Boundary element formulations for fracture mechanics problems

Wei-Liang Wu

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BOUNDARY ELEMENT FORMULATIONS FOR FRACTURE MECHANICS PROBLEMS

By
Wei-Liang Wu

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY AT UNIVERSITY OF WOLLONGONG NORTHFIELDS AVENUE, WOLLONGONG NSW 2522 AUSTRALIA NOVEMBER 2004
UNIVERSITY OF WOLLONGONG
SCHOOL OF
MATHEMATICS AND APPLIED STATICS

This thesis is submitted to the University of Wollongong, and has not been submitted for a higher degree to any other university or institution.

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November 2004
UNIVERSITY OF WOLLONGONG

Date: November 2004

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Title: Boundary Element Formulations for Fracture Mechanics Problems
School: Mathematics and Applied Statics
Degree: Ph.D.

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To My Parents with Love and Gratitude.
Acknowledgements

I would like to acknowledge the many people who have helped me in my life and who helped bring this thesis to fruition.

In particular, I want to thank my supervisor, Dr. Xiaoping Lu who had constantly been a source of knowledge and expertise on mathematical techniques which have been utilised in this thesis. I thank her for her constant support, encouragement and her willingness to help at any time despite her hectic schedule.

Above all, I want to thank my teachers – both in Taiwan and Australia. To those who helped me learn in the classroom. To those who helped me learn in the world beyond the classroom.

I would also like to thank the colleagues of the University of Wollongong who devoted time and effort to provide me with critical feedback on earlier drafts, pointing inaccuracies, suggesting better examples, and gently noting things that needed smoothing. I would like to mention especially Dr. Ahmed A. El-Feki, Dr. Anna Maria Milan, Dr. Bin Liu and Frank Bierbrauer.

In addition, my deepest thanks are owed to my family who have always inspired me to greater things particularly my parents for their love, sacrifice and support.
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