel and rail, and by the lateral force applied by the wheel flange on the rail. Buckling reaction forces are developed in the lateral directions of the rail due to high compressive stresses caused by high temperatures in rails.

In order to assess lateral forces in track, the Office of Research and Experiments (ORE, 1965, 1970) carried out test programs for speeds up to 200 km/hour. They found that the lateral track force is dependent only on the radius of curvature, and suggested the following empirical expression:

\[ P_H = 35 + \frac{7400}{R} \]  \hspace{1cm} (2.13)

where, \( P_H \) = lateral force at curved track (kN), and \( R \) = radius of curve (m).

Similar empirical formula for the lateral rail force is used in France (Key, 1998), where lateral track force is considered to increase with traffic load, as given below:

\[ H_s > 10 + \frac{P}{3} \]  \hspace{1cm} (2.14)

where, \( H_s \) = force (kN) required to initiate lateral displacement, and \( P \) = Axle load (kN).

### 2.4 FACTORS GOVERNING BALLAST BEHAVIOUR

The mechanical response of ballast is governed by four factors: (a) characteristics of constituting particles, (e.g. size, shape, surface roughness, particle crushing strength, resistance to attrition etc.), (b) bulk aggregate characteristics including particle size
distribution, void ratio or density and degree of saturation, (c) loading characteristics including current state of stress, previous stress history and applied stress path, and (d) particle degradation, which is a combined effect of grain properties, aggregate characteristics and loading. These factors are discussed in the following Sections.

2.4.1 Particle Characteristics

The physical and mechanical characteristics of individual constituting particles significantly influence the behaviour of ballast under both static and cyclic loading. In the following Sections, various characteristics of individual ballast grains and their influences on the mechanical behaviour of ballast are discussed.

2.4.1.1 Particle size

Typically, size of ballast grains varies in the range of 10-60 mm. Due to transportation, handling, placement and compaction of ballast, and also movement of heavy construction machines over the ballast layer, there are some changes in the asperities of ballast grains. Some particles may split or even crush into several small pieces. With the increase in the number of train passages, the ballast particles are further degraded and gradually decrease in size. However, even after these changes in size, more than 90% of ballast grains remain in the original range of 10-60 mm.

Several researchers have studied the effects of particle size on the mechanical behaviour of ballast and other coarse aggregates. However, there are some contradictions amongst their findings. Kolbuszewski and Frederick (1963) indicated that the angle of shear resistance increases with larger particle size. They concluded that increasing particle size increases the dilatancy component of the angle of shear resistance. In contrast,
Marachi et al. (1972) presented experimental evidence that the angle of internal friction decreases with an increase in maximum particle size (Figure 2.9). Indraratna et al. (1998) observed similar findings in their study and indicated that the peak friction angle decreased slightly with an increase in grain size at low confining pressure (< 300 kPa). They concluded that at high stress levels (> 400 kPa), the effect of particle size on friction angle becomes negligible.

Raymond and Diyaljee (1979) observed that larger ballast of uniform grading provided higher plastic strain compared to small-sized uniform ballast. Although smaller aggregates showed less deformation (i.e. higher resistance) under smaller cyclic loads.

Figure 2.9. Effect of particle size on friction angle (after Marachi et al., 1972)
(amplitudes), those specimens failed immediately after increasing the load amplitudes from 140 kPa to 210 kPa. In contrast, larger ballast continued to resist cyclic loading without any sign of failure even after increasing the load amplitudes from 140 kPa to 210 kPa. Raymond and Diyaljee (1979) concluded that smaller ballast deforms less if the stress level does not exceed a critical value. However, smaller ballast has a lower final compacted strength than larger ballast.

Figure 2.10. Effect of grain size on Resilient Modulus of Ballast (after Janardhanam and Desai, 1983)

In an attempt to investigate the influence of particle size on ballast behaviour, Janardhanam and Desai (1983) conducted a series of true triaxial tests under cyclic loading. They indicated that the particle size does not appear to significantly influence ballast strains at various stress levels. They also concluded that volumetric strain is not affected by particle size. However, grain size has a significant effect on the resilient modulus of ballast. The modulus increases with the mean grain size at all levels of confinement, and at low confining pressure the relationship is almost linear with the mean grain size (Figure 2.10). In contrast, Indraratna et al. (1998) presented
experimental evidence based on monotonic triaxial tests that larger ballast has a smaller deformation modulus and Poisson’s ratio compared to smaller aggregates.

Considering the advantages and disadvantages of varying particle size, Selig (1984) recommended that the ideal ballast should be of 10-50 mm size with some particles beyond this range. The larger particles stabilise the track and the smaller particles reduce the contact forces between particles and minimise breakage.

2.4.1.2 Particle shape

Unlike particle size, there is some consensus amongst researchers regarding the effects of grain shape on the mechanical response of ballast and other coarse aggregates. In general, angularity increases frictional interlock between grains of aggregate, which increases its shear strength (Holz and Gibbs, 1956; Leps, 1970; Indraratna et al., 1998). Holz and Gibbs (1956) concluded that the shear strength of highly angular quarried materials is higher than that of relatively sub-angular, or sub-rounded river gravels (Figure 2.11). Vallerga et al. (1957) provided clear evidence that the angle of internal friction is remarkably high for angular aggregates compared to sub-rounded aggregates (Figure 2.12), while others concluded that the angle of internal friction depends on grain angularity (Kolbuszewski and Frederick, 1963; Leps, 1970). Jeffs and Marich (1987) and Jeffs (1989) demonstrated that angular aggregates give less settlement than round aggregates. Chrismer (1985) indicated that as grain angularity increases, further dilation is required for particle movement, which increases its resistance to shear deformation.
Jeffs and Tew (1991) reported that the shape of ballast grains depends on the production process and the nature of deposits. Raymond (1985) indicated that most specifications restricted the percentage of flaky particles whose ratio of the longest to smallest dimension exceeds 3, and excluded particles exceeding the ratio 10. Because these long but very thin particles can align and form planes of weakness in both vertical and lateral directions, they cannot be used as ballast. The disadvantages of increased flakiness appear to be increased abrasion, increased breakage, increased permanent strain
accumulation under repeated load and decreased stiffness (Selig and Waters, 1994). Most specifications also limit the percentage of misshapen particles, where the term ‘misshapen particle’ means flat, elongated, and flat and elongated particles. However, there is uncertainty regarding the allowable percentage of misshapen particles (Jeffs and Tew, 1991). Raymond (1985) stated that cuboidal is the best shape for high quality ballast, an opinion supported by Jeffs and Tew (1991).

2.4.1.3 Surface roughness

Surface roughness is considered to be one of the key factors that govern the angle of internal friction and hence, the strength and stability of ballast. Each grain has more or less roughness on its surface. The phenomenon ‘friction and frictional force’ is based on the roughness of the loaded surface, and the shear resistance of ballast and other aggregates depends on this frictional force. Raymond (1985) concluded that particle shape and surface roughness are of utmost importance and have long been recognised as the major factors influencing track stability. Canadian Pacific Rails preferred surface roughness over particle shape as the key parameter for track stability, and have stringent controls on grain surface rather than direct restrictions on particle shape (Raymond, 1985). Thom and Brown (1988, 1989) reported an increase in resilient modulus with increasing surface friction of grains, and concluded that the resistance to plastic strain accumulation increases with increasing visible surface roughness.

Almost all specifications of ballast demand crushed or fractured particles, which are defined as grains having a minimum of three crushed faces (i.e. freshly exposed surfaces with a minimum of one third of the maximum particle dimension) (Selig and Waters, 1994). These specifications ensure minimum surface roughness of ballast
particles, and assume that freshly exposed surfaces have a higher roughness compared to old surfaces, which have been smoothened by mechanical attrition and weathering.

Due to internal attrition of grains under cyclic traffic loading, surface roughness of ballast deteriorates with time (i.e. increasing number of train passage). Internal attrition also produces powder like fines and is a source of ballast fouling. This reduction in surface roughness by internal attrition and breakage of sharp corners (as mentioned earlier) after millions of load cycles, causes the angle of internal friction and the shear strength of recycled ballast to decrease considerably. Therefore, it is conceivable that the surface roughness of individual particles significantly affects the mechanical behaviour of ballast and ultimately, track stability.

2.4.1.4 Parent rock strength

The strength of the parent rock is probably the most important factor directly governing ballast degradation, and indirectly, settlement and lateral deformation of the track. Rock strength includes both compressive and tensile strength. Under the same loading and boundary conditions, weak particles will result in more grain breakage and more plastic settlement than strong particles. Although the strength of the parent rock is not usually tested or required by most ballast specifications (e.g., TS 3402 of RIC, NSW), higher parent rock strength is ensured by the selection criteria, which includes petrological examination. High rock strength is also indirectly reflected in other tests such as ‘Aggregate crushing value’, ‘Los Angeles Abrasion value’ and ‘Wet attrition value’. These test results collectively indicate the durability of ballast and the strength of the parent rock. However, in order to enhance the control on the quality of ballast during selection, parent rock strength values may be included in the specifications.
2.4.1.5 Particle crushing strength

The individual particle crushing strength is an important factor governing particle degradation, which includes grain splitting and breakage of sharp corners under loading. Particle fracture plays a vital role in the behaviour of crushable aggregates (McDowell and Bolton, 1998). Particle crushing strength depends primarily on the strength of parent rock, the geometry of the grain, the loading point and loading direction. Fracture in rock grains initiates by tensile failure, and the fracture strength can be measured indirectly by diametral compression between flat platens (Jaeger, 1967). For a particle of diameter \( d \) under diametral compressive force \( F \), a characteristic tensile stress \( \sigma \) is induced within it (Jaeger, 1967), as given by Equation 2.15.

\[
\sigma = \frac{F}{d^2} \quad (2.15)
\]

It is relevant to mention here that Equation 2.15 is consistent with the definition of the tensile strength of concrete in the Brazilian test, where a cylinder of concrete is compressed diametrically and then split due to an induced tensile stress. Following Equation 2.15, McDowell and Bolton (1998) and Nakata et al. (2001) described the characteristic particle tensile strength \( \sigma_f \), as given by:

\[
\sigma_f = \frac{F_f}{d^2} \quad (2.16)
\]

where, the subscript \( f \) denotes failure.

It should be mentioned here that Equation 2.16 assumes a circular specimen, while most ballast grains are angular. Therefore, actual crushing strength of ballast at the contact points between a particle and the compression machine (top and bottom platens) and also between particles will be much higher than the value computed by Equation 2.16.
The actual shapes of individual particles and corresponding actual contact stresses are highly variable and difficult to measure with reasonable accuracy without the use of more complex shape functions and precise shape measurements. Moreover, complete splitting and crushing of a particle under compression (see Fig. 4.5) occurs at much higher load, which is generally obtained after the failure and breakage of small sharp corners between a particle and the platens, and the load is then distributed over a wider area of the particle. Therefore, in determining the tensile strength from a single particle crushing test, computation of average tensile strength of a single particle using Equation 2.16 is considered to be reasonable and acceptable.

Festag and Katzenbach (2001) categorised grain crushing into particle breakage (fracture) and grain abrasion. Particle breakage is the dissection of grains into parts with nearly the same dimension and generally occurs in high stress domain. On the other hand, abrasion is a phenomenon where very small particles break off from the grain surface, and is independent of the stress level. Abrasion takes place in granular materials when the particles slip or roll over each other during shear deformation, and may occur even at low stress level. Grain breakage may be absent if the stress level is low compared to particle strength, however, grain abrasion will continue at any stress level. The crushing strength of particles is also not required by many ballast specifications. However, it is reflected in the ‘Aggregate crushing value’ and other standard durability tests required by many ballast specifications.

2.4.1.6 Resistance to attrition and weathering

These properties of individual grains also govern ballast degradation under traffic loading and environmental changes. Usually, ballast particles are not individually
assessed for their capacity to resist attrition and weathering, rather, their resistance is collectively assessed for the aggregate mass. Several standard test methods for quantifying the resistance of ballast against attrition and weathering are available and used by different railway organisations. These tests include Los Angeles Abrasion (LAA) test, mill abrasion (MA), the Deval test and Sulphate Soundness test etc. (Selig and Waters, 1994). The Los Angeles Abrasion test, the mill abrasion (MA) and Deval tests are used in North America and Europe to measure the resistance of ballast to attrition. The Sulphate Soundness test is primarily used to assess resistance to the chemical action of Sodium Sulphate and Magnesium Sulphate (salt). High resistance to attrition and weathering is usually ensured by specifying certain values required for the above mentioned durability tests in ballast standards and specifications, as shown in Table 2.3 earlier.

2.4.2 Aggregate Characteristics

The characteristics of aggregate mass that govern ballast behaviour include particle size distribution (PSD), void ratio (or density) and degree of saturation. These characteristics are discussed in the following Sections.

2.4.2.1 Particle size distribution

The distribution of particle size (i.e. gradation) has a remarkable influence on the track deformation behaviour (Jeffs and Tew, 1991). Several researchers have studied the effects of particle gradation on the strength and deformation behaviour of aggregates. Thom and Brown (1988) conducted a series of repeated load triaxial tests on crushed dolomitic limestone aggregates with similar maximum particle size varying the particle size distribution from wide to uniform gradation. Each grading curve was characterised
by an exponent ‘n’ shown in Figure 2.13(a) where higher values of ‘n’ indicate greater uniformity of particle size. According to their results (Figures 2.13b-e) elastic shear stiffness (modulus) and permeability increase as the grading parameter ‘n’ increases, while density and friction angle decrease as the value of ‘n’ increases.

Figure 2.13. (a) Gradation of particles, and its effects on (b) friction angle, (c) density, (d) shear modulus and (e) permeability (after Thom and Brown, 1988)
Thom and Brown (1988) indicated that optimum dry density was achieved at about $n = 0.3$ for all types of compaction efforts (i.e. heavily compacted, lightly compacted and uncompacted), and also noted that particle size distribution does not significantly influence the angle of internal friction for uncompacted specimens. One significant finding of their research is that uniform gradation provides a higher stiffness compared to well-graded aggregates. In contrast, Raymond and Diyaljee (1979) demonstrated that well-graded ballast gives lower settlement compared to single sized ballast (Figure 2.14).

![Figure 2.14. Effects of gradation on vertical strain of ballast under cyclic loading (after Raymond and Diyaljee, 1979)](image)

It has been argued that single sized (uniform) ballast has larger void volume than broadly graded ballast (Raymond, 1985). Well-graded (broadly-graded) ballast is stronger due to its void ratio being less compared to uniformly graded ballast (Marsal, 1967; Jeffs and Tew, 1991; Raymond, 1985). However, ballast specifications generally demand uniformly graded aggregates to fulfil its drainage requirements. Since ballast
must be a free draining coarse medium, the optimum gradation would be between uniformly graded large aggregates that give almost instantaneous drainage and broadly graded (well-graded) aggregates that provide higher strength and less settlement. Optimum gradation should provide sufficient drainage capacity (hydraulic conductivity) along with high density, strength, and resilience.

2.4.2.2 Void ratio (or density)

Researchers have long recognised that the volume of voids in a porous medium (e.g. soil and rock aggregates) compared to its volume of solids (i.e. void ratio) significantly affects its mechanical behaviour (Tezaghi and Peck, 1948; Roscoe et al., 1958; Roscoe et al., 1963; Schofield and Wroth, 1968). It has been well established that an aggregate having a lower initial void ratio (i.e. higher initial density) is stronger and gives a smaller settlement compared to an aggregate with a higher initial void ratio (i.e. lower initial density). In widely accepted Critical State Soil Mechanics (CSSM), the significance of void ratio ($e$) in the mechanical behaviour of soil has been recognised by considering it as a governing state variable along with two other stress invariants: mean effective normal stress $p'$, and deviatoric stress $q$ (Roscoe et al., 1963; Schofield and Wroth, 1968).

As ballast is a porous granular medium, its strength and deformation behaviour is also governed by its void ratio (or degree of compaction) (Jeffs and Tew, 1991; Selig and Waters, 1994; Indraratna et al., 1997). All researchers investigating track stability concluded that an increase in ballast density (i.e. lower void ratio) enhances its strength and stability (Gaskin et al., 1978; Profillidis, 1995; Indraratna et al., 2000). Selig and Waters (1994) concluded that low-density ballast leads to high plastic strains. Indraratna
et al. (1998) indicated that the critical stage of ballast life is immediately after track construction or maintenance, when ballast is in its loosest state (i.e. highest void ratio). Track stability can be significantly improved by increasing the bulk density of the ballast bed by further compaction or by using broadly graded (well-graded) aggregates. However, a higher compaction effort also increases the risk of particle breakage, and an well-graded ballast decreases its drainage characteristics.

2.4.2.3 Degree of saturation

Ballast response to mechanical forces is affected by water and the degree of saturation. Water influences track settlement and particle breakage, and also leads to other serious problems. In saturated conditions, subgrade soils soften and mix with water to form a slurry which, under cyclic traffic loading, can be pumped up to the ballast layer, as mentioned earlier. Clay pumping is one of the major causes of ballast contamination (Selig and Waters, 1994; Indaratna et al., 2002a). Sowers et al. (1965) indicated that water entering micro-fissures at the contact points between particles increases local stress and leads to increased particle breakage.

Indaratna et al. (1997) conducted one-dimensional compression tests to investigate the effects of saturation on the deformation and degradation of ballast. They observed a sudden increase in ballast settlement by about 2.6 mm due to sudden flooding (Figure 2.15), and reported a further increase in settlement with time (creep) under saturated conditions. They concluded that saturation increased settlement by about 40% of that of dry ballast.
2.4.3 Loading Characteristics

The deformation and degradation behaviour of ballast is profoundly dependent on its loading characteristics. The confining pressure, previous load history, current state of stress, number of load cycles, frequency and load amplitudes are among the key loading parameters that govern track deformation. The effects of these loading variables are discussed in the following Sections.

2.4.3.1 Confining pressure

Researchers and engineers have recognised the significant effects of confining pressure on the strength and deformation behaviour of soils and granular materials from the earliest days of soil mechanics (Terzaghi and Peck, 1948; Drucker et al., 1957; Roscoe et al., 1958; Vesic and Clough, 1968). Marsal (1967) was one of the pioneers who closely studied the effect of confining pressure on the deformation behaviour and
particle breakage of rockfills. He tested basalt and granitic gneiss aggregates under high confining pressures (500-2500 kPa), and observed that the shear strength is not a linear function of the acting normal pressure. Charles and Watts (1980) and Indraratna et al. (1993) also reported a pronounced non-linearity of failure envelope for coarse granular aggregates at low confining pressure (Figure 2.16). Vesic and Clough (1968) studied the shear behaviour of sand under low to high pressures and concluded that a mean normal stress exists beyond which the curvature of the strength envelope vanishes and the shear strength is not affected by the initial void ratio. They called it ‘breakdown stress’ ($\sigma_B$), because it represents the stress level at which all dilatancy effects disappear and beyond which particle breakage becomes the only mechanism, in addition to simple slip, by which shear deformation takes place.

![Figure 2.16. Non-linear strength envelop at low confining pressure (after Charles and Watts, 1980)](image-url)
Well documented studies indicate that the angle of internal friction of granular mass decreases with increasing confining pressure (Marachi et al., 1972; Leps, 1970; Charles and Watts, 1980; Indraratna et al., 1993). Indraratna et al. (1998) presented laboratory experimental results of railway ballast (latite basalt), which revealed that as confining pressure increases from 1 kPa to 240 kPa, drained friction angle of ballast decreases from about 67° to about 46° (Figure 2.17). They concluded that the high values of apparent friction angle at low confining pressures are related to low contact forces well below the grain crushing strength and the ability of aggregates to dilate at low stress levels.

Marsal (1967) noticed that shearing of rockfill caused a significant amount of particle breakage, and indicated that the particle breakage of granitic gneiss increased with the increase in confining pressure. Vesic and Clough (1968) concluded that as the mean normal stress increases, the crushing becomes more pronounced and the dilatancy...
effects gradually disappear. Indraratna et al. (1993) indicated that the large reduction of friction angle at high confining pressures is probably associated with significant crushing of angular particles. Although ballast is subjected to low confinement in track, it also suffers particle breakage, crushing, attrition and wearing under cyclic traffic loading (Selig and Waters, 1994; Jeffs, 1989; Indraratna et al., 2000). Indraratna et al. (1998) presented experimental evidence that the breakage of latite ballast may increase by about 10 times as the confining pressure increases from 1 kPa to 240 kPa.

2.4.3.2 Load history

Until the late 1950’s, soil mass was considered as perfectly plastic solids. Drucker, Gibson and Henkel (1957) were probably the first, among others, who considered soils as work-hardening plastic materials. With their work-hardening theories, they explained the volume change behaviour of clays during loading, unloading and reloading in a consolidation test, and proposed possible yield surfaces for consolidation. Since publishing their remarkable development, soil is considered a work-hardening plastic material, and the researchers recognise the influence of previous load history on the deformation behaviour of soils.

Diyaljee (1987) conducted a series of laboratory cyclic tests to investigate the effects of stress history on ballast behaviour. In each test, he applied various cyclic deviatoric stresses (70-315 kPa) in several stages (10,000 cycles each) on identical ballast specimens (same gradation, density and confinement). He found that 2 specimens (T3 and T4, Figure 2.18a) in stage 2 loading (140 kPa) deformed almost the same as the specimens T5 and T6 in stage 1 with the same load (140 kPa) without any previous stress history, where specimens T3 and T4 had a previous stress history of 70 kPa cyclic
loading in stage 1. Stage 1 loading is 50% of stage 2 loading, and has an almost negligible influence on the accumulated plastic deformation occurring during stage 2 loading. In contrast, specimens T4 and T9 (Figure 2.18b) with a maximum load history of 210 kPa, showed a very small increase in plastic strain at 245 kPa cyclic stress, compared to specimen T13 at the same loading without any previous load history.

Figure 2.18. Effects of stress history on deformation of ballast under cyclic loading, (a) deviator stress up to 210 kPa, (b) cyclic stress above 210 kPa (after Diyaljee, 1987)
Diyaljee (1987) concluded that a previous stress history more than 50% of the currently applied cyclic deviator stress significantly decreases the plastic strain accumulation in ballast. However, a previous stress history less than 50% of the currently applied cyclic deviator stress does not contribute to plastic strain accumulation. His findings agree with the research previously carried out by the Office of Research and Experiments of the International Union of Railways (ORE, 1974).

2.4.3.3 Current stress state

The current state of stress also influences the deformation and degradation behaviour of ballast. The state of stress is defined by all nine components of stress tensor, \( \sigma_{ij} \), where, \( i=1,2,3; \quad j=1,2,3 \) (Chen and Saleeb, 1982). However, due to the difficulties and complexities arising from dealing with these stress elements and also their dependencies on axis rotation, the invariants of stress tensor are conventionally employed to describe the state of stress (Chen and Saleeb, 1982). In soil mechanics, the state of stress and the failure criteria are usually defined by two stress invariants: the mean effective normal stress \( p' \), and the deviator stress \( q \) (Roscoe et al, 1958; 1963).

Roscoe and co-researchers developed the first comprehensive stress-strain constitutive model for clay based on the plasticity theory and the critical states (Roscoe et al., 1963; Roscoe and Burland, 1968), and showed that the plastic strain increment depends on the state of stress and other factors. As the state of stress and another state variable (void ratio) of a soil element moves towards the critical state, the rate of plastic shear strain corresponding to any load increment becomes higher. At the critical state, the shear strain continues to increase at a constant stress and constant volume.
Poorooshasb et al. (1966) studied the yielding of sand under triaxial compression and showed that the slope of the plastic strain increment increases from a small value (or zero) to an extremely high value as the state of stress moves towards the failure line (Figure 2.19). At a stress state close to the failure line, the very high slope of the plastic strain increment indicates that the plastic shear strain increment is very high compared to the plastic volumetric strain increment. Other researchers also reported similar effects of stress state on the plastic deformation of soils and granular aggregates (Dafalias and Herrmann, 1982; Mroz and Norris, 1982; Lade, 1977).

![Figure 2.19. Effect of stress state on plastic strains (after Poorooshab et al., 1966)](image)

2.4.3.4 Number of load cycles

Engineers and researchers recognised the influence of number of load cycles on the accumulation of plastic deformation of ballast and other granular media. An increase in number of load cycles generally increases settlement and lateral deformation of granular
aggregates including ballast. However, the degree and rate of deformation at various load cycles are the important aspects that have been studied by various researchers.

Shenton (1975) reported that the track settlement immediately after tamping increased at a decreasing rate with the number of axles (Figure 2.20a). He also indicated that the track settlement may be approximated by a linear relationship with the logarithm of load cycles (Figure 2.20b). Raymond et al. (1975) also demonstrated that both axial and volumetric strains of dolomite ballast increased linearly with the logarithm of load cycles, irrespective of loading amplitude (Figure 2.21). Similar observations were also reported by other researchers (Raymond and Bathurst 1994; Selig and Waters, 1994). In contrast, Raymond and Diyaljee (1979) presented evidence, as shown in Figure 2.14 earlier, that the accumulated plastic strains of ballast may not be linearly related to the logarithm of load cycles for all ballast types, grading, and load magnitudes. Diyaljee (1987) reported that the plastic strain of ballast also increased non-linearly with an increase in logarithm of load cycles at a higher cyclic deviator stress (see Figure 2.18).

Figure 2.20. Settlement of track after tamping, (a) in plain scale, (b) in semi-logarithmic scale (after Shenton, 1975)
Shenton (1984) examined a wide range of track settlement data collected from different parts of the world and concluded that the linear relationship of track settlement with the logarithm of load cycles or total tonnage might be a reasonable approximation over a short period of time. However, this approximation can lead to a significant underestimation for a large number of axles (Figure 2.22).
Jeffs and Marich (1987) conducted a series of cyclic load tests on ballast and indicated a rapid increase in settlement initially, followed by a stabilised zone with a linear increase in settlement (Figure 2.23). They also noticed a sudden increase in the rate of settlement in the stabilised (post-compaction) zone, which they called ‘re-compaction’ of ballast.
Jeffs and Marich (1987) attributed this re-compaction to the failure of particle contact points within the ballast bed causing a sudden increase in settlement rate. The effect of re-compaction was noticed for about 100,000 load cycles, after which the rate of settlement became almost constant.

Ionescu et al. (1998) conducted a series of true triaxial tests on latite ballast and concluded that the behaviour of ballast is highly non-linear under cyclic loading (Figure 2.24). They also reported a rapid increase in initial settlement (similar to Jeffs and Marich, 1987) during the first 20,000 load cycles, followed by a consolidation stage up to about 100,000 cycles. Ionescu et al. (1998) indicated that the ballast bed stabilised during this first 100,000 load cycles, after which the settlement increased at a decreasing rate.

Figure 2.24. Settlement of ballast under cyclic loading (after Ionescu et al., 1998)
2.4.3.5 Frequency of loading

Because train speeds vary, it is important to study the influence of loading frequency on the ballast behaviour. Shenton (1975) carried out a series of cyclic loading tests, varying the frequency from 0.1 to 30 Hz, while maintaining other variables (e.g. confining pressures, load amplitude etc.) constant. Based on the test results (Figure 2.25), Shenton concluded that the frequency of loading does not significantly influence the deformation behaviour of ballast. However, he pointed out that these test findings should not be confused with track behaviour, where increased train speed increases the dynamic forces and imparts greater stresses on ballast.

Kempfert and Hu (1999) reported in-situ measurements of dynamic forces in track resulting from speeds up to 400 km/hour and found that a speed up to about 150 km/hour has an insignificant influence on the dynamic vertical stress (Figure 2.26). These field measurements appear to be consistent with Shenton’s (1975) laboratory findings. However, the measured data shows a linear increase in dynamic stress as the...
speed increases from 150 to about 300 km/hour. Beyond 300 km/hour and up to the maximum measured speed (400 km/hour), the effect of speed on dynamic stress becomes insignificant again.

![Figure 2.26. Effects of train speed on dynamic stresses (after Kempfert and Hu, 1999)](image)

### 2.4.3.6 Amplitude of loading

The amplitude of cyclic loading also plays a major role in ballast deformation. Stewart (1986) carried out a series of cyclic triaxial tests varying the load amplitudes at every 1,000 cycles to study the influence of load amplitude on ballast deformation. Figure 2.27(a) shows the test load amplitude and Figure 2.27(b) shows the vertical strain of ballast against the number of load cycles. Stewart (1986) indicated that the permanent strain in the first cycle increased significantly when the load amplitude was increased. He also noted that an increase in load amplitude beyond the maximum past stress level increased the settlement immediately and also increased the final (long term) cumulative strain. Diyaljee (1987) and Ionescu et al. (1998) reported similar findings in
their laboratory investigations. In contrast, decreasing the load amplitude does not contribute to the accumulated plastic strain (Stewart, 1986; Diyaljee, 1987). Stewart (1986) also concluded that the final cumulative strains obtained at the end of various staged, variable-amplitude loading tests (after 4,000 cycles), were independent of the order of applied stresses.

Figure 2.27. Effect of cyclic load amplitude on ballast deformation, (a) test load amplitude, and (b) ballast strain (after Stewart, 1986)
Recently, Suiker (2002) studied the effects of load amplitude on the ballast behaviour. He referred the cyclic load amplitude in terms of the ratio between the cyclic stress ratio and the maximum static stress ratio \[ n = \frac{q}{p}_{\text{cyc}}/\frac{q}{p}_{\text{stat, max}} \]. He concluded that at low cyclic stress level \( n < 0.82 \), the rate of plastic deformation of ballast is negligible (Figure 2.28). In other words, the response of ballast below this cyclic stress level becomes almost elastic. He called this phenomenon ‘shakedown’, and it will be discussed further in Section 2.8.1.

![Figure 2.28. Effect of cyclic stress level on ballast strain (after Suiker, 2002)](image)

2.4.4 Particle Degradation

The most important mechanical behaviour of granular materials such as stress-strain and strength behaviour, volume change and pore pressure developments, and variation in permeability, depend on the integrity of the particles or the amount of particle crushing that occurs from stress change (Lade et al., 1996). All granular aggregates subjected to stresses above normal geotechnical ranges exhibit considerable particle breakage (Terzaghi and Peck, 1948; Hirschfield and Poulos, 1963; Bishop, 1966; Marsal, 1967; Lee and Farhoomand, 1967; Lee and Seed, 1967; Vesic and Clough, 1968; Bilam, 1971;
Miura and O-hara, 1979; Hardin, 1985). Some researchers indicate that particle
breakage can even occur at low confining pressure (Miura and O-hara, 1979; Lade et al.
1996; Indraratna and Salim, 2002). The significance of particle degradation on the
mechanical behaviour of granular aggregates has been recognised by many engineers
and researchers (Marsal, 1967; Vesic and Clough, 1968; Bilam, 1971; Miura and O-
vara, 1979; Hardin, 1985; Indratana et al, 1998; Ueng and Chen, 2000). In the following
Sections, the methods of particle breakage quantification, factors affecting particle
breakage and the influence of particle breakage on the deformation behaviour of ballast
and other granular aggregates are discussed.

2.4.4.1 Quantification of particle breakage

Several investigators attempted to quantify particle breakage upon loading and proposed
their own techniques for computation (Lee and Farhoomand, 1967; Marsal, 1967;
Hardin, 1985), while others focused primarily on the probability of particle fracture
(McDowell et al. 1996; McDowell and Bolton, 1998). In most of these methods,
different empirical indices or parameters were proposed as indicators of particle
breakage. All breakage indices are based on changes in particle size after loading. While
some indices are based on change in a single particle size, others are based on changes
in overall grain-size distribution. Lade et al. (1996) summarised the most widely used
breakage indices for comparison.

Marsal (1967) and Lee and Farhoomand (1967) were the first, among others, who
developed independent techniques and indices for quantifying particle breakage. Marsal
(1967) noticed significant amount of particle breakage during large-scale triaxial tests
on rockfill materials and proposed an index of particle breakage ($B_g$). His method
involves changes in the overall grain-size distribution of aggregates after the load application. Marsal (1967) sieved all the specimens before and after each test. From the recorded changes in particle gradation, the difference in percentage retained on each sieve size ($\Delta W_k = W_{ki} - W_{kf}$) is computed, where, $W_{ki}$ represents the percentage retained on sieve size $k$ before the test and $W_{kf}$ is the percentage retained on the same sieve size after the test. He noticed that some of these differences were positive and some were negative. In fact, the sum of all positive values of $\Delta W_k$ must be theoretically equal to the sum of all negative values. He defined the breakage index $B_g$, as the sum of the positive values of $\Delta W_k$, expressed as a percentage. The breakage index $B_g$, has a lower limit of zero indicating no particle breakage, and has a theoretical upper limit of unity (100%) representing all particles broken to sizes below the smallest sieve size used.

Lee and Farhoomand (1967) measured the extent of particle breakage while investigating earth dam filter materials. They primarily investigated the effects of particle crushing on plugging of dam filters and proposed a breakage indicator expressing change in a single particle size (15% passing, $D_{15}$), which is a key parameter in filter design. Later on, Hardin (1985) defined two different quantities: the breakage potential $B_p$, and total breakage $B_t$, based on changes in grain-size distribution, and introduced the relative breakage index $B_r$ ($=B_t/B_p$), as an indicator of particle degradation. Hardin’s relative breakage $B_r$, has a lower limit of zero and an upper limit of 1 (unity). It is relevant to mention here that Hardin’s (1985) method requires a planimeter or numerical integration technique for computing $B_t$ and $B_p$. Lade et al. (1996) compares the above 3 methods of particle breakage measurements in a graphical form, as shown in Figure 2.29.
Miura and O-hara (1979) used the changes in grain surface area ($\Delta S$) as an indicator of particle breakage. Their concept is based on the idea that new surfaces will be generated as the particles are broken, and therefore, the changes in surface area can be used as a measure of particle breakage. In their method, the specific surface area of each particle size (i.e. sieve size) is computed assuming that all grains are perfectly spherical. The sieving data before and after the test along with the specific surface area are used to calculate the change in surface area, $\Delta S$. The parameter $\Delta S$ has a lower limit of zero and has no theoretical upper limit.

After considering the various methods of particle breakage quantification, Marsal’s (1967) breakage index $B_g$, has been adopted in this study, due to its simplicity in computation and ability to provide a perception regarding the degree of particle degradation from its numerical value.

Figure 2.29. Various definitions of particle breakage (after Lade et al., 1996)
2.4.4.2 Factors affecting particle breakage

Ballast breakage depends on several factors, including load amplitude, frequency, number of cycles, aggregate density, grain angularity, confining pressure and degree of saturation. However, the most significant factor governing ballast breakage is the fracture strength of its constituting particles (Indraratna and Salim, 2003). Lee and Farhoomand (1967) indicated that particle size, angularity, particle size distribution and magnitude of confining pressure affect particle degradation. They concluded that larger particle size, higher grain angularity and uniformity in gradation increase particle crushing. Marsal (1967) agreed with Lee and Farhoomand (1967) with respect to particle breakage, and pointed out additional key factors such as the average value of contact forces (stresses), strength of particles at contact points, and the number of contacts per particle. Marsal (1967) also indicated that the presence of micro-fissures in crushed rocks from blasting and crushing processes is another reason of particle breakage.

Bishop (1966) indicated that at high stress level, particle breakage during shearing is much higher than during the consolidation stage. Lade et al. (1996) pointed out that larger grains contain more flaws or defects and have a higher probability of disintegration. Smaller particles are generally created after fracturing along these defects. As fracturing continues, the subdivided particles contain fewer defects and are therefore, less prone to crushing. McDowell and Bolton (1998) reported that the tensile strength of a single particle decreases as the particle size increases. Lade et al. (1996) indicated that increasing mineral hardness decreases particle crushing.
2.4.4.3 Effects of particle breakage

Particle breakage influences the behaviour of ballast and other granular aggregates. As mentioned earlier in Section 2.4.4, many investigators observed change in particle size (particle degradation) due to change of stress. Some researchers only reported the amount of particle breakage in terms of breakage indices or factors, while a few others attempted to correlate the computed breakage indices with the strength, dilatancy, and friction angle. However, there is a lack of research on the specific effects of particle breakage on the mechanical behaviour of ballast and other granular aggregates.

Figure 2.30. Effect of particle breakage on principal stress ratio at failure (after Marsal, 1967)
In an attempt to correlate the strength of aggregates with particle breakage, Marsal (1967) plotted the peak principal stress ratio \( \sigma_1/\sigma_3 \) against the breakage index \( B_g \) (Figure 2.30), and concluded that the shear strength decreases with increasing particle breakage. Although no distinct correlation could be established between the principal stress ratio at failure and smaller values of particle breakage (< 15%), Marsal’s (1967) test data defined a lower bound of \( \sigma_1/\sigma_3 \) against breakage (Figure 2.30). Miura and O-hara (1979) defined the ratio of surface area increment to the plastic work increment \( (dS/dW) \) as the particle crushing rate and reported that the principal stress ratio at failure decreases linearly with increasing particle crushing rate at failure \( (dS/dW)_f \), as shown in Figure 2.31.

![Figure 2.31. Effect of particle crushing rate on principal stress ratio at failure (after Miura and O’hara, 1979)](image)

Indraratna et al. (1998) presented a correlation between the particle breakage index, principal stress ratio and peak friction angle of railway ballast, as shown in Figure 2.32. They indicated that both the peak principal stress ratio and peak friction angle of ballast decreased as the breakage index increased at higher confining pressure.
In the past, several investigators studied the response of ballast and other granular materials under both monotonic and cyclic loadings. Considering the coarse granular nature of ballast, its response to loading is expected to be analogous to other granular media (e.g. rockfills, and even coarse sands). In the following Sections, the mechanical behaviour of ballast in terms of strength, settlement, and particle breakage under monotonic and cyclic loadings is compared with other granular materials.
2.5.1 Monotonic Loading

Before studying the mechanical response of ballast under complex cyclic train loading, it would be prudent to investigate its behaviour under a simple monotonic loading. Once one understands the mechanisms of ballast deformation and degradation under monotonic loading, it would be easier to go through complex behaviour during cyclic loading, which is a combination of loading, unloading and reloading.

Study of ballast as a geotechnical material was initiated only a few decades ago, while research on the behaviour of sands started in the early era of geotechnical engineering (1940’s). Advance study on rockfills probably started in the late 1960’s. Most rockfills tested in the laboratory are similar in size of ballast, as also the shape and source (parent rock). Therefore, it is appropriate to compare ballast behaviour with rockfill.

One significant difference between the test condition of rockfill and ballast is the confining pressure. Since rockfill is used in dams and is usually under medium to high pressure, the mechanical behaviour of rockfill was previously studied at high confining pressures (2.5 - 4.5 MPa) (e.g. Marsal, 1967; Marachi et al., 1972). Subsequently, some researchers concentrated their study on rockfill at low to medium confining pressures (< 1 MPa), realising that normal stress on the critical failure surface of a rockfill dam would not be so high (Charles and Watts, 1980; Indraratna et al., 1993). In contrast, ballast on railway track is subjected to much less confining pressure. Raymond and Davies (1978) indicated that the lateral stress in ballast is unlikely to exceed 140 kPa.

Figures 2.33(a) and (b) show the comparison between the stress-strain and volume change behaviour of ballast and rockfill under monotonic loading. These figures
indicate that the stress-strain response of ballast and rockfill are comparable. However, at low confining pressure (< 100 kPa), ballast exhibits dilatant behaviour in triaxial compression (Raymond and Davies, 1978; Indraratna et al., 1998), whereas at higher confinement, both ballast and rockfill show overall contraction at failure (Raymond and Davies, 1978; Charles and Watts, 1980; Indraratna et al., 1993; Indraratna et al., 1998).

![Graph showing stress-strain-volume change behaviour for (a) latite ballast (after Indraratna et al., 1998), (b) greywacke rockfill (after Indraratna et al., 1993)]

Indraratna et al. (1998) also compared the peak principal stress ratio and shear strength envelopes of ballast and rockfills (Figure 2.34), and indicated that the peak principal stress ratio and the strength envelope of ballast follow the rockfill test results, which were reported by Marsal (1967; 1973), Marachi et al. (1972) and Charles and Watts (1980).
2.5.2 Cyclic Loading

Theyse (2000) reported the results of Heavy Vehicle Simulator (HVS) tests at 40 kN dual wheel load on unbound granular road pavement. The plastic strains on granular road base, subbase, and other layers observed in the HVS test (Theyse, 2000) were compared with the plastic settlement of ballast (Jeffs, 1989), as shown in Figures 2.35(a) and (b).

Figure 2.35 shows that the plastic deformation of railway ballast and road base aggregates under repetitive loads are very similar and comparable. Plastic settlements in both types of aggregates increased rapidly at the initial stage of cyclic loading (up to about 100,000 – 200,000 cycles), after that the aggregates became relatively stabilised and the settlement increased at a slower, almost constant rate with increasing number of load cycles.
Stewart et al. (1985) indicated that resilient modulus, as defined in Figure 2.36, is the appropriate elastic modulus for ballast and other granular aggregates subjected to repeated loading. Figure 2.37 shows a comparison between the resilient elastic modulus of ballast (Knutson and Marshall, 1977), and the dynamic shear modulus of remoulded sand (Assimaki et al., 2000). This figure shows that the modulus of both ballast and sand increases linearly with the first invariant of stresses (or mean confining pressure) in a log-log scale. Therefore, the resilient characteristics of ballast and other granular media are also comparable and depend on the applied pressure.
Figure 2.37. Elastic parameters under cyclic loading for (a) ballast (after Knutson and Marshall, 1977), (b) sand (after Assimaki et al., 2000)
2.6 TRACK MAINTENANCE

Rail tracks deform both vertically and laterally under repeated loads resulting from varying traffics and speeds, causing deviation from its design geometry. Although these deviations are apparently small, they are usually irregular in nature, deteriorate riding quality and increase dynamic loads, which in turn, further worsen the track geometry. In order to maintain the vertical and lateral alignment, riding quality and safety levels, rail tracks often need maintenance after their construction. Figure 2.38 shows a typical longitudinal track profile for a pre-maintenance and post-maintenance period.

Figure 2.38. Typical track longitudinal profile before and after maintenance (after Shenton, 1975)

Worldwide, track maintenance is a costly routine exercise, and a major portion of the maintenance budget is spent on geotechnical problems (Raymond et al., 1975; Shenton, 1975; Indraratna et al., 1998). Ballast is the only external constraint applied to the track for holding the running surface geometry (Shenton, 1975). Each year, hundreds of millions of dollars are spent on large terrains of rail track, particularly for ballast
maintenance, in different countries of the world, including the USA, Canada and Australia (Raymond et al., 1975; Indraratna et al., 1998). In the following Sections, the causes of track deterioration and the different techniques used for ballast maintenance are discussed.

2.6.1 Ballast Fouling

‘Fouling’ is the term used to indicate contamination of ballast by the presence of fines. Fouling of ballast over time is the primary reason why track geometry deteriorates. As the ballast is fouled, its performance decreases, resulting in higher settlement and poor drainage. It is expected that fresh and clean ballast will be placed in a track with fouling components not exceeding 2% by weight. Typically, porosity in ballast is in the order of 35-50%, so, fouling probably does not start to become significant until the fines increase to 10% or more (Selig, 1984). In order to quantify the degree of fouling numerically, Selig and Waters (1994) proposed ‘Fouling Index’, \( F_I \), defined as:

\[
F_I = P_4 + P_{200}
\]  

(2.17)

where, \( P_4 \) = percentage passing 4.75 mm (No. 4) sieve, and

\( P_{200} \) = percentage passing 0.075 mm (No. 200) sieve.

Selig and Waters (1994) classified the degree of fouling based on the value of fouling index (Equation 2.17), as shown in Table 2.4.
Table 2.4: Classification of ballast fouling (after Selig and Waters, 1994)

<table>
<thead>
<tr>
<th>Ballast Category</th>
<th>Fouling Index, $F_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Moderately clean</td>
<td>1 to &lt; 10</td>
</tr>
<tr>
<td>Moderately fouled</td>
<td>10 to &lt; 20</td>
</tr>
<tr>
<td>Fouled</td>
<td>20 to &lt; 40</td>
</tr>
<tr>
<td>Highly fouled</td>
<td>≥ 40</td>
</tr>
</tbody>
</table>

There are several sources of ballast fouling which may be classified into 5 categories (Selig and Waters, 1994):

(i) Ballast breakdown (particle breakage)

(ii) Subgrade infiltration

(iii) Infiltration from underlying granular layers (subbase)

(iv) Sleeper (tie) wear

(v) Infiltration from ballast surface.

Detailed sources of ballast fouling are given in Table 2.5. Selig and Waters (1994) summarised their field and laboratory investigations with the type and sources of ballast fouling in North America, and concluded that ballast breakdown or particle breakage is the most significant source of fouling (Figure 2.39).

Figure 2.39. Sources of ballast fouling in North America (after Selig and Waters, 1994)
Table 2.5: Sources of ballast fouling (after Selig and Waters, 1994)

<table>
<thead>
<tr>
<th>Category No.</th>
<th>Description</th>
<th>Reasons of Fouling</th>
</tr>
</thead>
</table>
| (i)          | Ballast breakdown                    | a) Rail traffics • Cyclic loadings  
                              | b) Vibration  
                              | c) Hydraulic action of subgrade slurry  
                              | b) Compaction machines  
                              | c) Handling  
                              | d) Tamping operations  
                              | e) Freezing and water in voids  
                              | f) Chemical weathering  
                              | g) Thermal stresses |
| (ii)         | Subgrade infiltration                | a) Poor subbase (filter) layer  
                              | b) Insufficient drainage  
                              | c) Saturation (presence of water)  
                              | d) Pumping action by cyclic loads |
| (iii)        | Infiltration from underlying granular layers | a) Old track bed breakdown  
                              | b) Migration of subballast particles due to poor filtering (inadequate gradation) |
| (iv)         | Sleeper (tie) wear                   | a) Attrition between sleeper and ballast due to lateral ballast deformation |
| (v)          | Infiltration from ballast surface    | a) Delivered with ballast  
                              | b) Dropped from passenger and freight trains  
                              | c) Wind blown  
                              | d) Water borne  
                              | e) Splashing from adjacent wet spots  
                              | f) Meteoric dirt |

2.6.2 Track Maintenance Techniques

2.6.2.1 Ballast tamping

Ballast tamping is routinely used all over the world to correct the track geometry. Tamping consists of lifting the track and laterally squeezing the ballast beneath the sleeper to fill the void spaces generated by the lifting operation, as shown in Figure 2.40. The sleepers thus retain their elevated positions.

Ballast tamping is an effective process for re-adjusting the track geometry. However, some detrimental effects, such as ballast damage, loosening of ballast bed and reduced
track resistance to lateral displacement and buckling, accompany it. Loosening of ballast by the tamping process causes high settlement in track. Tamping is eventually needed again over a shorter period of time (Figure 2.40c), and in the long run, ballast gradually becomes contaminated (fouled) by fines, which impairs drainage and its ability to hold the track geometry. Eventually fouled ballast will need to be replaced, or cleaned and re-used in track (Selig and Waters, 1994).

Figure 2.40. Ballast tamping, (a) and (b) sequence of operations, and (c) shortening of tamping cycles (after Selig and Waters, 1994)
2.6.2.2 Stoneblowing

‘Stoneblowing’ is a new mechanised method of reinstating railway track to its desired line and level (Anderson et al., 2001; Key, 1998). Before the mechanised tamping, track had been re-levelled by ‘hand shovel packing’, where the sleepers were raised and fine aggregates were shoveled into the void with minimum disturbance to the well-compacted ballast. The mechanised version of this process is known as ‘pneumatic ballast injection’ or ‘stoneblowing’ (Anderson et al., 2001). The stoneblowing machine lifts the sleeper and blows a predetermined amount of small single size stones into the void beneath the sleeper to create a two layer granular foundation for each sleeper. Figure 2.41 shows schematic operational steps of ballast maintenance by stoneblowing.

Figure 2.41. Schematic of stoneblowing operation (after Anderson et al., 2001)

Figure 2.42. Improvement of vertical track profile after stoneblowing (after Anderson et al., 2001)
Anderson et al. (2001) reported field track data measured in the UK before and after stoneblowing, and concluded that this technique improves the track profile significantly (Figure 2.42). Before stoneblowing the track was deteriorating over time, but afterwards the track quality (standard deviation in Figure 2.42) not only improved, it was also maintained for the measured period of time (9 months).

2.6.2.3 Ballast cleaning and ballast renewal

As mentioned earlier, when ballast gets excessively fouled (beyond a threshold value) its function is impaired even after using other maintenance techniques (e.g. tamping or stoneblowing). In that case, the contaminated ballast must be cleaned or replaced by fresh ballast. Ballast cleaning and renewal process is a costly and time consuming exercise. It also disrupts traffic flow, and therefore, is not frequently undertaken. Deciding which remedial measure would be appropriate to undertake depends on site examination and in-situ track investigation of foundation materials, including subgrade. Traditionally, investigation of foundation is carried out from a series of cross track trenches (Selig and Waters, 1994). However, sinking bore holes (Figure 2.43) will provide further information regarding its foundation condition.

Cleaning the fouled ballast is usually carried out by a track-mounted cleaner, as shown in Figure 2.44. The cleaner digs away the ballast below the sleepers by a chain with ‘excavating teeth’ attached, conveys it up to vibrating screens, which separate the dirts (fines) from the aggregates. The dirt is conveyed away to lineside or to spoil wagons for disposal and the cleaned ballast is returned for re-use.
Ideally, the ballast cleaner separates the fines from fouled ballast to provide a uniform depth of compacted and clean ballast resting on the geometrically smooth cut surface of a compacted subballast layer (Selig and Waters, 1994). However, past experience indicates that the cutter bar is not able to cut the geometrically smooth surface required for the compacted subballast layer, due to mechanical vibrations and operator dependent cutting depths.
When ballast is excessively dirty, it may need to be totally removed rather than on-track cleaning, and replaced with fresh ballast. In these instances, the cleaner cuts the ballast and conveys it into wagons. After removing fouled ballast, the conveyor/hopper wagons are moved to a discharge side for stockpiling and/or recycling. Figure 2.45 shows a typical large stockpile of waste ballast at a Sydney suburb (Chullora).

![Figure 2.45. Stockpiles of waste ballast at Chullora (Sydney)](image)

To minimise further quarrying for fresh ballast and protect the environment, and most importantly, to minimise the track construction and maintenance cost, discarded waste ballast may be cleaned and recycled to the track. The performance of recycled ballast requires detailed investigation, both in the laboratory and on track. Use of geosynthetics may improve the performance of track, and these issues are discussed in the following Sections.
2.7 APPLICATION OF GEOSYNTHETICS IN RAIL TRACK

2.7.1 Geosynthetics: Definition, Types and Functions

Geosynthetics is the collective term applied to thin, flexible sheets manufactured from synthetic materials (e.g. polyethylene, polypropylene, polyester etc.), which are used in conjunction with soils and aggregates to enhance its engineering characteristics (e.g. strength, hydraulic conductivity, filtration, separation etc.). Geosynthetics may be classified into two major groups: (a) geotextiles, and (b) geomembranes (Ingold, 1994). Geotextiles are basically textile fabrics, which are permeable to fluids (water and gas). There are some other synthetic products closely allied to geotextiles such as geogrids, geomeshes, geonets and geomats, which have all been used in geotechnical practice. All geotextiles and related products are permeable to fluids, whereas geomembranes are substantially impermeable to fluids and are primarily used for retention purposes. Figure 2.46 shows common types of geosynthetics used in geotechnical engineering. The functions of different types of geosynthetics are given in Table 2.6.

Figure 2.46. Types of geosynthetics, (a) woven geotextile, (b) non-woven geotextile, (c) geogrid, (d) geonet, (e) geomesh, (f) geomat, (g) geocell, (f) geocomposite (after Ingold, 1994)
Table 2.6: Functions of geosynthetics

<table>
<thead>
<tr>
<th>Type of Geosynthetic</th>
<th>Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geotextiles</td>
<td>Reinforcement</td>
</tr>
<tr>
<td></td>
<td>Filtration</td>
</tr>
<tr>
<td></td>
<td>Separation</td>
</tr>
<tr>
<td></td>
<td>Transmission of fluids</td>
</tr>
<tr>
<td>Woven</td>
<td></td>
</tr>
<tr>
<td>Non-woven</td>
<td></td>
</tr>
<tr>
<td>Geogrids</td>
<td>Reinforcement</td>
</tr>
<tr>
<td>Geomesh</td>
<td>Reinforcement</td>
</tr>
<tr>
<td></td>
<td>Filtration</td>
</tr>
<tr>
<td>Geonets</td>
<td>Transmission of fluids</td>
</tr>
<tr>
<td>Geomats</td>
<td>Reinforcement</td>
</tr>
<tr>
<td></td>
<td>Confinement</td>
</tr>
<tr>
<td>Geocells</td>
<td>Reinforcement</td>
</tr>
<tr>
<td></td>
<td>Confinement</td>
</tr>
<tr>
<td>Geocomposite</td>
<td>Reinforcement</td>
</tr>
<tr>
<td></td>
<td>Separation</td>
</tr>
<tr>
<td></td>
<td>Filtration</td>
</tr>
<tr>
<td></td>
<td>Transmission of fluids</td>
</tr>
<tr>
<td>Geomembranes</td>
<td>Isolation</td>
</tr>
<tr>
<td></td>
<td>Separation</td>
</tr>
<tr>
<td></td>
<td>Reinforcement</td>
</tr>
</tbody>
</table>

2.7.2 Geosynthetics in Track

Taking their functions into account, different types of geosynthetics have been used in track depending on their cost and the engineering properties of the substructure materials. Geosynthetics usually minimise the vertical track deformation by reducing lateral movement (through transferring lateral loads from ballast to geosynthetics by shear), dissipating excess pore pressures developed under fast cyclic loading, and keeping ballast relatively clean through separation and filtering functions.

Geotextiles have been frequently used to maintain track substructure, especially in localised mud problem areas, such as (a) wet cuts, (b) soft subgrade, (c) road grade crossings, (d) railroad track crossings, and (e) turnouts (Selig and Waters, 1994). An example of geotextiles in track is shown in Figure 2.47.
Amsler (1986) reported a case study in Geneva regarding track performance with and without geosynthetics. The left track (Figure 2.48a) was completely renewed in 1982, using a traditional design cross-section (without any geosynthetics). In 1983, the right track (Figure 2.48b) was renewed following a new design cross-section incorporating non-woven geotextiles at the subbase/subgrade interface. Both tracks were monitored by a track-quality measuring wagon both before and after the rehabilitation. The cross slope difference (per millimetre) between two rails of a track (warp) as a function of traveled distance was used as an indicator of stability and riding comfort. Figure 2.48 shows the pre- and post-renewal monitored data (warp) of both tracks along with their corresponding design cross-sections. The smaller values of the measured data after installation of the geotextiles on the right track (Figure 2.48b) clearly shows the benefits of using geosynthetics in rail track.
Amsler (1986) concluded that using geotextiles in foundation improved the track quality significantly and the improvement was also maintained for a longer period. Other researchers found similar improvements in track stabilisation with the use of geosynthetics (Ashpiz et al., 2002; Selig and Waters, 1994). Track rejuvenation without geosynthetics, however improves performance for a short time and then deteriorates almost to the pre-renewal level within about 1-2 years (Amsler, 1986).

Atalar et al. (2001) studied the effects of geogrids on the settlement behaviour of track foundation in a large-scale model apparatus. Their test equipment and the settlement results are shown in Figure 2.49. The results revealed that subbase settlement decreased significantly when only one layer of geogrid and geotextile combination was included at
the subbase/subgrade interface. Settlement decreased further when additional layers of geogrid were placed inside and on top of the subbase (Figure 2.49b). Bathurst and Raymond (1987) reported a similar decrease in permanent settlement when geogrid was included at different elevations inside the ballast layer.

![Figure 2.49](image-url)  
(a) Use of geogrids in ballast bed, (a) test set-up, (b) settlement results (after Atalar et al., 2001)

Railway engineers often express their concerns about the durability of geosynthetics in the harsh track environment due to the close contact with sharp angular ballast and heavy cyclic traffic loading. Several investigators studied the durability of geotextiles in ballast bed environment, and most of them reported favourably. Selig and Waters (1994) found that even after 3 years of service in a British Rail site, the extracted geogrid and geotextiles were in good condition (Figure 2.50).
Ashpiz et al. (2002) investigated the durability of spunbonded geotextile used in St. Petersburg-Moscow line, and reported only 0.2% and 0.3% surface damage after 1 year and 5 years of service, respectively. The retained strength was found to be about 74% and 72% after 1 year and 5 years period, respectively. Nancey et al. (2002) reported similar findings regarding the durability of a thick geotextile tested at 50 Hz frequency eccentric wheel loads in the ‘vibrogir’ model device. After 200 hours of cyclic loading (equivalent to 730 MGT loading), they found that the flow capacity, permeability, and puncture resistance of the thick geotextile were almost unaffected by the simulated
traffic. Raymond and Bathurst (1990) however, reported evidence of particle penetration holes in geotextiles extracted from 175 mm depth of rail track (Figure 2.51).

Ashpiz et al. (2002) reported some contamination of geotextiles when extracted from a track after 5 years of service (Figure 2.52). A visual inspection revealed that the contamination was mainly due to fines generated by abrasion and breakage of upper ballast aggregates. On the basis of laboratory test results of uncleaned geotextile with ballast, they concluded that contamination of geotextiles had little influence on the drainage capacity of the ballast-geotextile system.

Figure 2.52. Extracted geotextile from a track after 5 years of service (after Ashpiz et al., 2002)
CHAPTER 3
THEORETICAL BACKGROUND OF BALLAST MODELLING

3.1 INTRODUCTION

Until today, the vast majority of the railway engineers have regarded ballast as an elastic media. Although the accumulation of plastic deformation under cyclic traffic loading is evident, most researchers are primarily interested in modelling the dynamic resilient modulus of ballast. Limited research is conducted on modelling plastic deformation of ballast associated with cyclic loading, although some researchers have attempted to simulate the plastic deformation empirically. Despite spending a huge annual sum for constructing and maintaining railway tracks, its design is still predominantly empirical in nature (Suiker, 2002). A large number of researchers have modelled elasto-plastic deformation of sand and other granular media under both monotonic and cyclic loadings. As ballast comprised of coarse aggregates, these elasto-plastic deformation models may be useful for developing a specific model to simulate ballast behaviour including plastic deformation and particle breakage under cyclic loading.

3.2 PLASTIC DEFORMATION OF BALLAST

Various researchers have empirically modelled the permanent deformation of ballast under cyclic loading. Shenton (1975) represented the average vertical strain of a ballast layer at any number of load cycles with the strain at the first cycle of loading and the logarithm of the number of load cycles, as given below:
\[ \varepsilon_N = \varepsilon_1 (1 + 0.2 \log_{10} N) \]  

(3.1)

where, \( \varepsilon_N \) = average vertical strain of a ballast layer at load cycle \( N \), \( \varepsilon_1 \) = average vertical strain at load cycle 1, and \( N \) = number of load cycles.

A similar logarithmic function of load cycles was presented by Indraratna et al. (2002a, 2002b) when modelling the plastic deformation of ballast with/without geosynthetic reinforcement. It is given by:

\[ S = a + b \log N \]  

(3.2)

where, \( S \) = ballast settlement, \( N \) = number of load cycles, and \( a \) and \( b \) are empirical constants.

Stewart (1986) conducted a series of variable amplitude cyclic triaxial tests on ballast and concluded that the predictions based on the superposition of ballast strains using an equation similar to Equation 3.1 for various stress magnitudes agree well with the experimental results.

Subsequently, Shenton (1984) presented an empirical model for ballast settlement based on extensive field data, as given below:

\[ S = K_1 N^{0.2} + K_2 N \]  

(3.3)

where, \( S \) is the settlement of ballast at the sleeper/ballast interface; \( K_1, K_2 \) are empirical constants, and \( N \) = total number of axles (or cycles). Shenton considered that the settlement of ballast composed of two parts: the first component \((K_1 N^{0.2})\) predominates up to 1 million load cycles, and the second part \((K_2 N)\) is only a small portion of the settlement and becomes relatively insignificant above 1 million load cycles.
Raymond and Bathurst (1994) correlated the track settlement to the logarithm of total tonnage based on the available field settlement data, as shown below:

$$ S_e(t) = a_r + a'_0 \log \left( \frac{t}{t_r} \right) $$

where, $S_e(t) =$ mean settlement of a ballast layer over unit length at tonnage $t$, $a_r =$ settlement at the reference tonnage, $a'_0 =$ slope of the semi-logarithmic relation, $t_r =$ reference tonnage taken as 2 million ton, and $t =$ total tonnage.

Chrismer and Selig (1993) modelled the average vertical strain of a ballast layer as a power function of the number of load cycles:

$$ \varepsilon_N = \varepsilon_1 N^b $$

where, $\varepsilon_N =$ average vertical strain of a ballast layer after $N$ load cycles, $\varepsilon_1 =$ the average vertical strain at first load cycle, $b =$ a constant, and $N =$ the number of load cycles. They concluded that the power equation represents ballast strain better than the logarithmic models.

Similarly, Indraratna et al. (2001) and Ionescu et al. (1998) reported that a power function best represented their ballast settlement data, as given by:

$$ S = S_1 N^b $$

where, $S =$ ballast settlement after $N$ number of load cycles, $S_1 =$ ballast settlement after first cycle of loading, $b =$ empirical constant, and $N =$ number of load cycle.

Recently, Suiker (2002) developed a plastic deformation model for ballast, where both plastic ‘frictional sliding’ and ‘volumetric compaction’ mechanisms have been considered during cyclic loading. He called it ‘Cyclic Densification Model’, where the
plastic flow rule has been decomposed into a frictional contribution and a compaction component, as given by:

\[
\frac{d\varepsilon_{ij}^p}{dN} = \frac{d\kappa^p}{dN} m^f_{ij} + \frac{d\varepsilon_{vol,c}^p}{dN} m^c_{ij}
\]  

(3.7)

where, \(d\varepsilon_{ij}^p\) is the infinitesimal increment of plastic strain, \(d\kappa^p\) is the increment of plastic distortional strain, \(d\varepsilon_{vol,c}^p\) is the plastic volumetric strain increment due to cyclic compaction, \(m^f_{ij}\) and \(m^c_{ij}\) denote the flow directions for frictional sliding and volumetric compaction, respectively, and \(dN\) is the increment of load cycle.

Suiker (2002) divided the stress domain into four regimes:

(i) The shakedown regime where the cyclic response of ballast is fully elastic,
(ii) The cyclic densification regime where progressive plastic deformation occurs under cyclic loading,
(iii) The frictional failure regime where frictional collapse occurs due to cyclic stress level exceeding the static maximum strength, and
(iv) Tensile failure regime where non-cohesive granular materials disintegrate due to induced tensile stresses.

Figure 3.1. Four response regimes during cyclic loading (after Suiker, 2002)
These regimes are shown in Figure 3.1 in the $p$-$q$ plane, where, $p$ and $q$ are the mean effective normal stress and deviator stress (invariants), respectively.

Suiker’s (2002) cyclic densification model is an advanced step in modelling plastic deformation and plastic compaction of ballast under cyclic loading. However, particle breakage associated with cyclic loading, an important factor governing the plastic deformation and cyclic compaction of ballast, is not considered in Suiker’s (2002) model. Therefore, a new constitutive model has been developed in this study (Chapter 6), which will demonstrate the relevance of particle breakage in the plastic deformation of ballast.

3.3 OTHER PLASTIC DEFORMATION MODELS

There are a number of other plasticity models available in the literature and are relevant to the plastic deformation of ballast. These were initially developed to analyse the plastic deformation of clays, sands and gravels. However, because sands and gravels behave similar to ballast, these plasticity models are helpful in developing a model for simulating the deformation and degradation of ballast under cyclic loading.

3.3.1 Critical State Model

In the late 1950’s and 1960’s, Roscoe and his co-researchers developed a critical state model based on the theory of plasticity and soil behaviour at the critical states (Roscoe et al., 1958; 1963; Roscoe and Burland, 1968; Schofield and Wroth, 1968). Roscoe and his co-researchers were the first to successfully model plastic shear deformation and the associated volume change of soils during shearing. The mathematical model developed for the plastic deformation of clay is known as ‘Cam-clay’ (Roscoe et al., 1963;
Schofield and Wroth, 1968), and later known as the ‘modified Cam-clay’ (Roscoe and Burland, 1968).

The ‘critical state’ has been defined as the state at which soil continues to deform at constant stress and constant void ratio (Roscoe et al., 1958). The main features of the critical state model are:

a) All the possible states of a soil form a stable state boundary surface (SSBS), as shown in Figure 3.2(a).

b) Deformation of soil remains elastic until its stress state reaches the stable state boundary surface, i.e. yielding of soil initiates when the stress path meets the SSBS.

c) At the critical state, all the energy transmitted to a soil element across its boundary is dissipated within the soil element as frictional heat loss, without changing its stress or volume. Thus, at the critical state, $q = Mp$ (Figure 3.2b), where, $M$ is the coefficient of friction at the critical state.

d) The projection of the critical state line (CSL) on $e-p$ plane is parallel to the Normal Compression Line (NCL) obtained under isotropic compression (Figure 3.2c). The NCL and the projection of CSL become parallel straight lines when plotted in a semi-logarithmic $e-lnp$ scale (Figure 3.2d). The swelling and recompression are also assumed to be linear in $e-lnp$ plane.
Been et al. (1991) studied the critical state/steady state of sands and concluded that the critical state line is approximately bilinear in the $e$-$\log p$ plane, as shown in Figure 3.3. They found an abrupt change in the slope of the critical state line for Leighton Buzzard sand and Erksak sand at about 1 MPa. They attributed this sudden change in slope of the critical state line to be caused by grain breakage.

![Critical state model](image)

Figure 3.2. Critical state model, (a) state boundary surface, (b) projection of CSL in $q$-$p$ plane, (c) projection of CSL and NCL on $e$-$p$ plane, and (d) CSL and NCL plotted in $e$-$\ln p$ plane

![Bi-linear critical state line](image)

Figure 3.3. Bi-linear critical state line of sands (after Been et al., 1991)
Although the original critical state model (Roscoe et al., 1958; 1963) was based on extensive laboratory tests results of remoulded clay, some researchers attempted to model the deformation behaviour of sands and gravels similar to the critical state (Cam-clay) model. Schofield and Wroth (1968) presented a critical state model for gravels (Granta-gravel) neglecting the elastic component of the volumetric strain.

Jefferies (1993) stated that the Cambridge-type models (e.g. Granta-gravel) could not reproduce softening and dilatancy of sands, which are on the dense (dry) side of the critical state line. He pointed out that the inability of Cambridge-models to dilate is a large deficiency in modelling sand behaviour, as virtually all sands are practically denser than the critical and dilate during shearing. He proposed a critical state model for sand (Nor-Sand) assuming associated flow (normality) and infinity of the isotropic normal compression line (NCL). The initial density of sand was incorporated through the state parameter $\psi$, as defined by Been and Jefferies (1985). Jefferies (1993) employed the following dilatancy rule:

$$ D = \frac{M - \eta}{1 - N} $$

(3.8)

where, $D = \frac{\Delta \varepsilon_p}{\Delta \varepsilon_q}$ is a dilatancy function, $\varepsilon_p$ and $\varepsilon_q$ are strains corresponding to the stresses $p$ and $q$, a dot superscript represents incremental change, $M$ is the critical state friction coefficient, $\eta$ is shear stress ratio ($= q/p$) and $N$ is a density dependent material property.

Using Equation (3.8) and the normality condition, Jefferies (1993) formulated the yield surface for Nor-sand, as given by:
\[ \eta = \frac{M}{N} \left[ 1 + (N-1) \left( \frac{p}{p_i} \right)^{N/(1-N)} \right] \quad \text{if } N \neq 0 \quad (3.9a) \]

\[ \eta = M \left[ 1 + \ln \left( \frac{p}{p_i} \right) \right] \quad \text{if } N = 0 \quad (3.9b) \]

where, \( p_i \) is the mean stress at the image state defined by the condition \( \mathcal{G}_p = 0 \). A simple hardening rule was used by Jefferies (1993), as given below:

\[ \frac{\mathcal{G}_p}{\mathcal{G}_\eta} = h (p_{i,max} - p_i) \quad (3.10) \]

where, \( h \) is a proportionality constant and \( p_{i,max} \) is the maximum value of \( p_i \).

The Nor-sand (Jefferies, 1993) adequately modelled the behaviour of sand including dilatancy, post-peak strain softening, the effects of confining pressure and initial density. However, researchers question the assumption of normality (associated flow) in sand, and therefore, most other researchers used non-associated flow in their formulations (Lade, 1977; Pender, 1978; Mroz and Norris, 1982; Daffalias and Herrmann, 1980; 1982).

### 3.3.2 Elasto-plastic Constitutive Models

Lade (1977) presented an elasto-plastic constitutive model for cohesionless soils with curved yield surfaces based on the theory of plasticity, non-associated flow rule, and an empirical work-hardening law. He assumed that the total strain increments \( d\varepsilon_{ij} \), are composed of three components, (a) elastic increments \( d\varepsilon_{ij}^e \), (b) plastic collapse components \( d\varepsilon_{ij}^{pc} \), and (c) plastic expansive increments \( d\varepsilon_{ij}^{pe} \), such that:

\[ d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^{pc} + d\varepsilon_{ij}^{pe} \quad (3.11) \]
The elastic strain increments were computed from pressure dependent unloading-reloading elastic modulus, as given by:

\[ E_{ur} = K_{ur} p_a \left( \frac{\sigma_3}{p_a} \right)^n \]  

(3.12)

where, \( E_{ur} \) = unloading-reloading elastic modulus, \( K_{ur} \) = dimensionless modulus number (constant), \( p_a \) = atmospheric pressure, \( \sigma_3 \) = confining pressure and \( n \) is an exponent.

Lade (1977) expressed various yield surfaces and plastic potentials as functions of the stress invariants. He used identical formulation for the yield function and plastic potential in modelling the plastic collapse component of strain, and is given by:

\[ f_c = g_c = I_1^2 + 2I_2 \]  

(3.13)

where, \( f_c \) is the yield surface, \( g_c \) is the plastic potential, the subscript \( c \) indicates plastic collapse, and \( I_1 \) and \( I_2 \) are the 1st and 2nd invariants of stresses, respectively. In modelling the plastic expansive strain component, Lade employed two different functions for the yield surface and the plastic potential (i.e. non-associated flow), as given by:

\[ f_p = \left( I_3 / I_3 - 27 \right) \left( I_3 / p_a \right)^m \]  

(3.14a)

\[ g_p = I_1^3 - \left[ 27 + \eta_2 \left( p_a / I_1 \right)^m \right] I_3 \]  

(3.14b)

where, \( I_3 \) is the third invariant of stresses, \( \eta_2 \) is a constant for the given values of \( f_p \) and \( \sigma_3 \), and \( m \) is an exponent.

Lade (1977) also employed an isotropic work-hardening and softening law, as given by:

\[ W_p = F_p \left( f_p \right) \]  

(3.15)

where, \( W_p \) = plastic work done and \( F_p \) is a monotonically increasing or decreasing
positive function. The behaviour of cohesionless soils including dilatancy, strain-hardening and post-peak strain-softening, was predicted very well by Lade’s model. However, the capability of Lade’s model to predict shear behaviour from an anisotropic initial stress state was neither verified nor discussed. His model was verified only for shearing from isotropic initial stress state. In employing a stress-strain constitutive model to cyclic loading, where stresses are often changing from non-isotropic stress states, the model must be capable of predicting shear behaviour from both isotropic and anisotropic initial stress states.

Pender (1978) successfully overcame the limitations of Lade’s (1977) formulation and developed a constitutive model for the shear behaviour of overconsolidated soils based on the critical state framework, non-associated flow, and the theory of plasticity. He assumed constant stress ratio yield loci and parabolic undrained stress paths, as given by:

\[f = q - \eta_j p = 0\] (3.16)

\[
\left(\frac{\eta - \eta_0}{A \eta - \eta_0}\right)^2 = \frac{p_{cs}}{p} \left[1 - \frac{P_o}{p}\right] \left[1 - \frac{P_o}{p_{cs}}\right] \] (3.17)

where, \(f\) = yield function,

\(\eta\) = a given stress ratio (= \(q/p\)),

\(\eta_0\) is the initial stress ratio,

A is +1 for loading towards the critical state in compression, and –1 for extension,

\(p_{cs}\) is the value of \(p\) on the critical state line corresponding to the current void ratio,

\(p_o\) is the intercept of the undrained stress path with the initial stress ratio line, and
$M$ is the slope of the critical state line in $p$-$q$ plane.

Pender (1978) assumed the ratio of plastic distortional strain increment ($d\varepsilon_p^p$) to plastic volumetric strain increment ($d\varepsilon_v^p$), as given by:

$$
\frac{d\varepsilon_p^p}{d\varepsilon_v^p} = \frac{(AM - \eta_o)^2}{(AM)^2 \left( \frac{p_o}{p_{cs}} - 1 \right) \left( (AM - \eta_o) - (\eta - \eta_o) \frac{p}{p_{cs}} \right)}
$$

The general constitutive relationship for incremental plastic strain is given by Hill (1950) as:

$$
d\varepsilon_{ij}^p = h \frac{\partial g}{\partial \sigma_{ij}} df
$$

Combining Equations (3.16-3.19), Pender (1978) formulated the following expression for incremental plastic strains:

$$
d\varepsilon_s^p = \frac{2\kappa \left( \frac{p}{p_{cs}} \right) (\eta - \eta_o) d\eta}{(AM)^2 (1 + e) \left( \frac{2p_o}{p} - 1 \right) \left( (AM - \eta_o) - (\eta - \eta_o) \frac{p}{p_{cs}} \right)}
\quad (3.20)
$$

$$
d\varepsilon_v^p = \frac{2\kappa \left( \frac{p_o}{p_{cs}} - 1 \right) \left( \frac{p}{p_{cs}} \right) (\eta - \eta_o) d\eta}{(AM - \eta_o)^2 (1 + e) \left( \frac{2p_o}{p} - 1 \right)}
\quad (3.21)
$$

where, $\kappa$ is the slope of the swelling/recompression line in $e$-$\ln p$ plot.

Pender’s (1978) model was able to predict non-linear stress-strain behaviour, dilatancy, strain-hardening and post-peak strain-softening aspects of overconsolidated soils during shearing. His model can also be applied to shearing from an initial stress of both isotropic and anisotropic states, which is essential for modelling deformation behaviour.
under cyclic loading. In this study, ballast behaviour under monotonic loading has been modelled following Pender’s (1978) modelling technique, along with a new plastic dilatancy rule incorporating particle breakage. This is presented in detail in Chapter 6.

Later on, Pender (1982) introduced a cyclic hardening parameter to capture the cyclic stress-strain behaviour of soils and extended his previous formulation, as given by:

\[
d e_s^p = \frac{2\kappa \left( \frac{p}{P_{cs}} \right) (\eta - \eta_0)^{\xi} d\eta}{(AM)^2 (1 + e) \left( \frac{2P_{o}}{P} - 1 \right) (AM - \eta_0) \left( \frac{p}{P_{cs}} - (\eta - \eta_0) \right)}
\]

\[
\xi = \left\{ \frac{q_p}{P_{cs}} \right\}^{\hat{\alpha}} \left( H^{\hat{\beta}} - 1 \right)
\]

where, \( \xi \) is the cyclic hardening index, 
\( q_p \) is the change in \( q \) in the previous half cycle, 
\( H \) is the number of half cycles, and 
\( \hat{\alpha}, \hat{\beta} \) are soil parameters for cyclic hardening.

Pender (1982) considered that the value of the cyclic hardening index (\( \xi \)) would increase with an increase in number of half cycles and approached the hardening index (\( \xi \)) in an empirical way. He did not relate cyclic hardening index with cyclic compaction (densification), which is observed in cyclic tests. Ballast usually hardens under cyclic loading due to plastic volumetric compaction (Suiker, 2002), which is absent in Pender’s (1982) cyclic deformation model.

Tatsuoka et al. (2003) presented a cyclic stress-strain model for sand in plane strain loading. They have expressed the stress-strain relationship of plane strain compression
and plane strain extension in terms of an empirical hyperbolic equation, as given by:

\[ y = \frac{x}{1 + \frac{x}{C_1} + \frac{x}{C_2}} \]  

(3.24)

where, \( y = \frac{\tau}{\tau_{\text{max}}} \)

\( x = \frac{\varepsilon}{\varepsilon_{\text{ref}}} \)

\( \tau = \sigma_{\text{vertical}} - \sigma_{\text{horizontal}} = \text{shear stress}, \quad \tau_{\text{max}} = \text{maximum shear stress}, \)

\( \varepsilon_{\sigma} = \varepsilon_{\text{vertical}} - \varepsilon_{\text{horizontal}} = \text{shear strain}, \quad \varepsilon_{\text{pref}} = \text{reference shear strain}, \) and

\( C_1 \) and \( C_2 \) are the fitting parameters, which also depend on the strain level, \( x \).

Tatsuoka et al. (2003) described a set of rules (e.g. proportional rule, external and internal rules, drag rule etc.) to simulate the hysteretic stress-strain relationship of sand under cyclic loading. They proposed a drag parameter, which is a function of plastic shear strain. The drag parameter was employed in the model to simulate the evolution of the stress-strain hysteretic loop as the number of cycle increases. Tatsuoka et al. (2003) used the following equations to model plastic dilatancy in plane strain cyclic loading:

\[ d = \frac{s(1 + 1/K') + (1 - 1/K')}{s(1 - 1/K') + (1 + 1/K')} \]  

for loading  

(3.25a)

\[ d = \frac{s(1 + 1/K') - (1 - 1/K')}{s(1 - 1/K') + (1 + 1/K')} \]  

for unloading  

(3.25b)

where, \( d = -d\varepsilon_{\text{vol}}/d\varepsilon_{\sigma} \)

\( s = \sin \phi_{\text{mob}} \)

\( K' = \text{model constant} \)

\( \phi_{\text{mob}} = \text{mobilised friction angle} \).
Although their model was based on empirical formulation, Tatsuoka et al. (2003) successfully simulated the stress-strain and volume change behaviour of sand under plane strain cyclic loading, as shown in Figure 3.4. One limitation of their model is that the hyperbolic stress-strain formulation (Equation 3.24) is independent of the plastic volumetric strain resulting from the dilatancy equations (Equations 3.25a and 3.25b), while many other researchers indicate that volumetric strain significantly affects the stress-strain behaviour of soils and granular aggregates (Roscoe et al., 1963; Schofield and Wroth, 1968; Indraratna et al., 1998).

Figure 3.4. Model simulation of sand under plane strain cyclic loading, (a) stress-strain, and (b) volume change behaviour (after Tatsuoka et al., 2003)
3.3.3 Bounding Surface Plasticity Models

To realistically model the stress-strain behaviour of soils under cyclic loading, some researchers introduced the concept of bounding surface plasticity in their formulations (Dafalias and Herrmann, 1980; 1982; Mroz and Norris, 1982). The simple elasto-plastic or non-linear elastic models may be used for soils under monotonic loading to simulate deformation with sufficient accuracy. However, for complex loading systems involving loading, unloading and repetitive actions of loads, more complex hardening rules should be examined to simulate cyclic deformation behaviour more realistically (Mroz and Norris, 1982).

Figure 3.5. Schematic illustration of bounding surface (after Dafalias and Herrmann, 1982)

The ‘bounding surface’ concept was originally introduced by Dafalias and Popov (1975, 1976), and simultaneously and independently by Krieg (1975), in conjunction with an enclosed yield surface for metal plasticity. Both name and concept were inspired from observing that stress-strain curves converge to specific ‘bounds’ at a rate, which depends on the distance of the stress point from the bounds. Dafalias and Herrmann (1980) presented two different direct bounding surface formulations within the framework of critical state soil plasticity for the quasi-elastic range in triaxial stress space. Dafalias and Herrmann (1982) subsequently extended their previous
formulations and presented a generalised bounding surface plasticity model in a three-dimensional stress space in terms of stress invariants. Figure 3.5 shows the schematic representation of the bounding surface.

Mroz and Norris (1982) examined the qualitative response of a two surface plasticity model and a model with infinite number of loading surfaces under cyclic loading, and then developed their formulations in triaxial stress space. The general expression of the plastic strain increment vector of their model is given by:

\[
\dot{\varepsilon}^p = \frac{1}{K} n_g \left( n_f^T \sigma \right)
\]  \hspace{1cm} (3.26)

where, \( \dot{\varepsilon}^p \) is the plastic strain increment vector, \( \sigma \) is the stress increment vector, \( n_g \) and \( n_f \) are the unit vectors normal to the plastic potential surface and yield surface, respectively, and \( K \) is a scalar hardening modulus.

Mroz and Norris (1982) considered that the hardening modulus \( K \), (Equation 3.26) evolves from an initial value on yield surface \( K_y \), at point P (Figure 3.6) to a bounding value \( K_R \), at point R on the consolidation surface. The point R on the consolidation
surface is a conjugate point of \( P \) such that the direction of the unit vector normal to the yield surface at point \( P \) is the same as the direction of unit vector normal to the consolidation surface at point \( R \). The evolution of modulus \( K \), depends on the distance between the current stress point \( P \) and its conjugate point \( R \). The maximum distance between the yield and consolidation surfaces is given by:

\[
K = K_R + \left( K_y - K_R \right) \left( \frac{\delta}{\delta_0} \right)^\gamma \tag{3.27}
\]

\[
\delta = f' \left( \sigma^*_{R} - \sigma^*_{P} \right)^{1/2} \tag{3.28}
\]

\[
\delta_0 = 2(a_e - a_0) \tag{3.29}
\]

where, \( a_c \) and \( a_0 \) are the semidiameters of the consolidation and yield surfaces, respectively (Figure 3.6), and \( \gamma \) is a constant parameter. Mroz and Norris (1982) indicated that the value of \( \delta_0 \) changes only slightly due to change in density, while \( \delta \) changes with the changes in stress and depends on the instantaneous positions of the yield and consolidation surfaces.

For the plastic model with infinite number of loading surfaces, Mroz and Norris (1982) employed a plastic hardening modulus \( K \), almost similar to Equation (3.27), as given by:

\[
K = K_R + \left( K_y - K_R \right) \left( R_1 \right)^\gamma \tag{3.30}
\]

\[
R_1 = \frac{a_e - a_{l1}}{a_e} \tag{3.31}
\]

where, \( a_{l1} \) is the semidiameter of the first loading surface, \( f_{l1} = 0 \) (Figure 3.7).
Although Mroz and Norris (1982) had not quantitatively modelled any particular soil, the qualitative aspects of soil behaviour under cyclic loading were well predicted (Figure 3.8). In this study, the concept of varying hardening modulus within the bounding surface (Mroz and Norris, 1982) has been used to model the deformation behaviour of ballast under cyclic loading, and is discussed in detail in Chapter 6.

### 3.4 MODELLING OF PARTICLE BREAKAGE

Many researchers have recognised that particle breakage during a stress change in granular geomaterial affects its deformation behaviour significantly (Marsal, 1967;
Limited number of researchers focused on modelling the breakage of particles during shear deformation. Some investigators attempted to quantify the degree of particle breakage, while others correlated the measured breakage indicator with various engineering properties of ballast and other granular aggregates.

McDowell et al. (1996) and McDowell and Bolton (1998) developed a conceptual and analytical model based on the probability of fracture for the evolution of particle size in granular medium under one-dimensional compression. They considered that the probability of particle fracture is a function of applied stress, particle size and coordination number (number of contacts with neighbouring particles), and postulated that plastic hardening is due to an increase in specific surface, which must accompany irrecoverable compression caused by particle breakage. McDowell and co-researchers indicated that when particles fracture, the smallest particles are geometrically self-similar in configurations under increasing stress (Figure 3.9), and that a fractal geometry evolves with successive fracture of the smallest grains.

Figure 3.9. Crushing of a triangular particle into two geometrically similar pieces (after McDowell et al., 1996)
McDowell et al. (1996) and McDowell and Bolton (1998) also added a fracture energy term to the well-known Cam-clay plastic work equation (Roscoe et al. 1963; Schofield and Wroth, 1968), as given by:

\[ q\delta\varepsilon_q^p + p'\delta\varepsilon_v^p = M\delta\varepsilon_q^p + \frac{\Gamma_s dS}{V_s(1+e)} \]  (3.32)

where, \( \delta\varepsilon_q^p \) is the increment of plastic shear strain, \( \delta\varepsilon_v^p \) is the increment of plastic volumetric strain, \( dS \) is the increase in surface area of volume \( V_s \) of solids distributed in a gross volume of \( V_s(1+e) \), \( e \) is the void ratio and \( \Gamma_s \) is the ‘surface free-energy’.

Although McDowell and co-researchers added this surface energy term to the plastic work equation during shear deformation (Equation 3.32), they did not examine the applicability of their formulation nor verify the equation for shearing with available test data. They restricted their study to the volume change behaviour of aggregates caused by particle breakage in one-dimensional compression.

Ueng and Chen (2000) studied the effects of particle breakage on the shear behaviour of sands, and formulated a useful relationship between the principal stress ratio, rate of dilation, angle of internal friction and the energy consumption due to particle breakage per unit volume during triaxial shearing. Their formulation is given by:

\[ \frac{\sigma'_1}{\sigma'_3} = \left(1 + \frac{d\varepsilon_v}{d\varepsilon_1}\right)\tan^2\left(45^\circ + \frac{\phi_f}{2}\right) + \frac{dE_B}{\sigma'_3 d\varepsilon_1} \left(1 + \sin \phi_f\right) \]  (3.33)

where, \( \sigma'_1 \) is the major principal stress, \( \sigma'_3 \) is minor principal stress, \( d\varepsilon_v \) is volumetric strain increment, \( d\varepsilon_1 \) is the major principal strain increment, \( \phi_f \) is the angle of internal friction and \( dE_B \) is the increment of energy consumption per unit volume caused by particle breakage during shearing.
Ueng and Chen (2000) used the increase in specific surface area per unit volume ($dS_v$) as the indicator of particle breakage, and correlated the rate of energy consumption due to particle breakage at failure ($dE_B/d\varepsilon_1$), with the rate of increase in surface area at failure ($dS_v/d\varepsilon_1$), as given by:

\[ dE_B = kdS_v \]  

(3.34)

where, $k$ is a proportionality constant.

Ueng and Chen’s (2000) formulation is a significant development in modelling particle breakage during shearing. However, its application is limited to the strength of geomaterials in terms of principal stress ratio in triaxial shearing. It cannot be used directly to predict plastic deformations of ballast under monotonic and cyclic loadings and the associated particle breakage. Ueng and Chen’s (2000) techniques have been employed in this study (Chapter 6) to model the plastic deformation of ballast including particle breakage.
CHAPTER 4
LABORATORY EXPERIMENTAL INVESTIGATIONS

4.1 INTRODUCTION

The mechanical response of ballast under monotonic and cyclic loadings has been investigated via a series of laboratory experiments. In order to study the strength, deformation and degradation characteristics of ballast (both fresh and recycled), several monotonic loading tests were conducted in a large-scale triaxial apparatus. The crushing strength of fresh and recycled ballast grains was studied in a separate series of single particle crushing tests. In order to investigate the deformation and degradation behaviour of fresh and recycled ballast under cyclic loading, a small section of track was simulated in a prismatic triaxial chamber in the laboratory. Representative field lateral stresses were applied to the ballast specimens and a cyclic vertical load equivalent to a typical 25 ton/axle train load was applied to the specimens. To enhance the engineering performance of recycled ballast in track, an attempt was made to stabilise recycled ballast in the laboratory model using various types of geosynthetics. The whole experimental programme of this research study has been divided into three categories: (a) monotonic triaxial tests, (b) single grain crushing tests, and (c) cyclic triaxial tests. The details of the equipments, test materials, specimen preparation and test procedures are described in the following Sections.
4.2 MONOTONIC TRIAXIAL TESTS

The strength, deformation, and degradation behaviour of ballast under monotonic loading has been investigated using a large-scale triaxial apparatus. Consolidated drained triaxial shearing tests were conducted on ballast specimens at various effective confining pressures. The conventional triaxial apparatus is one of the most versatile and widely used laboratory methods for obtaining the deformation and strength characteristics of geomaterials (Indraratna et al., 1998). Despite its wide acceptance as the principal geotechnical testing apparatus, it is impractical and almost impossible to conduct shear test on a ballast specimen in the conventional triaxial apparatus, because of large grain size. According to AS 2758.7 (1996), ballast grains can be 63.0 mm maximum, while the diameters of the conventional triaxial specimens are 37-50 mm. Therefore, to conduct a shear test on a ballast specimen, one needs to either scale down the ballast grains to fit within a conventional triaxial apparatus or fabricate a larger testing rig.

Many researchers indicated that the strength and deformation of aggregates are influenced by particle size (Marsal, 1967; Marachi et al., 1972; Indraratna et al., 1993). Because of the inevitable size-dependent dilation and particle crushing mechanism, the disparity between the actual size of ballast in track and scaled down aggregates for testing in a conventional triaxial apparatus may give misleading or inaccurate results and strength parameters (Indraratna et al., 2000). To overcome this problem, it is essential and imperative to conduct large-scale triaxial testing of field-size ballast so that realistic strength-deformation and degradation characteristics are obtained. This is why a large-scale triaxial facility was designed and built at the University of Wollongong (Indraratna, 1996).
4.2.1 Large-scale Triaxial Apparatus

The large-scale triaxial apparatus (Figure 4.1) can accommodate specimens of 300 mm diameter and 600 mm high. The main components of the apparatus are: (a) cylindrical triaxial chamber, (b) axial loading unit, (c) cell pressure control unit in combination of air and water pressure, (d) cell pressure and pore pressure measurement system, (e) axial deformation measuring device, and (f) volumetric change measurement unit. The change of volume of a specimen during consolidation and drained shearing is measured by a coaxial piston located within a small cylindrical chamber connected to the main cell, in which the piston moves up or down depending on the increase or decrease in volume.

Figure 4.1. Large-scale triaxial apparatus built at the University of Wollongong, (a) triaxial cell and loading frame, and (b) control panel board
Figure 4.2. (a) Schematic illustration of large-scale triaxial rig. (b) details of the triaxial chamber (modified after Indraratna et al., 1998)
Confining pressure is applied to the test specimen using a combination of air and water. Any change in specimen volume during shearing will affect the cell water pressure, which is minimised by compressed air in the pressure control chamber. Cell pressure can be decreased by opening an exhaust valve and increased by a control valve, which allows compressed air into the pressure control chamber.

A vertical load is applied via a pump connected to the hydraulic loading unit (Figure 4.2), and measured by a pressure transducer connected to the loading unit. Cell and pore water pressures are measured by two transducers. Vertical deformation of the specimen and movement of the co-axial piston of the volumetric measurement device are measured by two linear variable differential transducers (LVDT). The details of the triaxial apparatus are shown in Figure 4.2.

4.2.2 Characteristics of Test Ballast

4.2.2.1 Source of ballast

Fresh and recycled ballast specimens were tested under monotonic drained shearing in the large triaxial apparatus. Fresh ballast was collected from Bombo quarry (NSW), which is a major source for the Rail Infrastructure Corporation (RIC) of NSW. Recycled ballast was collected from Chullora stockpiles (Sydney), where discarded waste ballast was screened and the fine particles separated from coarse grains in a recycling plant.

4.2.2.2 Fresh ballast characteristics

As fresh ballast was part of the ballast delivered to the track site, its particle size, gradation, and other index properties were as specified by RIC (TS 3402, 2001), and
represents sharp angular coarse aggregates of crushed volcanic basalt (latite). The basalt is a fine-grained, dense-looking black rock, with the essential minerals being plagioclase (feldspar) and augite (pyroxenes).

Although a variety of parent rocks are used as the source of ballast in different parts of the world, igneous and sedimentary rocks are most widely used because they generally have high hardness and compressive strength, and are resistant to weathering. The common mineral groups are pyroxenes, quartz and feldspar. The specific minerals constituting parent rock govern the physical and mechanical properties of ballast. The durability, shape and strength of fresh ballast used in this study are summarized in Table 4.1. The grain size distribution (both fresh and recycled) including the RIC specification is shown in Figure 4.3. The selected grain size distribution used in this study (Figure 4.3) is typical of ballast gradations used by RIC.

Table 4.1: Characteristics of fresh ballast (after Indraratna et al., 1998)

<table>
<thead>
<tr>
<th>Characteristics test</th>
<th>Test result</th>
<th>Recommendations by Australian Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Aggregate crushing value</td>
<td>12%</td>
<td>&lt; 25%</td>
</tr>
<tr>
<td>• Los Angeles Abrasion</td>
<td>15%</td>
<td>&lt; 25%</td>
</tr>
<tr>
<td>• Wet attrition value</td>
<td>8%</td>
<td>&lt; 6%</td>
</tr>
<tr>
<td>Strength</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Ponit load index</td>
<td>5.39 MPa</td>
<td>-</td>
</tr>
<tr>
<td>Shape</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Flakiness</td>
<td>25%</td>
<td>&lt; 30%</td>
</tr>
<tr>
<td>• Misshapen particles</td>
<td>20%</td>
<td>&lt; 30%</td>
</tr>
</tbody>
</table>
To avoid the influence of particle size and gradation on experimental results, a single particle size distribution (Figure 4.3) was selected within the given range of RIC ballast specification (TS 3402). The same gradation curve was followed when preparing the test specimens, both fresh and recycled. The sample size ratio is defined by the ratio between the diameter of triaxial specimen and maximum particle size. Many researchers argued that as the sample size ratio approaches 6, the effect of sample size becomes negligible (Marachi et al., 1972; Indraratna et al., 1993; 1998). A maximum ballast size of 53 mm was selected in the current monotonic triaxial program, the corresponding sample size ratio becoming 5.7, which was considered to be reasonable to have a negligible effect on the test results.
4.2.2.3 Recycled ballast characteristics

Discarded ballast removed from the track during the renewal operation had been stockpiled in the specified yard. With the volume of waste ballast increasing daily, Rail Infrastructure Corporation (RIC) considered recycling some ballast partly to road construction and other projects, and some back to the track. With this objective in mind, RIC installed a recycling plant at their Chullora yard. Recycled ballast used in this research study was collected from Chullora after screening off the fine particles by the recycling plant.

A physical examination indicated that about 90% of the recycled ballast was semi-angular crushed rock fragments, while the remaining 10% consisted of semi-rounded river gravels and other impurities (cemented materials, sleeper fragments, nuts, bolts, fine particles etc.) (Indraratna et al., 2002a). Most of the semi-angular rock particles were almost the same size and shape as fresh ballast, while the obvious difference was that these were less angular, had less asperity, and were dirtier. Fine particles were clearly visible around recycled ballast grains even after passing through the screening operation. It is anticipated that its strength, bearing capacity and resiliency will be less due to less angularity, more heterogeneity and more impurities than fresh ballast.

4.2.3 Preparation of Ballast Specimens

All load cells, pressure transducers and LVDTs were calibrated before preparing the test specimens. To prepare the specimen for triaxial testing, a 5 mm thick cylindrical rubber membrane was placed around the pedestal of the base plate and clamped with 2 steel bands. The membrane was stiff enough to stand by itself. The membrane was then temporarily supported by a steel cylindrical split mould clamped together with nuts and
The ballast was carefully sieved using standard sieves, and different proportions of particle size were mixed together as per the selected gradation curve shown in Figure 4.3. The mixed ballast was then placed inside the rubber membrane and compacted.

![Triaxial chamber, split mould and a ballast specimen](image)

The ballast was compacted with a hand-held vibratory hammer in four layers, each approximately 150 mm thick. All specimens were prepared and compacted in an identical way to obtain a consistent initial compacted density/void ratio, as the strength of a granular assembly is highly sensitive to its initial density. In this study, the bulk unit weights of the test specimens were 15.4 - 15.6 kN/m³, which represent typical ballast density in the field. To minimise particle breakage during vibration, a 5 mm thick rubber pad was placed underneath the vibrator. After compaction, a steel cap was placed on top of the specimen and the membrane was clamped securely to the top cap with 2 steel bands. The split mould was then removed (Figure 4.4). The specimens were standing themselves inside the rubber membrane without the need of any vacuum loading. The triaxial cylinder was then placed around the specimen and connected to
Table 4.2: Monotonic triaxial test program

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Type of Ballast Tested</th>
<th>Effective Confining Pressure (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FB-10</td>
<td>Fresh Ballast</td>
<td>10</td>
</tr>
<tr>
<td>FB-50</td>
<td>Fresh Ballast</td>
<td>50</td>
</tr>
<tr>
<td>FB-100</td>
<td>Fresh Ballast</td>
<td>100</td>
</tr>
<tr>
<td>FB-200</td>
<td>Fresh Ballast</td>
<td>200</td>
</tr>
<tr>
<td>FB-300</td>
<td>Fresh Ballast</td>
<td>300</td>
</tr>
<tr>
<td>RB-10</td>
<td>Recycled Ballast</td>
<td>10</td>
</tr>
<tr>
<td>RB-50</td>
<td>Recycled Ballast</td>
<td>50</td>
</tr>
<tr>
<td>RB-100</td>
<td>Recycled Ballast</td>
<td>100</td>
</tr>
<tr>
<td>RB-200</td>
<td>Recycled Ballast</td>
<td>200</td>
</tr>
<tr>
<td>RB-300</td>
<td>Recycled Ballast</td>
<td>300</td>
</tr>
</tbody>
</table>

4.2.4 Test Procedure

After preparing the specimen, the triaxial cell was placed inside the loading frame, and the specimen was filled with water through the base plate. The triaxial chamber was also filled with water and left overnight to saturate the specimen. Consolidation of the specimen was commenced after achieving the Skempton’s pore pressure parameter \( B > 95\% \) (Skempton, 1954). The test specimens were consolidated to preselected confining pressures of 10 to 300 kPa (measured at the top of the triaxial chamber) before shearing, to investigate the influence of confining pressure on the strength, deformation and degradation of ballast. Raymond and Davies (1978) indicated that lateral stress in rail track is unlikely to exceed 140 kPa, as mentioned earlier in Section 2.5.1 (Chapter 2). Nevertheless, the behaviour of ballast was investigated in this study over a wider range of confining pressures. The range of confining pressures (10-300 kPa) applied in this study is expected to cover all possible lateral stresses in track, and is consistent with the
previous research (e.g. Indraratna et al., 1998; Raymond and Davies, 1978). The monotonic triaxial test program carried out in this study is shown in Table 4.2.

The consolidation pressure was applied in several steps, except FB-10 and RB-10, where only a single step was followed. The change in volume of the specimen was recorded after each step. After consolidating a specimen to its preselected pressure (see Table 4.2), the vertical load was increased using a hydraulic pump to commence shearing. Fully drained compression tests were conducted at an axial strain rate of 0.25% per minute, which allowed excess pore pressure to dissipate completely. The pressure transducers and LVDTs were connected to the digital panel board and a datalogger (DT800), supported by a host computer. All load, pressure and displacement measurements were recorded by the datalogger. The shearing was continued until the vertical strain of ballast reached about 20%. Additional triaxial tests were conducted on fresh ballast terminating the shearing at 0%, 5% and 10% axial strains to study the variation of ballast breakage with increasing strains. The ballast specimens were recovered at the end of each test, then dried and sieved, and the changes in particle size were recorded. All vertical and lateral stress measurements were corrected for membrane effect as per Duncan and Seed’s (1967) procedure.

4.3 SINGLE GRAIN CRUSHING TESTS

As mentioned in Section 2.4.1.5 (Chapter 2), the crushing strength of individual particles is a key parameter governing ballast degradation. To assess crushing strength characteristics, single grain crushing tests were conducted on various sizes of fresh and recycled ballast. The schematic illustration of the grain crushing test is shown in Figure 4.5, where a single grain was placed between the top and bottom platens of a
compression machine. The initial particle diameter ($d$) was measured before applying any compression. The maximum load at which a particle fractured ($F_f$) was recorded and the corresponding tensile strength was calculated using Equation 2.16 (Chapter 2).

$$F_f = \text{Fracture force}$$

Figure 4.5. Schematic of ballast grain fracture test (after Indraratna and Salim, 2003)

### 4.4 CYCLIC TRIAXIAL TESTS

Ideally, ballast should be tested in real track under actual loading conditions. However, these tests are costly, time consuming, and disrupt traffics. Moreover, many variables, which affect the proper formulation of definitive ballast relationship, are often difficult to control in the field (Jeffs and Marich, 1987). Therefore, laboratory experiments simulating field load and boundary conditions are usually carried out on ballast specimens. With the assistance of Rail Services Australia (currently, amalgamated with RIC), a large-scale prismoidal triaxial apparatus was designed and built at the University of Wollongong to investigate the response of a ballasted track under cyclic loading.

Several investigators have used large testing chambers with rigid and fully restrained walls to study ballast behaviour under cyclic loading (e.g. Atalar et al., 2001; Raymond
The lateral movement of ballast in real railway tracks is not fully restrained, particularly in the direction perpendicular to the rails (Indraratna et al., 2001). The confinement offered by fully restrained cell walls is, therefore, a major shortcoming in physical modelling of ballast in the laboratory. Consequently, some investigators developed semi-confined devices for ballast modelling (Jeffs and Marich, 1987; Norman and Selig, 1983). To simulate lateral deformation of ballast occurring in the real track situations, the vertical walls of the prismatic triaxial rig were designed and built to allow free lateral movements under imparted loadings.

### 4.4.1 Large Prismatic Triaxial Apparatus

The large prismatic triaxial rig used in this study can accommodate specimens 800 mm long, 600 mm wide, and 600 mm high. Figure 4.6(a) shows the prismatic triaxial chamber and Figure 4.6(b) is a schematic of the triaxial apparatus including specimen set-up. This is a true triaxial apparatus where three independent principal stresses can be applied in three mutually orthogonal directions. A system of hinge and ball bearings enables the vertical walls to move laterally. Since each wall of the rig can move independently in the lateral directions, the ballast specimen is free to deform laterally under cyclic vertical load and lateral pressures. The lateral confinement offered by the shoulder and crib ballast in an actual track is not sufficient to restrain lateral movement of ballast, hence, the prismatic triaxial rig with unrestrained sides provides an ideal facility for physical modelling of ballast under cyclic loading. Although the actual stress states may not be exactly simulated in the regions of lateral boundaries, this particular design of the chamber reasonably simulates realistic track boundary conditions.
The cyclic vertical load ($\sigma_1$) is provided by a servo-hydraulic actuator and the load is transmitted to the ballast through a 100 mm diameter steel ram and a rail/sleeper arrangement (Figure 4.7a). Intermediate and minor principal stresses ($\sigma_2$ and $\sigma_3$, respectively) are applied via hydraulic jacks, and are measured by attached load cells (Figure 4.7b).
Sleeper settlement and lateral deformations of the vertical walls are measured by 18 electronic potentiometers. Two pressure cells (150 mm X 150 mm X 22 mm each), one just beneath the sleeper and the other at the ballast/capping interface, are placed inside the chamber to monitor ballast stress. The volume of these pressure cells were taken into account during computation of ballast density/void ratio. Eight settlement plates are installed at each of the sleeper/ballast and ballast/capping interfaces to measure vertical strain. To get high quality real time data, all load cells, pressure cells and electronic
potentiometers are connected to a data logger, and supported by a host computer. This fully instrumented equipment can measure all vertical and lateral loads and associated deformations.

### 4.4.2 Materials Tested

#### 4.4.2.1 Ballast, capping and clay characteristics

As mentioned earlier, fresh and recycled ballast specimens were tested under representative cyclic loading. The properties of fresh and recycled ballast were discussed earlier in Section 4.2.2. A thin layer of compacted clay was used in the laboratory model to simulate subgrade of a real track. A capping layer comprising sand-gravel mixture was used between the ballast and clay layers. The particle size distribution of ballast (both fresh and recycled) and the capping materials, including RIC specification (TS 3402) are shown in Figure 4.8. Table 4.3 shows the grain size characteristics of fresh and recycled ballast and the capping materials used in cyclic tests.

![Figure 4.8. Particle size distribution of ballast and capping materials](image-url)
Table 4.3: Grain size characteristics of ballast and capping materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Particle shape</th>
<th>$d_{\text{max}}$</th>
<th>$d_{\text{min}}$</th>
<th>$d_{10}$</th>
<th>$d_{30}$</th>
<th>$d_{50}$</th>
<th>$d_{60}$</th>
<th>$C_u$</th>
<th>$C_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fresh ballast</td>
<td>Highly angular</td>
<td>63.0</td>
<td>19.0</td>
<td>24.0</td>
<td>30.0</td>
<td>35.0</td>
<td>38.0</td>
<td>1.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Recycled ballast</td>
<td>Semi-angular</td>
<td>63.0</td>
<td>19.0</td>
<td>24.0</td>
<td>30.0</td>
<td>35.0</td>
<td>38.0</td>
<td>1.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Capping</td>
<td>Angular to rounded</td>
<td>19.0</td>
<td>0.05</td>
<td>0.07</td>
<td>0.17</td>
<td>0.26</td>
<td>0.35</td>
<td>5.0</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Remoulded alluvial soft clay from Sydney was used to represent track subgrade in the laboratory model. Table 4.4 shows the index properties of the clay used in this study. The clay has been classified as CH (high plasticity clay) based on the Casagrande Plasticity Chart.

Table 4.4: Soil properties of clay used in cyclic test specimens (after Redana, 1999)

<table>
<thead>
<tr>
<th>Soil Properties</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay content (%)</td>
<td>40-50</td>
</tr>
<tr>
<td>Silt content (%)</td>
<td>45-60</td>
</tr>
<tr>
<td>Water content, $w$ (%)</td>
<td>40</td>
</tr>
<tr>
<td>Liquid limit, $w_L$ (%)</td>
<td>70</td>
</tr>
<tr>
<td>Plastic limit, $w_P$ (%)</td>
<td>30</td>
</tr>
<tr>
<td>Plasticity Index, $PI$ (%)</td>
<td>40</td>
</tr>
<tr>
<td>Unit weight, $\gamma$ (t/m³)</td>
<td>1.7</td>
</tr>
<tr>
<td>Specific Gravity, $G_s$</td>
<td>2.6</td>
</tr>
</tbody>
</table>

4.4.2.2 Characteristics of geosynthetics

Three types of geosynthetics were used in this study to stabilise recycled ballast in the laboratory model. These are: (a) geogrid, (b) woven-geotextile, and (c) geocomposite, a combination of geogrid and non-woven geotextile bonded together. The physical, structural and geotechnical characteristics of these geosynthetics are described below.
Geogrid

The geogrid used in this study was TENAX LBO SAMP 330 bi-oriented geogrid supplied by Polyfabrics Australia Pty Ltd. TENAX geogrid (Figure 4.9) is made of polypropylene, and manufactured by extrusion and biaxial orientation to enhance its tensile properties. It is generally used for soil stabilisation and embankment reinforcement. This geogrid has high tensile strength, high elastic modulus, and strong resistance to construction damage and environmental exposure. With its large apertures (>25 mm), geogrid provides strong mechanical interlock with coarse ballast grains. The physical, strength and technical characteristics of TENAX geogrid are given in Table 4.5 and the typical load-deformation behaviour is shown in Figure 4.10.

![Figure 4.9. TENAX LBO SAMP 330 geogrid](image-url)
Table 4.5: Properties of TENAX geogrid (courtesy, Polyfabrics Australia Pty Ltd)

<table>
<thead>
<tr>
<th>Physical Characteristics</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>Bi-oriented geogrid</td>
</tr>
<tr>
<td>Mesh Type</td>
<td>Rectangular apertures</td>
</tr>
<tr>
<td>Standard Colour</td>
<td>Black</td>
</tr>
<tr>
<td>Polymer Type</td>
<td>Polypropylene</td>
</tr>
<tr>
<td>Carbon Black Content</td>
<td>2%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimensional Characteristics</th>
<th>Unit</th>
<th>LBO 330 SAMP</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperture size MD</td>
<td>mm</td>
<td>40</td>
<td>b,d</td>
</tr>
<tr>
<td>Aperture size TD</td>
<td>mm</td>
<td>27</td>
<td>b,d</td>
</tr>
<tr>
<td>Mass per unit area</td>
<td>g/m²</td>
<td>420</td>
<td>b</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Technical Characteristics</th>
<th>Unit</th>
<th>LBO 330 SAMP</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength at 2% strain</td>
<td>KN/m</td>
<td>10.5</td>
<td>b,c,d</td>
</tr>
<tr>
<td>Tensile strength at 5% strain</td>
<td>KN/m</td>
<td>21</td>
<td>b,c,d</td>
</tr>
<tr>
<td>Peak tensile strength</td>
<td>KN/m</td>
<td>30</td>
<td>a,c,d</td>
</tr>
<tr>
<td>Yield point elongation</td>
<td>%</td>
<td>11</td>
<td>b,c,d</td>
</tr>
</tbody>
</table>

Notes:

a) 95% lower confidence limit values, ISO 2602
b) Typical values
c) Tests performed using extensometers
d) MD: machine direction (longitudinal to the roll)
   TD: transverse direction (across roll width)

Figure 4.10. Typical load-deformation response of TENAX geogrid (courtesy, Polyfabrics Australia Pty Ltd)
Woven-geotextile

ProPex 2044 polypropylene geotextile supplied by Amoco Chemicals Pty Ltd, Australia, was used in this study. ProPex 2044 (Figure 4.11) is a high strength woven-geotextile having a tensile strength of over 80 kN/m. It has good particle retention characteristics and high flow capacity. The physical, strength and geotechnical properties of ProPex 2044 woven-geotextile are summarised in Table 4.6.

Table 4.6: Characteristics of ProPex 2044 woven-geotextile (courtesy, Amoco Chemicals Pty Ltd, Australia)

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Unit</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>g/m²</td>
<td>&gt; 450</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>kN/m</td>
<td>&gt; 80</td>
</tr>
<tr>
<td>Pore size</td>
<td>mm</td>
<td>&lt; 0.30</td>
</tr>
<tr>
<td>Flow rate</td>
<td>litres/m²/sec</td>
<td>&gt; 30</td>
</tr>
</tbody>
</table>
Geocomposite (geogrid + non-woven geotextile)

TENAX GT 330 geogrid-geotextile geocomposite supplied by Ployfabrics Australia Pty Ltd, was used in recycled ballast. TENAX GT 330 is manufactured by bonding TENAX LBO SAMP 330 geogrid and non-woven polypropylene geotextile together. Adding a non-woven geotextile to geogrid enables this composite to provide filtering and separating functions. Due to large apertures (> 25 mm, see Table 4.5), geogrid alone cannot provide these functions effectively. In a geocomposite, the geogrid makes a strong mechanical interlock with the ballast grains, provides reinforcement, while the non-woven geotextile filters, separates and allows partial in-plane drainage. Figure 4.12 shows the TENAX GT 330 geocomposite used in this study. The physical and mechanical characteristics of the geocomposite are given in Table 4.7.
Table 4.7: Characteristics of TENAX GT 330 geocomposite (courtesy, Polyfabrics Australia Pty Ltd)

<table>
<thead>
<tr>
<th>Geogrid Physical Characteristics</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>Bi-oriented geogrid</td>
</tr>
<tr>
<td>Mesh Type</td>
<td>Rectangular apertures</td>
</tr>
<tr>
<td>Standard Colour</td>
<td>Black</td>
</tr>
<tr>
<td>Polymer Type</td>
<td>Polypropylene</td>
</tr>
<tr>
<td>Carbon Black Content</td>
<td>2%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Geotextile physical Characteristics</th>
<th>Unit</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass per unit area</td>
<td>g/m²</td>
<td>140</td>
</tr>
<tr>
<td>Polymer type</td>
<td>-</td>
<td>Polypropylene</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimensional Characteristics</th>
<th>Unit</th>
<th>GT 330</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geogrid Aperture size MD</td>
<td>mm</td>
<td>40</td>
<td>b,d</td>
</tr>
<tr>
<td>Geogrid Aperture size TD</td>
<td>mm</td>
<td>27</td>
<td>b,d</td>
</tr>
<tr>
<td>Mass per unit area</td>
<td>g/m²</td>
<td>560</td>
<td>b</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Technical Characteristics</th>
<th>Unit</th>
<th>LBO 330 SAMP</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>MD</td>
<td>MD</td>
<td>30</td>
<td>a,c,d</td>
</tr>
<tr>
<td>TD</td>
<td>TD</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Yield point elongation</td>
<td>%</td>
<td>11</td>
<td>b,c,d</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
- a) 95% lower confidence limit values, ISO 2602
- b) Typical values
- c) Tests performed using extensometers
- d) MD: machine direction (longitudinal to the roll)  
  TD: transverse direction (across roll width)

4.4.3 Preparation of Test Specimens

A small track section including subgrade, capping, ballast, sleeper and rail was simulated inside the triaxial chamber (see Figure 4.6b) to represent a real track in the laboratory. A compacted clay layer of 50 mm thick was placed at the bottom of the triaxial chamber to model subgrade of a real track. Only a thin layer of clay was used in the current laboratory model due to limited height of the triaxial chamber. Although no experimental investigation has been made within the scope of this study to examine the
influence of subgrade thickness on ballast behaviour, it is expected that a thicker subgrade of a specific thickness will equally affect the deformation and degradation response of various ballast specimens. Moreover, the vertical strains of ballast are computed by excluding the deformation of the capping and subgrade layers. In this respect, the thickness of clay layer used in the laboratory model is expected to have an insignificant influence on the test results, especially when comparing the response of different ballast specimens with and without geosynthetic inclusion. It is relevant to note here that subgrade vertical stresses were not measured during cyclic loading tests.

A capping layer (100 mm) of sand-gravel mixture was used above the clay layer to represent subballast of a track. Both the load bearing ballast (300 mm thick) and crib ballast (150 mm thick) layers consisted of either fresh or recycled ballast. The load bearing ballast was placed above the compacted capping layer. An assembly of timber sleeper and rail section was placed above the compacted load bearing ballast, and the space between the sleeper and vertical walls was filled with crib ballast. One layer of geosynthetics (geogrid, woven-geotextile or geocomposite) was placed at the ballast/capping interface (i.e. the weakest interface) to improve the performance of recycled ballast. To completely recover the load bearing ballast after the test, 2 layers of thin, loose, geotextiles were placed above and below the ballast layer for isolation purpose only.

A vibratory hammer was used to compact the ballast and capping layers. To achieve representative field density, compaction was carried out in several layers, each about 75 mm thick. A 5 mm thick rubber pad was used beneath the vibrator to minimise particle breakage. Each test specimen was compacted to the same initial density. The bulk unit
weights of the compacted ballast and capping layers were about 15.3 kN/m$^3$ and 21.3 kN/m$^3$, respectively. The initial void ratio ($e_o$) of ballast layer was 0.74.

### 4.4.4 Cyclic Load Test Program

In this study, 10 cyclic triaxial tests were carried out on fresh and recycled ballast, with and without geosynthetic inclusion. To study the effect of saturation, 5 specimens were tested dry and 5 were tested wet, with all specimens having identical loading and boundary conditions. Table 4.8 gives the details of the cyclic triaxial test program.

#### Table 4.8: Cyclic triaxial test program

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Type of Ballast</th>
<th>Type of Geosynthetics Used</th>
<th>Test Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>FB(dry)</td>
<td>Fresh Ballast</td>
<td>-</td>
<td>Dry</td>
</tr>
<tr>
<td>FB(wet)</td>
<td>Fresh Ballast</td>
<td>-</td>
<td>Wet</td>
</tr>
<tr>
<td>RB(dry)</td>
<td>Recycled Ballast</td>
<td>-</td>
<td>Dry</td>
</tr>
<tr>
<td>RB(wet)</td>
<td>Recycled Ballast</td>
<td>-</td>
<td>Wet</td>
</tr>
<tr>
<td>RB-GD(dry)</td>
<td>Recycled Ballast</td>
<td>TENAX Geogrid</td>
<td>Dry</td>
</tr>
<tr>
<td>RB-GD(wet)</td>
<td>Recycled Ballast</td>
<td>TENAX Geogrid</td>
<td>Wet</td>
</tr>
<tr>
<td>RB-GT(dry)</td>
<td>Recycled Ballast</td>
<td>ProPex Geotextile</td>
<td>Dry</td>
</tr>
<tr>
<td>RB-GT(wet)</td>
<td>Recycled Ballast</td>
<td>ProPex Geotextile</td>
<td>Wet</td>
</tr>
<tr>
<td>RB-GCOM(dry)</td>
<td>Recycled Ballast</td>
<td>TENAX Geocomposite</td>
<td>Dry</td>
</tr>
<tr>
<td>RB-GCOM(wet)</td>
<td>Recycled Ballast</td>
<td>TENAX Geocomposite</td>
<td>Wet</td>
</tr>
</tbody>
</table>

In addition to the cyclic tests, one slow repeated load test was conducted on recycled dry ballast without any geosynthetic inclusion. The repeated load test was carried out at various pre-selected load cycles (i.e., before applying any cyclic load, after 100,000 load cycles, and after 500,000 load cycles). This test was performed to study the stress-strain response of ballast for a number of load cycles, and also to examine how the stress-strain evolves during the course of cyclic loading.
4.4.5 Magnitude of Cyclic Load

The maximum sleeper/ballast contact stress must be ascertained before commencing any cyclic load test. As discussed earlier in Section 2.3.1 (Chapter 2), the maximum sleeper/ballast contact stress depends on many factors, including wheel static load and train speed. The static axle load depends on the type of vehicle, and may vary from 70-350 kN (Esveld, 2001). For establishing the maximum sleeper/ballast contact stress to apply in the laboratory cyclic load tests, a nominal axle load of 250 kN was assumed, which corresponds to a static wheel load of 125 kN.

Following the design method proposed by Li and Selig (1998), the design wheel load for a train speed of 100 km/hour with a wheel diameter of 0.97 m, was computed to be 192 kN (Equation 2.5). Atalar et al. (2001) reported that part of this wheel load is transmitted to the adjacent sleepers, and 40-60% of the wheel load is resisted by the sleeper directly beneath the wheel. Assuming 50% of the design wheel load as the rail seat load and $F_2 = 1$, $l = 2.5$ m, $W = 0.26$ m, Equation 2.12 gives an average contact pressure of 443 kPa. Assuming $a = 0.5$ m and sleeper width = 0.26 m, the stress distribution shown in Figure 2.8, gives an average sleeper/ballast contact stress of 369 kPa.

Based on the above estimations, the maximum cyclic vertical stress for the current laboratory investigation was selected to be 460 kPa. The corresponding maximum vertical load for the laboratory model translated to 73 kN, which is consistent with the previous study (Ionescu et al., 1998).
4.4.6 Test Procedure

Small lateral pressures ($\sigma_2 = 10$ kPa and $\sigma_3 = 7$ kPa) were applied to the triaxial specimens through hydraulic jacks to simulate field confinement. In a real track, the confinement is generally developed by the weight of crib and shoulder ballast, along with particle frictional interlock. An initial vertical load of 10 kN was applied to the specimens to stabilise the sleeper and the ballast, and to serve as a reference for all settlement and lateral movement measurements. At this state, initial readings of all load cells, pressure cells, potentiometers, and settlement plates were taken.

The cyclic vertical load was applied by a dynamic actuator with a maximum load of 73 kN. The cyclic tests were conducted at a frequency of 15 Hz. Although some researchers indicated that loading frequency has insignificant influence on ballast deformation (Shenton, 1975), some others concluded from field measurements that the dynamic stress increased linearly as the train speed increased from 150 to 300 km/hour (Kempfert and Hu, 1999), as mentioned earlier in Section 2.4.3.5 (Chapter 2). In this study, a single loading frequency (15 Hz) was used in all cyclic load tests, varying the type of ballast and type of geosynthetics. It is believed that higher frequency will impart higher dynamic stresses on ballast bed, and therefore, will influence deformation and degradation of ballast. Further study of ballast behaviour varying the loading frequency is ongoing at the University of Wollongong under the auspices of Cooperative Research Center for Railway Engineering and Technology (Rail-CRC). The total number of load cycles applied in each test was half a million. The cyclic load was halted at selected load cycles to record settlement, lateral displacement and load magnitude readings. For wet tests (see Table 4.8), the ballast specimens were gradually flooded with water before applying the cyclic load and water was added during cyclic loading to maintain
100% saturation. At the end of each test, the ballast specimens were recovered, sieved, and any change in particle size was recorded for breakage computation.

The repeated load test was carried out using the prismoidal triaxial rig at selected interval of load cycles, including the start of cyclic loading. In this test, the vertical load was slowly increased from the initial value to the maximum 73 kN, and then decreased to its initial value. This loading-unloading procedure was repeated for several cycles. During the repeated load test, all load and deformation measurements were continuously recorded using the datalogger (DT800).
CHAPTER 5
EXPERIMENTAL RESULTS AND DISCUSSIONS

5.1 INTRODUCTION

The strength, deformation and degradation behaviour of fresh and recycled ballast have been studied in a series of monotonic triaxial shearing tests using a large-scale triaxial apparatus. The effects of confining pressure on friction angle, dilatancy, stress-ratio and particle breakage were particularly examined. The cyclic stress-strain and particle breakage behaviour of fresh and recycled ballast have also been investigated in a large prismatic triaxial chamber simulating a small track section inside the apparatus. The stabilisation aspect of recycled ballast by the inclusion of various types of geosynthetics has been studied using the laboratory model apparatus. To quantify ballast degradation, each specimen was sieved before and after the tests. The crushing strength of ballast grains was examined by conducting a series of single particle crushing tests. The results of these laboratory experiments are presented and discussed in the following Sections.

5.2 MONOTONIC TRIAXIAL TEST RESULTS

5.2.1 Stress-strain Behaviour

A series of five isotropically consolidated drained triaxial shearing tests was conducted on fresh and recycled ballast each, with the effective confining pressure varying from 10 to 300 kPa. As mentioned earlier in Section 4.2.4, triaxial shearing was continued up to
an axial strain of about 20%. No distinct failure plane was observed in these tests even after 20% axial straining. The variations of deviator stresses \( q = \sigma'_1 - \sigma'_3 \) and volumetric strains \( \varepsilon_v = \varepsilon_1 + 2\varepsilon_3 \) with the shear strains \( \varepsilon_s = \frac{2}{3}(\varepsilon_1 - \varepsilon_3) \) of fresh and recycled ballast under monotonic triaxial loading, are shown in Figures 5.1 and 5.2, respectively. The major and minor principal effective stresses are represented by \( \sigma'_1 \) and \( \sigma'_3 \), respectively, and the corresponding strains are denoted by \( \varepsilon_1 \) and \( \varepsilon_3 \), respectively.

![Figure 5.1. Stress-strain and volume change behaviour of fresh ballast in isotropically consolidated drained shearing](image-url)
Figures 5.1 and 5.2 clearly show that the shear behaviour of ballast, both fresh and recycled, is non-linear. It is evident that an increase in confining pressure increases the deviator stress. At low confinement (≤ 100 kPa), the volume of the ballast specimen increases (dilation, represented by negative $\varepsilon_v$) during drained shearing. Higher confining pressure tends to shift the overall volumetric strain towards contraction (i.e. $\varepsilon_v$ becomes positive). A state of peak deviator stress ($\sigma'_1 - \sigma'_3$), can be regarded as ‘failure’ for ballast. At low confining pressures, a peak deviator stress (i.e. failure) is evident, followed by a post–peak strain softening associated with volume increase (i.e.
The shear behaviour of recycled ballast under monotonic triaxial shearing is generally similar to the fresh ballast.

In order to compare the stress-strain behaviour of fresh and recycled ballast under triaxial compression, the stress-strain and volume change data of Figures 5.1 and 5.2 were re-plotted together, as shown in Figure 5.3. For clarity, only 3 sets of test data at 10, 100 and 300 kPa confining pressures are shown in this figure.

![Figure 5.3. Comparison of stress-strain and volumetric behaviour between fresh and recycled ballast under triaxial drained shearing](image)
Figure 5.3 shows that recycled ballast generally has a lower shear strength \((\sigma'_1 - \sigma'_3)_p\), compared to the fresh ballast. Owing to the sharp corners breaking off under previous traffic loading cycles, the recycled ballast has less angularity than the fresh ballast. Less angularity and fine dust around the grains of recycled ballast reduce its frictional interlock and lower its shear strength. Figure 5.3 also shows that fresh ballast dilates more than the recycled ballast at low confinement (e.g. 10 kPa), which is attributed to the higher angularity of fresh ballast. Dilatancy is suppressed at higher confinement (e.g. 300 kPa), and both fresh and recycled ballast continue to contract at a decreasing rate as the shear strain increases.

Figures 5.4 and 5.5 show the variation of deviator stress ratio \((\eta = q/p')\) with the increasing shear strain \((\varepsilon_s)\) for the fresh and recycled ballast, respectively. These figures reveal that the deviator stress ratio increases rapidly to a peak value at low confinement, and then decreases gradually as the shear strain increases. At higher confining pressures \((\geq 200 \text{ kPa})\), however, the deviator stress ratio increases at a decreasing rate with the increasing shear strain, and reaches a stable value at higher strain levels. It was noted in these figures that all stress ratio-strain data approached a common stress ratio \((\eta)\) value as the shear strain increased, irrespective of the confining pressures. Apparently, both the fresh and recycled ballast exhibit similar variation of deviator stress ratio with the increasing shear strain.
Figure 5.4. Stress ratio ($\eta$) versus shear strain plots for fresh ballast under drained shearing

Figure 5.5. Stress ratio ($\eta$) versus shear strain plots for recycled ballast under drained shearing
Figures 5.6 and 5.7 show the variation of effective principal stress ratio ($\sigma'_1/\sigma'_3$) with increasing shear strain for the fresh and recycled ballast, respectively. These test results clearly demonstrate that at low confining pressure, both fresh and recycled ballast exhibit higher principal stress ratio ($\sigma'_1/\sigma'_3$). A peak value of the principal stress ratio is clearly evident at low confinement, followed by strain softening. In contrast, no clear peak principal stress ratio is visible at a higher confinement. However, it was noted that at a higher confining pressure, the principal stress ratio increased at a decreasing rate towards a stable value as the shear strain increased (Figures 5.6 - 5.7). Apparently, the peak principal stress ratio decreases with increasing confining pressure, and this behaviour is attributed to the absence of dilatancy at higher confinement. It was also noted in Figures 5.6 - 5.7 that irrespective of the confining pressures, all principal stress ratio data moved towards a common value with increasing shear strains.

In order to compare the principal stress ratio ($\sigma'_1/\sigma'_3$) of fresh and recycled ballast, the data of Figures 5.6 - 5.7 were re-plotted together, as shown in Figure 5.8. Only 3 sets of test data at 10, 100 and 300 kPa confining pressures are shown in this figure for clarity. Figure 5.8 reveals that the fresh ballast exhibits a higher stress ratio compared to recycled ballast, especially at low confining pressure. The difference between the principal stress ratio of fresh and recycled ballast at high confining pressure (e.g. 300 kPa) becomes insignificant, primarily due to the absence of dilatancy.
Figure 5.6. Variation of effective principal stress ratio with shear strain for fresh ballast

Figure 5.7. Variation of effective principal stress ratio with shear strain for recycled ballast
Figure 5.8. Comparison of principal stress ratio between fresh and recycled ballast in drained triaxial shearing

5.2.2 Shear Strength and Stiffness

Figure 5.9 shows the shear strength results of fresh and recycled ballast in terms of principal stress ratio at failure $(\sigma'_1/\sigma'_3)_f$, plotted against the effective confining pressure. Selected rockfill data from previous studies (Marsal, 1967; Marachi et al., 1972; Charles and Watts, 1980) were also plotted in this figure for comparison. Since the current triaxial tests were conducted at relatively low stress levels compared to the previous studies on rockfill, fresh ballast exhibits a relatively higher stress ratio at failure compared to the rockfills. Figure 5.9 also demonstrates that the failure stress ratio of recycled ballast is significantly lower than the fresh ballast at low confinement. It was also noted that in general, the principal stress ratio at failure decreased with the increasing confining pressure both for the ballast and rockfills.
The peak friction angles ($\phi_p$) of fresh and recycled ballast obtained from drained triaxial shearing were plotted against the effective confining pressures, as shown in Figure 5.10. The $\phi_p$ values of other crushed basalt (rockfill) obtained at relatively higher confining pressures by the previous researchers, were also plotted in this figure for comparison. Figure 5.10 reveals that the peak friction angle of both fresh and recycled ballast decreases with increasing confining pressure, which is attributed primarily to the decrease in dilatancy at elevated pressure. This behaviour of ballast is consistent with the findings of previous research on rockfill (Marsal, 1967; Marachi et al., 1972; Charles and Watts, 1980). The influence of dilatancy and particle breakage on the friction angle of ballast at various confining pressures is discussed further in Section 6.2.2 of Chapter 6. Figure 5.10 also confirms that recycled ballast has a lower frictional strength compared to fresh ballast. The current test results reveal that the peak friction angle of fresh and recycled ballast varies from $69^\circ$ to $46^\circ$ and $54^\circ$ to $43^\circ$, respectively, as the effective confining pressure increases from 10 to 300 kPa.
Figure 5.10. Variation of peak friction angle of fresh and recycled ballast with effective confining pressure

The variation of initial elastic modulus ($E_i$) of fresh and recycled ballast with the effective confining pressure is presented in Figure 5.11. The test results reveal a linear relationship between the initial deformation modulus and the effective confining pressure for both fresh and recycled ballast within the stress range used in this study. In general, the initial elastic modulus of ballast increases with increasing confining pressure. Obviously, fresh ballast exhibits a higher elastic modulus compared to recycled ballast (Figure 5.11), due to higher angularity and better frictional interlock in fresh aggregates.
5.2.3 Particle Breakage in Triaxial Shearing

As mentioned earlier in Section 4.2.4 (Chapter 4), changes in grain size resulting from shearing were recorded in each test. Figure 5.12 shows the change in ballast gradation plotted in the conventional gradation curves. It is obvious from this figure that small changes of ballast size cannot be clearly illustrated in the conventional gradation plots. Therefore, an alternative technique extending the method proposed by Marsal (1967; 1973) was adopted, where the differences in percentage retained before and after testing ($\Delta W_k$) were plotted against the sieve size. Figures 5.13(a) and (b) show the variation of $\Delta W_k$ with sieve size for fresh and recycled ballast, respectively.
Figure 5.12. Change in particle size of ballast shown in conventional gradation plots

Figure 5.13. Alternative method showing the change in particle size under triaxial shearing, (a) fresh ballast, and (b) recycled ballast

Figures 5.13(a) and (b) indicate that the breakage of particles under triaxial compression increases with increasing confining pressure. It is evident that larger particles (> 30 mm) are more vulnerable to breakage than smaller grains, for both fresh and recycled
ballast. To compare the degradation characteristics between fresh and recycled ballast, the data from Figures 5.13(a) and (b) were re-plotted together, as shown in Figure 5.14. Only three sets of experimental data (at 10, 100 and 300 kPa) are presented here for clarity. Figure 5.14 shows that the recycled ballast suffers higher particle breakage compared to fresh ballast. A large number of hairline micro-cracks in recycled ballast grains resulting from previous loading cycles, is believed to be a major reason for this behaviour. The presence of these micro-cracks decreases the crushing strength of recycled ballast, which was confirmed later by the single grain crushing tests (Section 5.3 of this Chapter). Recycled ballast is therefore, more vulnerable to degradation, and requires external reinforcing agents to strengthen its resistance against breakage so that it can compete with fresh ballast as a potential construction material. The breakage indices $B_g$, (Marsal, 1967; 1973) of both fresh and recycled ballast tested under monotonic triaxial shearing, are presented in Table 5.1.

![Figure 5.14. Comparison of particle breakage between fresh and recycled ballast](image_url)
Table 5.1: The breakage indices ($B_g$) of ballast under monotonic loading

<table>
<thead>
<tr>
<th>Effective confining pressure (kPa)</th>
<th>Breakage Index, $B_g = \Sigma$ Positive $\Delta W_k$</th>
<th>Fresh ballast</th>
<th>Recycled ballast</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.34</td>
<td>2.99</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>4.74</td>
<td>5.77</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>6.64</td>
<td>7.60</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>10.69</td>
<td>11.95</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>14.29</td>
<td>15.68</td>
<td></td>
</tr>
</tbody>
</table>

The influence of strain level on the degree of particle breakage was investigated by terminating shearing at pre-selected axial strains (e.g. 0%, 5%, 10%, and 20%) and computing the breakage index from the measurements of grain size changes, as mentioned earlier in Section 4.2.4 (Chapter 4). Figure 5.15 shows the variation of $B_g$ values with the increasing axial strains. The failure strains ($\varepsilon_{1f}$) are indicated on this plot, and the locus of the failure strains is also shown in the figure.

Figure 5.15. Variation of particle breakage of fresh ballast with axial strain (after Indraratna and Salim, 2002)
Figure 5.15 shows that the degree of particle breakage increases non-linearly with increasing axial strain, and the magnitude of breakage also increases with higher confining pressure. The trend lines of breakage indices are shown as the solid lines in Figure 5.15. It was noted that the breakage of ballast continued to increase even after the peak deviator stress (or failure). The test results also indicate that the rate of particle breakage $dB_g/d\varepsilon_1$, (i.e. slope) is high initially, and decreases with increasing confining pressure towards a constant.

5.2.4 Critical State of Ballast

The variation of deviator stress $q$, with the mean effective stress $p'$, for fresh and recycled ballast under triaxial drained shearing are shown in Figures 5.16(a) and (b), respectively. Apparently, an increase in confining pressure increases the mean effective stress, which leads to higher deviator stress. The test results show that at the end of drained shearing, the states of stress ($p', q$) of all ballast specimens, which were consolidated to various confining pressures, lie approximately on a straight line. In other words, irrespective of the confining pressures, the stress state of ballast moves during triaxial shearing towards unique (i.e. critical) states, which are related to each other linearly in the $p'-q$ plane. The test results reveal that the slopes of the critical state lines ($M$) for the fresh and recycled ballast used in this research study are approximately 1.90 and 1.67, respectively.
Figure 5.16. Variation of $p'$ and $q$ in drained triaxial shearing, (a) fresh ballast, and (b) recycled ballast.
The variations of void ratio ($e$) with the mean effective stress ($p'\lambda$) during drained shearing were plotted in a semi-logarithmic scale, as shown in Figures 5.17(a) and (b) for the fresh and recycled ballast, respectively. These figures show that in drained shearing, the void ratio of ballast (both fresh and recycled) changes as such that the
states of the specimens at large shear strain levels relate to each other in a very specific way. Irrespective of the confining stresses, all ballast specimens move towards the critical state. Figure 5.17 indicates that an increase in void ratio (i.e. dilation) is associated with drained shearing when the effective confining pressure is low (≤ 100 kPa). In contrast, overall volumetric contraction occurs when the confining pressure is high (200 kPa and above). The critical state lines for the fresh and recycled ballast are also shown in these figures. The test results reveal that the slopes of the critical state lines in $e$-ln$p'$ plane ($\lambda$) for the fresh and recycled ballast are approximately 0.19 and 0.16, respectively.

5.3 SINGLE PARTICLE CRUSHING TEST RESULTS

Angularity, coarseness, uniformity of gradation, lower particle strength, stress level and anisotropy promote grain crushing (Bohac et al. 2001). However, the most important factor is the resistance of grains to fracture (i.e. crushing strength). As indicated in Section 2.4.1.5 (Chapter 2), fracture in a particle initiated by tensile failure, and the tensile strength of rock grains is represented by Equation 2.16. The tensile strength of various sized fresh and recycled ballast grains obtained from a series of single particle crushing tests is shown in Figure 5.18.

Figure 5.18 shows that in general, fresh ballast grains have a higher tensile strength than recycled ballast, especially the smaller grains. Since recycled ballast has undergone millions of loading cycles in the past, as mentioned earlier, it contains more micro-cracks than fresh ballast, hence it is expected to be more prone to crushing. Regression analysis of the single particle strength data indicates that recycled ballast generally has about 35% lower tensile strength than fresh aggregates. This lower crushing strength of
recycled ballast is directly responsible for its higher particle breakage under monotonic triaxial shearing compared to the fresh ballast (see Figure 5.14).

Figure 5.18. Single grain crushing strength of fresh and recycled ballast

Figure 5.18 also indicates that for both fresh and recycled ballast, the tensile strength decreases linearly with the increasing grain size. Mcdowell and Bolton (1998) and Nakata et al. (2001) observed a similar trend for sand and limestone. This is because larger particles contain more flaws and have a higher probability of defects (Lade et al., 1996). Fracturing larger particles along these defects creates smaller particles. The subdivided particles contain fewer defects and are less likely to fracture. In other words, smaller grains are more resistant to crushing and larger grains are more vulnerable to breakage. The grain crushing test findings are also consistent with the breakage results shown earlier (see Figure 5.13), where larger grains exhibited higher particle breakage. Figure 5.18 indicates that the degree of scatter of the strength data from its best-fit line is higher for recycled ballast than for fresh aggregate. This is attributed to the heterogeneity of recycled ballast grains (obtained from different sources and mixed together), whereas, fresh ballast contains relatively homogeneous minerals.
5.4 CYCLIC LOAD TEST RESULTS

5.4.1 Settlement Response of Ballast

The response of fresh and recycled ballast under cyclic loading was investigated in a laboratory model apparatus in both dry and wet states. Figures 5.19(a) and (b) show the settlements of fresh and recycled ballast, both dry and wet, and with and without inclusion of geosynthetics. As expected, fresh dry ballast gives the least settlement (Figure 5.19a). It is believed that the relatively higher angularity of fresh ballast contributes to better particle interlock and therefore, gives less settlement. Recycled ballast without any geosynthetic inclusion exhibits significantly higher settlement compared to fresh ballast, especially when wet (saturated). The reason for this is that reduced angularity of recycled ballast results in less friction angle (see Figure 5.10) and lower deformation modulus (Figure 5.11) compared to fresh ballast. The test results show that wet recycled ballast (without any geosynthetic inclusion) gives the highest settlement (Figure 5.19b), because water acts as a lubricant, which reduces frictional resistance.

Figure 5.19 also shows the benefits of using geosynthetics in recycled ballast (both dry and wet). Each of the three types of geosynthetics used in this study decreases the settlement considerably. However, the geocomposite (geogrid bonded with a non-woven geotextile) stabilises recycled ballast remarkably well, as revealed in the test results (Figure 5.19). The combination of reinforcement by the geogrid and the filtration and separation provided by the non-woven geotextile (of the geocomposite), minimises the lateral spreading and fouling of recycled ballast, especially when wet. The non-woven geotextile also prevents the fines moving up from the capping and subgrade layers, and keeps the recycled ballast relatively clean.
Figure 5.19. Settlement response of fresh and recycled ballast under cyclic loading, (a) in dry condition, and (b) in wet condition

In contrast, the geogrid can only stabilise recycled ballast marginally, especially in wet conditions, because its high apertures (> 25 mm) cannot prevent the fines migrating from the capping and subgrade layers. The woven-geotextile decreases the settlement of recycled ballast effectively when dry (Figure 5.19a). However, owing to its limited filtration capability with the aperture size less than 0.30 mm, the woven-geotextile is not as effective as the geocomposite when wet (Figure 5.19b). Despite these differences in the settlement behaviour, Figure 5.19 shows one common feature; initially the
settlement increases rapidly in all specimens. It was also noted that all ballast specimens stabilized within about 100,000 load cycles, beyond which the settlement increase was marginal.

The settlement data of Figure 5.19 were re-plotted in a semi-logarithmic scale, as shown in Figure 5.20. The highly non-linear variation of ballast settlement with the increasing load cycles (Figure 5.19) becomes almost linear in the semi-logarithmic plot (Figures

Figure 5.20. Settlement of fresh and recycled ballast plotted in semi-logarithmic scale (a) dry specimens, (b) wet specimens
5.20a-b). The linear trend lines of the settlement data are shown as solid lines in these figures. Figure 5.20 reveals that the inclusion of geosynthetics in recycled ballast decreases its settlement in the first cycle of loading, and also decreases the overall settlement rate. It was noted that although fresh dry ballast showed less settlement than the recycled dry ballast, the rate of settlement of fresh ballast seems to be slightly higher than recycled ballast, especially at smaller load cycles (Figure 5.20a). This is attributed to the faster deterioration and breakage of sharp corners and small asperities in fresh ballast. At a large number of load cycles (> 100,000), the rate of settlement of fresh ballast becomes almost the same as that of recycled ballast, because increased particle breakage at higher load cycles causes increasingly more fouling to fresh ballast and at very large number of load cycles (say a few millions) these 2 types of ballast will become practically the same materials. However, since recycled ballast gives higher overall settlement, it poses higher risk of differential track settlement, which is a major concern for the stability and safety of trains. Figure 5.20 clearly shows that inclusion of geosynthetics significantly decreases the overall settlement including the rate of settlement of recycled ballast, especially when wet, and these favourable effects are more pronounced in case of a bonded geocomposite inclusion.

The test results also indicate that the ballast settlement under cyclic loading may be represented by the following semi-logarithmic relationship:

\[ S = a + b(\ln N) \]  

(5.1)

where, \( S \) is the ballast settlement, \( N \) is the number of load cycles, and \( a \) and \( b \) are two empirical constants, depending on the type of ballast, type of geosynthetics used, initial density and the degree of saturation.
5.4.2 Strain Characteristics

The difference between the ballast settlement and the settlement of the ballast/capping interface (measured by the settlement plates) was used to calculate the average vertical strain (major principal strain, \( \varepsilon_1 \)) of the load bearing ballast layer. Figures 5.21(a) and (b) show the average vertical strain of ballast against the number of load cycles plotted in a semi-logarithmic scale for the dry and wet specimens, respectively. These figures also demonstrate appreciable reductions in the vertical strain of recycled ballast when various geosynthetics are included. In particular, all the three types of geosynthetics used in this study decreased the vertical strain of recycled ballast in dry state (Figure 5.21a). However, in wet conditions, the geocomposite appears to be the most effective, where the vertical strain of recycled ballast decreased close to that of fresh ballast without any geosynthetics (Figure 5.21b). The reasons were explained earlier in Section 5.4.1. The geogrid alone decreased the vertical strain of recycled ballast marginally when wet, and the woven-geotextile stabilised the recycled ballast moderately in saturated condition.

Figure 5.21 shows that the vertical strain of ballast increases linearly with the logarithm of load cycles, and may be expressed by a function similar to Equation 5.1, as given by:

\[
\varepsilon_1 = c + d(\ln N)
\]  

(5.2)

where, \( \varepsilon_1 \) is the major (vertical) principal strain, \( N \) is the number of load cycles, and \( c \) and \( d \) are two empirical constants.
The lateral strains of ballast (intermediate principal strain \( \varepsilon_2 \), and minor principal strain \( \varepsilon_3 \)) were calculated from the lateral deformation measurements of the vertical walls and the initial lateral dimensions of the test specimens. The lateral strain perpendicular to the sleeper (i.e. parallel to the rails) is the intermediate principal strain (\( \varepsilon_2 \)), which corresponds to the intermediate principal stress (\( \sigma_2 \)). The strain parallel to the sleeper is the minor principal strain (\( \varepsilon_3 \)), and corresponds to the minor principal stress (\( \sigma_3 \)).
The intermediate principal strains ($\varepsilon_2$) of ballast were plotted against the logarithm of load cycles, as shown in Figures 5.22(a) and (b). The test results show that the recycled ballast (without stabilisation) gives a higher lateral strain initially because of its less angularity and friction compared to the fresh ballast. The test results indicate that at increased load cycles, the intermediate principal strains of both fresh and recycled ballast almost converge to one value. Geosynthetics in recycled ballast decreased the
intermediate principal strain in both dry and wet conditions. The superiority of the geocomposite over the other two types of geosynthetics, in terms of minimising lateral strain of recycled ballast, is obvious, as revealed in Figure 5.22. It was also noted in this figure that the use of geosynthetics in recycled ballast decreased $\varepsilon_2$ below that of fresh ballast at a higher number of load cycles. The current test results indicate that the intermediate principal strain may also be represented by a semi-logarithmic function, similar to Equation 5.2.

The variations of the minor principal strains ($\varepsilon_3$) of ballast with increasing load cycles are shown in Figures 5.23(a) and (b) in a semi-logarithmic scale. These figures reveal that both the geocomposite and woven-geotextile decrease the minor principal strain of recycled ballast effectively, whether dry or wet. In contrast, the geogrid decreases the lateral strain of recycled ballast only slightly. Figure 5.23(b) shows that recycled ballast gives a significantly higher lateral strain ($\varepsilon_3$) compared to fresh ballast in saturated condition. The test results also indicate that the minor principal strain of recycled ballast decreases appreciably when stabilised with a geocomposite. A decrease in the rate of lateral strain in recycled ballast (i.e. slope) by the use of woven-geotextile or geocomposite is clearly evident (Figure 5.23a). More significantly, recycled ballast stabilised with the geocomposite or woven-geotextile, exhibited lateral strain ($\varepsilon_3$) less than the fresh ballast (without any geosynthetics) at a higher number of load cycles. This has significant bearing in the maintenance of rail tracks. The reduction in the lateral movement of ballast with the inclusion of geocomposite decreases the need for additional layers of crib and shoulder ballast during maintenance operation.
In order to quantify particle breakage under cyclic loading, the load bearing ballast layer was isolated from the crib ballast and capping layer by thin loose geotextiles placed above and below the ballast layer. These loose geotextiles did not resist any lateral movement, they were separators only, and useful in the recovery of complete ballast specimens at the end of testing. Each specimen was sieved before and after the test and
changes in ballast grading were recorded. Figure 5.24 shows the change in the particle size distribution in a conventional gradation plot. Only one specimen data (dry recycled ballast) is shown in this figure for clarity.

![Conventional plot of particle size distribution of ballast before and after test](image)

**Figure 5.24.** Conventional plot of particle size distribution of ballast before and after test

Since small changes in particle size cannot be illustrated clearly in the conventional gradation plots (Figure 5.24), an alternative method was adopted, as explained earlier in Section 5.2.3. Figures 5.25(a) and (b) show the variations of $\Delta W_k$ with sieve size for the dry and wet specimens, respectively.
Figure 5.25. Change in particle size of ballast under cyclic loading, (a) in dry condition, and (b) in wet condition.

Figure 5.25 indicates that recycled ballast alone suffers higher particle breakage than fresh ballast, either wet or dry. Use of geosynthetics decreased the degradation of recycled ballast almost to that for fresh ballast without any geosynthetics. It is also clear from Figure 5.25 that larger particles are more vulnerable to breakage. This observation is in agreement with the lower tensile strength of larger grains found in single particle crushing tests (see Figure 5.18). The breakage indices $B_g$, (Marsal, 1967) of ballast with and without inclusion of geosynthetics are shown in Table 5.2 and Figure 5.26 for comparison.
Table 5.2: Breakage indices of ballast under cyclic loading

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Type of Ballast</th>
<th>Type of Geosynthetics Used</th>
<th>Test Condition</th>
<th>$B_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FB(dry)</td>
<td>Fresh Ballast</td>
<td>-</td>
<td>Dry</td>
<td>1.50</td>
</tr>
<tr>
<td>FB(wet)</td>
<td>Fresh Ballast</td>
<td>-</td>
<td>Wet</td>
<td>1.63</td>
</tr>
<tr>
<td>RB(dry)</td>
<td>Recycled Ballast</td>
<td>-</td>
<td>Dry</td>
<td>2.96</td>
</tr>
<tr>
<td>RB(wet)</td>
<td>Recycled Ballast</td>
<td>-</td>
<td>Wet</td>
<td>3.19</td>
</tr>
<tr>
<td>RB-GD(dry)</td>
<td>Recycled Ballast</td>
<td>Geogrid</td>
<td>Dry</td>
<td>1.70</td>
</tr>
<tr>
<td>RB-GD(wet)</td>
<td>Recycled Ballast</td>
<td>Geogrid</td>
<td>Wet</td>
<td>1.88</td>
</tr>
<tr>
<td>RB-GT(dry)</td>
<td>Recycled Ballast</td>
<td>Woven Geotextile</td>
<td>Dry</td>
<td>1.56</td>
</tr>
<tr>
<td>RB-GT(wet)</td>
<td>Recycled Ballast</td>
<td>Woven Geotextile</td>
<td>Wet</td>
<td>1.64</td>
</tr>
<tr>
<td>RB-GCOM(dry)</td>
<td>Recycled Ballast</td>
<td>Geocomposite</td>
<td>Dry</td>
<td>1.54</td>
</tr>
<tr>
<td>RB-GCOM(wet)</td>
<td>Recycled Ballast</td>
<td>Geocomposite</td>
<td>Wet</td>
<td>1.60</td>
</tr>
</tbody>
</table>

Figure 5.26. Particle breakage of ballast with and without geosynthetics

It may be concluded from Table 5.2 and Figure 5.26 that particle breakage in recycled ballast is 95-97% higher than the fresh ballast. Saturation increases ballast degradation slightly (about 8%). Geosynthetics (either geogrid, woven-geotextile or geocomposite) decrease the breakage of recycled ballast by 40-48%, which means its breakage index ($B_g$) comes down close to the value of fresh ballast without any geosynthetics.
5.5 REPEATED LOAD TEST RESULTS

The stress-strain plots of the repeated load test performed on dry, recycled ballast specimen in the prismoidal triaxial chamber at different intervals of cyclic loading are shown in Figure 5.27. Before the cyclic load was applied, the stiffness of the recycled ballast was relatively low. This is because the ballast was relatively loose (initial bulk unit weight = 15.3 kN/m$^3$) at the beginning of loading. With an increase in load and associated deformation during the first cycle of repeated load (Figure 5.27), the aggregates could re-arrange themselves, therefore, the void ratio decreased (ballast compacted), which resulted in higher stiffness.

![Stress-strain plots in repeated load test at various stages of cyclic loading (after Indraratna and Salim, 2003)](image)

The unloading path (Figure 5.27) indicates a non-linear resilient behaviour with some strain recovery, while the plastic strain remains significant when the unloading was completed. The reloading path apparently becomes almost linear with increasing strain, while the subsequent unloading path remains non-linear. Each loading-unloading path
generates a hysteresis loop. The area covered in the loop represents the amount of energy dissipated during that loading-unloading stage. Figure 5.27 also indicates that during the initial stage of cyclic loading (cycles 1-5), the mean slope of the hysteresis loop increases rapidly with the higher number of cycles. This confirms that the resilient modulus of ballast increases with the increase in load repetition. As the load cycle increases, the resilient modulus increases further (Figure 5.27), as a result of cyclic shakedown and densification (Festag and Katzenback, 2001; Suiker, 2002).
CHAPTER 6
A NEW CONSTITUTIVE MODEL FOR BALLAST

6.1 INTRODUCTION

Researchers and practicing engineers have long recognised that the ballast bed accumulates plastic deformation under cyclic loading. Despite this, little or no effort has been made to develop realistic constitutive stress-strain relationships, particularly modelling plastic deformation and particle degradation of ballast under cyclic loading. Several researchers attempted to model the constitutive behaviour of soils and granular aggregates under monotonic loading (e.g. Roscoe et al., 1963; Schofield and Wroth, 1968; Lade, 1977; Pender, 1978), and various approaches were made to simulate the cyclic response of granular media. Some are quite innovative and successful to a limited extent. Nevertheless, constitutive modelling of geomaterials under cyclic loading still remains a challenging task.

In case of railway ballast, the progressive change in particle geometry due to internal attrition, grinding, splitting and crushing (i.e. degradation) under cyclic traffic loads, further complicates the stress-strain relationship. There is a lack of constitutive model, which includes the effect of particle breakage during shearing. In this study, a new stress-strain and particle breakage model has been developed, first for monotonic loading, and then extended for the more complex cyclic loading. In the following Sections, modelling of particle breakage and formulation of the new stress-strain relationships for monotonic and cyclic loadings are described in detail.
6.2 MODELLING OF PARTICLE BREAKAGE

Since triaxial testing is considered to be one of the most versatile and useful laboratory methods for evaluating the fundamental strength and deformation properties of geomaterials, a triaxial specimen was considered as the basis for developing a relationship between the stress, strain and particle breakage. The axisymmetric triaxial specimen has one advantage that two of its principal stresses (and also strains) are equal, which reduces the number of independent stress-strain parameters governing shear deformation. Figure 6.1(a) shows an axisymmetric triaxial specimen of ballast subjected to drained compression loading, while Figure 6.1(b) shows the details of contact forces and the relative deformation between two typical particles of the specimen in an enlarged scale.

Figure 6.1. Triaxial compression of ballast, (a) specimen under stresses and saw-tooth deformation model, (b) details of contact forces and deformations of two particles at contact (after Indraratna and Salim, 2002)
The vertical force $F_{1i}$, and the horizontal force $F_{3i}$, are acting at contact $i$ between the two particles, which are sliding relative to each other under the applied stresses (major effective principal stress $\sigma_1^\prime$, and minor effective principal stress $\sigma_3^\prime$). It is assumed that the sliding plane makes an angle of $\beta_i$ with the major principal stress, $\sigma_1^\prime$ (Figure 6.1a).

If $N_i$ and $S_i$ are the normal force and shear resistance, respectively, then by resolving the forces $F_{1i}$ and $F_{3i}$, it can be shown that:

\begin{align*}
N_i &= F_{ui} \sin \beta_i + F_{3i} \cos \beta_i \\
S_i &= F_{ui} \cos \beta_i - F_{3i} \sin \beta_i
\end{align*}

(6.1) (6.2)

Assuming no cohesion (i.e. $c = 0$) between the ballast particles, the shear resistance $S_i$, can be expressed by the Mohr-Coulomb theory, as given by:

$$S_i = N_i \tan \phi_u$$

(6.3)

where, $\phi_u$ is the friction angle between the two particles. Assuming $\delta u_i$ is the incremental displacement at contact $i$ in the direction of sliding, the horizontal and vertical displacement components $\delta x_i$ and $\delta y_i$, can be expressed as:

\begin{align*}
\delta x_i &= \delta u_i \sin \beta_i \\
\delta y_i &= \delta u_i \cos \beta_i \\
\delta x_i &= \delta y_i \tan \beta_i
\end{align*}

(6.4) (6.5) (6.6)

If any particle breakage is accompanied by sliding during shear deformation, it is reasonable to assume that the total work done by the applied forces $F_{1i}$ and $F_{3i}$ at the contact $i$, is spent on overcoming frictional resistance and particle breakage, hence:

$$F_{ui} \delta y_i - F_{ui} \delta x_i = N_i \tan \phi_u \delta u_i + \delta E_{bi}$$

(6.7)

where, $\delta E_{bi}$ is the incremental energy spent on particle breakage at contact $i$ due to the
deformation $\delta u_i$. The energy term $(F_{3i}\delta x_i)$ on the left hand side of Equation 6.7 is shown to be negative due to the fact that the direction of the displacement component $\delta x_i$ is opposite to the direction of applied force $F_{3i}$.

Substituting Equations 6.1, 6.5 and 6.6 into Equation 6.7 gives:

$$F_{3i}\delta y_i - F_{3i}\delta y_i \tan \beta_i = F_{3i}\delta y_i \tan \beta_i \tan \phi_\mu + F_{3i}\delta y_i \tan \phi_\mu + \delta E_{\beta_i}$$

(6.8)

If the average number of contacts per unit length in the directions of three principal stresses $\sigma'_1$, $\sigma'_2$ and $\sigma'_3$ are denoted by $n_1$, $n_2$ and $n_3$, respectively, then the average contact forces and the vertical displacement component can be expressed as:

$$F_{3i} = \frac{\sigma'_i}{n_2 n_3}$$

(6.9)

$$F_{3i} = \frac{\sigma'_3}{n_1 n_2}$$

(6.10)

$$\delta y_i = \frac{\delta \epsilon_i}{n_i}$$

(6.11)

where, $\delta \epsilon_1$ is the major principal strain increment.

Replacing Equations 6.9-6.11 into Equation 6.8 gives:

$$\left(\frac{\sigma'_1}{n_2 n_3}\right)\left(\frac{\delta \epsilon_i}{n_i}\right) - \left(\frac{\sigma'_3}{n_1 n_2}\right)\left(\frac{\delta \epsilon_i}{n_i}\right) \tan \beta_i = \left(\frac{\sigma'_1}{n_2 n_3}\right)\left(\frac{\delta \epsilon_i}{n_i}\right) \tan \beta_i \tan \phi_\mu + \left(\frac{\sigma'_3}{n_1 n_2}\right)\left(\frac{\delta \epsilon_i}{n_i}\right) \tan \phi_\mu + \delta E_{\beta_i}$$

(6.12)

Multiplying both sides by $n_1 n_2 n_3$ gives:

$$\sigma'_1 \delta \epsilon_i - \sigma'_3 \delta \epsilon_i \left(\frac{n_1}{n_i}\right) \tan \beta_i = \sigma'_1 \delta \epsilon_i \tan \beta_i \tan \phi_\mu + \sigma'_3 \delta \epsilon_i \left(\frac{n_3}{n_i}\right) \tan \phi_\mu + \delta E_{\beta_i} \left(n_1 n_2 n_3\right)$$

(6.13)

where, the product $n_1 n_2 n_3$ represents the total number of contacts in a unit volume of ballast.
Let $\delta E_b = \delta E_{bi}(n_1n_2n_3)$ represent the incremental energy spent on particle breakage per unit volume of ballast during the strain increment $\delta \varepsilon_1$, and $r_n = (n_3/n_1)$. Then, Equation 6.13 can be re-written as:

$$\sigma'_1\delta \varepsilon_1 - \sigma'_3\delta \varepsilon_1 r_n \tan \beta_i = \sigma'_1\delta \varepsilon_1 \tan \beta_i \tan \phi_\mu + \sigma'_3\delta \varepsilon_1 r_n \tan \phi_\mu + \delta E_b$$  \hspace{1cm} (6.14)

The conventional triaxial stress invariants, $p'$ (mean effective normal stress) and $q$ (deviator stress), are given by:

$$p' = \frac{(\sigma'_1 + 2\sigma'_3)}{3}$$  \hspace{1cm} (6.15)

$$q = q' = \sigma'_1 - \sigma'_3$$  \hspace{1cm} (6.16)

Solving Equations 6.15 and 6.16, the stresses $\sigma'_1$ and $\sigma'_3$ can be written as:

$$\sigma'_1 = p' + \frac{2q}{3}$$  \hspace{1cm} (6.17)

$$\sigma'_3 = p' - \frac{q}{3}$$  \hspace{1cm} (6.18)

Substituting Equations 6.17 and 6.18 into Equation 6.14 gives:

$$\left( p' + \frac{2q}{3} \right) \delta \varepsilon_1 - \left( p' - \frac{q}{3} \right) \delta \varepsilon_1 r_n \tan \beta_i = \left( p' + \frac{2q}{3} \right) \delta \varepsilon_1 \tan \beta_i \tan \phi_\mu + \left( p' - \frac{q}{3} \right) \delta \varepsilon_1 r_n \tan \phi_\mu + \delta E_b$$  \hspace{1cm} (6.19)

Re-arranging Equation 6.19, the deviator stress ratio becomes:

$$\frac{q}{p'} = \frac{r_n \tan (\beta_i + \phi_\mu) - 1}{\left[ \frac{2}{3} + \frac{1}{3} r_n \tan (\beta_i + \phi_\mu) \right]^2} + \frac{\delta E_b}{p' \delta \varepsilon_1 \left[ \frac{2}{3} + \frac{1}{3} r_n \tan (\beta_i + \phi_\mu) \right] \left[ 1 - \tan \beta_i \tan \phi_\mu \right]}$$  \hspace{1cm} (6.20)
In case of infinitesimal increments (e.g. $\delta \varepsilon_1 \to 0$), the major principal strain increment $\delta \varepsilon_1$, should be replaced by the differential increment $d\varepsilon_1$, and similarly the other finite increments $\delta E_B$, $\delta y_i$ and $\delta x_i$ should be substituted by the corresponding differentials $dE_B$, $dy_i$ and $dx_i$, respectively. Thus, for the limiting case ($\delta \varepsilon_1 \to 0$), the term $(\delta E_B/\delta \varepsilon_1)$ on the right hand side of Equation 6.20 becomes the derivative $dE_B/d\varepsilon_1$, which represents the rate of energy consumption due to particle breakage during shear deformation.

Rowe (1962) studied the effect of dilatancy on the friction angle of granular aggregates and concluded that the interparticle friction angle $\phi_{\mu}$ should be replaced by $\phi_f$, which is the friction angle of aggregates after correction for dilatancy. The friction angle $\phi_f$ varies from $\phi_{\mu}$ at very dense state to $\phi_{cv}$ at very loose condition, where deformation takes place at a constant volume. The energy spent on the rearrangement of particles during shearing has been attributed to the difference between $\phi_f$ and $\phi_{\mu}$.

Rowe (1962) also concluded that dense assemblies of cohesionless particles deform in such a way that the minimum rate of internal energy (work) is absorbed in frictional heat. According to this principle, shear deformation occurs in ballast when at each contact $i$, the energy ratio ($ER_i$) of the work done by $F_{1i}$ to that by $F_{3i}$ (i.e., $ER_i = F_{1i}\delta y_i/F_{3i}\delta x_i$) is the minimum. By expanding the expression of $ER_i$ and letting the derivative $d(ER_i)/d\beta_i = 0$, one can determine the critical direction of sliding at contact $i$ (i.e. $\beta_i = \beta_c$) for the minimum energy ratio condition. In other words, $\beta_i = \beta_c$, when $ER_i = ER_{min}$ (minimum energy ratio).

Using the minimum energy ratio principle, Ueng and Chen (2000) showed the following two expressions for the ratio $r_n (= n_3/n_1)$ and the critical sliding angle $\beta_c$:
\[ r_n = \frac{1 - \frac{d\varepsilon_v}{d\varepsilon_1}}{\tan \beta_c} \quad (6.21) \]

\[ \beta_c = 45^\circ - \frac{\phi_f}{2} \quad (6.22) \]

where, \( d\varepsilon_v \) is the volumetric strain increment (compression is taken as positive) of the triaxial specimen corresponding to \( d\varepsilon_1 \).

Substituting Equations 6.21 - 6.22, \( \phi_b \) by \( \phi_f \) and \( \beta_i = \beta_c \) into Equation 6.20, and using the differential increment terms, the deviator stress ratio becomes (Indraratna and Salim, 2002):

\[
q = \frac{\left(1 - \frac{d\varepsilon_v}{d\varepsilon_1}\right)\tan^2\left(45^\circ + \frac{\phi_f}{2}\right) - 1}{\left[\frac{2}{3} + \frac{1}{3} \left(1 - \frac{d\varepsilon_v}{d\varepsilon_1}\right)\tan^2\left(45^\circ + \frac{\phi_f}{2}\right)\right]} + \frac{\left(1 + \sin \phi_f\right)}{\left[\frac{2}{3} + \frac{1}{3} \left(1 - \frac{d\varepsilon_v}{d\varepsilon_1}\right)\tan^2\left(45^\circ + \frac{\phi_f}{2}\right)\right]}
\]

(6.23)

In the current study, \( \phi_f \) is considered as the basic friction angle, which excludes the effects of both dilatancy and particle breakage.

It is interesting to note that Equation 6.23 simplifies to the well-known critical state equation when particle breakage is ignored. In critical state soil mechanics (Schofield and Wroth, 1968), particle breakage during shearing was not taken into account. At the critical state, soil mass deforms continuously at constant stress and constant volume. If the breakage of particles is ignored (i.e. \( dE_b = 0 \)) at the critical state (i.e. \( dp' = dq = d\varepsilon_v = 0 \) and \( \phi_f = \phi_c \)), then Equation 6.23 is reduced to the following critical state relationship:
6.2.1 Evaluation of $\phi$ for Ballast

In order to evaluate the basic friction angle ($\phi$) for the ballast used in this study, the last term of Equation 6.23 containing the energy consumption due to particle breakage was set to zero. The resulting apparent (equivalent) friction angle was denoted by $\phi_{fb}$, which naturally included the contribution of particle breakage but excluded the effect of dilation. Thus, Equation 6.23 was simplified to:

$$\left(\frac{q}{p'}\right)_{cs} = \frac{\tan^2\left(45^\circ + \frac{\phi_{cs}}{2}\right) - 1}{2 + \frac{1}{3}\tan^2\left(45^\circ + \frac{\phi_{cs}}{2}\right)} = \frac{6\sin\phi_{cs}}{3 - \sin\phi_{cs}} = M$$  \hspace{1cm} (6.24)

Using the laboratory experimental results of deviator stress ratio at failure ($q/p'$)$_f$ and corresponding value of $(1 - d\epsilon_v/d\epsilon_1)_f$ into Equation 6.25, the value of $\phi_{fb}$ was easily computed. The calculated values of $\phi_{fb}$ were plotted against the effective confining pressure (Figure 6.2), and also against the rate of particle breakage at failure $(dB_g/d\epsilon_1)_f$, as shown in Figure 6.3. The values of $(dB_g/d\epsilon_1)_f$ for fresh ballast were obtained from the laboratory experimental results (Figure 5.15, Chapter 5).
It is evident from Figure 6.2 that the angle $\phi_b$, increases at a decreasing rate with the increasing confining pressure. At an elevated confining pressure, the degree of particle
breakage is higher (see Figure 5.15, Chapter 5), which means increased energy consumption for higher particle breakage, which is clearly reflected in the increased values of $\phi_b$. Figure 6.3 reveals that $\phi_b$ also increases non-linearly with the rate of particle breakage at failure ($dB_g/d\varepsilon_1$). By extrapolating this relationship back to zero rate of particle breakage [i.e. ($dB_g/d\varepsilon_1$) = 0], the basic friction angle $\phi_f$, excluding the effect of particle breakage, can be estimated. In this study, the value of $\phi_f$ for the fresh ballast was found to be approximately 44° (Figure 6.3) based on the current test results.

### 6.2.2 Contribution of Particle Breakage on Friction Angle

The peak friction angle ($\phi_p$) of ballast and other granular aggregates is conveniently calculated from the triaxial test results of peak principal stress ratio, by re-arranging the Mohr-Coulomb failure criterion, as given in the following relationship:

$$\frac{\sigma'_1}{\sigma'_3} = \frac{1 + \sin \phi_p}{1 - \sin \phi_p}$$

(Equation 6.26)

Equation 6.26 relates the peak friction angle ($\phi_p$) with the peak value of principal stress ratio ($\sigma'_1/\sigma'_3$)$_p$, hence provides an obvious upper bound for the internal friction angle of aggregates. In contrast, the basic friction angle ($\phi_f$) evaluated at zero dilatancy and at zero particle breakage, provides a lower bound (see Figure 6.4), and is considered to be independent of the confining pressure (Indraratna and Salim, 2002). Therefore, the basic friction angle ($\phi_f$) may be considered to be the same as the angle of repose of the material. As explained earlier, the apparent friction angle $\phi_{fb}$, includes the effect of particle breakage, but excludes dilatancy.

Figure 6.4 illustrates the various angles of friction ($\phi_f$, $\phi_{fb}$ and $\phi_p$) computed for fresh ballast, and the values were plotted against increasing confining pressure. This figure
shows that the difference between $\phi_p$ (Equation 6.26) and $\phi_{fb}$ (Equation 6.25) at low confinement is very high because of higher dilatancy. At low stresses, the degree of particle breakage is also low, and therefore, the difference between $\phi_{fb}$ and $\phi_f$ is also small. As confining pressure increases, the difference between $\phi_{fb}$ and $\phi_f$ increases, which is attributed to the higher rate of particle degradation (i.e. increased energy consumption for particle breakage). At increased confining pressure, a higher rate of particle breakage contributes to an increase in friction angle; however, dilatancy is suppressed, and volumetric contraction adversely affects the friction angle. The peak friction angle ($\phi_p$) computed from the laboratory triaxial test results can be viewed as the summation of basic friction angle $\phi_f$, and the effects of dilatancy and particle breakage, as illustrated in Figure 6.4. It was also noted in this figure that the peak friction angle decreased with increasing confining pressure, an observation consistent with the previous studies (Marsal, 1967; Charles and Watts, 1980; Indraratna et al., 1998).

Figure 6.4. Effect of particle breakage and dilatancy on friction angle (after Indraratna and Salim, 2002)
Bolton (1986) studied the strength and dilatancy of sand, and modelled the dilatancy-related component of friction angle ($\phi_{\text{max}} - \phi_{\text{crit}}$), as a function of relative dilatancy index, which depends on the initial density and effective mean stress at failure. Bolton (1986) used the notation $\phi_{\text{crit}}$ to indicate the friction angle at a critical state (i.e. at zero dilation). If the value of $\phi_f$ estimated in Figure 6.3 is considered as the value of $\phi_{\text{crit}}$ for fresh ballast, then Bolton’s model can be used to predict its maximum friction angle ($\phi_{\text{max}}$). The predicted $\phi_{\text{max}}$ can be obtained by adding the dilatancy component to $\phi_{\text{crit}}$. It should be mentioned here that Bolton’s (1986) model does not incorporate particle breakage. While this is acceptable for fine granular media such as sand, where particle breakage may be insignificant, Bolton’s model may not be appropriate for coarser, angular aggregates like ballast, where particle degradation can be significant. Nevertheless, the predicted $\phi_{\text{max}}$ (Bolton, 1986) for fresh ballast is shown in Figure 6.4 for comparison. This figure indicates that Bolton’s model predicts $\phi_{\text{max}}$, which agrees with $\phi_f$ at low confining pressures where particle breakage is small. However, it seems that Bolton’s model overpredicts $\phi_{\text{max}}$ (or dilatancy-related friction component) for ballast at higher confining pressures.

Equation 6.23 explains the mechanism behind the frictional strength of ballast and other granular aggregates, particularly with regard to particle breakage during shearing. It may be helpful to distinguish between the effect of particle breakage, dilatancy, and the basic friction component of shear strength for ballast and other granular media.
6.3 CONSTITUTIVE MODELLING FOR MONOTONIC LOADING

6.3.1 Stress and Strain Parameters

To develop a constitutive stress-strain and particle breakage model in a generalised stress space, a three-dimensional Cartesian coordinate system \((x_i, j=1,2,3)\) was used to define the stress and strains in ballast. Since ballast is a free draining granular medium, all the stresses used in this study were considered to be effective.

For a three-dimensional ballast element under stresses (Figure 6.5), the following stress and strain invariants were used to formulate a relationship between the stress, strain, and particle breakage:

\[
q = \sqrt{\frac{3}{2}} s_{ij} s_{ij} = \frac{1}{2} \left[ (\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 \right] + 3(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2) \quad (6.27)
\]

\[
p = \frac{1}{3} \sigma_{kk} = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}) \quad (6.28)
\]

where, \(q\) is the distortional stress (invariant), \(p\) is the mean effective normal stress (invariant), \(\sigma_{ij}\) is the stress tensor \((i = 1,2,3, \text{ and } j = 1,2,3)\) and \(s_{ij}\) is the stress deviator tensor, as defined below:
Chapter 6

\[ s_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \]  
(6.29)

where, \( \delta_{ij} \) is the Kronecker delta (i.e. \( \delta_{ij} = 1 \) if \( i = j \), and \( \delta_{ij} = 0 \) if \( i \neq j \)). The usual summation convention over the repeated indices was adopted in these notations.

The complementary strain invariants are the distortional strain \( \varepsilon_s \), and volumetric strain \( \varepsilon_v \), respectively, as defined below:

\[
\varepsilon_s = \sqrt{\frac{2}{3} \varepsilon_{ij} \varepsilon_{ij} = \sqrt{\frac{2}{9} \left[ (\varepsilon_{11} - \varepsilon_{22})^2 + (\varepsilon_{22} - \varepsilon_{33})^2 + (\varepsilon_{33} - \varepsilon_{11})^2 \right] + \frac{4}{3} (\varepsilon_{12}^2 + \varepsilon_{23}^2 + \varepsilon_{31}^2)} \]  
(6.30)

\[
\varepsilon_v = \varepsilon_{kk} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \]  
(6.31)

where, \( \varepsilon_{ij} \) is the strain tensor, and \( \varepsilon_{ij} \) is the strain deviator tensor, which is defined as:

\[
\varepsilon_{ij} = \varepsilon_{ij} - \frac{1}{3} \varepsilon_{kk} \delta_{ij} \]  
(6.32)

For the special case of an axisymmetric triaxial specimen (where, \( \sigma_2 = \sigma_3 \) and \( \varepsilon_2 = \varepsilon_3 \)), the above stress and strain invariants simplify to the following well-known functions:

\[
q = \sigma_1 - \sigma_3 \]  
(6.33)

\[
p = \frac{1}{3} (\sigma_1 + 2 \sigma_3) \]  
(6.34)

\[
\varepsilon_s = \frac{2}{3} (\varepsilon_1 - \varepsilon_3) \]  
(6.35)

\[
\varepsilon_v = \varepsilon_1 + 2 \varepsilon_3 \]  
(6.36)

### 6.3.2 Incremental Constitutive Model

In classical soil plasticity, the total strains \( \varepsilon_{ij} \), are usually decomposed into elastic (recoverable) and plastic (irrecoverable) components \( \varepsilon'_{ij} \) and \( \varepsilon''_{ij} \), respectively:
\[ \varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p \]  \hspace{1cm} (6.37)

where, the superscript \( e \) denotes elastic component, and \( p \) indicates plastic component.

Accordingly, the strain increments are also divided into elastic and plastic components:

\[ d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p \]  \hspace{1cm} (6.38)

Similarly, the increments of strain invariants are also separated into elastic and plastic components, as given below:

\[ d\varepsilon_s = d\varepsilon_s^e + d\varepsilon_s^p \]  \hspace{1cm} (6.39)

\[ d\varepsilon_v = d\varepsilon_v^e + d\varepsilon_v^p \]  \hspace{1cm} (6.40)

The elastic components of the strain increment can be computed using the theory of elasticity, where the elastic distortional strain increment \( d\varepsilon_s^e \) is given by:

\[ d\varepsilon_s^e = \frac{dq}{2G} \]  \hspace{1cm} (6.41)

where, \( G \) is the elastic shear modulus.

The elastic volumetric strain increment \( d\varepsilon_v^e \), can be determined using the swelling/recompression constant \( \kappa \), and is given by (Roscoe et al. 1963, Schofield and Wroth, 1968):

\[ d\varepsilon_v^e = \frac{\kappa}{1 + e_i} \left( \frac{dp}{p} \right) \]  \hspace{1cm} (6.42)

where, \( e_i \) is the initial void ratio at the start of shearing.

In developing Equation 6.23 (Section 6.2), the special case of axisymmetric triaxial shearing \( (\sigma_2 = \sigma_3 \text{ and } \varepsilon_2 = \varepsilon_3) \) was considered (see Figure 6.1), and only the plastic
components of strain increment were taken into account. Thus, the strain increments \( d\varepsilon_v \) and \( d\varepsilon_1 \) in Equation 6.23 refer to the plastic strain increments \( d\varepsilon_v^p \) and \( d\varepsilon_1^p \), respectively. Equation 6.23 can be extended to a generalised stress-strain formulation by replacing the principal strain increment with a combination of strain invariants. The principal strain increments of an axisymmetric specimen can easily be replaced with the incremental strain invariants using Equations 6.35 - 6.36; and hence, it can be shown that:

\[
1 - \frac{d\varepsilon_1^p}{d\varepsilon_1^v} = -2 \frac{d\varepsilon_1^p}{d\varepsilon_1^v} = \frac{d\varepsilon_1^p - \frac{2}{3} d\varepsilon_1^v}{d\varepsilon_1^v + \frac{1}{3} d\varepsilon_1^v} \tag{6.43}
\]

Substituting \( d\varepsilon_1 \) by \( d\varepsilon_1^p \), \( d\varepsilon_v \) by \( d\varepsilon_v^p \) and Equation 6.43 into Equation 6.23 gives:

\[
q = \frac{\left( \frac{d\varepsilon_1^p - \frac{2}{3} d\varepsilon_1^v}{d\varepsilon_1^v + \frac{1}{3} d\varepsilon_1^v} \right) \tan^2 \left( 45^\circ + \frac{\phi_f}{2} \right) - 1}{\left( \frac{2}{3} + \frac{1}{3} \frac{d\varepsilon_1^p - \frac{2}{3} d\varepsilon_1^v}{d\varepsilon_1^v + \frac{1}{3} d\varepsilon_1^v} \right) \tan^2 \left( 45^\circ + \frac{\phi_f}{2} \right) + \frac{1}{1 + \sin \phi_f}}
\]

\[
p\left( \frac{d\varepsilon_1^p + \frac{1}{3} d\varepsilon_1^v}{\frac{2}{3} + \frac{1}{3} \frac{d\varepsilon_1^p - \frac{2}{3} d\varepsilon_1^v}{d\varepsilon_1^v + \frac{1}{3} d\varepsilon_1^v} \tan^2 \left( 45^\circ + \frac{\phi_f}{2} \right) + \frac{1}{1 + \sin \phi_f}} \right)
\]

Critical State Line (CSL) and the critical state parameters are often employed in modelling plastic deformations of soils. Critical state parameters are the fundamental properties of a soil including a granular assembly. In case of a granular medium where progressive particle breakage occurs under imparted loading, the critical state line of the aggregates also changes gradually. However, in the current formulation, it was assumed
that the critical state line of ballast remains unchanged (i.e. fixed) in the $p$-$q$-$e$ space. Considering the small change in particle size distribution after testing (see Figures 5.12 and 5.24), it is expected that the change in critical state line during loading would be small and the errors in model computation resulting from this simplified assumption will be negligible.

Using the critical state friction ratio $M = 6 \sin \phi_f / (3 - \sin \phi_f)$, it can be shown that,

$$\tan^2 \left( \frac{45^\circ + \phi_f}{2} \right) = \frac{1 + \sin \phi_f}{1 - \sin \phi_f} = \frac{3 + 2M}{3 - M}$$

(6.45)

$$1 + \sin \phi_f = \frac{6 + 4M}{6 + M}$$

(6.46)

Replacing Equations 6.45 - 6.46 and $q/p = \eta$ (stress ratio) into Equation 6.44 gives:

$$\eta \left[ \frac{2}{3} + \frac{1}{3} \left( \frac{d e_v^p - \frac{2}{3} d e_v^p}{d e_v^p + \frac{1}{3} d e_v^p} \right) \left( \frac{3 + 2M}{3 - M} \right) \right] = \left( \frac{d e_v^p - \frac{2}{3} d e_v^p}{d e_v^p + \frac{1}{3} d e_v^p} \right) \left( \frac{3 + 2M}{3 - M} \right) - 1 + \frac{d E_B}{p \left( d e_v^p + \frac{1}{3} d e_v^p \right)} \left( \frac{6 + 4M}{6 + M} \right)$$

(6.47)

Re-arrangement of Equation 6.47 gives:

$$\frac{\eta}{3} \left[ 9 d e_v^p - 2M d e_v^p \right] = -3 d e_v^p + 3M d e_v^p - M d e_v^p + \frac{d E_B}{p} \left[ \frac{(3 - M)(6 + 4M)}{6 + M} \right]$$

(6.48)

Equation 6.48 can be further re-arranged to give the ratio between the plastic volumetric and distortional strain increments, as given below:

$$\frac{d e_v^p}{d e_s^p} = \frac{9(M - \eta)}{9 + 3M - 2\eta M} + \frac{d E_B}{p d e_s^p} \left( \frac{9 - 3M}{9 + 3M - 2\eta M} \right) \left( \frac{6 + 4M}{6 + M} \right)$$

(6.49)
Equation 6.49 captures: (a) plastic volumetric strain increment associated with the plastic distortional strain increment, and (b) corresponding energy consumption for particle breakage during shear deformation. It is relevant to note here that Equation 6.49 becomes undefined when $d\varepsilon_p^p$ becomes zero under isotropic stress condition; hence, it is only valid for shearing where stresses are anisotropic. In Equation 6.49, the rate of energy consumption per unit volume of ballast ($dE_B/d\varepsilon^p_1$) must be determined first. The incremental energy consumption due to particle breakage per unit volume $dE_B$ (Equation 6.49), can be related to the increment of breakage index $dB_g$, where the breakage index can be measured in the laboratory, as explained earlier.

![Figure 6.6. Relationship between the rate of energy consumption and rate of particle breakage (after Salim and Indraratna, 2004)](image)

The experimental values of $(q/p')$, $(1-d\varepsilon_v/d\varepsilon_1)$, and the basic friction angle of fresh ballast ($\phi_f$) were substituted into Equation 6.23, and the values of $dE_B/d\varepsilon_1$ were then back calculated. From the experimental results (Figure 5.15, Chapter 5), the rates of particle breakage $dB_g/d\varepsilon_1$, at various axial strains and confining pressures, were
determined. The computed \( \frac{dE_B}{d\varepsilon_1} \) values were then plotted against these experimental \( \frac{dB_g}{d\varepsilon_1} \) values, as shown in Figure 6.6. This figure indicates that \( \frac{dE_B}{d\varepsilon_1} \) and \( \frac{dB_g}{d\varepsilon_1} \) are linearly related to each other. Therefore, it may be assumed that the incremental energy consumption due to particle breakage per unit volume is proportional to the corresponding increment of breakage index (i.e., \( dE_B = \beta dB_g \), where \( \beta \) is a constant of proportionality). Therefore, Equation 6.49 becomes:

\[
\frac{dE_b^p}{d\varepsilon_1^p} = \frac{9(M - \eta)}{9 + 3M - 2\eta M} + \frac{\beta dB_g}{pdc_s} \left( \frac{9 - 3M}{9 + 3M - 2\eta M} \right) \left( \frac{6 + 4M}{6 + M} \right)
\]

The experimental data of Figure 5.15 (Chapter 5) were re-plotted as \( B_g \) versus distortional strain \( \varepsilon_s \), as shown in Figure 6.7. These breakage data were also re-plotted in a modified scale as \( ln \{ p_{cs(i)}/p_{(i)} \} B_g \) versus \( \varepsilon_s \), as shown in Figure 6.8, where, \( p_{cs} \) is the value of \( p \) on the critical state line at the current void ratio, and the subscript \( (i) \) indicates the initial value at the start of shearing. The definition of \( p_{cs} \) is illustrated in Figure 6.9 for clarity. Figure 6.8 shows that the wide variations of \( B_g \) values (see Figure 6.7) due to varying confining pressures are practically eliminated in this technique, and all breakage data fall close to a single line (non-linear). Thus, the breakage of particles under triaxial shearing may be represented by a single non-linear function, as given by:

\[
B_g = \frac{\theta \left[ 1 - \exp(-\nu \varepsilon_s) \right]}{\ln \left( \frac{p_{cs(i)}}{p_{(i)}} \right)}
\]

where, \( \theta \) and \( \nu \) are two material constants relating to the breakage of ballast.
Figure 6.7. Variation of particle breakage of fresh ballast with distortional strain and confining pressure (re-plotted from Figure 5.15)

Figure 6.8. Modelling of ballast breakage during triaxial shearing (after Salim and Indraratna, 2004)
The values of $dB_g/d\varepsilon_p$ at various distortional strains and confining pressures can be obtained easily from Figure 6.7. These breakage rates were then plotted as $\ln\{p_{cs(i)}/p_{(i)}\} dB_g/d\varepsilon_p$ versus $(M-\eta^*)$, as shown in Figure 6.10, where $\eta^* = \eta(p/p_{cs})$.

Figure 6.10 indicates that the values of $ln\{p_{cs(i)}/p_{(i)}\} dB_g/d\varepsilon_p$ are related with $(M-\eta^*)$ linearly, irrespective of the confining pressures. Thus, a linear relationship between the
The rate of particle breakage \( (dB_g/dε^p) \) and \((M-η^*)\) is proposed, as given below:

\[
\frac{dB_g}{dε^p} = \chi + \mu(M - η^*) \ln \left( \frac{P_{cr(i)}}{P_{(i)}} \right)
\]

(6.52)

where, \( χ \) and \( μ \) are two material constants relating to the rate of ballast breakage (see Figure 6.10).

Substituting Equation 6.52 into Equation 6.50 gives:

\[
\frac{dε^p}{dε^p} = \frac{9(M - η)}{9 + 3M - 2ηM} + \left( \frac{β}{p} \right) \left[ \chi + μ(M - η^*) \ln \left( \frac{P_{cr(i)}}{P_{(i)}} \right) \right]
\]

\[
\left( \frac{9 - 3M}{9 + 3M - 2ηM} \right) \left( \frac{6 + 4M}{6 + M} \right)
\]

(6.53)

Equation 6.53 can be represented by:

\[
\frac{dε^p}{dε^p} = \frac{9(M - η)}{9 + 3M - 2ηM} + \left( \frac{B}{p} \right) \left[ \chi + μ(M - η^*) \right]
\]

(6.54)

where,

\[
B = \frac{β}{\ln \left( \frac{P_{cr(i)}}{P_{(i)}} \right)} \left[ \frac{9 - 3M}{6 + M} \right] = \text{constant}
\]

(6.55)

Equation 6.54 is the governing differential equation for the plastic strain increment incorporating particle breakage. The plastic components of strain increment can be computed by employing Equation 6.54 along with the general incremental constitutive relationship given by Hill (1950):
\[ d\varepsilon^p_{ij} = h \frac{\partial g}{\partial \sigma_{ij}} df \]  

(6.56)

where, \( h \) is a hardening function, \( g \) is a plastic potential function, and \( df \) is the differential of a function \( f = 0 \) that defines yield locus.

The plastic potential, \( g \)

Equation 6.56 can be employed to express the plastic volumetric and distortional strain increments, and it can be shown that:

\[ \frac{d\varepsilon^p_v}{d\varepsilon^p_s} = \frac{\partial g}{\partial p} \quad \frac{d\varepsilon^p_s}{d\varepsilon^p_v} = \frac{\partial g}{\partial q} \]  

(6.57)

By definition, the plastic strain increment vector is normal to the plastic potential surface. Thus, at any point \((p, q)\) on the plastic potential \( g = g(p, q) \),

\[ \frac{d\varepsilon^p_v}{d\varepsilon^p_s} = -\frac{dq}{dp} \]  

(6.58)

Substituting Equation 6.58 into Equation 6.54 gives:

\[ -\frac{dq}{dp} = \frac{9(M - \eta)}{9 + 3M - 2\eta M} + \left( \frac{B}{p} \right) \left[ \chi + \mu(M - \eta^*) \right] \]  

(6.59)

Equation 6.59 can be re-written in the following form:

\[ \frac{dq}{dp} + \frac{9(Mp - q) + B\{\chi + \mu(M - q / p_{eq})\}}{(9 + 3M)p - 2qM} = 0 \]  

(6.60)
This is a first-order linear differential equation of \( q \). The solution of Equation 6.60 gives the plastic potential function \( g(p, q) \). It is pertinent to mention here that it only requires partial derivatives of \( g \) with respect to \( p \) and \( q \), rather than the explicit function of \( g \), to derive expressions for the plastic strain increments. Since Equation 6.60 is linear in \( q \),

\[
\frac{\partial g}{\partial q} = 1
\]  

\[
\frac{\partial g}{\partial p} = \frac{9(Mp - q) + B\left(\chi + \mu(M - q/p_{cs})\right)}{(9 + 3M)p - 2qM} = \frac{9(M - \eta) + (B/p)\left(\chi + \mu(M - \eta^*)\right)}{9 + 3M - 2\eta M}
\]  

The derivation technique of Equations 6.61 and 6.62 from the differential equation (Equation 6.60) is shown in Appendix A, based on a simple example. It is relevant to note that substitution of Equations 6.61 and 6.62 into Equation 6.57 satisfies the governing differential equation (Equation 6.54).

To formulate the yield and hardening functions, the following assumption and postulates were made with regard to railway ballast:

**Assumption:** As shear deformation increases, ballast moves towards its critical state.

The critical state was defined earlier in Section 3.3.1 (Chapter 3). It is commonly assumed in the critical state soil mechanics that the projections of critical state line on \( e - \ln p \) and \( p-q \) planes are straight lines, which are also implied in the current formulation. Indraratna and Salim (2001) presented experimental evidence that ballast, like other soils, moves towards a common (critical) state as the shear deformation increases, irrespective of initial states and confining pressures (see Figures 5.16 - 5.17, Chapter 5).
**Postulate 1:** Ballast deforms plastically, if and only when there is a change in stress ratio, $q/p (= \eta)$.

A hypothesis similar to the above postulate was made by Pender (1978) for overconsolidated soils. The implication of this postulate is that it specifies the yield function $f$, for ballast. Within the common range of stresses ($< 1$ MPa) encountered in railway tracks, Postulate 1 is only valid for time-independent situations (i.e. no creep effects).

Plastic deformation occurs in ballast resulting from grain slippage, particle rolling, grain attrition, fracture and crushing, and the resulting rearrangement of particles. Under isotropic stress (i.e. $q = 0$, $\eta = 0$), it is believed that the above mentioned mechanisms of grain rearrangement are insignificant in coarse aggregates like ballast, hence no apparent plastic deformation (Salim and Indraratna, 2004). However, a small increase in stress ratio (and corresponding distortional stress, $q$) brings the ballast specimen closer to its critical state, activates the grain rearrangement mechanisms, and leads to incremental shear distortion (irrecoverable) and associated plastic volumetric strain. It is believed that under stress levels approaching the crushing strength of aggregates, time-dependent (creep) effects will also lead to additional particle breakage and associated plastic deformation. At very high values of $p$ where the grains may crush and even pulverise, Postulate 1 needs to be modified to incorporate a capped-type yield surface, which is more appropriate for clays and sands. However, within the scope of this study, creep has not been incorporated, rather the research has focussed on ballast deformation and particle breakage alone under imparted loading.
In the current model (non-capped), the yield loci are represented by constant stress ratio ($\eta = \text{constant}$) lines in the $p-q$ plane (Figure 6.11). The yield locus moves kinematically along with its current stress ratio as the stress changes. Mathematically, the yield function $f$, specifying the yield locus for the current stress ratio $\eta_j$, is expressed by Pender (1978) as:

$$f = q - \eta_j, p = 0$$

(6.63)

Figure 6.11. Yield loci represented by constant stress ratio lines in $p, q$ plane (after Salim and Indraratna, 2004)

Figure 6.12 shows the direction of plastic strain vectors (Equation 6.49) for different yield loci. Each plastic strain increment vector can be separated into a volumetric component and a distortional component, as mentioned earlier. It is usually assumed that the plastic distortional strain increment ($d\varepsilon_p^p$) is positive when $d\eta$ is positive. If the effect of particle breakage on the direction of plastic strain increment is small and $d\varepsilon_p^p$ is positive, then according to Equation 6.49, the plastic volumetric strain increment will be either positive, zero, or negative, depending primarily on the sign of the term ($M-\eta$), i.e. on the position of current yield locus relative to the critical state line (CSL) in the $p-q$
plane. If $\eta < M$ (i.e. the current yield locus is below the CSL), the direction of the plastic strain increment will be such that its volumetric component becomes positive (i.e. contraction, see Figure 6.12). In contrast, if $\eta > M$ (i.e. the current yield locus is above the CSL), the increment of plastic volumetric strain will be negative (i.e. dilation). Thus, the $p-q$ plane is considered to be divided into two distinct regimes by the CSL. The area above the CSL is the plastic dilation regime, and the area below the CSL is the plastic contraction regime (Figure 6.12).

![Figure 6.12. Plastic strain increment vectors for different yield loci (after Salim and Indraratna, 2004)](image)

Differentiating Equation 6.63, and substituting $dq = \eta_j dp + pd\eta$,

$$df = dq - \eta_j dp = pd\eta$$  \hspace{1cm} (6.64)

The hardening function ($h$) is formulated based on an undrained stress path where the volumetric strain remains zero. The second postulate is made regarding the shape of the undrained stress path, as given below.
Postulate 2: The undrained stress paths are parabolic in the $p$-$q$ plane and are expressed by the following relationship (Pender, 1978):

$$
\left( \frac{\eta}{M} \right)^2 = \frac{p_{cs}}{p} \left[ 1 - \frac{P_o}{P} \right] \left[ 1 - \frac{P_o}{P_{cs}} \right]^{-1}
$$

(6.65)

where, $p_{cs}$ is the value of $p$ on the critical state line corresponding to the current void ratio, as illustrated in Figure 6.9. Thus, $p_{cs} = \exp\{(\Gamma - e)/\lambda_{cs}\}$, $\Gamma = $ void ratio on the CSL at $p = 1$, and $\lambda_{cs}$ is the slope of the projection of CSL on the $e - \ln p$ plane.

$p_o$ is the value of $p$ at the intersection of the undrained stress path with the initial stress ratio line.

Figure 6.13 shows the parabolic undrained stress paths (Equation 6.65) in $q/p_{cs}$ and $p/p_{cs}$ plane. In this figure, $p$ and $q$ were normalised by $p_{cs}$. No undrained test on ballast was carried out in this study to verify Postulate 2. However, previous experimental results reported by various researchers (e.g. Roscoe et al., 1963; Ishihara et al., 1975) indicate that undrained stress paths may be reasonably approximated by parabolic curves. If an undrained shearing (compression) starts from an initial stress of $p/p_{cs}$ less than 1, the stress path will move towards the right (i.e. towards $p/p_{cs} = 1$ at the critical state) following Equation 6.65. In contrast, the stress path will move towards the left (i.e. towards the critical state point), if undrained compression starts from an initial stress of $p/p_{cs}$ greater than 1 (which is very unlikely for ballast).
Hardening function, $h$

A hardening function was derived based on the undrained stress path, where the total volume change of a specimen is constrained to zero. Schofield and Wroth (1968) explained that although the total volumetric strain in an undrained shearing is zero, there is an elastic (recoverable) volumetric strain increment associated with an increase in $p$, and an equal and opposite plastic volumetric strain component counters the elastic volumetric strain increment. Thus, in an undrained shearing,

$$de_v^e + de_v^p = d\varepsilon_v = 0$$ (6.66)

Substituting Equation 6.42 into Equation 6.66, and writing an expression for the plastic volumetric strain increment following Equation 6.56, it can be shown that:

$$\frac{\hat{h}}{\hat{\tilde{g}}} \frac{\partial g}{\partial p} p d\eta + \frac{\kappa dp}{p(1 + e_v)} = 0$$ (6.67)

Differentiating Equation 6.65, an alternative differential form of the undrained stress path is obtained:
Substituting Equations 6.62 and 6.68 into Equation 6.67, and re-arranging, the hardening function becomes:

$$h = \frac{2\kappa \left( \frac{p_o}{p_{cs}} - 1 \right) (9 + 3M - 2\eta \mu \eta)}{M^2 (1 + e_i) \left( \frac{2p_o}{p} - 1 \right) p_{cs} \left[ 9(M - \eta) + \frac{B}{p} \left( \chi + \mu(M - \eta^*) \right) \right]}$$  \hspace{1cm} (6.69)$$

It should be mentioned here that the above hardening function (Equation 6.69) clearly depends on $p$, $p_o$ and $p_{cs}$, besides other parameters. The parameter $p$ represents the current mean stress, while $p_{cs}$ is the image of current void ratio in terms of stress on the critical state line (see Figure 6.9). Thus, the hardening function (Equation 6.69) correctly incorporates the effect of current void ratio (or density) relative to the critical state void ratio. The above expression of hardening function $h$, gives a positive value if a ballast specimen is in a state looser than the critical (i.e. $p_o > p_{cs}$). In the normal range of stresses, ballast and other coarse aggregates remain in states denser than the critical (i.e. $p_o < p_{cs}$), and therefore, the sign of the hardening function (Equation 6.69) should be reversed (Salim and Indraratna, 2004). Substituting Equation 6.69 with a negative sign and also Equations 6.61 and 6.64 into Equation 6.56, the plastic distortional strain increment becomes:

$$d{\varepsilon}_p^p = \frac{2\kappa \left( \frac{p}{p_{cs}} - 1 \right) \eta d\eta}{M^2 (1 + e_i) \left( \frac{2p_o}{p} - 1 \right) p_{cs} \left[ 9(M - \eta) + \frac{B}{p} \left( \chi + \mu(M - \eta^*) \right) \right]}$$  \hspace{1cm} (6.70)$$
Equation 6.70 is based on the strain hardening function derived from an undrained stress path where both $p_o$ and $p_{cs}$ remain constant throughout. Therefore, the factor $(1-p_o/p_{cs})$ in the numerator remains a constant during an undrained test and may be considered as a function of the initial state of ballast at the start of shearing. In a drained shearing, the value of $p_{cs}$ varies as the void ratio ($e$) changes. The parameter $p_o$ is re-defined for a drained test as the value of $p$ at the intersection of the initial stress ratio line with an imaginary undrained stress path, which passes through the current stress ($p$, $q$) point and current ($p_{cs}$, $M_{p_{cs}}$) point corresponding to the current void ratio (Pender 1978). This definition of $p_o$ in a drained test is graphically illustrated in Figure 6.14.

![Figure 6.14. Definition of $p_o$ in a drained shearing (after Pender, 1978)](image)

Since the void ratio ($e$) varies during drained shearing, the corresponding $p_{cs}$ (see Figure 6.9) changes, as mentioned earlier. Therefore, the imaginary undrained stress path (Equation 6.65), which is a function of $p_{cs}$, also varies during drained test, resulting in a variable $p_o$ value (see Figure 6.14). For drained shearing, the plastic distortional strain
increment may be expressed by modifying Equation 6.70, as given below:

\[
d e^p_s = \frac{2\alpha k \left( \frac{p}{p_{cs}} \right) \left[ 1 - \frac{p_{o(i)}}{p_{cs(i)}} \right] (9 + 3M - 2\eta M) \eta d\eta}{M^2 \left( 1 + e_i \right) \left\{ \frac{2p_o}{p} - 1 \right\} \left[ 9(M - \eta) + \frac{B}{p} \{ \chi + \mu(M - \eta^*) \} \right]} \tag{6.71}
\]

where, \( \alpha \) is a model constant relating to the initial stiffness of ballast, and \( p_{o(i)} \) and \( p_{cs(i)} \) are the initial values of \( p_o \) and \( p_{cs} \), respectively.

Numerical implementation of the above model indicated that in a stress-controlled computation, as the stress ratio (\( \eta \)) increased and approached close to the value of \( M \) (i.e. \( \eta \approx M \)), the computed plastic distortional strain increment (Equation 6.71) became extremely high because of the small value of the term \( (B/p \{ \chi + \mu(M - \eta^*) \}) \) related to particle breakage. Similarly, in a strain-controlled computation, as the plastic distortional strain increased at a stress ratio (\( \eta \)) close to \( M \), the corresponding increment in stress ratio became very small (close to zero), and the resulting total stress ratio practically remained the same as its value. Thus, it is clear that Equation 6.71 doesn’t allow the stress ratio to exceed \( M \). However, experimental results of ballast indicate that the stress ratio exceeds \( M \) at low confining pressure (see Figures 5.4, 5.5 and 5.16, Chapter 5). To capture these experimental observations where the stress ratio (\( \eta \)) may exceed the value of \( M \) at low confinement, the following modifications to Equations 6.71 and 6.54 are proposed (Salim and Indraratna, 2004):

\[
d e^p_s = \frac{2\alpha k \left( \frac{p}{p_{cs}} \right) \left[ 1 - \frac{p_{o(i)}}{p_{cs(i)}} \right] (9 + 3M - 2\eta M) \eta d\eta}{M^2 \left( 1 + e_i \right) \left\{ \frac{2p_o}{p} - 1 \right\} \left[ 9(M - \eta) + \frac{B}{p} \{ \chi + \mu(M - \eta^*) \} \right]} \tag{6.72}
\]

\[
\frac{d e^p_s}{d e^o_s} = \frac{9(M - \eta)}{9 + 3M - 2\eta M} + \left( \frac{B}{p} \right) \left[ \chi + \mu(M - \eta^*) \right] \tag{6.73}
\]
The term \((M - \eta^*)\) in the denominator of Equation 6.72 will now vary from a positive value to zero as the distortional strain increases. The stress ratio \(\eta\) may increase to a value equal to or higher than \(M\) (at small strain), but the value of \((M - \eta^*)\) remains substantially greater than zero, providing an acceptable value of \(d\varepsilon^p_s\).

It is necessary to conduct a strain-controlled computation to predict the post-peak behaviour of ballast. For the strain-controlled prediction, Equation 6.72 can be re-written in the following form:

\[
d\eta = \frac{M^2(1 + e_i)\left(\frac{2p_o}{p} - 1\right)\left[9(M - \eta^*) + \frac{B}{p} \chi + \mu(M - \eta^*)\right]d\varepsilon^p_s}{2\alpha\kappa\left(\frac{P}{P_{cs}}\right)\left[1 - \frac{p_o(i)}{p_{cs(i)}}\right](9 + 3M - 2\eta^* M)\eta}
\]

(6.74)

### 6.4 CONSTITUTIVE MODELLING FOR CYCLIC LOADING

In section 6.3, a constitutive model for ballast incorporating particle breakage has been presented for monotonic loading, where the shear stress is increased from an isotropic initial stress state (i.e. initial stress ratio is zero). In case of cyclic loading, stress can increase or decrease from any state, isotropic or even anisotropic. Therefore, in order to formulate a constitutive model for cyclic loading, a stress-strain and particle breakage model must be developed first for shearing from an anisotropic initial stress state, where shearing may commence from an initial stress ratio, \(\eta\). The model should cover shearing from both isotropic \((\eta = 0)\) and anisotropic \((\eta \neq 0)\) initial stress states.
6.4.1 Shearing from an Anisotropic Initial Stress State

To extend the above constitutive model (Section 6.3.2) for shearing from an anisotropic initial stress state where the initial stress ratio is \( \eta_i \), the previous Postulate 2 needs to be amended as follows:

**Postulate 2a:** The generalised undrained stress path from an initial stress ratio of \( \eta_i \), is assumed to be parabolic, and is given by:

\[
\left( \frac{\eta - \eta_i}{M - \eta_i} \right)^2 = \frac{p_{cs}}{p} \left[ \frac{1 - \frac{p_o}{p}}{1 - \frac{p_o}{p_{cs}}} \right]
\]

(6.75)

where, \( p_o \) and \( p_{cs} \) are the same as defined earlier.

Postulate 2a is a modified form of a hypothesis proposed by Pender (1978). Differentiating Equation 6.75 with respect to \( p \) and re-arranging gives:

\[
\frac{2(\eta - \eta_i) \left( 1 - \frac{p_o}{p_{cs}} \right) \left( \frac{p}{p_{cs}} \right)}{(M - \eta_i)^3 \left( \frac{2p_o}{p} - 1 \right)} d\eta = \frac{dp}{p}
\]

(6.76)

The plastic potential function \( (g) \) used for shearing from an isotropic initial stress state, is also used for shearing from an anisotropic initial stress state. Substituting Equations 6.62 and 6.76 into Equation 6.67, and solving for the hardening function, it can be shown that:
Substituting Equations 6.61, 6.64 and 6.77 into Equation 6.56, the plastic distortional strain increment can now be written as:

\[
d\varepsilon^p_s = \frac{2\kappa \left( \frac{1}{p_{cs}} \right) \left( \frac{P_o}{p_{cs}} - 1 \right) (9 + 3M - 2\eta M)(\eta - \eta_i)}{(M - \eta_i)^2 (1 + e_i) \left( \frac{2p_o}{p} - 1 \right) \left[ 9(M - \eta) + (B / p)[\chi + \mu(M - \eta^*)] \right]} d\eta
\]  

(6.78)

In Section 6.3.2, it was pointed out that the theoretical formulation of plastic distortional strain increment (Equation 6.71, similar to Equation 6.78) could not predict the stress-strain behaviour of ballast well, especially at low confining pressures, where the theoretical model (Equation 6.71) underpredicted the stress and the shear strength. To capture the experimental observations that the stress ratio \( \eta \) can exceed the critical state value \( M \) at low confining pressures, Equation 6.78 has also been amended with a modified stress ratio \( \eta^* \) (similar to Equation 6.72), and the following modified form of the plastic distortional strain increment is proposed:

\[
d\varepsilon^p_s = \frac{2\alpha \kappa \left( \frac{P}{p_{cs}} \right) \left( 1 - \frac{P_o(i)}{p_{cs(i)}} \right) (9 + 3M - 2\eta^* M)(\eta - \eta_i)}{(M - \eta_i)^2 (1 + e_i) \left( \frac{2p_o}{p} - 1 \right) \left[ 9(M - \eta^*) + (B / p)[\chi + \mu(M - \eta^*)] \right]} d\eta
\]  

(6.79)

where, \( \eta^* = \eta(p/p_{cs}) \), as shown earlier.

The relationship between the plastic volumetric strain increment and plastic distortional strain increment remains the same, as given by Equation 6.73, and particle breakage is
also simulated as before (Equation 6.51). The modified plastic hardening function corresponding to Equation 6.79 is given by:

\[
h = \frac{2\alpha \kappa \left( \frac{1}{P_{cs}} \right) \left( 1 - \frac{P_{o(i)}}{P_{cs(i)}} \right) \left[ 9 + 3M - 2\eta^* M \right] \left( \eta - \eta_i \right)}{(M - \eta_i) \left( 1 + e_i \right) \left( \frac{2P_o}{p} - 1 \right) \left[ 9(M - \eta^*) + (B / p) \left( \chi + \mu(M - \eta^*) \right) \right]}
\]  

(6.80)

### 6.4.2 Cyclic Loading Model

A common shortcoming of many stress-strain constitutive models for geomaterials is that these were developed for specific requirements and applicable only to specific loading conditions. This limitation in constitutive modelling becomes pronounced when an artificial distinction is made between monotonic and cyclic loadings for practical purposes (Dafalias and Herrmann, 1982). In reality, cyclic loading is a sequence of several monotonic ones, a combination of loading, unloading, and reloading. Therefore, the constitutive laws should be based on a more fundamental framework so that they are applicable to all types of loading, whether monotonic, cyclic or any other combination.

The classical theory of plasticity provides such a framework, and significant advances have been made in the past 40 years, especially after the development of the critical state theory (Roscoe et al, 1963; Schofield and Wroth, 1968). These theories can adequately and accurately simulate the deformation response of geomaterials under monotonic loading. However, some important aspects of deformation behaviour, particularly in cyclic loading, cannot be adequately modelled with these theories. One of the main reasons is that in the classical concept of yield surface, there is little flexibility in varying the plastic modulus when the loading directions are changed. This implies a purely elastic stress domain, which is contrary to reality for many
geomaterials (Dafalias and Herrmann, 1982). Therefore, the classical theory of plasticity is unable to simulate, even qualitatively, the accumulation of plastic strains with increasing load cycles.

To overcome these limitations, a new concept of plasticity called ‘bounding surface plasticity’ was introduced by Dafalias and Popov (1975; 1976) and Krieg (1975), as mentioned earlier in Section 3.3.3 (Chapter 3). The salient features of the bounding surface plasticity theory are: (a) plastic deformation may occur for stress changes within the surface, and (b) the possibility of having a very flexible plastic modulus. These are the clear advantages over the classical yield surface plasticity theory.

The most difficult part of constitutive modelling, especially in cyclic loading, is the formulation of proper evolution of hardening modulus, because the memory of particular loading events and progressive cyclic hardening or softening phenomena should be included in the model (Mroz and Norris, 1982). One possibility is to consider a smaller yield surface within a larger bounding surface and vary the plastic (or hardening) modulus depending on the distance of the current stress point relative to its conjugate point on the bounding surface, as examined by Mroz and Norris (1982) and Dafalias and Herrmann (1982), among others. Their novel approach was considerably successful, at least qualitatively, for predicting different aspects of soil behaviour under loading, both monotonic and cyclic.

To simulate the response of ballast under cyclic loading, the concept of bounding surface plasticity along with varying hardening function was adopted in this study, as described in the following Sections.
6.4.2.1 Conceptual model

In the current study, it was assumed that during cyclic reloading, ballast would deform plastically but at a smaller scale, and that these small plastic deformations would also be governed by linear kinematic yield surfaces (same as Equation 6.63) within a bounding yield surface. During virgin loading where the stress state remains on the bounding surface, the plastic deformations are same as in the monotonic shearing (Section 6.4.2). The plastic deformations under cyclic loading are generally computed by formulating an appropriate plastic hardening function that varies with the state of geomaterials (i.e. \( p \), \( q \) and \( e \)) and the previous stress history.

Before formulating an appropriate varying hardening function for a general cyclic loading, the evolution of the plastic hardening function during a simple loading-unloading and reloading path ‘a-b-c-d’ (Figure 6.15) was considered first. The constant stress ratio lines (OA, OB, OC, OD etc.) shown in Figure 6.15 represent yield loci, as mentioned earlier in Postulate 1. The dotted curves shown in Figure 6.15 represent possible caps of the yield loci at very high stress levels, which could not be ascertained within the scope of this study. However, it is anticipated that at very high stress levels, ballast will yield in both isotropic compression (\( \eta = 0 \)) and shearing (i.e. \( d|\eta| > 0 \)), and that the degree of particle breakage will be very high at those stresses.

According to Postulate 1, the line OA (Figure 6.15) connecting the initial stress point ‘a’ and the origin of stresses ‘O’, represents the initial yield locus. The line OA’ represents a similar yield locus for negative \( q \) (i.e. in extension). In triaxial extension, \( q \) is often considered to be negative. If the stress ratio at point ‘a’ represents the maximum past stress ratio of ballast, then the line OA forms its current bounding surface. It is
assumed that if the stress state is on the current bounding surface and the change of stress is directed towards the exterior of the bounding surface (i.e. away from the $\eta = 0$ line, or $d|\eta| > 0$), it represents ‘loading’, which causes plastic deformation, in addition to elastic strain. The plastic deformation associated with this ‘loading’ will be governed by the bounding hardening function $h_{\text{bound}}$, which is the same as given by Equation 6.80.

In contrast, if any change of stress is directed towards the interior of bounding surface (i.e. towards the $\eta = 0$ line, or $d|\eta| < 0$), it represents ‘unloading’ and causes only elastic recovery of strain. There is no plastic deformation associated with ‘unloading’. If the change of stress commences from a point interior to the bounding surface and is directed towards the bounding surface ($d|\eta| > 0$), it represents ‘reloading’ and also

Figure 6.15. Bounding surface for a simplified stress path ‘a-b-c-d’ under cyclic loading
causes plastic deformation, but at a considerably smaller scale. The plastic deformation associated with this ‘reloading’ will be governed by a new hardening function, $h_{\text{int}}$. The mathematical formulation of $h_{\text{int}}$ is given later in Section 6.4.2.2.

In stress path ‘a-b’ (Figure 6.15), since the stress point ‘a’ is on the current bounding surface (OA) and the direction of stress change is towards the exterior of the current bounding surface, the plastic hardening function for this ‘loading’ is given by Equation 6.80. At the end of stress path ‘a-b’, a new bounding surface is formed by the line OB, (connecting the stress point ‘b’ and the origin ‘O’). During the stress path ‘b-c’, since the stress change is directed towards the interior of current bounding surface (i.e. towards $\eta = 0$ line), the deformation corresponding to this ‘unloading’ is purely elastic.

During the stress path ‘c-d’, since the stress change starts from a point (‘c’), which is inside the current bounding surface (OB), and the stress change is directed towards the bounding surface, the plastic hardening function for this ‘reloading’ will be $h_{\text{int}}$. It is also assumed that the hardening function $h_{\text{int}}$, starts with an initial value at the beginning of reloading (e.g. point ‘c’ in Figure 6.15), and gradually evolves to the bounding value as the stress path meets the current bounding surface.

The essential features of the current cyclic constitutive model are summarised below:

- Plastic deformations are associated with all ‘loading’ and ‘reloading’, in addition to elastic strains
- ‘Unloading’ causes only elastic recovery of strain
- ‘Loading’ is defined by: $\eta = \eta_{\text{bound}}$ and $d|\eta| > 0$
- ‘Unloading’ is defined by: $d|\eta| < 0$
‘Reloading’ is defined by: $|\eta| < |\eta_{\text{bound}}|$ and $d|\eta| > 0$

- If $\eta = \eta_{\text{bound}}$, $h = h_{\text{bound}}$
- If $|\eta| < |\eta_{\text{bound}}|$, $h = h_{\text{int}}$

where, $h_{\text{bound}}$ = hardening function at the bounding surface given by Equation 6.80,

$\eta_{\text{bound}}$ = stress ratio at the bounding surface.

### 6.4.2.2 Mathematical model

The mathematical expressions of the initial hardening function $h_{\text{int}(i)}$ and the evolution of plastic hardening function $h_{\text{int}}$, within the bounding surface are given below:

\[
h_{\text{int}(i)} = h_i e^{-\xi_1 e^p} \tag{6.81}
\]

\[
h_{\text{int}} = h_{\text{int}(i)} + (h_{\text{bound}} - h_{\text{int}(i)}) R e^{-\xi_2 e^p} \tag{6.82}
\]

\[
R = \frac{\eta - \eta_i}{\eta_{\text{bound}} - \eta_i} \tag{6.83}
\]

where, $h_i$ = initial hardening function at the start of cyclic loading (e.g. $h_i = h$ at point ‘a’ in Figure 6.15),

$h_{\text{int}}$ = hardening function at the interior of bounding surface (for ‘reloading’)

$h_{\text{int}(i)}$ = initial value of $h_{\text{int}}$ for ‘reloading’,

$\xi_1$, $\xi_2$ and $\gamma$ are dimensionless parameters, and the first two are related to cyclic hardening.

The function $h_{\text{int}}$ for the first ‘reloading’ is modelled by Equation 6.82. For the second and subsequent ‘reloadings’, $h_{\text{int}}$ is given by:

\[
h_{\text{int}} = h_{\text{int}(i)} + (h_{\text{bound}} - h_{\text{int}(i)}) R e^{-\xi_3 e_{\nu 1}} \tag{6.84}
\]

where, $\xi_3$ is another dimensionless parameter related to cyclic hardening, and $e_{\nu 1}$ is the
accumulated plastic volumetric strain since the end of the first load cycle.

The plastic distortional strain increment corresponding to any ‘loading’ is given by Equation 6.79, and for a ‘reloading’, $d\varepsilon_s^p$ is given by:

$$d\varepsilon_s^p = h_{in} \eta \int p \, d\eta$$  \hspace{1cm} (6.85)

Equation 6.73 gives the plastic volumetric strain increment, as in monotonic shearing, and Equation 6.51 models particle breakage. Although actual breakage process depends on the cyclic loading and the fatigue failure of ballast grains, however, in the current study, particle breakage has been modelled as a function of distortional strain $\varepsilon_s$, initial mean stress $p^{(i)}$ and the initial void ratio represented by the parameter $p_{cs}^{(i)}$, based on the experimental findings (see Figure 6.7). Each load increment during loading and reloading causes an increase in stress ratio $d\eta$, resulting in an increase in plastic distortional and volumetric strains (Equations 6.85 and 6.73, respectively) and these strains are accumulated with increasing load cycles, although there is no net change in $q$ for cyclic loading with a constant load amplitude. These increase in distortional strain and the resulting induced internal stresses cause internal attrition, grinding, breakage of sharp corners and asperities, and even splitting and crushing of weaker grains, which are represented together in the breakage index ($B_g$), as modelled by Equation 6.51. Thus, the effect of cyclic loading on the particle breakage process has been adequately simulated in the current model.

The implementation of the above constitutive model has been carried out numerically and the verification of the model is discussed in the following Chapter 7.
CHAPTER 7
MODEL VERIFICATION AND DISCUSSION

7.1 INTRODUCTION

The new stress-strain and particle breakage constitutive model developed and explained in Chapter 6 has been employed to predict ballast response under both monotonic and cyclic loadings. The model parameters were evaluated based on the experimental results. The analytical predictions were compared with the laboratory experimental data to examine and verify the model quantitatively. Additionally, ballast specimens under triaxial stress were analysed by finite element method (FEM) using ABAQUS, and the numerical predictions were also compared with the analytical model. This Chapter describes the numerical techniques adopted to implement the constitutive model, the evaluation of model parameters, and comparison of the analytical and numerical predictions with the test data. The analytical predictions using the monotonic loading model (Section 6.3) were compared with the triaxial test results of fresh ballast, while the predictions using the cyclic loading model (Section 6.4) were verified against the prismoidal triaxial test results of fresh ballast.

7.2 NUMERICAL METHOD

To implement the constitutive model developed in this study, a simple numerical procedure was adopted to solve the differential Equations 6.41-6.42, 6.73, 6.79 and 6.85, which could not be integrated directly. For monotonic model predictions, a strain-controlled computation was conducted adopting the following equation:
\[(\eta)_{n+1} = (\eta)_n + \left( \frac{d\eta}{d\varepsilon^p} \right)_n \delta\varepsilon^p \]  
(7.1)

where, the subscript ‘\(n\)’ represents a current value and the subscript ‘\(n+1\)’ indicates a value after the increment.

For cyclic model predictions, a stress-controlled computation was carried out following the equation:

\[(\varepsilon^p)_n = (\varepsilon^p)_n + \left( \frac{d\varepsilon^p}{d\eta} \right)_n \delta\eta \]  
(7.2)

For both monotonic and cyclic model predictions, the numerical values of \(\varepsilon^p\), \(\varepsilon^e\), and \(\varepsilon^v\) were computed by:

\[(\varepsilon^p)_n = (\varepsilon^p)_n + \left( \frac{d\varepsilon^p}{d\varepsilon^s} \right)_n \delta\varepsilon^s \]  
(7.3)

\[(\varepsilon^e)_n = (\varepsilon^e)_n + \left( \frac{d\varepsilon^e}{dq} \right)_n \delta q \]  
(7.4)

\[(\varepsilon^v)_n = (\varepsilon^v)_n + \left( \frac{d\varepsilon^v}{dp} \right)_n \delta p \]  
(7.5)

Equation 6.79 was used for the derivatives \(\frac{d\eta}{d\varepsilon^p}\) and \(\frac{d\varepsilon^p}{d\eta}\) of Equations 7.1 and 7.2, respectively. Equations 6.73, 6.41, and 6.42 were used for the derivatives \(\frac{d\varepsilon^p}{d\varepsilon^s}\), \(\frac{d\varepsilon^e}{dq}\), and \(\frac{d\varepsilon^e}{dp}\) of Equations 7.3, 7.4, and 7.5, respectively, for both monotonic and cyclic model predictions.
Chapter 7

7.3 EVALUATION OF MODEL PARAMETERS

The monotonic shearing model (Section 6.3) contains 11 parameters, which can be evaluated using conventional drained triaxial test results together with the measurements of particle breakage, as explained below. The critical state parameters ($M$, $\lambda_{cs}$ and $\Gamma$, and $\kappa$) can be determined from a series of drained triaxial compression tests at various effective confining pressures. The slope of the line connecting the critical state points in $p$-$q$ plane gives the value of $M$, and that in $e$-$\ln p$ plane gives $\lambda_{cs}$. The void ratio ($e$) of the critical state line at $p = 1$ kPa is the value of $\Gamma$. The parameter $\kappa$ can be determined from an isotropic (hydrostatic) loading-unloading test with the measurements of volume change. The slope of the unloading part of isotropic test data plotted in $e$-$\ln p$ plane gives the value of $\kappa$. The elastic shear modulus $G$, can be evaluated from the unloading part of stress-strain ($q$-$\varepsilon_s$) plot in triaxial shearing.

The model parameter $\beta$ (Equation 6.55) can be evaluated by measuring the particle breakage ($B_g$) at various strain levels, as explained earlier in Section 6.3.2 (see Figure 6.6, Chapter 6). The parameters $\theta$ and $\nu$ can be determined by replotting the breakage data as $\ln \{p_{\alpha(i)}/p(i)\} B_g$ versus $\varepsilon_s^p$ (see Figure 6.8), and finding the coefficients of the non-linear function (Equation 6.51) that best represent the test data. The parameters $\chi$ and $\mu$ can be evaluated by plotting the rate of particle breakage data in terms of $\ln \{p_{\alpha(i)}/p(i)\} dB_g/d\varepsilon_s^p$ versus $(M-\eta^*)$ (see Figure 6.10) and determining the values of the intercept and slope of the best-fit line. The parameter $\alpha$ is used in the current model to match the initial stiffness of the analytical predictions with the experimental results, and can be evaluated by a regression analysis or a trial and error process comparing model predictions with a set of experimental data.
The cyclic loading model (Section 6.4) has 4 parameters in addition to the above. These 4 parameters can be evaluated from the stress-strain measurements for a number of load cycles during a cyclic test. The parameter $\xi_1$ can be determined from the initial re-loading data, while the parameters $\xi_2$ and $\xi_3$ can be evaluated from the remaining parts of the first re-loading and the following re-loading data, respectively. The model parameter $\gamma$ can also be evaluated from any re-loading stress-strain data. The determination of the above model parameters (both for monotonic and cyclic models) from laboratory experimental test results are explained in Appendix B.

7.4 Verification of the Model and Discussion

7.4.1 Model Predictions for Monotonic Loading

The deformation response of ballast under monotonic loading was predicted using the current constitutive model (Section 6.3), and then compared with the experimental results. In predicting ballast behaviour using the current model, the following model parameters were used: $M = 1.9$, $\lambda_{cs} = 0.188$, $\Gamma = 1.83$, $\kappa = 0.007$, $G = 80$ MPa, $\alpha = 28$, $\beta = 0.0029$ kN-m/m$^3$, $\chi = 0.21$, $\mu = 0.50$, $\theta = 0.125$, and $\nu = 10.5$. Ten of the above 11 parameters were evaluated from drained triaxial compression test results, as explained earlier in Section 7.3. The value $\alpha = 28$ was determined by initial stiffness matching of analytical predictions with several test results of ballast (Salim and Indraratna, 2004). A typical example of numerical computation using the current constitutive model is shown in Appendix C.
Figure 7.1. Analytical prediction of stress-strain of ballast with and without particle breakage compared to test data

Figure 7.2. Volume change predictions with and without particle breakage compared to test data
The analytical predictions were made following a strain-controlled computation. For a given initial state of ballast \((p, q, \text{ and } e)\), a small plastic distortional strain increment was assumed and the corresponding new stress ratio was computed as per the numerical procedure shown earlier (Section 7.2). The corresponding plastic and elastic volumetric strains were computed using Equations 7.3 and 7.5, while the elastic distortional strain increment was obtained from Equation 7.4. The breakage index \((B_\text{g})\) at the end of strain increment was computed using Equation 6.51.

Figure 7.1 shows the stress-strain predictions for ballast, while Figure 7.2 depicts the predicted volume change compared to current experimental data and the previous test data of ballast, as reported by Indraratna et al. (1998). The analytical predictions without any particle breakage (i.e. using \(\beta = 0\) in Equation 6.55) are also shown in these figures for comparison. Excellent agreement is found between the model predictions and the experimental data, especially with particle breakage. Since the confining pressures used in the laboratory experiments were small (300 kPa maximum) compared to the compressive strength of the parent rock of about 130 MPa (Indraratna et al., 1998), only a small fraction of the imparted energy was consumed in particle breakage. Therefore, the difference between the model predictions with and without particle breakage is small (Figures 7.1 – 7.2). As seen in Figure 7.2, the gap between the predicted curves with and without breakage increases as the confining pressure increases (e.g. \(\sigma_3 = 300\) kPa), where particle breakage becomes increasingly more significant. It is anticipated that at very high confining pressures (\(> 1\) MPa), particle breakage will be high and particle crushing will dominate the deformation behaviour of ballast, especially the volumetric changes.
Figure 7.3 shows the model prediction of particle breakage ($B_g$) compared with the experimental data. It shows that the predicted breakage values are close to the measured data. Figure 7.3 verifies that the current analytical model predicts the breakage of ballast to an acceptable accuracy.

As mentioned earlier, the postulates made in the current model are comparable to the hypotheses made by Pender (1978) for overconsolidated soils. Despite these similarities, there are some significant differences between these two approaches. Pender (1978) assumed that all soils, which are denser than the critical (i.e. $p_o < p_{oc}$), would exhibit plastic dilation during shear deformation. He adopted a function for the ratio between plastic strain increments, $\frac{d\varepsilon_v^p}{d\varepsilon_s^p}$, which makes the plastic volumetric strain...
increment negative (i.e. dilation) for all soils denser than the critical. However, Indraratna and Salim (2001) reported that at a relatively high confinement (> 200 kPa), plastic volumetric contraction occurs during shearing of ballast, which is still on the denser side of the critical state line (CSL). This aspect of ballast behaviour is well captured in the current model. Equation 6.73 (Chapter 6) provides positive plastic volumetric strain (i.e. contraction) for ballast, which is denser than the critical, as long as the stress ratio (\( \eta \)) does not exceed \( M \).

In contrast, Pender's (1978) hypothesis always provides plastic dilation (negative \( d\varepsilon^p \)) for all stress ratios if the soil is on the denser side of the CSL (i.e. \( p_o < p_{cs} \)). Other major difference between the two models is the incorporation of particle breakage, which is absent in Pender’s (1978) model. Any particle breakage will consume part of the imparted energy, and therefore, a reduced amount of energy will be spent on frictional deformation and the resulting plastic distortional strain increment will be smaller. This is clearly reflected in the denominators of Equations 6.72 and 6.79, which include the breakage term. Moreover, particle breakage will contribute to an increase in plastic volumetric strain (contraction), an aspect that is correctly represented in the current model (Equation 6.73).

An interesting point to note is that Equation 6.73 of the current model always governs the plastic volumetric strain (positive or negative) towards the critical state. At the initial stage of shearing (\( \eta < M \)), Equation 6.73 provides plastic volumetric contraction (\( d\varepsilon^p \) positive) so that ballast hardens, and as a result, it can sustain additional shear stress (i.e. \( \eta \) increases towards \( M \)). If the stress ratio \( \eta \) exceeds \( M \) (under low confinement), Equation 6.73 provides negative \( d\varepsilon^p \) (or dilation) when the value of the
breakage related term is small, and therefore, the material softens, and the stress ratio gradually decreases towards the critical state value $M$.

### 7.4.2 Analytical Model Compared to FEM Predictions

In this study, the current analytical model predictions were also compared with the results of finite element analysis employing ABAQUS. The finite element code ABAQUS is a powerful tool and commercially available for analysing a wide range of engineering problems including geomechanics. In this Section, the analytical model predictions and the ABAQUS finite element predictions are compared with the experimental data.

![Figure 7.4](image)

Figure 7.4. (a) Ballast specimen, (b) discretisation and mesh used in finite element modelling of the ballast specimen
Finite element analyses were carried out for a cylindrical ballast specimen (Figure 7.4a) using axisymmetric elements. As $\sigma_2 = \sigma_3$ and $\varepsilon_2 = \varepsilon_3$ in triaxial shearing (i.e. axisymmetric), the shaded area of the specimen (Figure 7.4a) was discretised, as illustrated in Figure 7.4(b). The left boundary of Figure 7.4(b) represents the central specimen axis, which does not move laterally under triaxial loading, hence the roller supports to restrain lateral movement (i.e. vertical degree of freedom only).

In ABAQUS, the extended Drucker-Prager model with hardening was used to simulate inelastic deformation of granular materials (Hibbit, Karlsson and Sorensen, Inc., 2002). Figures 7.5(a) and (b) show the FEM stress-strain and volume change predictions compared with the analytical predictions. The experimental results were also plotted in these figures for convenience and comparison. A typical output of numerical analysis of ballast using the finite element code ABAQUS is given in Appendix D.

Figure 7.5(a) indicates that both the analytical and FEM models predict the stress-strain response of ballast fairly well, but the writer’s constitutive model is slightly better. In contrast, Figure 7.5(b) clearly shows that the FEM model (ABAQUS) could not simulate the volumetric response of ballast well, especially at high confining pressures (e.g. 200 and 300 kPa). In particular, the finite element simulation could not predict the specimen contraction at high stresses. Apart from restrained lateral displacements at high confining pressures, particle breakage is also increasingly more significant, as discussed earlier, hence, the subsequent overall contraction of the specimen is inevitable.
Figure 7.5. Analytical model predictions of ballast compared with FEM analysis results and experimental data, (a) stress-strain, and (b) volume change behaviour.

Particle breakage was not taken into account in the constitutive model of ABAQUS. Moreover, the plastic volumetric deformation of geomaterials is simulated in ABAQUS by a single value of dilation angle, which restricts the volumetric contraction in the finite element simulation. Therefore, it is not surprising that acceptable volumetric
matching could not be achieved in the ABAQUS simulation. As the writer’s constitutive model incorporates the effect of particle breakage on both volumetric and distortional strains, and also correctly simulates the plastic volumetric response associated with shearing (Equation 6.73), better predictions of volumetric behaviour using the model were achieved (Figure 7.5a).

7.4.3 Model Predictions for Cyclic Loading

The qualitative prediction of cyclic stress-strain using the present constitutive model (Section 6.4) is shown in Figure 7.6. In addition to 11 model parameters used earlier in Section 7.4.1, the following values of 4 other model parameters were used: \( \xi_1 = 1400 \), \( \xi_2 = 25 \), \( \xi_3 = 3400 \), and \( \gamma = 2 \). Figure 7.7 shows the cyclic load-deformation test results of ballast as reported by Key (1998). Comparing Figures 7.6 and 7.7, it may be concluded that the qualitative stress-strain model prediction is comparable to the experimental data. The qualitative model prediction (Figure 7.6) also shows that as the load cycle increases, the plastic strain accumulates at the decreasing rate, which is a key feature in cyclic deformation of many geomaterials. It also depicts that the plastic strain is high in the first cycle of loading, then gradually decreases with increasing load cycles, a typical behaviour of ballast under cyclic loading (Key, 1998).
Figure 7.6. Qualitative model prediction of cyclic stress-strain of ballast

Figure 7.7. Cyclic load test results of ballast (after Key, 1998)
The model predictions of distortional strain ($\varepsilon_s$) and volumetric strain ($\varepsilon_v$) of fresh ballast (wet) under a system of cyclic vertical stress and lateral confinement similar to that applied in the prismoidal triaxial tests, were compared with the experimental data, as shown in Figures 7.8 - 7.9. In this study, 2 other cyclic stress-strain models (Tatsuoka et al., 2003 and Pender, 1982) were also employed to predict the cyclic response of ballast, and their predictions were also compared with the current model. Since the model parameters were evaluated from triaxial results of fresh ballast specimens, which were saturated in drained shearing, cyclic model predictions using those parameters were compared with the results of fresh ballast tested in a wet state. The typical numerical computations of ballast response under cyclic loading using the current model, Tatsuoka et al. (2003) and Pender’s (1982) models are shown in Appendix C.

Tatsuoka et al. (2003) simulated the stress-strain hysteretic loop in plane strain cyclic loading based on an empirical hyperbolic relationship (Equation 3.24), and the evolution of the stress-strain with increasing load cycles was governed by a set of rules in their technique, as mentioned earlier in Section 3.3.2. In contrast, Pender’s (1982) model was formulated based on the critical state framework and the classical theory of plasticity. Since there is little flexibility in the classical plasticity theory in varying the plastic modulus when loading direction is reversed, as mentioned earlier in Section 6.4.2, Pender (1982) adopted a cyclic hardening index $\xi$ (Equation 3.23) in his model to overcome this limitation. On the other hand, the current model was developed based on the critical state framework and the bounding surface plasticity concept, rather than the classical plasticity theory. The current model also incorporates particle breakage under loading.
The following parameters were used for analysing ballast behaviour using Tatsuoka et al. (2003): $\gamma_{ref} = 1.61\%$, $\beta_{max} = 0.024$, $F = 0.14$, $M_o = 2000$, $K' = 0.45$ for loading in the first cycle, $K' = 0.24$ for reloading and $K' = 0.24106$ for unloading. The parameter $\gamma_{ref}$ was evaluated from the monotonic shearing results of ballast ($q_{max}/G$). The parameter $\beta_{max}$ represents the maximum drag in Tatsuoka et al. (2003), and is related to the plastic shear strain in cyclic loading. The parameter $M_o$ was evaluated from the initial stiffness of $sin\phi_{mob}-\gamma$ relationship. As Tatsuoka et al. (2003) did not indicate the evaluation technique for the model parameters $F$ and $K'$, the above values of these parameters were used in this study to give the best possible predictions.

The following parameters were used for the prediction of ballast behaviour using Pender’s (1982) model: $M = 1.90$, $\lambda = 0.188$, $\kappa = 0.007$, $G = 80$ MPa, $\hat{\alpha} = 0.05$ and $\hat{\beta} = 0.10$. The first 4 parameters of Pender’s (1982) model (i.e. $M$, $\lambda$, $\kappa$ and $G$) are the same as in the current model. Pender (1982) did not show the evaluation technique for the model parameters $\hat{\alpha}$ and $\hat{\beta}$. In this study, the above values of $\hat{\alpha}$ and $\hat{\beta}$ were used to give the best possible predictions using Pender’s (1982) model.

Figure 7.8 shows that Pender’s (1982) model slightly underpredicts distortional strain at smaller load cycles ($< 100,000$) but overpredicts slightly at higher load cycles ($> 200,000$). In contrast, Tatsuoka et al. (2003) slightly overpredicts distortional strain at smaller load cycles ($< 200,000$). At higher load cycles ($> 200,000$), Tatsuoka et al. (2003) gives improved matching of distortional strain with the experimental data. Figure 7.8 clearly shows that the prediction of distortional strain using the current model closely matches with the laboratory measured data.
Figure 7.8. Model prediction of ballast distortional strain compared with experimental data

Figure 7.9. Model prediction of volumetric strain of ballast compared with test data
Figure 7.9 shows that Tatsuoka et al. (2003) slightly underpredicts the volumetric strain of ballast at smaller load cycles (< 300,000) and the rate of volumetric strain with increasing load cycles is slightly higher than the laboratory observations. Although the stress-strain was simulated for plane strain cyclic loading (i.e. $\varepsilon_2 = 0$), Tatsuoka et al. (2003) generally gives reasonable volumetric strain under triaxial cyclic loading (Figure 7.9). However, it is anticipated that as $\varepsilon_2 = 0$ (i.e. plane strain), Tatsuoka et al. (2003) will give excessive lateral strains ($\varepsilon_3$).

In contrast, Pender’s (1982) model clearly underpredicts volumetric strain of ballast. Since Pender (1982) considered that all soils denser than the critical would dilate plastically during shear deformation, his model was unable to simulate cyclic densification (i.e. volumetric contraction) of ballast, which was observed in the current study and also in the previous studies (Key, 1998; Suiker, 2002). In the current model, plastic volumetric strain increment is positive (i.e. contraction, rather than dilation) if the stress ratio ($\eta$) is less than $M$, as explained earlier in Section 6.3.2 (Chapter 6). This plastic volumetric contraction is accumulated with increasing load cycles, causing cyclic densification in ballast. Thus, the current model correctly simulates the volumetric response of ballast under cyclic loading, as revealed in Figure 7.9.

Figure 7.10 shows the predicted particle breakage ($B_g$) of ballast using the current model compared with experimental data. Since Tatsuoka et al. (2003) and Pender (1982) did not consider any breakage of particles during shearing, these models were unable to simulate the breakage of ballast under cyclic loading, and therefore, not shown in this figure. Tatsuoka et al. (2003) and Pender (1982) developed their models primarily for sands and overconsolidated fine-grained soils, where particle breakage is insignificant.
In rail tracks, particle breakage is the main source of ballast fouling, as mentioned earlier in Section 2.6.1, and also affects the strength and deformation behaviour of ballast. In the current model, the particle breakage was incorporated in the incremental stress-strain formulations appropriately. Figure 7.10 shows that the predicted breakage of ballast increases rapidly up to about 50,000 load cycles, beyond which the increase in breakage becomes marginal. The close agreement between the model prediction and the experimental data (Figure 7.10) verifies that the current model can predict ballast breakage under cyclic loading to an acceptable accuracy.
CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 CONCLUSIONS

8.1.1 General Observations

The deformation and degradation behaviour of both fresh and recycled ballast under monotonic and cyclic loading was investigated in this study using a large cylindrical and a prismatic triaxial apparatus. A series of consolidated drained triaxial compression tests was conducted on both types of ballast at various effective confining pressures. Particle breakage of ballast resulting from triaxial shearing was measured by sieving the specimens before and after each test. Additionally, a series of single grain crushing tests was carried out on fresh and recycled ballast of various sizes.

A series of cyclic triaxial tests simulating a typical axle load of 25 ton was conducted on fresh and recycled ballast. The cyclic tests were carried out in dry and wet states to examine the effect of saturation. Three types of geosynthetics (geogrid, woven-geotextile, and geocomposite) were used in the current study to stabilise recycled ballast. This study confirms that recycled ballast has a lower shear strength (peak deviator stress), lower peak stress ratio, lower internal friction angle, and lower elastic modulus than fresh ballast. Reduced angularity caused by sharp corners breaking off in previous load cycles is the key reason for its inferior engineering properties. The study also reveals that the effective confining pressure significantly affects ballast behaviour. Ballast usually exhibits a high peak stress ratio and a high peak friction angle associated
with dilatancy at a low confining pressure. As the confining pressure increases, dilatancy is suppressed, leading to an overall volumetric contraction and a reduced peak friction angle.

Particle breakage measurement indicates that recycled ballast is more vulnerable to fracture than fresh ballast. Presence of micro-fractures in recycled ballast exacerbates degradation. Recycled ballast grains have approximately 35% lower tensile strength than the fresh ones. The tensile strength of ballast grains generally decreases as the particle size increases. Particle breakage increases at a decreasing rate towards a constant as the axial strain increases, and continues to increase even after the peak deviator stress. The test results also reveal that ballast breakage increases with increase in confining pressure, and that the larger aggregates are more vulnerable to degradation.

The findings of this study clearly show that ballast settlement increases non-linearly with increasing load cycles. Recycled ballast gives approximately 40-60% increase in vertical strain than fresh ballast, while saturation increases vertical strain by approximately 20-40%. Inclusion of a geogrid in recycled ballast decreases its settlement considerably in a dry state. In wet conditions, geogrid only stabilises recycled ballast marginally. The woven-geotextile effectively stabilises recycled ballast when it is dry, but is not as effective as a geocomposite when it is wet. It is concluded from this study that a geocomposite comprising a geogrid and a non-woven geotextile bonded together stabilises recycled ballast effectively, in both dry and wet conditions. The non-woven geotextile element of the geocomposite provides effective filtration to recycled ballast, prevents fines moving up from the subgrade, thereby keeping the ballast relatively clean. It also separates coarse ballast layer from the underlying
capping and subgrade layers, and provides in-plane drainage. The geogrid component of the geocomposite provides adequate reinforcement and restricts lateral expansion of the ballast bed upon loading. These functions of the geocomposite effectively minimise the settlement and lateral deformation of recycled ballast, especially when wet.

The study also indicates that particle breakage in recycled ballast subject to cyclic loading is almost double that of fresh ballast, while saturation only increases particle breakage slightly. Geosynthetics decrease the breakage of recycled ballast quite significantly. All three types of geosynthetics used in this study decreased the breakage index of recycled ballast close to the value of fresh ballast without geosynthetics.

8.1.2 Modelling Aspects

In this study, a new stress-strain and particle breakage constitutive model has been developed for ballast based on the critical state framework and the bounding surface plasticity concept. A non-associated flow and a kinematic type yield locus (constant stress ratio) were adopted in the model. Particle breakage was modelled by a single non-linear function of distortional strain and the initial state of ballast, and then incorporated in a plastic flow rule. In the current model, the $p$-$q$ plane is divided into two distinct regimes (plastic volumetric contraction and plastic dilation). Any stress ratio below the critical state value ($M$) will produce plastic volumetric contraction, and a stress ratio above $M$ may induce plastic dilation. The overall elasto-plastic behaviour of ballast is described through 15 model parameters (constants), 5 of which are specifically associated with particle breakage and 3 are related to cyclic hardening.
The model captures the strain-hardening and post-peak strain-softening features of ballast adequately. The cyclic densification and cyclic hardening features of ballast behaviour are appropriately simulated in the current model. Conventional stress-strain constitutive models do not consider particle breakage during shearing. In this new model, the effects of particle breakage on the plastic distortional and volumetric strains have been incorporated.

The formulations of the current model can be employed along with a set of triaxial test data to compute the contribution of particle breakage to the friction angle of ballast and other granular coarse aggregates. The current study reveals that the contribution of particle breakage to the friction angle of ballast increases at a decreasing rate as the confining pressure becomes higher, while the dilatancy component of friction angle decreases with increasing confinement. This study confirms that the peak friction angle \( \phi_p \) represents the summation of basic friction angle \( \phi_f \) and the effects of dilatancy and particle breakage during shearing.

Analytical predictions of ballast behaviour employing the current constitutive model have been compared with the experimental results. Cylindrical ballast specimens under triaxial stress were also analysed numerically using a finite element code (ABAQUS), and compared with the observations and analytical model. The results of this study indicate that the current analytical model gives better predictions, especially for volumetric behaviour under higher confining pressure, compared to FEM analysis. The absence of particle breakage and simplified plastic dilatancy simulation in the constitutive model of ABAQUS are believed to be the primary reasons for the smaller volumetric contractions predicted by the FEM analysis. The incorporation of particle
breakage and the appropriate formulation of plastic volumetric strain in the writer’s constitutive model are the key reasons for the better volumetric matching, especially at high confining pressures.

The deformation and degradation response of ballast under cyclic loading was predicted using the new cyclic constitutive model developed in this study, and compared with the test results. Additionally, 2 other cyclic stress-strain models presented by the previous researchers (Tasuoka et al., 2003 and Pender, 1982) were used in this study to predict ballast response under cyclic loading, and those predictions were also compared with the current model. This study reveals that Tatsuoka et al. (2003) predict volume change of ballast under cyclic loading reasonably well, while Pender’s (1982) model underpredicts volumetric contraction of ballast. The analysis clearly shows that the current model accurately predicts the stress-strain and volume change of ballast, and particularly, particle breakage under both monotonic and cyclic loadings.

### 8.2 RECOMMENDATIONS FOR FUTURE STUDY

The study carried out within the scope of this research has generated several new aspects of ballast behaviour that need to be investigated in more detail. The following issues in particular, are recommended for future research on the behaviour of ballast and rail track, including the use of geosynthetics.

1. Deformation and particle breakage behaviour of fresh ballast subjected to the inclusion of different types of geosynthetics: This study will examine the
potential of geosynthetics in improving fresh ballast performance and the possible extension of current track maintenance cycle, as well as reducing the thickness of ballast bed.

2. Behaviour of a composite bed comprising a fresh ballast layer, a recycled ballast layer, and 1 or 2 layers of geosynthetics at the intersections of fresh/recycled ballast and recycled ballast/capping layer: This study may result in an improved track foundation to carry much higher loads and decrease the demand for fresh ballast. However, the maintenance of track may become more complicated due to increased layering.

3. Effects of particle size distribution of ballast on its strength, deformation, particle breakage and hydraulic conductivity (drainage): This study will help to develop an optimum particle size distribution, which will provide enhanced stability, reduced deformation and breakage, while ensuring adequate drainage in track.

4. An extension of the current constitutive model to include the effect of geosynthetics at the interfaces (reduced displacement boundary).

5. Evolution of the critical states of ballast with increasing particle degradation under cyclic loading and the incorporation of these critical states into a more comprehensive constitutive model.
6. Yielding of ballast under isotropic loading at high stress levels, extension of the currently used open-ended yield loci to include a cap, and developing a generalised constitutive model applicable to a wider range of loading, including very high dynamic stresses and associated impact loading.
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APPENDIX A

Derivation of Partial Derivatives of $g(p, q)$ With Respect To $p$ and $q$ From A First Order Linear Differential Equation:

A simple first order linear differential equation is considered, as given by:

$$\frac{dq}{dp} + pq = 0 \quad (A1)$$

After separating the variables and integrating, the solution of the differential equation (Equation A1) is given by:

$$\ln q + \frac{p^2}{2} + c = 0 \quad (A2)$$

Equation A2 can be re-written in the following form:

$$q - e^{-(p^2/2+c)} = 0 \quad (A3)$$

If Equation A3 represents the function $g = g(p, q)$, then,

$$g(p, q) = q - e^{-(p^2/2+c)} = 0 \quad (A4)$$

Differentiating $g$ with respect to $q$ and $p$ partially gives:

$$\frac{\partial g}{\partial q} = 1 \quad (A5)$$

$$\frac{\partial g}{\partial p} = (-p)\!\left\{e^{-\left(p^2/2+c\right)}\right\} = pq \quad (A6)$$
APPENDIX B

Determination of Model Parameters from Laboratory Experimental Results

(1) For Monotonic Loading Model:

The current monotonic loading model contains 11 parameters, which can be determined from drained triaxial compression tests with the measurements of particle breakage, as explained below:

The critical state parameters ($M$, $\lambda_{cs}$, $\kappa$ and $\Gamma$) can be evaluated from the critical state line, which is determined from the results of a series of drained triaxial compression tests, as shown in Figures B1(a) - (b).

![Critical state line](image1)

![Critical state line](image2)

Figure B1. Determination of model parameters $M$, $\lambda_{cs}$, $\kappa$ and $\Gamma$) from laboratory experimental results
The elastic shear modulus $G$, can be evaluated from unloading stress-strain data of triaxial shearing, as shown in Figure B2. The slope of the unloading part of $q$-$\varepsilon_s$ plot gives the value of the parameter $G$.

The model parameter $\beta$ can be evaluated by plotting the computed $dE_B/d\varepsilon_1$ values against the experimental $dB_B/d\varepsilon_1$ values, as shown in Figure B3. The values of $dE_B/d\varepsilon_1$ can be computed from Equation 6.23 after substituting the experimental values of $q$, $p'$, $(1-d\varepsilon_v/d\varepsilon_1)$, and the basic friction angle, $\phi_f$. The values of $dB_B/d\varepsilon_1$ at various strain levels can be determined from the plot of experimental measurements of breakage index $B_B$ (see Figure 5.15). The slope of the linear best-fit line of the plot $dE_B/d\varepsilon_1$ versus $dB_B/d\varepsilon_1$ gives the value of $\beta$ (Figure B3).
Figure B3. Determination of model parameter $\beta$ from the measurements of particle breakage in triaxial compression tests.

The parameters $\theta$ and $\nu$ can be evaluated by re-plotting the particle breakage data ($B_g$) in a modified scale of $\ln \{ p_{ci}(\theta) / p(\theta) \} B_g$ versus $\varepsilon$, as shown in Figures B4(a) - (b). Figure B4(b) shows that the variations of particle breakage ($B_g$) with increasing distortional strains and confining pressures (as shown in Figure B4a) can be effectively represented by a single function (Equation 6.51), and the coefficients of the exponential function (Equation 6.51) gives the values of $\theta$ and $\nu$. 
Figure B4. Determination of model parameters $\theta$ and $\nu$ from breakage measurements, (a) variation of $B_g$ with strains and confining pressures, (b) modelling of particle breakage

The model parameters $\chi$ and $\mu$ can be determined by plotting the rate of particle breakage $dB_g/d\varepsilon_s^p$ at various distortional strains and confining pressures in terms of $\ln \{p_{cs(i)}/p(i)\} dB_g/d\varepsilon_s^p$ versus $(M-\eta^*)$, as shown in Figure B5, where $\eta^* = \eta(p/p_{cs})$. The intercept and the slope of the best-fit line of this plot give the values of $\chi$ and $\mu$, respectively.
The parameter $\alpha$ can be evaluated by matching the initial stiffness of analytical predictions with a set of experimental results, as shown in Figures B6(a)-(c). The analytical predictions of stress-strain of ballast using $\alpha = 10$, $\alpha = 50$ and $\alpha = 28$ compared with the test data are shown in Figures B6(a), (b) and (c), respectively. Figure B6(a) shows that the initial stiffness of the stress-strain prediction for $\alpha = 10$ is higher compared to the experimental results. In contrast, $\alpha = 50$ gives the stress-strain predictions with a lower initial stiffness than the test data. Figure B6(c) clearly shows that a value of $\alpha = 28$ gives a very good matching between the analytical predictions and the laboratory measurements.
Figure B6. Determination of model parameter $\alpha$ by stiffness matching between analytical predictions and test data using a value of (a) $\alpha = 10$, (b) $\alpha = 50$ and (c) $\alpha = 28$
(2) **For Cyclic Loading Model:**

The cyclic loading model presented in this study contains additional 4 parameters, which can be evaluated from the laboratory measured data of cyclic stress-strain, as explained in the following:

Figure B7(a) shows a typical stress-strain ($q$-$\varepsilon_s$) plot under cyclic loading. The cyclic stress-strain data of Figure B7(a) can be re-plotted as distortional stress versus plastic distortional strain ($q$-$\varepsilon_s^p$), as shown in Figure B7(b), by subtracting the elastic component (using Equation 6.41) from the total distortional strain.

The value of the hardening function $h$ (Equation 6.80) at the start of cyclic loading (i.e. point ‘i’ in Figure B7(b) gives the value of $h_i$ (Equation 6.81). The value of $h_{int(i)}$ (Equation 6.81) for the first reloading ‘bc’ (Figure B7b) can be computed by substituting the test values of $d\varepsilon_s^p$, $p$ and $d\eta$ for the first incremental load ‘bb$_1$’ of this reloading into Equation 6.85. The cyclic model parameter $\xi_1$ can then be evaluated by substituting $h_i$, $h_{int(i)}$ and the value of $\varepsilon_v^p$ at the start of first ‘reloading’ into Equation 6.81.

Similarly, the values of $h_{int}$ for the following load increments (‘b$_1$b$_2$’, ‘b$_2$b$_3$’ etc.) can be computed by substituting the values of $d\varepsilon_s^p$, $p$ and $d\eta$ for the corresponding load increments into Equation 6.85. The value of $h$ (Equation 6.80) at point ‘a’ gives the value of $h_{bound}$ for the reloading ‘bc’ (Figure B7b). The model parameters $\xi_2$ and $\gamma$ can be evaluated by a trial and error process after substituting a set of known values of $h_{int}$, $h_{int(i)}$, $h_{bound}$, $R$ (from Equation 6.83) and $\varepsilon_v^p$ for the load increments (‘b$_1$b$_2$’, ‘b$_2$b$_3$’, ‘b$_3$b$_4$’ etc.) of ‘bc’ into Equation 6.82.
Figure B7. Determination of cyclic model parameters $\xi_1$, $\xi_2$, $\xi_3$ and $\gamma$ from laboratory test data, (a) cyclic stress-strain plot, and (b) cyclic stress-plastic strain plot.

In a similar way, the value of $h_{int(i)}$ for the following reloading ‘de’ (Figure B7b) and the values of $h_{int}$ for the load increments (‘d1d2’, ‘d2d3’, etc.) of ‘de’ can be computed. The model parameter $\xi_3$ can then be evaluated by substituting the values of $h_{int}$, $h_{int(i)}$, $h_{bound}$, $R$, $\gamma$ and $e''_{yj}$ for the load increment ‘d1d2’ or ‘d2d3’ into Equation 6.84.
APPENDIX C

(1) Typical Numerical Computation Employing The Current Model (Monotonic Loading) Using Excel Spreadsheet:

<table>
<thead>
<tr>
<th>Deformation and degradation model (Fatigue response)</th>
<th>4 x 4 Matrix</th>
<th>5 x 5 Matrix</th>
<th>6 x 6 Matrix</th>
<th>7 x 7 Matrix</th>
<th>8 x 8 Matrix</th>
<th>9 x 9 Matrix</th>
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<tbody>
<tr>
<td>E1</td>
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<td>0.016</td>
<td>0.017</td>
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</tr>
<tr>
<td>E4</td>
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<td>0.022</td>
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<td>0.024</td>
</tr>
<tr>
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<td>0.030</td>
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**Note:** The table above shows the deformation and degradation model for various matrices with increasing size. The values in the table represent the strain values for each matrix.
APPENDIX C (Contd.)

(2) Typical Numerical Computation Employing The Current Model (Cyclic Loading) Using Excel Spreadsheet:

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<td>0.0018</td>
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</tr>
</tbody>
</table>

Note: The table continues with similar entries for subsequent cycles.
APPENDIX C (Contd.)

(3) Typical Numerical Computation Employing Tatsuoka et al. (2003) Model Using Excel Spreadsheet:

| Cycle | $k^*$ | $k^*$ | $f' | f'' | $s' | $s'' | $h_{n+1}' | $h_{n+1}'' | $h_{n+1}' | $h_{n+1}'' | $h_{n+1}' | $h_{n+1}'' | $h_{n+1}' | $h_{n+1}'' | $h_{n+1}' | $h_{n+1}'' | $h_{n+1}' | $h_{n+1}'' | $h_{n+1}' | $h_{n+1}'' | $h_{n+1}' | $h_{n+1}'' |
|-------|-------|-------|-----|-----|-----|-----|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1     |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 2     |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 3     |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 4     |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 5     |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 6     |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 7     |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 8     |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 9     |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 10    |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 11    |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 12    |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 13    |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 14    |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 15    |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 16    |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| 17    |       |       |     |     |     |     |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |

...
### APPENDIX C (Contd.)


| 1999  | 99 | 1982 | 74 | 1990 | -2.062 | 7.322 | | | | | | | | | | | | | | | | | | | |
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| 1999  | 99 | 1982 | 74 | 1990 | -2.062 | 7.322 | | | | | | | | | | | | | | | | | | | |
APPENDIX D

Typical Output of Numerical Analysis for Ballast Using The Finite Element Code ABAQUS:

1

ABAQUS VERSION 6.3-1                                      DATE 29-Jul-2004
TIME 18:04:20         PAGE     1

This program has been developed by

Hibbitt, Karlsson & Sorensen, Inc.
1080 Main Street
Pawtucket, R.I. 02860

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E-mail: abaqus@worley.com.au

PROCESSING PART, INSTANCE, AND ASSEMBLY INFORMATION
*******************************************************
END PROCESSING PART, INSTANCE, AND ASSEMBLY INFORMATION
***********************************************************

OPTIONS BEING PROCESSED
***************************

*HEADING
CYLINDRICAL TRIAXIAL TEST
*NODE
*NSET, NSET=NBOTTOM, GENERATE
Appendix D

*NSET, NSET=NTOP, GENERATE
*NSET, NSET=NLEFT, GENERATE
*NSET, NSET=NOUTR, GENERATE
*ELEMENT, TYPE=CAX4R
*ELSET, ELSET=ALLEL
*ELSET, ELSET=ELBOT, GENERATE
*ELSET, ELSET=ELTOP, GENERATE
*ELSET, ELSET=ELRIGHT, GENERATE
*ELSET, ELSET=EL148
*MATERIAL, NAME=BALLAST
*POROUS ELASTIC, SHEAR=G
*DRUCKER PRAGER
*DRUCKER PRAGER HARDENING
*INITIAL CONDITIONS, TYPE=RATIO
*INITIAL CONDITIONS, TYPE=STRESS
*INITIAL CONDITIONS, TYPE=RATIO
*INITIAL CONDITIONS, TYPE=STRESS
*SOLID SECTION, ELSET=ALLEL, MATERIAL=BALLAST
*BOUNDARY
*SOLID SECTION, ELSET=ALLEL, MATERIAL=BALLAST
*SOLID SECTION, ELSET=ALLEL, MATERIAL=BALLAST
*INITIAL CONDITIONS, TYPE=RATIO
*INITIAL CONDITIONS, TYPE=STRESS
*INITIAL CONDITIONS, TYPE=RATIO
*INITIAL CONDITIONS, TYPE=STRESS
*INITIAL CONDITIONS, TYPE=RATIO
*INITIAL CONDITIONS, TYPE=STRESS
*AMPLITUDE, NAME=LINEAR1, DEFINITION=TABULAR
*AMPLITUDE, NAME=STEP1, DEFINITION=TABULAR
*STEP, INC=1
*STEP, INC=1000
*STEP, INC=1
*STEP, INC=1000
*STEP, INC=1
*STATIC
*CONTROLS, PARAMETERS=FIELD, FIELD=DISPLACEMENT
*DLOAD
*EL PRINT, ELSET=EL148, FREQUENCY=1, SUMMARY=NO
*END STEP
*STEP, INC=1000
*STATIC
*CONTROLS, PARAMETERS=FIELD, FIELD=DISPLACEMENT
*CONTROLS, PARAMETERS=TIME INCREMENTATION
*DLOAD, AMPLITUDE=STEP1
*BOUNDARY, AMPLITUDE=LINEAR1
*EL PRINT, ELSET=EL148, FREQUENCY=1, SUMMARY=NO
*END STEP
*BOUNDARY
*STEP, INC=1
*STATIC
*NODE PRINT, NSET=NTOP, SUMMARY=NO
*END STEP
*STEP, INC=1000
*STATIC
*BOUNDARY, AMPLITUDE=LINEAR1
*NODE PRINT, NSET=NTOP, FREQUENCY=1, SUMMARY=NO, TOTALS=NO
*NODE PRINT, NSET=NOUTR, FREQUENCY=1, SUMMARY=NO, TOTALS=YES
*END STEP

PROBLEM SIZE

NUMBER OF ELEMENTS IS 120
NUMBER OF NODES IS 144
NUMBER OF NODES DEFINED BY THE USER 144
TOTAL NUMBER OF VARIABLES IN THE MODEL: 288
(DEGREES OF FREEDOM PLUS ANY LAGRANGE MULTIPLIER VARIABLES)

END OF USER INPUT PROCESSING

JOB TIME SUMMARY
USER TIME (SEC) = 6.2700
SYSTEM TIME (SEC) = 0.68000
TOTAL CPU TIME (SEC) = 6.9500
WALLCLOCK TIME (SEC) = 8

ABAQUS VERSION 6.3-1
DATE 29-JUL-2004
TIME 18:04:30
PAGE 1
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CYLINDRICAL TRIAXIAL TEST
STEP 1  INCREMENT 1
TIME COMPLETED IN THIS STEP 0.

STEP 1  STATIC ANALYSIS

AUTOMATIC TIME CONTROL WITH -
A SUGGESTED INITIAL TIME INCREMENT OF 1.00
AND A TOTAL TIME PERIOD OF 1.00
THE MINIMUM TIME INCREMENT ALLOWED IS 1.000E-05
THE MAXIMUM TIME INCREMENT ALLOWED IS 1.00

MEMORY AND DISK ESTIMATE
SUMMARY FOR CURRENT NODE ORDERING (STEP 1 TO STEP 2)
(NOTE THAT IF NODE ORDERING CHANGES THE SIZE ESTIMATES FOR THE STEPS WILL CHANGE)

<table>
<thead>
<tr>
<th>STEP</th>
<th>MAXIMUM DOF</th>
<th>FLOATING PT</th>
<th>MINIMUM MEMORY</th>
<th>MEMORY TO MINIMIZE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>REQUIRED DISKSPACE</td>
<td>WAVEFRONT</td>
<td>OPERATIONS</td>
<td>(MBYTES)</td>
</tr>
<tr>
<td></td>
<td>I/O</td>
<td>PER ITERATION</td>
<td>(KBYTES)</td>
<td>(MBYTES)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>46</td>
<td>1.53E+05</td>
<td>14.60</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>46</td>
<td>1.53E+05</td>
<td>14.60</td>
</tr>
<tr>
<td></td>
<td>----</td>
<td>-------</td>
<td>-----------</td>
<td>-----------</td>
</tr>
<tr>
<td>MAX</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95.91</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table above provides a step by step summary of some basic sizing information
for the problem. Some further description of the parameters given follows:
(1) Maximum DoF Wavefront - Size of the biggest front in the equation solver. Provides a basic sizing of the most memory intensive segment of solver.

(2) Floating point operations per iteration - Measure of the number of floating point operations required for a single solver pass. On a given platform, the time required for a solver pass will be roughly a linear function of this value.

Note - except for the first step, the value in this table does not include additional floating point operations that are required in multiple load case. This situation causes a reordering to be performed, at which time an upper bound on the number of additional floating point operations will be calculated and included under the estimate summaries for the relevant steps (see note below on reordering).

(3) Minimum memory required - Minimum possible memory value for standard_memory that enables ABAQUS to solve the problem. Use of memory will be minimized by writing as much information as possible to disk which will increase I/O time.

(4) Memory to minimize I/O - Value of standard_memory that allows ABAQUS to keep all significant scratch files in memory. This will minimize I/O time for the user with access to a large amount of memory.

(5) Required disk space - Amount of disk required for scratch files. These will be deleted at the end of the analysis.

Note - whenever possible the user should set standard_memory to be less than the physical memory on the machine. Unless necessary users should not make use of virtual memory even in an attempt to keep scratch files in memory.

Note - if a reordering is performed (this will generally be done only for 3D-3D large sliding problems), the size estimates done at this time will no longer be valid. The estimates will be redone in the event of a reordering and a new summary will be printed for the remaining steps.

Size estimates for current step

<table>
<thead>
<tr>
<th>Size Estimate</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of equations</td>
<td>288</td>
</tr>
<tr>
<td>Max DoF wavefront</td>
<td>46</td>
</tr>
<tr>
<td>Floating point ops per solver iteration</td>
<td>1.53E+05</td>
</tr>
<tr>
<td>Memory used for step</td>
<td>14.66 MBYTES</td>
</tr>
</tbody>
</table>

Estimated file sizes
FILE           KWORDS           KBYTES
..fct             6.036           47.156
..ncr             0.960            7.500
..opr             5.280           41.250
-------        -------          -------
TOTAL          12.276           95.906

INCREMENT     1 SUMMARY

TIME INCREMENT COMPLETED 1.00 , FRACTION OF STEP COMPLETED 1.00
STEP TIME COMPLETED 1.00 , TOTAL TIME COMPLETED 1.00

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>148</td>
<td>1</td>
<td>-3.0000E+05</td>
<td>-3.0000E+05</td>
<td>-3.0000E+05</td>
<td>6.4176E-05</td>
</tr>
</tbody>
</table>

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT FOOT-</th>
<th>E11</th>
<th>E22</th>
<th>E33</th>
<th>E12</th>
</tr>
</thead>
</table>

NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>151</td>
<td>3.4280E-09</td>
</tr>
<tr>
<td>152</td>
<td>2.6748E-09</td>
</tr>
<tr>
<td>153</td>
<td>1.7930E-09</td>
</tr>
<tr>
<td>154</td>
<td>1.0005E-09</td>
</tr>
<tr>
<td>155</td>
<td>4.6324E-10</td>
</tr>
<tr>
<td>156</td>
<td>-1.5963E-12</td>
</tr>
<tr>
<td>157</td>
<td>-5.3404E-10</td>
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<tr>
<td>158</td>
<td>-1.0048E-09</td>
</tr>
<tr>
<td>159</td>
<td>-1.5306E-09</td>
</tr>
</tbody>
</table>
CYLINDRICAL TRIAXIAL TEST
STEP 2 INCREMENT 1

TIME COMPLETED IN THIS STEP 0.

STEP 2 STATIC ANALYSIS

AUTOMATIC TIME CONTROL WITH -
A SUGGESTED INITIAL TIME INCREMENT OF 1.000E-03
AND A TOTAL TIME PERIOD OF 1.00
THE MINIMUM TIME INCREMENT ALLOWED IS 1.000E-03
THE MAXIMUM TIME INCREMENT ALLOWED IS 0.100

MEMORY AND DISK ESTIMATE

SIZE ESTIMATES FOR CURRENT STEP

NUMBER OF EQUATIONS 288
MAX DOF WAVEFRONT 46
FLOATING POINT OPS PER SOLVER ITERATION 1.53E+05
MEMORY USED FOR STEP 14.66 MBYTES

ESTIMATED FILE SIZES

FILE     KWORDS  KBYTES
--------  -------  -------
.fct     6.036    47.156
.nck     0.960    7.500
.opr     5.280    41.250
--------  -------  -------
TOTAL    12.276   95.906

INCREMENT 1 SUMMARY

TIME INCREMENT COMPLETED 1.000E-03, FRACTION OF STEP COMPLETED 1.000E-03
STEP TIME COMPLETED 1.000E-03, TOTAL TIME COMPLETED 1.00

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOTE</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT-</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>151</td>
<td></td>
<td>-1.2000E-04</td>
</tr>
<tr>
<td>152</td>
<td></td>
<td>-1.2000E-04</td>
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<tr>
<td>153</td>
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<td>-1.2000E-04</td>
</tr>
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<td>-1.2000E-04</td>
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<td>158</td>
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<td>-1.2000E-04</td>
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<tr>
<td>159</td>
<td></td>
<td>-1.2000E-04</td>
</tr>
</tbody>
</table>

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT-</th>
<th>U1</th>
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<tbody>
<tr>
<td>36</td>
<td></td>
<td>5.3881E-06</td>
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<td>66</td>
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<td>5.3759E-06</td>
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<tr>
<td>126</td>
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<td>5.3885E-06</td>
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<tr>
<td>TOTAL</td>
<td></td>
<td>2.1528E-05</td>
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</tbody>
</table>

INCREMENT 2 SUMMARY

TIME INCREMENT COMPLETED 1.000E-03, FRACTION OF STEP COMPLETED 2.000E-03
STEP TIME COMPLETED 2.000E-03, TOTAL TIME COMPLETED 1.00

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET E1148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT</th>
<th>FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
</table>

NODE OUTPUT
THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
<th>U2</th>
<th>NOTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>151</td>
<td>-2.4000E-04</td>
<td></td>
</tr>
<tr>
<td>152</td>
<td>-2.4000E-04</td>
<td></td>
</tr>
<tr>
<td>153</td>
<td>-2.4000E-04</td>
<td></td>
</tr>
<tr>
<td>154</td>
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<td></td>
</tr>
<tr>
<td>155</td>
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<td></td>
</tr>
<tr>
<td>157</td>
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<td>159</td>
<td>-2.4000E-04</td>
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</tr>
</tbody>
</table>

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
<th>U1</th>
<th>NOTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
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<tr>
<td>66</td>
<td>1.0911E-05</td>
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<tr>
<td>96</td>
<td>1.0911E-05</td>
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</tr>
<tr>
<td>126</td>
<td>1.0954E-05</td>
<td></td>
</tr>
</tbody>
</table>

TOTAL | 4.3731E-05 |

INCREMENT 3 SUMMARY

TIME INCREMENT COMPLETED 1.500E-03, FRACTION OF STEP COMPLETED 3.500E-03
STEP TIME COMPLETED 3.500E-03, TOTAL TIME COMPLETED 1.00

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT PT FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
</table>

NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
<th>U2</th>
<th>NOTE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT-</th>
<th>U1</th>
</tr>
</thead>
<tbody>
<tr>
<td>151</td>
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<td>-4.2000E-04</td>
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</tr>
</tbody>
</table>

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>148</td>
<td>1</td>
<td>-3.0867E+05</td>
<td>-4.0171E+05</td>
<td>-3.2057E+05</td>
<td>-1.1355E+04</td>
</tr>
</tbody>
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THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT-</th>
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</tr>
</thead>
<tbody>
<tr>
<td>151</td>
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THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
<thead>
<tr>
<th>NODE</th>
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<th>U1</th>
</tr>
</thead>
<tbody>
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<td>36</td>
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<tr>
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<td>3.2196E-05</td>
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<td>96</td>
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<td>3.2196E-05</td>
</tr>
<tr>
<td>126</td>
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<td>3.2413E-05</td>
</tr>
</tbody>
</table>

TOTAL: 1.2922E-04

INCREMENT 5 SUMMARY

TIME INCREMENT COMPLETED 3.375E-03, FRACTION OF STEP COMPLETED 9.125E-03
STEP TIME COMPLETED 9.125E-03, TOTAL TIME COMPLETED 1.01

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT</th>
<th>FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>148</td>
<td>1</td>
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<td>-3.1037E+05</td>
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<td>-3.3164E+05</td>
<td>-1.3636E+04</td>
</tr>
</tbody>
</table>

NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT-</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
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<tr>
<td>159</td>
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<td>-1.0950E-03</td>
</tr>
</tbody>
</table>

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR
NODE FOOT- U1
NOTE
36    6.7061E-05
66    6.9779E-05
96    6.9779E-05
126   6.7062E-05
TOTAL 2.7368E-04

INCREMENT 6 SUMMARY
TIME INCREMENT COMPLETED 5.063E-03, FRACTION OF STEP COMPLETED 1.419E-02
STEP TIME COMPLETED 1.419E-02, TOTAL TIME COMPLETED 1.01

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET E1148

ELEMENT PT FOOT- S11 S22 S33 S12
NOTE
148 1 -3.1650E+05 -4.6447E+05 -3.4808E+05 -2.0918E+04

NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

NODE FOOT- U2
NOTE
151  -1.7025E-03
152  -1.7025E-03
153  -1.7025E-03
154  -1.7025E-03
155  -1.7025E-03
156  -1.7025E-03
157  -1.7025E-03
158  -1.7025E-03
159  -1.7025E-03

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

NODE FOOT- U1
NOTE
36    1.2334E-04
66    1.2539E-04
96    1.2539E-04
THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT</th>
<th>FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>148</td>
<td>1</td>
<td></td>
<td>-3.2489E+05</td>
<td>-5.1279E+05</td>
<td>-3.6956E+05</td>
<td>-3.0745E+04</td>
</tr>
</tbody>
</table>

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT-</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>151</td>
<td></td>
<td>-2.6138E-03</td>
</tr>
<tr>
<td>152</td>
<td></td>
<td>-2.6138E-03</td>
</tr>
<tr>
<td>153</td>
<td></td>
<td>-2.6138E-03</td>
</tr>
<tr>
<td>154</td>
<td></td>
<td>-2.6138E-03</td>
</tr>
<tr>
<td>155</td>
<td></td>
<td>-2.6138E-03</td>
</tr>
<tr>
<td>156</td>
<td></td>
<td>-2.6138E-03</td>
</tr>
<tr>
<td>157</td>
<td></td>
<td>-2.6138E-03</td>
</tr>
<tr>
<td>158</td>
<td></td>
<td>-2.6138E-03</td>
</tr>
<tr>
<td>159</td>
<td></td>
<td>-2.6138E-03</td>
</tr>
</tbody>
</table>

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT-</th>
<th>U1</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td></td>
<td>2.0952E-04</td>
</tr>
<tr>
<td>66</td>
<td></td>
<td>2.1471E-04</td>
</tr>
<tr>
<td>96</td>
<td></td>
<td>2.1471E-04</td>
</tr>
<tr>
<td>126</td>
<td></td>
<td>2.0952E-04</td>
</tr>
</tbody>
</table>

TOTAL 8.4847E-04
## Element Output

The following table is printed at the integration points for element type CAX4R and element set EL148.

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT FOOT- NOTE</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>148</td>
<td>1</td>
<td>-3.3607E+05</td>
<td>-5.6965E+05</td>
<td>-3.9720E+05</td>
<td>-4.3598E+04</td>
</tr>
</tbody>
</table>

### Node Output

The following table is printed for nodes belonging to node set NTOP.

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT- U2</th>
<th>NOTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>151</td>
<td></td>
<td>-3.9806E-03</td>
</tr>
<tr>
<td>152</td>
<td></td>
<td>-3.9806E-03</td>
</tr>
<tr>
<td>153</td>
<td></td>
<td>-3.9806E-03</td>
</tr>
<tr>
<td>154</td>
<td></td>
<td>-3.9806E-03</td>
</tr>
<tr>
<td>155</td>
<td></td>
<td>-3.9806E-03</td>
</tr>
<tr>
<td>156</td>
<td></td>
<td>-3.9806E-03</td>
</tr>
<tr>
<td>157</td>
<td></td>
<td>-3.9806E-03</td>
</tr>
<tr>
<td>158</td>
<td></td>
<td>-3.9806E-03</td>
</tr>
<tr>
<td>159</td>
<td></td>
<td>-3.9806E-03</td>
</tr>
</tbody>
</table>

The following table is printed for nodes belonging to node set NOUTR.

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT- U1</th>
<th>NOTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td></td>
<td>3.4612E-04</td>
</tr>
<tr>
<td>66</td>
<td></td>
<td>3.4962E-04</td>
</tr>
<tr>
<td>96</td>
<td></td>
<td>3.4962E-04</td>
</tr>
<tr>
<td>126</td>
<td></td>
<td>3.4612E-04</td>
</tr>
</tbody>
</table>

**Total**: 1.3915E-03

## Increment 9 Summary

Time increment completed: 1.709E-02, Fraction of step completed: 5.026E-02

Step time completed: 5.026E-02, Total time completed: 1.05
THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>148</td>
<td>1</td>
<td>-3.5345E+05</td>
<td>-6.5430E+05</td>
<td>-4.3518E+05</td>
<td>-6.3727E+04</td>
</tr>
</tbody>
</table>

**Node Output**

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>151</td>
<td>-6.0309E-03</td>
</tr>
<tr>
<td>152</td>
<td>-6.0309E-03</td>
</tr>
<tr>
<td>153</td>
<td>-6.0309E-03</td>
</tr>
<tr>
<td>154</td>
<td>-6.0309E-03</td>
</tr>
<tr>
<td>155</td>
<td>-6.0309E-03</td>
</tr>
<tr>
<td>156</td>
<td>-6.0309E-03</td>
</tr>
<tr>
<td>157</td>
<td>-6.0309E-03</td>
</tr>
<tr>
<td>158</td>
<td>-6.0309E-03</td>
</tr>
<tr>
<td>159</td>
<td>-6.0309E-03</td>
</tr>
</tbody>
</table>

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
<th>U1</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>5.5386E-04</td>
</tr>
<tr>
<td>66</td>
<td>5.5133E-04</td>
</tr>
<tr>
<td>96</td>
<td>5.5133E-04</td>
</tr>
<tr>
<td>126</td>
<td>5.5386E-04</td>
</tr>
<tr>
<td>TOTAL</td>
<td>2.2104E-03</td>
</tr>
</tbody>
</table>

**Increment 10 Summary**

TIME INCREMENT COMPLETED 2.563E-02, FRACTION OF STEP COMPLETED 7.589E-02
STEP TIME COMPLETED 7.589E-02, TOTAL TIME COMPLETED 1.08

**Element Output**

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET
**Appendix D**

**ELEMENT OUTPUT**

The following table is printed for nodes belonging to node set `NTOP`.

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>151</td>
<td>-9.1064E-03</td>
</tr>
<tr>
<td>152</td>
<td>-9.1064E-03</td>
</tr>
<tr>
<td>153</td>
<td>-9.1064E-03</td>
</tr>
<tr>
<td>154</td>
<td>-9.1064E-03</td>
</tr>
<tr>
<td>155</td>
<td>-9.1064E-03</td>
</tr>
<tr>
<td>156</td>
<td>-9.1064E-03</td>
</tr>
<tr>
<td>157</td>
<td>-9.1064E-03</td>
</tr>
<tr>
<td>158</td>
<td>-9.1064E-03</td>
</tr>
<tr>
<td>159</td>
<td>-9.1064E-03</td>
</tr>
</tbody>
</table>

The following table is printed for nodes belonging to node set `NOUTR`.

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
<th>U1</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>8.7081E-04</td>
</tr>
<tr>
<td>66</td>
<td>8.6056E-04</td>
</tr>
<tr>
<td>96</td>
<td>8.6056E-04</td>
</tr>
<tr>
<td>126</td>
<td>8.7081E-04</td>
</tr>
</tbody>
</table>

**TOTAL**  
3.4627E-03

**INCREMENT 11 SUMMARY**

- Time increment completed: 3.844E-02
- Fraction of step completed: 0.114
- Step time completed: 0.114
- Total time completed: 1.11

**ELEMENT OUTPUT**

The following table is printed at the integration points for element type `CAX4R` and element set `EL148`.

<table>
<thead>
<tr>
<th>ELEMENT PT FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>148 1</td>
<td>-4.0445E+05</td>
<td>-9.1866E+05</td>
<td>-5.5422E+05</td>
<td>-1.2231E+05</td>
</tr>
</tbody>
</table>
THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOTE</td>
<td></td>
</tr>
<tr>
<td>151</td>
<td>-1.3720E-02</td>
</tr>
<tr>
<td>152</td>
<td>-1.3720E-02</td>
</tr>
<tr>
<td>153</td>
<td>-1.3720E-02</td>
</tr>
<tr>
<td>154</td>
<td>-1.3720E-02</td>
</tr>
<tr>
<td>155</td>
<td>-1.3720E-02</td>
</tr>
<tr>
<td>156</td>
<td>-1.3720E-02</td>
</tr>
<tr>
<td>157</td>
<td>-1.3720E-02</td>
</tr>
<tr>
<td>158</td>
<td>-1.3720E-02</td>
</tr>
<tr>
<td>159</td>
<td>-1.3720E-02</td>
</tr>
</tbody>
</table>

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
<th>U1</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOTE</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>1.3595E-03</td>
</tr>
<tr>
<td>66</td>
<td>1.3480E-03</td>
</tr>
<tr>
<td>96</td>
<td>1.3480E-03</td>
</tr>
<tr>
<td>126</td>
<td>1.3595E-03</td>
</tr>
</tbody>
</table>

TOTAL 5.4150E-03

INCREMENT 12 SUMMARY

TIME INCREMENT COMPLETED 5.767E-02, FRACTION OF STEP COMPLETED 0.172
STEP TIME COMPLETED 0.172, TOTAL TIME COMPLETED 1.17

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOTE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>148</td>
<td>1</td>
<td>-4.5018E+05</td>
<td>-1.1273E+06</td>
<td>-6.4883E+05</td>
<td>-1.7281E+05</td>
</tr>
</tbody>
</table>
### Appendix D

The following table is printed for nodes belonging to node set NTOP

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT-</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>151</td>
<td></td>
<td>-2.0639E-02</td>
</tr>
<tr>
<td>152</td>
<td></td>
<td>-2.0639E-02</td>
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<tr>
<td>153</td>
<td></td>
<td>-2.0639E-02</td>
</tr>
<tr>
<td>154</td>
<td></td>
<td>-2.0639E-02</td>
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<tr>
<td>155</td>
<td></td>
<td>-2.0639E-02</td>
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<tr>
<td>156</td>
<td></td>
<td>-2.0639E-02</td>
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<tr>
<td>157</td>
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<td>-2.0639E-02</td>
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<td>158</td>
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<td>-2.0639E-02</td>
</tr>
<tr>
<td>159</td>
<td></td>
<td>-2.0639E-02</td>
</tr>
</tbody>
</table>

The following table is printed for nodes belonging to node set NOUTR

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT-</th>
<th>U1</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td></td>
<td>2.1066E-03</td>
</tr>
<tr>
<td>66</td>
<td></td>
<td>2.0672E-03</td>
</tr>
<tr>
<td>96</td>
<td></td>
<td>2.0672E-03</td>
</tr>
<tr>
<td>126</td>
<td></td>
<td>2.1066E-03</td>
</tr>
</tbody>
</table>

TOTAL 8.3478E-03

Increment 13 Summary

Time increment completed 8.650E-02, fraction of step completed 0.258
Step time completed 0.258, total time completed 1.26

Element Output

The following table is printed at the integration points for element type CAX4R and element set E1148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT</th>
<th>FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>148</td>
<td>1</td>
<td></td>
<td>-4.8895E+05</td>
<td>-1.3937E+06</td>
<td>-7.6565E+05</td>
<td>-2.2129E+05</td>
</tr>
</tbody>
</table>

Node Output

The following table is printed for nodes belonging to node set NTOP

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT-</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>151</td>
<td></td>
<td>-3.1019E-02</td>
</tr>
</tbody>
</table>

281
THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT-</th>
<th>U1</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td></td>
<td>3.2278E-03</td>
</tr>
<tr>
<td>66</td>
<td></td>
<td>3.2528E-03</td>
</tr>
<tr>
<td>96</td>
<td></td>
<td>3.2528E-03</td>
</tr>
<tr>
<td>126</td>
<td></td>
<td>3.2278E-03</td>
</tr>
</tbody>
</table>

TOTAL 1.2961E-02

INCREMENT 14 SUMMARY

TIME INCREMENT COMPLETED 0.100 , FRACTION OF STEP COMPLETED 0.358
STEP TIME COMPLETED 0.358 , TOTAL TIME COMPLETED 1.36

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>148</td>
<td>1</td>
<td>-5.0334E+05</td>
<td>-1.5649E+06</td>
<td>-8.4976E+05</td>
<td>-2.4198E+05</td>
</tr>
</tbody>
</table>

NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
<th>NODE</th>
<th>FOOT-</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>151</td>
<td></td>
<td>-4.3019E-02</td>
</tr>
<tr>
<td>152</td>
<td></td>
<td>-4.3019E-02</td>
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<td>153</td>
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<td>-4.3019E-02</td>
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<td>154</td>
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<td>155</td>
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<td>-4.3019E-02</td>
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<td>156</td>
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<td>-4.3019E-02</td>
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<td>157</td>
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<td>-4.3019E-02</td>
</tr>
<tr>
<td>158</td>
<td></td>
<td>-4.3019E-02</td>
</tr>
</tbody>
</table>
THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
<th>U1</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOTE</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>4.5235E-03</td>
</tr>
<tr>
<td>66</td>
<td>4.7361E-03</td>
</tr>
<tr>
<td>96</td>
<td>4.7361E-03</td>
</tr>
<tr>
<td>126</td>
<td>4.5235E-03</td>
</tr>
<tr>
<td>TOTAL</td>
<td>1.8519E-02</td>
</tr>
</tbody>
</table>

INCREMENT 15 SUMMARY

TIME INCREMENT COMPLETED 0.100 , FRACTION OF STEP COMPLETED 0.458
STEP TIME COMPLETED 0.458 , TOTAL TIME COMPLETED 1.46

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOTE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>148</td>
<td>1</td>
<td>-5.1792E+05</td>
<td>-1.6835E+06</td>
<td>-9.1032E+05</td>
<td>-2.5880E+05</td>
</tr>
</tbody>
</table>

NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
<th>U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOTE</td>
<td></td>
</tr>
<tr>
<td>151</td>
<td>-5.5019E-02</td>
</tr>
<tr>
<td>152</td>
<td>-5.5019E-02</td>
</tr>
<tr>
<td>153</td>
<td>-5.5019E-02</td>
</tr>
<tr>
<td>154</td>
<td>-5.5019E-02</td>
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<tr>
<td>155</td>
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</tr>
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<td>-5.5019E-02</td>
</tr>
<tr>
<td>157</td>
<td>-5.5019E-02</td>
</tr>
<tr>
<td>158</td>
<td>-5.5019E-02</td>
</tr>
<tr>
<td>159</td>
<td>-5.5019E-02</td>
</tr>
</tbody>
</table>

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
<th>U1</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOTE</td>
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</tr>
</tbody>
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NOTE

36  5.8297E-03
66  6.2363E-03
96  6.2363E-03
126 5.8297E-03
TOTAL 2.4132E-02

INCREMENT 16 SUMMARY

TIME INCREMENT COMPLETED 0.100, FRACTION OF STEP COMPLETED 0.558
STEP TIME COMPLETED 0.558, TOTAL TIME COMPLETED 1.56

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
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<th>FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
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<tbody>
<tr>
<td>148</td>
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<td>-5.2060E+05</td>
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<td>-9.5338E+05</td>
<td>-2.6441E+05</td>
</tr>
</tbody>
</table>

NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
<th>NODE</th>
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<th>U2</th>
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<tr>
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<td>-6.7019E-02</td>
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THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
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<tr>
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TOTAL 2.9825E-02

INCREMENT 17 SUMMARY

TIME INCREMENT COMPLETED 0.100 , FRACTION OF STEP COMPLETED 0.658
STEP TIME COMPLETED 0.658 , TOTAL TIME COMPLETED 1.66

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
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<th>S33</th>
<th>S12</th>
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NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
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</table>

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
<thead>
<tr>
<th>NODE</th>
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<tbody>
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TOTAL 3.5605E-02

INCREMENT 18 SUMMARY

TIME INCREMENT COMPLETED 0.100 , FRACTION OF STEP COMPLETED 0.758
STEP TIME COMPLETED 0.758, TOTAL TIME COMPLETED 1.76

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT FOOT- NOTE</th>
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<th>S22</th>
<th>S33</th>
<th>S12</th>
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<tbody>
<tr>
<td>148</td>
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<td>-2.5904E+05</td>
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NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
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<tbody>
<tr>
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THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
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<tbody>
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<td>1.1314E-02</td>
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TOTAL 4.1405E-02

INCREMENT 19 SUMMARY

TIME INCREMENT COMPLETED 0.100, FRACTION OF STEP COMPLETED 0.858
STEP TIME COMPLETED 0.858, TOTAL TIME COMPLETED 1.86

ELEMENT OUTPUT
THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
</thead>
<tbody>
<tr>
<td>148</td>
<td>1</td>
<td>-5.0602E+05</td>
<td>-1.9221E+06</td>
<td>-1.0221E+06</td>
<td>-2.5495E+05</td>
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</table>

NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
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THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
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<tbody>
<tr>
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INCREMENT 20 SUMMARY

TIME INCREMENT COMPLETED 0.100 , FRACTION OF STEP COMPLETED 0.958
STEP TIME COMPLETED 0.958 , TOTAL TIME COMPLETED 1.96

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148
THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
<thead>
<tr>
<th>NODE FOOT-</th>
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THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

<table>
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<th>NODE FOOT-</th>
<th>U1</th>
<th>NOTE</th>
</tr>
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<tbody>
<tr>
<td>36</td>
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</table>

TOTAL 5.3099E-02

INCREMENT 21 SUMMARY

TIME INCREMENT COMPLETED 4.151E-02, FRACTION OF STEP COMPLETED 1.00
STEP TIME COMPLETED 1.00, TOTAL TIME COMPLETED 2.00

THE FOLLOWING TABLE IS PRINTED AT THE INTEGRATION POINTS FOR ELEMENT TYPE CAX4R AND ELEMENT SET EL148

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>PT FOOT-</th>
<th>S11</th>
<th>S22</th>
<th>S33</th>
<th>S12</th>
</tr>
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<tr>
<td>148</td>
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<td>-2.3890E+05</td>
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</table>
THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NTOP

<table>
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THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET NOUTR

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<tbody>
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<tr>
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THE ANALYSIS HAS BEEN COMPLETED

ANALYSIS COMPLETE

JOB TIME SUMMARY
USER TIME (SEC) = 15.200
SYSTEM TIME (SEC) = 0.94000
TOTAL CPU TIME (SEC) = 16.140
WALLCLOCK TIME (SEC) = 16