Active seismic response control of tall buildings based on reduced order model

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Du, Haiping; Boffa, John; and Zhang, Nong: Active seismic response control of tall buildings based on reduced order model 2006.
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Disciplines
Physical Sciences and Mathematics

Publication Details

This conference paper is available at Research Online: https://ro.uow.edu.au/infopapers/3666
Active Seismic Response Control of Tall Buildings Based on Reduced Order Model

Haiping Du, John Boffa, and Nong Zhang

Abstract—This paper applies the dynamic model reduction method to obtain a reduced order model of an experimental tall building which has twenty floors and is 2.5m high. The experimental model is designed to imitate a real tall building with an active mass damper, and is used to study the available modelling and active control strategies for real tall buildings. Based on the dynamic model reduction method, a reduced order model is used within a $H_\infty$ controller design to suppress excessive vibrations induced by seismic excitation. The reduced order model effectively describes only those frequency characteristics from the full order model that are of interest. The controller design based on the low order model can be directly applied to the full order model without introducing the control and observer spillover problems. Numerical simulations confirm that the low order model is acceptable at describing the relevant dynamic characteristics of the full order model and that it can be used effectively to mitigate the vibration caused by seismic disturbances.

I. INTRODUCTION

Active control of civil engineering structures to reduce the excessive vibration caused by strong winds or earthquakes, has received considerable attention in recent years [1]. Various control strategies, such as $H_2$ (LQG) and $H_\infty$ control, neural network control, fuzzy logic control, adaptive control, sliding mode control, independent modal space control etc., have been proposed and developed to attenuate the effects of structural vibration. Even though some robust controllers can tolerate minor structural and parametric uncertainties, it is always necessary to obtain the most accurate model of the plant as possible, in order for the maximization of the control effectiveness. Normally, model based control strategies such as the $H_2$ (LQG), need an accurate plant model, which can easily be approximated by using high order modeling techniques, but is obviously restricted by the unavailability of the infinite dimensional model. Based on these high order plant models, feasible controllers are very difficult to find to fulfill the necessary performance requirements. In many cases, even higher order controllers are designed to deal with the high order plant, and it becomes evident that the size of the model should be reduced, to decrease the cost in hardware realization and to increase the computational efficiency for real-time applications. Hence, the development of an accurate yet low order model for civil structures is necessary for both analysis and control purposes.

A low order controller can be obtained either by the reduction of a high order controller (which is designed based on high order plant model), or by directly designing a low order controller from the high order plant model. Alternatively, a low order controller can be indirectly designed by obtaining a reduced (low order) plant model in the first place. As mentioned above, designing a high order controller directly from the high order plant has had some success, particularly when heuristic approaches are used [3], but it still faces many challenges. Therefore, the indirect method of designing a low order controller, by first reducing the order of the plant model, is still the most practical and effective method at present.

In control engineering, model reduction in terms of balanced truncation and Hankel norm approximation algorithms for state-space modeling is often used. However, in vibration control of civil structures, especially for tall buildings, these methods still require excessive computational efforts. Therefore, condensation techniques for plant model reduction of tall structures in second order form are often used. The static condensation method is presented in [4]. This method is only exact for static analysis, and often lacks accuracy for dynamic analysis, especially for the high frequency range. A mode-displacement method was presented in [5]. This method can produce a lower order plant model in terms of retained model coefficients, natural frequencies and a few modal coordinates. However, this method can only work well when the complete plant system remains in principal coordinates, and is inapplicable to a real physical plant. Zhang [6] presented a dynamic condensation
method, which retains the dynamic characteristics of the
original plant in an accurate manner, by selectively keeping
only those few modes that are of interest. Three different
model reduction methods are compared in [7] by computer
simulation and it is concluded that the dynamic model
reduction method performs in a superior manner.

In this paper, we will first use the dynamic model reduction
method to obtain a reduced order plant model of the 20-storey
building; we will then design a \( H_\infty \) controller based on the
reduced plant model; and finally, we will apply the designed
low order controller to the full order building model to reduce
the excessive vibration excited by a seismic disturbance.
Because the lower frequencies of the full order model can be
described exactly by the reduced order model, the \( H_\infty \)
controller designed from the low order model can be used to
control the full order model without influencing the high
residual modes that cause the control and observer spillover
problems. The results of numerical simulations are presented
at the end of this paper, to validate the acceptance of the
reduced order model and the performance of the designed
controller.

II. REDUCED ORDER MODEL

A 20-storey experimental tall building model was set up in
the laboratory to simulate a real tall building structure as
shown in Fig. 1. An active mass damper was installed on the
top storey to supply the active control force. The building
model consists of 20 lumped mass floors which are each
separated equally and are supported by two elastic steel
columns. The two steel columns supply the stiffness and
damping of the structure and are more representative of a real
building when compared to a previous single column design.
The Finite Element Method (FEM) was used to create the full
(high) order mathematical model of the building (plant) with
two degrees of freedom per floor. The influence coefficient
method (for continuous structures) was also employed in this
study, to obtain very similar results as the FEM, with the
advantage that it uses only half of the degrees of freedom that
the FEM requires. The linearized one-dimensional equation
of motion for the 20 degree-of-freedom (DOF) structure
equipped with active mass damper and subjected to
earthquake excitation can be written as

\[
\ddot{X}(t) + \dddot{X}(t) + \Phi \Lambda \Phi \dot{X}(t) = H \dot{u}(t) + E \dddot{x}_e(t)
\]

where \( \dot{X}(t) \) is the relative displacement of each floor with
respect to ground; \( u(t) \) is the control force; \( H \) defines the
location of the control force; \( \ddot{x}_e(t) \) is the earthquake ground
acceleration; \( E \) denotes the influence of earthquake
excitation; \( \Phi \), \( \Lambda \), and \( \Phi \) are the mass, damping, and stiffness
matrices of the building model, respectively.

The equation of motion can then be converted to a
state-space equation as

\[
\tau(t) = \Phi \ddot{X}(t) + \Phi \dot{u}(t) + \Phi \ddot{u}(t)
\]

where

\[
\Phi = \begin{bmatrix} 0 & -\Phi^{-1} \Lambda \Phi^{-1} \\
-\Phi^{-1} \Lambda \Phi^{-1} & \Phi^{-1} \end{bmatrix}, \quad \Phi = \begin{bmatrix} 0 & 0 \\
0 & \Phi^{-1} \end{bmatrix}, \quad \Phi = \begin{bmatrix} 0 & 0 \\
0 & \Phi^{-1} \end{bmatrix},
\]

\[
\tau(t) = \begin{bmatrix} \dot{X}(0) \\
\dot{x}_e(t) \end{bmatrix}, \quad w(t) = \dot{x}_e(t).
\]

For the obtained high order model in equation (2), which is
a 40 DOF state-space equation without active mass damper, a
feasible controller could not be found after several trials due
to the large size of the model. In practice, it is very difficult to
obtain a high order model for a real plant exactly. Even if one
manages to obtain a high order model, it cannot be expected
to control all modes of the plant especially the highest ones.
Therefore, the design of a high order controller based on a
high order plant model is not a practical solution in this case.
In addition to this, the high order control model cannot be
reduced based on its state-space equation (by using the
state-space equation model reduction approach), such as
balanced truncation method and Hankel norm approximation
method, due to the computational challenge. Therefore, a
model reduction method in terms of physical parameters such
as mass, damping and stiffness parameters must be used first,
before the state-space equation can be used. For doing so,
Guyan [4] presented a static model reduction method to
reduce the size of mass and stiffness matrices. However, the
reduced mass matrix produced by this method does not
preserve its accuracy while the reduced stiffness matrix does.
In [5], a mode-displacement method was presented. It can
produce a lower order plant model in terms of retained model
coefficients, natural frequencies and a few of the modal
coordinates. The mode-displacement works well when the
complete plant system remains in principal coordinates, but it
is inapplicable to a real physical plant model. Zhang [6]
presented a dynamic model reduction method that can
produce the reduced model formulated from condensed mass,
damping and stiffness coefficient matrices and retain a small
number of lowest modes of the original system. After
comparing the three different model reduction methods with
experimental validation in [8], the dynamic model reduction
method is proven to be applicable in both the theoretical
analysis and the experimental test. Therefore, this method
will be used here to obtain the reduced order model for our
control purposes.

In most engineering applications, it is recommended that
the lowest few natural frequencies and the corresponding
modes of the original structural system be kept in the reduced
model. For the system described in (1), if we choose \( n_c \) degrees of freedom of the original structural system to be
retained in the condensed model, the mass matrix of the
condensed model can be determined as [6]

\[
M_c = X_c^{-1} \left( \omega^2 I + B_c \right)^{-1},
\]

where \( X_c \) is the displacement vector of order \( n_c \) at the
chosen master coordinates; \( \omega \) is the excitation frequency;
\( B_c = \Phi \Lambda \Phi^{-1} \).
where $A = \begin{bmatrix} \lambda_1^2 & 0 & \cdots & 0 \\ 0 & \lambda_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n^2 \end{bmatrix}$, 
$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix}$,

and $\lambda_i$ represents the $i$th natural frequency, $\phi_{j,j}$ represents the modal coefficient at $j$th master coordinate. Consequently, the condensed stiffness matrix is determined as $K_c = M_cB_c$. As damping always exists in actual structural systems and is difficult to be modelled accurately, the level of modal damping is determined by experience or by experimental modal testing on the system. Then, the original system in equation (1) is reduced to

$$M_c\ddot{X}_c(t) + C_c\dot{X}_c(t) + K_cX_c(t) = H_cu(t) + E_c\ddot{x}_c(t)$$

(4)

where $M_c$, $C_c$, $K_c$, $H_c$, and $E_c$ are corresponding condensed matrices; $X_c(t)$ is the condensed displacement variable. Hence, the equation of motion for the reduced order model (4) is converted to a state-space equation as

$$\dot{x}(t) = Ax(t) + Bu(t) + B_2u(t)$$

(5)

where $A = \begin{bmatrix} 0 & I \\ -M_c^{-1}K_c & -M_c^{-1}C_c \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ M_c^{-1}E_c \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ M_c^{-1}H_c \end{bmatrix}$, $x(t) = \begin{bmatrix} X_c(t) \\ \dot{X}_c(t) \end{bmatrix}$.

In this paper, the three lowest natural frequencies and the corresponding modes of the 20-storey building model were chosen to be retained in the reduced order model. The 7th, 13th, and 20th original coordinates were chosen as the master coordinates. For comparison, we list the calculated natural frequencies of the lowest three modes by different methods in Table I. In which, ‘Full Order Model’ represented the full order model obtained by FEM; ‘DMRM’ indicates the reduced order model obtained by the dynamic model reduction method [6]; ‘Guan’ indicates the reduced order model obtained by the method presented in [4]; and ‘Mode-Displacement’ indicates the reduced order model obtained by the method presented in [5]. From Table I we can see that the dynamic model reduction method can obtain the accurate low frequencies compared with the full order model and has no problems with the coordinate transformation. The open-loop dynamic response to seismic excitation for the low order model and the full order model are also compared (the results are discussed under Section IV). It is made clear by the results that the low order (reduced) plant model produces very similar response output in terms of displacement, velocity and acceleration, with those produced by the full order plant model. This confirms that the reduced order plant model is representative of the full order plant model in the low frequency range and can be successfully used for the controller design task.

III. CONTROLLER DESIGN

In this paper, the $H_\infty$ controller will be designed based on the reduced order model and will then be applied to the full order model to evaluate its performance. Due to its robustness, $H_\infty$ control has been applied to many areas and disciplines. For seismic excited civil engineering structures, the $H_\infty$ control using static full state feedback, dynamic output feedback, and static output feedback have all been studied. Due to space limitations, a comparison of control performances of these different control strategies will not be presented here, only that of the $H_\infty$ dynamic output feedback controller. The $H_\infty$ control theory is well known and the detailed derivations can be referred to in literature, therefore only the essential contents will be presented here.

The $H_\infty$ control aims to reduce the effect of disturbance, e.g. seismic excitation $w(t)$, on the interested control output $z(t)$ such that the ratio of the $L_2$ norm of the control output $z(t)$ to the $L_2$ norm of the disturbance $w(t)$, with zero initial conditions, is smaller than $\gamma > 0$, that is, $\|z(t)\|_2 < \gamma \|w(t)\|_2$, where $\gamma$ is the disturbance attenuation performance, and the $L_2$ norm is defined by

$$\int_0^\infty z(t)z(t)^Tdt < \int_0^\infty w(t)w(t)^Tdt$$

For this study, we define the top floor acceleration as the control output, $z(t)$, i.e.,

$$z(t) = C_1x(t) + D_{11}w(t) + D_{12}u(t)$$

(6)

where constant matrices $C_1$, $D_{11}$, $D_{12}$ are drawn from matrices $A$, $B_1$, and $B_2$ to make $z(t) = \ddot{x}_{20}(t)$. The measured outputs are the relative displacements and velocities of the 7th, 13th and 20th floors with respect to ground, and the relative displacement and velocity of the active mass with respect to top floor. The output then becomes

$$y(t) = C_2x(t) + D_{21}w(t) + D_{22}u(t)$$

(7)

where $C_2 = I_{8 \times 8}$, $D_{21} = D_{22} = 0_{8 \times 4}$. Therefore, together with (5), we use the system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

(8)

to design a controller with the form of

$$y(t) = C_2x(t)$$

...
\[ \ddot{\zeta}(t) = A_k \zeta(t) + B_k y(t), \]
\[ u(t) = C_k \zeta(t) + D_k y(t) \]  \hspace{1cm} (9)

where \( \zeta \) is controller state variable; \( A_k, B_k, C_k, D_k \) are controller matrices to be designed, such that the closed-loop system

\[ \begin{align*}
\dot{x}_{cl}(t) &= \tilde{A} x_{cl}(t) + \tilde{B} w(t), \\
z(t) &= \tilde{C} x_{cl}(t) + \tilde{D} w(t)
\end{align*} \]  \hspace{1cm} (10)

where \( x_{cl}(t) = [z(t) \ \zeta(t)]^T \), \( \tilde{A} = \begin{bmatrix} A + B_2 D_c C_2 & B_2 C_k \\ B_k C_2 & A_k \end{bmatrix} \), \( \tilde{B} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \), \( \tilde{C} = \begin{bmatrix} C_1 + D_{12} D_k C_2 & D_{12} C_k \end{bmatrix} \), \( \tilde{D} = D_{11} \),

is stable, and the \( H_\infty \) norm of the closed-loop transfer function from \( w(t) \) to \( z(t) \) is

\[ \|F_{zw}\|_\infty = \|z(t)\|_{\infty} < \gamma \]  \hspace{1cm} for \( \gamma > 0 \). If

\( \gamma \) is minimized, then controller (9) is the optimal \( H_\infty \) controller.

By virtue of the Bounded Real Lemma, \( \tilde{A} \) is stable and \( \|F_{zw}\|_\infty < \gamma \) if and only if there exists a symmetric matrix \( P > 0 \) with

\[ \begin{bmatrix} AP + PA^T & \tilde{B} & PC \tilde{C} \\ \tilde{B}^T & -\gamma I & \tilde{D}^T \\ \tilde{C} P & \tilde{D} & -\gamma I \end{bmatrix} < 0 \]  \hspace{1cm} (11)

This is a linear matrix inequality (LMI) for \( P > 0 \) and can be easily resolved by the Matlab LMI toolbox. Then, the controller (9) can be obtained from the solution of LMI [9].

IV. NUMERICAL RESULTS

In this section, the 20-storey building model is used to illustrate the application of the reduced order control for the seismic excited structures.

The parameters for the 20-storey building model are designed as follows: total lumped mass of each floor is 29 kg; the length, width, and height of the total lumped mass per floor is 354mm, 228mm, and 50mm respectively; the two columns are made from 100mm × 5mm bright flat steel, and they are placed 100mm apart; the unclamped length of the columns, i.e. the effective height of the building model, is 2.5m; the distance between each floor is 76mm; the active mass is approximately 22kg. The active mass is connected to the top floor by a linear motor. The linear motor forms part of the twentieth floor of the building and it provides the control force between itself and the active mass (21st floor).

After obtaining the abovementioned building full order mathematical model (40 DOF) by the finite element method, or the 20 DOF model, by the influence coefficient method, the reduced order model (3 DOF) can be formed. As discussed previously, it only includes the lowest three modes of the original model, and is obtained by using the dynamic model reduction method presented in Section II. The \( H_\infty \) controller, which aims to minimize the top floor acceleration when the building is subjected to seismic excitation, is then designed based on this reduced order model. Finally, the low order controller is applied to the full order plant model to mitigate the effects of earthquake disturbances.

The open-loop and closed-loop frequency responses from the ground acceleration to the top floor acceleration for the full order model (20 DOF) and the reduced order model (3 DOF) are plotted in Fig. 2(a) and Fig. 2(b), respectively. It can be seen clearly in Fig. 2, that the designed controller effectively provides active damping to the lowest three resonance frequencies for both the full order model and the reduced order model. However, the controller does not influence the high order frequencies of the full order model too much and the control spillover problem is consequently avoided.

For the simulations of the time responses for both of open-loop and closed-loop systems, the recorded El Centro earthquake data was used. The original data had many dominate low frequency components and was sampled at 50 Hz. In order to shift the dominate frequency components to a broad range, the original earthquake data was re-sampled with sampling rate 400 Hz. The re-sampled earthquake signal is shown in Fig. 3.

Due to space limitations, only the time responses of displacement, velocity, and acceleration due to earthquake excitation, for the top floor, are plotted in Figs. 4-6, respectively. To confirm that the reduced order model retains accuracy in response output with those of the full order model, the time responses for the reduced order model are plotted as well. It can be seen in Figs. 4(a)-6(a) that the open-loop responses of the reduced order model and of the full order model are nearly the same, especially the displacement and velocity responses. The bigger difference existed in the top floor acceleration is mainly induced by the high order mode frequencies. Comparing the closed-loop responses shown in Figs. 4(a)-6(a) with the open-loop responses shown in Figs. 4(b)-6(b), we can see that the closed-loop responses are significantly reduced due to the application of active damping. Hence, the seismic excited vibration is successfully suppressed.

The peak response quantities of the 20-storey building model with and without the active mass damper are presented in Table II, denoted by ‘open-loop’ and ‘closed-loop’, respectively. In Table II, ‘Fl’ denotes the floor number, ‘Disp’ denotes the peak displacement, ‘Vel’ denotes the peak velocity, and ‘Acc’ denotes the peak acceleration, respectively, of the building model subject to simulated earthquake ground acceleration. As can be seen in Table II, a significant reduction in displacement, velocity, and acceleration is achieved when the building model is equipped
with an active mass damper. This confirms again that the low order controller is effective in controlling the vibration of the full order plant model.

V. CONCLUSION

This paper studies the vibration suppression problem for a 20-storey building model subjected to seismic excitation. Due to the high order of the building model, a feasible controller cannot be found by using LMIs, and model reduction methods such as balanced truncation and Hankel norm approximation. The dynamic model reduction method was applied successfully to obtain a reduced order model for the 20-storey building model. A $H_\infty$ controller was then designed based on this low order plant model, and was applied to attenuate the vibration of the original plant model while it was subjected to seismic excitation. Numerical simulations prove that this low order controller can control the higher order plant model very well. The reduced order plant model describes the lowest three modes of the original plant model very accurately. So much so, that the controller design based on the low order plant model can work well with the original model and no control and observer spillover problems are induced. Experimental validation will be done in the next step work.

REFERENCES

Fig. 3. El Centro earthquake ground acceleration

Fig. 4. Displacement of top floor

Fig. 5. Velocity of top floor

Fig. 6. Acceleration of top floor

TABLE I

COMPARISON OF THREE LOWEST FREQUENCIES

<table>
<thead>
<tr>
<th>Model</th>
<th>1st Mode Frequency (rad/sec)</th>
<th>2nd Mode Frequency (rad/sec)</th>
<th>3rd Mode Frequency (rad/sec)</th>
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<td>Full Order Model</td>
<td>14.1548</td>
<td>88.4742</td>
<td>247.1603</td>
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<tr>
<td>DMRM</td>
<td>14.1548</td>
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<td>247.1603</td>
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<td>14.1594</td>
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<td>252.9634</td>
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<td>Mode-Displacement</td>
<td>14.1548</td>
<td>88.4742</td>
<td>247.1603</td>
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</table>

TABLE II

PEAK QUANTITIES FOR EVERY FLOOR

<table>
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<tr>
<th>Fl</th>
<th>Disp (mm)</th>
<th>Vel (m/s)</th>
<th>Acc (m/s²)</th>
<th>Disp (mm)</th>
<th>Vel (m/s)</th>
<th>Acc (m/s²)</th>
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