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REPUDIATION, RETALIATION, AND THE SECONDARY MARKET PRICE OF SOVEREIGN DEBTS

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This note is concerned with the relationship between the secondary-market price of sovereign debts and the possibility of repudiation and retaliation. Using the distinction between "good" and "bad" states of world, the debtor countries optimal repudiation rate as well as the creditor's optimal retaliation and reservation price for any repudiation rate are analyzed under the assumption that the debtor and creditor are risk averse and expected-utility maximizers. Matching both debtor and creditor's considerations, the paper presents and discusses the properties of the equilibrium repudiation and retaliation rates and the secondary market price of a sovereign debt.
1. INTRODUCTION

During the 1970's and the 1980's the aggregate indebtedness of the developing countries grew very rapidly leading to a heightened concern about their ability to repay their debts. Policy measures that were adopted to alleviate the debt crisis, such as cuts in subsidies and wages, raised instead the income inequality and poverty levels in many of the developing countries. Furthermore, the large external debts tended to adversely affect the developing countries' growth prospects. In this respect Krugman (1989) and Sachs (1988, 1989) argue that high governmental debt-service payments require high tax rates that discourage capital formation and repatriation of flight capital. Similarly, Dornbusch (1988) argues that the fact that the government is the main maker of debt-service payments in most of the heavily indebted countries moderates the improving effect of devaluation on the trade balance.

These income-depressing effects of the external debt raise the probability of default when an adverse shock arrives. Moreover, country's external debt is a sovereign debt; and hence, when the potential penalties are insufficient, a rise in the country's level of indebtedness increases its incentive to repudiate (Kenen, 1990). Repudiation, in turn, might lead to a retaliation which takes the form of a seizure of the debtor country's reserves and other assets abroad.

A mechanism that may relieve the developing countries' debt burden and increase the expected debt repayments is the secondary market for sovereign debt. In this market, the debtor country can buy back its debt at a large discount, reflecting the creditors' belief that the debtor country will not meet its obligations fully. In this respect, Berg and Sachs (1988), who fitted a tobit model to developing countries' debt discounts, concluded that higher income inequality leads to a greater discount, while higher outward orientation, agricultural share, and GNP per capita squared tend to decrease the secondary market discount.

The aim of this paper is to provide an analytical explanation to the relationship between the secondary market price of sovereign debt and the possibility of repudiation and retaliation under the assumption that both creditor and debtor have an aversion toward risk and maximize expected utility. The analysis assumes that the creditor's retaliation is
limited to seizing the debtor's assets abroad and restrained from escalating into a trade warfare. It is assumed further that the debtor country is relatively small. In this case, the creditor and debtor should not be viewed as being engaged in an implicit contract such as a reputation contract:

"A good reputation for repaying foreign loans does not enhance a small country's ability to borrow abroad. As long as the country faces competitive foreign investors, then any service provided by the current lender can equally well be provided by a new investor." (Bulow and Rogoff, 1989, p.47.)

The paper continues as follows. Section 2 analyzes the debtor country's optimal repudiation rate. Section 3 inquires the creditor's optimal retaliation and presents the creditor's reservation price for any rate of repudiation. Section 4 matches the creditor's considerations with those of the debtor country, and presents the equilibrium repudiation and seizure rates, and the secondary market price of sovereign debt.

2. DEBTOR COUNTRY'S OPTIMAL REPUDIATION RATE

The formulation of the debtor country's behavior is based on the common distinction between "good" and "bad" states of world (Cf. Krugman, 1988). It is assumed that in a "good" state of world the debtor country enjoys relatively high revenues and is able to repay its loan; whereas in a "bad" state of the world the debtor country's revenues are moderate and consequently it repudiates part of the debt.

The debtor country's decision problem is postulated as setting the repudiation rate to maximize expected utility from the returns on domestic assets and asset holdings abroad, taking into account, on the one hand, that loan repayments reduce the consumption and investment budget; but on the other hand, that in the case of a default, creditors retaliate by seizing part of the asset holdings abroad. It is also assumed that creditors provide signals about the potential retaliation to any level of repudiation; and that these signals enable the debtor country to accurately assess the seizure rate function s(r). For simplicity sake, the rates of return on domestic and foreign assets are assumed to be identical.
In this setting the distribution of the debtor country's consumption and investment budget, $x$, is given by

$$
x = \begin{cases} 
(K + A)v - D & 1 - p \\
\left[K + (1 - s(r))A\right] \delta v - (1 - r)D & p 
\end{cases}
$$

where,

- $D$ = the debtor country's external debt;
- $A$ = the debtor country's assets abroad;
- $K$ = the debtor country's domestic assets;
- $p$ = the probability of a "good" state of world;
- $v$ = the return per unit of asset under a "good" state of world;
- $1 - p$ = the probability of a "bad" state of world;
- $\delta$ = the ratio of the debtor country's revenues under a bad state of world to those obtainable under a good state of world ($0 < \delta < 1$);
- $r$ = the debtor country's repudiation rate when a bad state of world is realized; and
- $s(r)$ = the seizure rate function, i.e., the share of the debtor's assets holding abroad seized by the creditor in retaliation to any given rate of repudiation.

The distribution of $x$ displayed above implies that the mean and variance of the debtor country's consumption and investment budget are

$$
E(X) = [(K + A)v - D] (1 - p) + \left[[K + (1 - s(r))A]\right] \delta v - (1 - r)D]p
$$

and

$$
\text{var}(x) = E(x^2) - (E(x))^2
$$

$$
= (1 - p) [(K + A)v - D]^2 + p\left[[K + (1 - s(r))A]\right] \delta v - (1 - r)D]^2
$$

$$
+ \left[[K + A)v - D] (1 - p) + \left[(K + (1 - s(r))A) \delta v - (1 - r)D]\right]p\right]^2.
$$

It is assumed for tractability that the debtor country has an expected utility function of the following flexible form:
\[ E[u(X)] = E(X) - 0.5R_d \text{var}(X) \]  \hspace{1cm} (4)

This form can be viewed as a second-order Taylor's approximation of the general expected utility function, indicating that the debtor country's utility from the expected consumption and investment is reduced by the costs of risk bearing stemming from the uncertainty about the state of the world and revenues' level. The costs of risk bearing are considered to be proportional to the variance of the consumption and investment budget, and are amplified by the debtor's degree of absolute risk aversion, \( R_d \). (See Freund, 1956, for a rigorous development of the mean-variance expected utility function, and Hammond, 1974, for a discussion of the generality of this framework.)

By substituting 2 and 3 into 4, the debtor country's decision problem can be rendered as

Maximizing  

\[
\begin{align*}
\text{Maximizing} \quad & \{(K + A)v - D\} (1 - p) + \{(K + (1 - s(r))A) \delta v - (1 - r)D\}p \\
- & 0.5R_d \{(1 - p) [(K + A)v - D]^2 + p[(K + (1 - s(r))A) \delta v - (1 - r)D]^2 \\
+ & \{(K + A)v - D\}(1 - p) + \{(K + (1 - s(r))A) \delta v - (1 - r)D\}p\}^2).
\end{align*}
\]

with respect to \( r \) and subject to the repudiation constraint

\[ r \leq 1. \]  \hspace{1cm} (5)

The Khun-Tucker conditions for maximum expected utility from repudiation are:

\[
\begin{align*}
p[D - s'(r) \delta vA][1 - R_d [(K + (1 - s(r))A) \delta v - (1 - r)D] (1 + p) \\
- R_d [(K + A)v - D](1 - p)] - \mu & \leq 0 \tag{6a}
\end{align*}
\]

\[
\begin{align*}
r[p[D - s'(r) \delta vA][1 - R_d [(K + (1 - s(r))A) \delta v - (1 - r)D] (1 + p) \\
- R_d [(K + A)v - D](1 - p)] - \mu = 0 \tag{6b}
\end{align*}
\]

\[
\begin{align*}
r & \leq 1 \tag{6c}
\end{align*}
\]

\[
\begin{align*}
\mu & \geq 0 \tag{6d}
\end{align*}
\]

where \( \mu \) is the Lagrange multiplier representing the shadow price of the repudiation constraint.
It is assumed throughout the rest of the paper that in a "bad" state of world the debtor country refrains from practising a complete repudiation and that it is worthy for the country to repay at least part of its loans (i.e., \(\mu = 0\) and hence \(r < 1\)). This is the case where the debtor country's assets holding abroad are sizable and/or the returns on these assets are sufficiently large, and the penalty is perceived to significantly increase with the repudiation rate. Under this assumption there is an interior solution to the debtor country's decision problem -- the optimal repudiation rate from the debtor country’s perspectives should obey the following equality:

\[
1 - R_d \{ [(K + (1 - s (r))A] \delta v - (1 - r)D](1 + p) - [(K + A) v - D](1 - p)\} = 0. \tag{7}
\]

That is, a change in the costs of risk-bearing from an infinitesimal increase in the repudiation rate should be offset by the change in the expected consumption and investment budget.

3. CREDITOR'S OPTIMAL SEIZURE RATE AND SECONDARY MARKET RESERVATION PRICE

Using the aforementioned distinction between "good" and "bad" states of world; in which the debtor country fully repays its loans or partially repays its loan, respectively; the returns for the creditor, \(y\), are distributed as follows:

\[
y = \begin{cases} 
D & (1 - p) \\
(1 - r)D + sA & p
\end{cases} \tag{8}
\]

It is assumed that the creditor's preferences over these random returns can be represented by the mean-variance expected utility function:

\[
E[u (y)] = E(y) - 0.5R_c \text{ var } (y) \tag{9}
\]

where \(R_c\) denotes the creditor's degree of absolute risk aversion.

Given that the creditor's subjective distribution of returns is as displayed by Eq. 8, the mean and variance of \(y\) are:

\[
E(y) = (1 - rp) D + psA \tag{10}
\]
\[ \text{var}(y) = (1 - p)D^2 + p[(1 - r)D + sA]^2 - [(1 - rp)D + psA]^2 \]
\[ = p (1 - p) [r^2 D^2 - 2rsAD + s^2 A^2]. \quad (11) \]

The substitution of Eq. 10 and Eq. 11 into Eq. 9 implies that the creditor's expected utility from the returns on the developing country's debt can be rendered as:

\[ E[u(y)] = (1 - rp)D + psA - 0.5R_c p (1 - p) [r^2 D^2 - 2rsAD + s^2 A^2]. \quad (12) \]

Obviously, it is the creditor's interest to maximize this expression by an appropriate choice of the seizure rate, \( s \), that will be applied once the debtor country defaults. This choice is made subject to the collateral constraint that \( s \leq 1 \).

The Kuhn-Tucker conditions for maximum constrained-expected utility from debt repayment and seizure of the debtor country's foreign assets are

\[ pA + R_c p(1 - p)(rAD - sA^2) - \Gamma \leq 0 \quad (13a) \]
\[ s[pA + R_c p(1 - p)(rAD - sA^2) - \Gamma] = 0 \quad (13b) \]
\[ s \leq 0 \quad (13c) \]
\[ \Gamma \geq 0 \quad (13d) \]

where \( \Gamma \) is the Lagrange multiplier. These conditions imply that in the case of repudiation \( (r > 0) \), the optimal seizure rate is given by

\[ s^* = \begin{cases} 
[(1 - p)R_c A]^{-1} + (D/A)r & \text{if } \Gamma = 0 \\
1 & \text{if } \Gamma > 0 
\end{cases} \quad (14) \]

The above solution to the creditor's decision problem indicates that if the collateral constraint was not binding (i.e., \( A \) is relatively large in comparison to \( D \) and hence \( \Gamma = 0 \)), the creditor's best policy is to adopt a "tit for tat" rule by which the seizure rate is smaller than unity and linearly increases with the debtor's repudiation rate according to the debtor's effective leverage \( (D/A) \). In this case the optimal seizure rate also increases with the creditor's subjective probability of default, and decreases with the
creditor's degree of absolute risk aversion. It is important to note that the Kuhn-Tucker conditions imply further that even in a "good" state of world in which the debtor country is expected to repay its debt fully (i.e., \( r = 0 \)), it is still optimal for the creditor to exercise a positive seizure rate in order to compensate for the risk-bearing costs stemming from the \textit{a priori} level of uncertainty about the state of world:

\[
s^* = \frac{1}{(1 - p)R_cA}.
\]  

(15)

Obviously, when the collateral constraint is binding, the creditor's best policy is a total seizure of the debtor country's assets holding abroad.

The creditor's reservation price \( (P_c) \) of recycling the debtor country's debt can now be found by substituting the optimal seizure rate into Eq. 12 and dividing the resultant expected utility by the debt level. When the collateral constraint is not binding, the creditor's reservation price of recycling the debtor country's debt is

\[
P_c = 1 + \frac{0.5p}{(1 - p)R_c D}.
\]  

(16)

In this case the creditor's reservation price increases with the creditor's subjective probability of default, and decreases with the creditor's degree of absolute risk aversion. It is important to note further that in this case where the collateral constraint is not binding, the creditor enjoys a powerful retaliatory edge over the debtor country and hence the creditor's reservation price is greater than unity.

However, it is more likely that the debtor country's foreign assets are significantly smaller than its external liabilities and hence the collateral constraint is binding. In this case, the creditor's reservation price is found by substituting a unit seizure rate in the creditor's expected utility and dividing the resultant by D:

\[
P_c = 1 - (r + 0.5R_c (1 - p) [r^2 - 2rA/D + (A/D)^2]D - A/D) p.
\]  

(17)

Note that since the effective leverage \( (A/D) \) is smaller than \( 1+2r \), the creditor's reservation price increases with the debtor country's effective leverage. Furthermore, in this case \( P_c \) decreases with the perceived repudiation rate \( (r) \) that would be implemented by the debtor country when a "bad" state of world is realized as long as
That is, the likelihood that a rise in the perceived repudiation rate reduces the creditor's reservation price increases with the debtor country's effective leverage and the creditor's degree of absolute risk aversion, and declines with the creditor's subjective probability that the debtor country would default.

4. \textbf{EQUILIBRIUM RATE OF REPUDIATION, THE SECONDARY MARKET PRICE AND CONCLUDING REMARKS}

The analysis of the secondary market equilibrium is carried out under the assumption that both debtor country and creditor have identical assessments of the distribution of the states of nature (p). The analysis considers the most likely case in which the debtor country's effective collateral (A) is very limited and thus leading the creditor to set the seizure rate to be equal to one in the case of a default. It is assumed further that this tentative and conditional seizure rate is signaled to the debtor country. Under this assumption the debtor country's optimum condition (Eq. 7) for maximum expected utility becomes

\begin{equation}
1 - R_d \left\{ [K_0 v - (1 - r)D] (1 + p) + [(K + A) v - D] (1 - p) \right\} = 0. \quad (19)
\end{equation}

Consequently, the debtor country sets the repudiation rate to be

\begin{equation}
r^* = \frac{2 + 1/R_d D - [(1 + p)\delta v + (1 - p)v] (K/D) - (1 - p)v (A/D)}{1 + p}. \quad (20)
\end{equation}

Eq. 20 indicates that the equilibrium rate of repudiation ($r^*$) decreases with the debtor country's degree of absolute risk aversion, level of indebtedness, and domestic and foreign assets. As can be expected, $r^*$ declines with the ratio of the debtor country's revenues under a "bad" state of world to those obtainable under a "good" state of world. However, the effect of the probability of a "bad" state of world on the equilibrium repudiation rate is not clear \textit{a-priori}:

\begin{equation}
\frac{dr^*}{dp} \gtrless 0 \text{ as } r^* \gtrless \left[(1 - \delta) K/D + A/D\right] v.
\end{equation}
Furthermore, the greater the debtor country’s effective leverage the greater its equilibrium rate of repudiation. This can be easily shown by differentiating $r^*$ with respect to $D/A$:

$$\frac{d r^*}{d(D/A)} = \left(\frac{1 - p}{1 + p}\right) \frac{\nu}{(D/A)^2} > 0. \quad (21)$$

Finally, the substitution of $r^*$ into the creditor’s reservation price equation (17) yields, in turn, the secondary market equilibrium price ($P^*$) of the sovereign debt:

$$P^* = 1 - \{r^* + 0.5R_c (1 - p) [r^*^2 - 2r^*A/D + (A/D)^2]D - A/D\}p. \quad (22)$$

Note that if inequality 18 holds, $P^*$ is inversely related to $r^*$ and hence the secondary equilibrium price of sovereign debt increases with the debtor country’s levels of absolute risk aversion and domestic assets. Note further that the effect of the debtor country’s effective leverage on the secondary market price of its debt is not clear a-priori. By differentiating both sides of Eq. 22 with respect to $D/A$ and reconsidering Eq. 21 it can be shown that

$$\frac{dP^*}{d(D/A)} (D/A) > \frac{2p + R_c D(1 - p)p r^*}{(D/A)^2} \left[ p + R_c D(1 - p)p r^* \right] < 0$$

as

$$p + R_c D(1 - p)p r^* > \frac{-1}{D/A} \left[ \frac{dP^*}{dr} (r^* - \frac{1}{(D/A)^2}) + \frac{dP^*}{d(D/A)} (\frac{1 - p}{1 + p}) \frac{\nu}{(D/A)^2} - R_c D(1 - p)p \right]. \quad (23)$$

Since $\frac{dP^*}{dr} < 0$, it can be concluded that for sufficiently small effective leverage’s values, the secondary market price of the country’s external debt rises as the country’s effective leverage increases; whereas for sufficiently high values of effective leverage, the secondary market price of the country’s external debt declines as its effective leverage rises.
Furthermore, the greater the debtor country’s effective leverage the greater its equilibrium rate of repudiation. This can be easily shown by differentiating $r^*$ with respect to $D/A$:

$$\frac{d\ r^*}{d(D/A)} = \left( \frac{1 - p}{1 + p} \right) \frac{v}{(D/A)^2} > 0.$$  \hfill (21)

Finally, the substitution of $r^*$ into the creditor’s reservation price equation (17) yields, in turn, the secondary market equilibrium price ($P^*$) of the sovereign debt:

$$P^* = 1 - \left[ r^* + 0.5R_c (1 - p) [r^*^2 - 2r^*A/D + (A/D)^2]D - A/D \right]p. \hfill (22)$$

Note that if inequality 18 holds, $P^*$ is inversely related to $r^*$ and hence the secondary equilibrium price of sovereign debt increases with the debtor country’s levels of absolute risk aversion and domestic assets. Note further that the effect of the debtor country’s effective leverage on the secondary market price of its debt is not clear \textit{a-priori}. By differentiating both sides of Eq. 22 with respect to $D/A$ and reconsidering Eq. 21 it can be shown that

$$\frac{dP^*}{d(D/A)} \stackrel{<}{\sim} 0$$

as

$$p + R_c D(1 - p)r^* \stackrel{>}{\sim} \frac{-1}{D/A} \left[ \frac{\partial P^*}{\partial r} \left( \frac{1 - p}{1 + p} \right) \frac{v}{(D/A)^2} - R_c D(1 - p)p \right]. \hfill (23)$$

Since $\frac{\partial P^*}{\partial r} < 0$, it can be concluded that for sufficiently small effective leverage’s values, the secondary market price of the country’s external debt rises as the country’s effective leverage increases; whereas for sufficiently high values of effective leverage, the secondary market price of the country’s external debt declines as its effective leverage rises.
REFERENCES


