Estimation for state space models: quasi-likelihood and asymptotic quasi-likelihood approaches

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Estimation for State Space Models: Quasi-likelihood and Asymptotic Quasi-likelihood Approaches

A thesis submitted in fulfilment of the requirements for the award of the degree of

Doctor of Philosophy

from

THE UNIVERSITY OF WOLLONGONG

by

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March 2008
Dedicated to

My Parents
Abstract

In this thesis, parameter estimation for multivariate heteroscedastic models with unspecified correlations is considered. Two alternative approaches for estimating the state variables and unknown parameters in nonlinear and non-Gaussian state space models with unspecified correlations are developed; these are the quasi-likelihood and asymptotic quasi-likelihood methods.

Quasi-likelihood and asymptotic quasi-likelihood approaches have been found to be useful in parameter estimation, especially when the underlying system dynamic probability distribution cannot be fully specified. The quasi-likelihood method relaxes the distributional assumptions and only assumes knowledge of the first two conditional moments. A limitation of quasi-likelihood is that the nature of the conditional covariance matrix is not always known. An alternative approach, asymptotic quasi-likelihood, is described. Semiparametric treatment is considered in this thesis. It involves a nonparametric kernel approach to estimating the unknown covariance matrix and it integrates the estimation of the covariance matrix with the standard quasi-likelihood approach.

Applications of the quasi-likelihood and asymptotic quasi-likelihood approaches to state space models are demonstrated via simulation studies. One simulation study based on the Poisson model and another based on the basic stochastic volatility model are presented. Results from these simulation studies show that the performance of the quasi-likelihood and asymptotic quasi-likelihood approaches are com-
parable to other methods, with no knowledge required of the underlying system’s probability structure.

Sensitivity of the quasi-likelihood and asymptotic quasi-likelihood estimation procedures to initial values assigned to state variables and starting parameters is investigated. A suggestion on choosing the initial value of state variables, without knowing the system’s probability structure is given. Standard steps used to improve the grid search method for obtaining better estimation of unknown parameters, in nonlinear and non-Gaussian state space models, are established.

Finally techniques developed in this thesis are applied to real data. Three datasets are considered: (i) the daily exchange rate of US Dollar/Australian Dollar and British Pound/Australian Dollar for the period from 1/1/2003 to 1/1/2006; (ii) the monthly number of cases of poliomyelitis reported by the U.S. Center for Disease Control for the years 1970 to 1983 and (iii) the exchange rates of Pound/Dollar from 1/10/81 to 28/6/85. Results, including residual analysis for the various modelling approaches, demonstrate that techniques developed in this thesis are applicable and have advantages when compared with other technical approaches.
Certification

I, Raed Ahmad Hasan Al zghool, declare that this thesis, submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Mathematics and Applied Statistics, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

_________________________

Raed Ahmad Hasan Al zghool

31 March 2008
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List of Publications

The following publications have been published by the author during his studies.


Part of materials of Papers 2, 4 and 5 are presented in Chapter 4 and part of Chapter 5 and 6. The material of paper 3 is given in Chapter 3 and Section 6.1 in Chapter 6.
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