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T K. Ho  
*Hong Kong Polytechnic University*, markho@uow.edu.au

C W. Tsang  
*Hong Kong Polytechnic University*

K H. Ip  
*Hong Kong Polytechnic University*

K S. Kwan  
*Hong Kong Polytechnic University*

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Keywords
open, railway, timetabling, optimisation, swarm, service, particle, train, markets

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Train Service Timetabling in Railway Open Markets by Particle Swarm Optimisation

T.K. Ho*, C.W. Tsang, K.H. Ip and K.S. Kwan
Department of Electrical Engineering, Hong Kong Polytechnic University
Hung Hom, Kowloon, Hong Kong
* Corresponding author
Tel: +(852) 2766 6146, Fax: +(852) 2330 1544, email:eetkho@polyu.edu.hk

Abstract
Railway timetabling is an important process in train service provision as it matches the transportation demand with the infrastructure capacity while customer satisfaction is also considered. It is a multi-objective optimisation problem, in which a feasible solution, rather than the optimal one, is usually taken in practice because of the time constraint. The quality of services may suffer as a result. In a railway open market, timetabling usually involves rounds of negotiations among a number of self-interested and independent stakeholders and hence additional objectives and constraints are imposed on the timetabling problem. While the requirements of all stakeholders are taken into consideration simultaneously, the computation demand is inevitably immense. Intelligent solution-searching techniques provide a possible solution. This paper attempts to employ a particle swarm optimisation (PSO) approach to devise a railway timetable in an open market. The suitability and performance of PSO are studied on a multi-agent-based railway open-market negotiation simulation platform.

Keywords: Intelligent transportation systems, railway open markets, timetabling, particle swarm optimisation

1. Introduction
The basic purpose of train timetabling in a railway network is to accommodate train service provisions according to the traffic demands, constraints imposed by physical network and the needs
to maintain and renew. It also involves locomotive/rolling stock and train crew planning. In busy networks, there are tens of stations with a few hundred trains per day while the track often contains complicated layouts, multiple intersecting in-line, out-line, one-way or two-way through-platforms. On the other hand, trains come in different types, station waiting times, inter-station run-times, headways, origins, destinations, preferred lines and platforms. Thus, train scheduling is a massive exercise the railway operators have to deal with in the pursuit of service quality.

Only a couple of decades ago, procedures to produce train schedules still rely heavily on experienced planners (Wren, 1995). Having taken details of infrastructure and train characteristics into account, the timetable planner drafts a schedule according to given train service specifications. The draft schedule is in graphical form to enable easy overlap of proposed train journeys for compatibility evaluation. Train services are put on the graph in the order of service priority. Given the number of permutations, the process usually takes weeks, sometimes months, to complete. This slow ad hoc process does not allow the planner to explore alternatives. With the availability of computer-aided tools in recent years, not only does timely formulation of complicated timetables become possible (Kroon et al, 2009; Caprara & Kroon, 2007), various aspects of timetable quality, such as robustness and station routing (Vansteenwegen & Van Oudheusden, 2007; Zwaneveld et al, 2001), are also addressed. With the regulated railways, the infrastructure owner and service provider are of the same organisation and they share the same business objectives. Conflicts on train service timetabling are resolved internally and no penalty is imposed if either party is unable to adhere to the timetable during operation.

For the last two decades, the railway reforms in many countries (ECMT, 1998; BTRE, 2003; Jahanshahi, 1998) have introduced deregulation on train service provisions and led to the privatisation of operations and the formation of open markets. The reforms aim to induce competition and hence improve service quality. Railway open markets come with different extents of separation of infrastructure from operations, infrastructure access pricing, government (or state) involvement and competition levels. In general, the infrastructure owners and service providers in
railway open markets are physically, financially and administratively independent entities who possess different business objectives, corporate strategies, service commitments and marketing plans, which implies more constraints to the timetabling problem. Further, more flexibility and frequent updates on the service timetables are often required to cope with competitions and customer needs. The demand on a more reliable, efficient and objective means to devise train schedules is more evident than ever. This study investigates the application of Particle Swarm Optimisation (PSO) techniques to incorporate the negotiations among the stakeholders in railway open markets.

This paper is organised as follows. The environment and service timetable negotiations are described in Section 2 and the design and formulation of PSO timetabling is given in Section 3. Section 4 presents the simulation results and discusses the feasibility and limitations of the proposed approach. Conclusions are then stated in Section 5.

2. Train service timetabling in open market

2.1 Open markets

In recent years, extensive regulatory reforms in railways have been implemented in many countries where the primary objective is to introduce intra-modal competition in rail transportation. These reforms are also encouraged by an adjustment in economic principles on railways (Watson, 2001; Gibson et al, 2002) which argues that competition is possible for train operations (above-rail activities) even though infrastructure provisions (below-rail activities) may prove more elusive. The barrier to intra-modal competition may therefore be lowered by allowing multiple train service operators to gain access to the infrastructure from a common provider. This forms the basis of a railway open market.

A railway open market, in its simplest form, consists of a group of Train Service Providers (TSPs) and an independent Infrastructure Provider (IP). Further, the ancillary services of rolling stock and maintenance provisions are also separately offered by the Rolling Stock leasing
Providers (RSPs) and the Maintenance Service Providers (MSPs) respectively (ECMT, 2005). A railway open market therefore involves multiple stakeholders arranged as a supply-chain through which railway resources (e.g. track capacity and rolling stock) are supplied to the TSPs to enable the ultimate train service provisions to the end-consumers.

The competing TSPs are classified by their types of service provisions. On the first level, train services are divided into freight or passenger. Freight services are further grouped by the nature of commodities being bulk (e.g. coal, petrochemicals) or non-bulk (e.g. foodstuffs, postal, parcels) (Johansson & Nilsson, 2004). On the other hand, passenger services are categorised as regional, intercity, or inter-continental according to the distance travelled. In an open market, these rail services are operated by different stakeholders while on-rail and off-rail competitions are introduced.

On-rail competition (DOTARS, 2003) refers to the competition of capacity and customers. Regardless of the service types, all TSPs are required to obtain track and station access from the IP as a result of their common operations on the same infrastructure. Such competition is anticipated to improve network utilisation. On the other hand, direct competition for customers between train operators usually occurs in the freight market. However, similar competition is less apparent in passenger services. It is unlikely to find two passenger operators competing for identical routes because of the limited demand. A moderate competition is still possible between partially overlapping routes or services of different train speeds.

On the other hand, off-rail competition (DOTARS, 2003) is originated from the social demand for benchmarking the quality of services among the existing operators, even if their services are running in different regions of the network. If the operators fail to provide an acceptable level of service, the rights of operation in the network may be terminated and acquired by other potential operators. The existing operators are thus pressed to respond to the market demand, and preferably, develop innovative plans to explore new demands. Under this market structure, negotiation becomes the critical process for the stakeholders to attain the resources of track access.
2.2 Timetabling

2.2.1 Stakeholder Negotiations

The negotiation between an IP and the TSPs is the core interaction in a railway open market. The main objective of this interaction is to form a track access rights agreement, which specifies the access price and capacity allocated to the TSP, as well as a service timetable. However, the requirements of the stakeholders are likely to be in conflicts and it is essential to resolve these disputes during the formation of an agreement. Access charge pricing and infrastructure capacity utilisation are the key issues of negotiation in track access rights agreement.

The IP is responsible for formulating a conflict-free and preferably efficient track access allocation plan and thus a timetable for the TSPs. Since the TSPs are independently managed, they occasionally request services with conflicting routes and timings. The IP has the responsibility to resolve their disputes in rights-of-way allocation.

As there are multiple TSPs in the open market, the IP is thus required to conduct multiple transactions with the TSPs before the entire service timetable can be produced. In other words, in addition to handling the individual negotiations of the TSPs, the IP needs to decide how these multiple negotiations are managed in order to increase its potential benefits. The process of formulating a timetable is illustrated by the example in UK (Dodgson, 1994). As given in Fig. 1, timetable generation begins with the TSPs tabling their service bids with track access requirements to the IP. After collecting the requests, the IP resolves the operational conflicts and produces a draft timetable. With this initial timetable available, the negotiation period begins, during which the IP and TSPs take turns to resolve further operational differences. Because of the substantial amount of time required for formulating the new offers, the negotiation process may only have a few rounds before the final timetable is produced. Afterwards, there is a period of time before the timetable is actually in operation. During this period, any ad hoc services (particularly for those operated by freight TSPs) may fill up the spare capacity.
2.2.2 Timetable generation

During the modification of the draft timetable, the IP has two possible approaches in allocating the track capacity and formulating the timetable, sequential and combinatorial timetable generation, as illustrated in Fig. 2. In the former, the IP conducts the individual negotiations with the TSPs in turn. The IP negotiates with the TSPs one by one and the complexity of the problem is substantially reduced. The negotiation between the IP and one of the TSPs has been modelled as a multi-dimensional constrained optimisation problem, which was solved initially by exhaustive enumeration (Tsang & Ho, 2006), and later improved by a branch-and-bound algorithm (Tsang & Ho, 2008). Nevertheless, the IP is required to determine the order in which the individual negotiations are to be conducted; and inevitably the resulting timetables vary with the sequences of TSPs negotiating with the IP. Sequential timetable generation on the basis of first-come-first-serve and TSPs’ highest-willingness-to-pay are the most commonly adopted approaches. However, a previous study (Tsang, 2007) pointed out that, with such simple rule-based methods, the cumulative revenue of the IP, the track capacity utilization and the resulting timetable is not necessarily be optimal. A fuzzy ranking approach has also been employed to enable the IP to compare TSP bids of multiple parameters and produce a negotiation sequence for the IP to match its business and operation objectives (Ho et al, 2009). Only the IP’s interests are maximised in this study, the TSPs’ service bids and their requirements are not thoroughly considered.

In the latter approach, the IP collects the service bids from all TSPs and determines the optimal allocations to meet all service bids. If the TSPs decide to reject the offers produced by the IP, they can revise their bids and submit them to the IP in the next round of negotiation. The process repeats until the track access agreements are secured, or the stakeholders withdraw from the negotiation. This approach involves the simultaneous scheduling of a substantial amount of train services and it has been argued as a NP-hard problem (Cai & Goh, 1994). Combinatorial generation is thus a complex and time-consuming process (Boyer, 1998) which may impose practical difficulty when considering the deadline of the final timetable production. However, the
combinatorial generation allows the objectives of both IP and TSPs to be taken into account in the timetabling process. Intelligent searching techniques are the potential tools to tackle the shortcomings of high demand of computation and thus slow solution process.

3. Problem formulation and design

Because of the complexity of the timetabling process; the lack of analytical models to explicitly relate operation constraints to the stakeholders’ interests; and the inevitably huge and irregular solution space, the traditionally mathematical programming techniques may not be the most practical solution for timetabling in railway open market. Particle swarm optimisation (PSO) is proposed here to address this combinatorial optimisation problem.

3.1 Particle swarm optimisation

Since its introduction more than a decade ago, particle swarm optimisation has evolved rapidly to substantially enhance its efficiency and applicability (Kennedy & Eberhart, 1995). PSO is a population-based metaheuristic. A particle swarm represents a group of potential solutions to the optimisation problem, in which the locations of the particles denote the potential solutions. Each particle is initialised randomly within the solution space and it flies through the solution space to locate the best position according to a specified fitness function. Its movement is directed by its own best position (local best) and that attained by all particles in the swarm (global best) so far. PSO shares similarity with genetic algorithms (GA) as both start with population of random solutions and search for the optimal solution by the best solution attained so far. However, the chromosomes share information on solution with others in GA while the particles in PSO only provide information on the global-best and local-best solutions for others. As a result, PSO tends to converge to the optimal solution more quickly. The formulation of the PSO is relatively simple and it has found numerous successful applications in various engineering problems (Lee et al, 2009;
3.2 Track Access Rights

The IP-TSP negotiation is an iterative process upon which the IP and TSP settle on a mutually agreed price for the required track access rights on the TSP’s preferred services. The TSP submits the initial service bid which contains the details of the track access rights requirements and the two parties then attempt to reach a deal through bargaining and concession.

A track access rights specifies the conditions for track usage by the TSP. It includes a service schedule describing the service times and the type of rolling stocks to be operated on the track. In order to facilitate the negotiation between IP and TSP, a parameter ‘flex’ is adopted to denote the flexibility with which the TSP is willing to adjust the schedule time (Gibson et al., 2002). Flex is defined as a set of levels where the lowest and highest levels refer to the minimum and maximum flexibilities to shift the time schedule respectively. The levels may be given in qualitative terms, implicitly indicating the progressive willingness of the TSP to make concessions during negotiation. Flex levels on different parameters, such as service timings, in the track access rights are allowed. The TSP also indicates an amount of payment, track access charge (TAC), in order to obtain the permission for operating its train services on the track.

A track access rights $P$ is thus defined as in Eq. (1), where $c \in \{1, 2, \ldots, \infty \}$ is the TAC; $\Psi$ is the train service schedule as defined in Eq. (2); $\omega \in \{ \omega_i | i = 1, \ldots, n_{\omega} \}$ is the rolling stock selected for operation ($n_{\omega}$ is the total number of types of rolling stock); and $\phi \in \{ \phi_i | i = 1, \ldots, n_{\phi} \}$ is the chosen flex level ($n_{\phi}$ is the total number of available flex levels).

$$P = \langle c, \Psi, \omega, \phi \rangle$$  \hspace{1cm} (1)

A train service schedule $\Psi$ is a set of IDs $S = \{ s_i | i = 1, \ldots, n_s \}$ identifying the sequence of stations to be visited ($n_s$ is the total number of train stations). The movement of train in time is
described by the service commencement time (i.e. the arrival time at the first station) $\zeta$ (in hh:mm), the dwell times at each station $T_D = \{ t_{Di} \mid i = 1, \ldots, n_s \}$ (in min), and the inter-station runtimes $T_R = \{ t_{Ri} \mid i = 1, \ldots, n_s - 1 \}$ (in min) between adjacent stations. Hence, $\Psi$ is formally defined as a 4-duple as follows.

$$\Psi = \langle S, \zeta, T_D, T_R \rangle$$

(2)

It is possible to include other parameters, such as safety and service quality records, and even the credibility of the TSPs, in the track access rights bids. More parameters inevitably impose further constraints on the service timetabling problem.

### 3.3 PSO in service timetabling

#### 3.3.1 Optimisation problem

If there are $n$ TSPs and one IP in the open market and the services to be timetabled cover $m$ stations, the IP has to deal with the following independent variables in the service timetabling with combinatorial timetable generation.

- **Commencement times:** $\zeta_{(k)}$, $k \in \{1, 2, \ldots, n\}$
- **Dwell times:** $t_{Di(k)}$, $i \in \{1, 2, \ldots, m\}$, $k \in \{1, 2, \ldots, n\}$
- **Inter-station run-times:** $t_{Ri(k)}$, $i \in \{1, 2, \ldots, m - 1\}$, $k \in \{1, 2, \ldots, n\}$
- **Rolling stocks:** $\omega_{(k)}$, $k \in \{1, 2, \ldots, n\}$
- **Flex level:** $\phi_{(k)}$, $k \in \{1, 2, \ldots, n\}$

The dimension of the optimisation problem is $5n + 2m$. The above variables then derive the dependent variables below.

- **Track access charge:** $c_{(k)}$, $k \in \{1, 2, \ldots, n\}$
- **Track capacity utilisation:** $\eta$
- **Departure times at stations:** $D_{i(k)}$, $i \in \{1, 2, \ldots, m\}$, $k \in \{1, 2, \ldots, n\}$
Arrival times at stations: $A_{i(k)}$, $i \in \{1, 2, \ldots, m\}$, $k \in \{1, 2, \ldots, n\}$

The objective of the IP is to maximise the sum of total track access charge and the total capacity utilisation. The objective function of the optimisation problem is given as follows.

$$\max U = \sum_{k=1}^{n} c_{(k)} - w_{q} \eta \quad (3)$$

$w_{q}$ is the valuation of per unit capacity consumption (in $\$) and $\eta$ is the capacity consumed by the train service (no unit). The term $-w_{q} \eta$ implies a minimisation of capacity usage by each TSP’s train service.

The optimisation is subject to the sets of constraints below. The basic domains of variables are:

- $\zeta_{(k)} \in \{1, 2, \ldots, \infty\}$
- $t_{p_{i(k)}} \in \{1, 2, \ldots, \infty\}$
- $t_{r_{i(k)}} \in \{1, 2, \ldots, \infty\}$
- $\omega_{(k)} \in \{1, 2, \ldots, n_{w}\}$
- $\phi_{(k)} \in \{1, 2, \ldots, \phi_{\omega}\}$
- and $c_{(k)} \in \{1, 2, \ldots, \infty\}$.

Service headway constraint:

If $D_{i(j)} + h_{\text{min}} \leq D_{i(k)}$, then $A_{i(j)} + h_{\text{min}} \leq A_{i(k)}$, $\forall j, k$

where $h_{\text{min}}$ is the minimum headway of the train services, which indicates the minimum separation between trains while maintaining safety.

3.3.2 Optimisation outcomes

There are two possible outcomes on the solution of the timetabling optimisation problem.

a) A feasible solution exists so that the track access rights are allocated to the all TSPs, i.e. a timetable of $n$ train services.

b) No solution exists and thus a timetable of $n$ services is not attained.

As a result, the IP asks all TSPs to relax their constraints and re-submit their bids before making another attempt to derive a timetable. However, it is not a realistic mechanism for combinatorial timetable generation because whenever one of the train services cannot be timetabled, the IP requests all TSPs to relax constraints. In practice, if $p$ train services cannot be scheduled, only the
corresponding TSPs should be requested to relax constraints and the IP offers the track access to the remaining \((n-p)\) TSPs.

In order to avoid the above all-or-nothing approach in timetabling, it is proposed to conduct sub-optimisations with respect to the different numbers of services to be timetabled. As illustrated in Fig. 3, the optimisations of all combinations of services, with number of services going from 1 to \(n\), are conducted. The combination with the maximum objective function value is thus determined and the corresponding TSPs are given the track access allocation accordingly. The remaining TSPs are requested to relax their constraints and the process repeats with a smaller number of TSPs in the subsequent rounds until all TSP services are given track access or no revised bid is required. In the first round, the number of sub-optimisations is given by

\[
S = \sum_{r=1}^{n} \frac{n!}{r!(n-r)!}.
\]

The total number of sub-optimisations in all rounds of timetabling is substantial but the scale of each optimisation is mostly of manageable size. Fast searching technique is still required here and, given the high number of sub-optimisations, the setup overhead of each sub-optimisation should be as simple as possible. PSO is thus employed to search for the solutions in the sub-optimisations. Fig. 4 depicts the process in each round of timetabling.

### 3.3.3 PSO

For a \(d\)-dimensional optimisation problem with PSO, the position and velocity of a particle \(i\) is denoted by the vectors \(x_i = [x_{i1}, x_{i2}, \ldots, x_{id}]^T\) and \(v_i = [v_{i1}, v_{i2}, \ldots, v_{id}]^T\). Let \(x^\#\) be the global best solution found (by all particles), and \(x_i^*\) be the local best solution attained (by particle \(i\)) thus far. The updated position and speed of the \(j^{th}\) component of particle \(i\) are computed by:

\[
v_j(t+1) = w v_j(t) + c_1 r_1 (x^\#(t) - x_j(t)) + c_2 r_2 (x_i^*(t) - x_j(t))
\]

\[
x_j(t+1) = x_j(t) + v_j(t+1)
\]
where $w$ is the inertia factor; $r_1$ and $r_2 \in [0, 1]$ are random numbers generated from a uniform distribution; $c_1$ and $c_2$ are the coefficients of self-recognition component and social component; $w$ is used to determine the effect of the last velocity on the new one. A large inertia facilitates global exploration (searching new areas) while a small one tends to focus on local exploration (fine-tuning the current search area). $r_1$, $r_2$, $c_1$ and $c_2$ determine the tendency of the particle movement towards the local best and global best solutions.

The dimension of the optimisation problem, and thus that of the position space of a particle in PSO, depends on the number of TSPs and also the number of parameters in the TSP’s bids. The latter is determined by the complexity of the services, such as the number of stations served. As more TSPs are in the market and the services cover more stations, the dimension of the problem escalates.

Track access charge (TAC) is an important component in the objective function to determine the global and local best and it is calculated by the updated position of the particle. The calculated TAC value is first compared to the TAC constraint submitted by each TSP, and if the calculated TAC value is found higher, the solution is deemed invalid and the objective is set to zero. Otherwise, the solution is accepted. Further, it is also necessary to check the right-of-way constraint in such a way that the trains scheduled on the track satisfy the minimum headway (i.e. time interval between two trains). The PSO process is illustrated by the flowchart in Fig. 5.

4. Results and discussions

4.1 Simulation Setup

To investigate the performance of the PSO approach for timetabling in open railway market and analyse the possible limitations, a hypothetical open market is set up to allow negotiations among the stakeholders to derive a timetable for a number of train services commencing over a time-span of an hour (7:00-8:00am). There are one IP and five TSPs in this open market and they provide freight, intercity and regional services over a section of track with five stations (A-E) across 85 km.
The services come with different preferred service requirements and operation conditions. Of the five TSPs, one operates freight service and another provides intercity service; and they stop at selected stations only. The remaining three TSPs run regional services which stop at all or most stations. Tables 1 show the inter-station distances.

4.1.1 Evaluation indices

In this study, the resulting timetables are evaluated in terms of revenue intake and service quality and there are a few pre-defined evaluation indices. The first index consists of two components. The first one is the total IP utility, $IP_{UT}$, which is the sum of revenue to be collected by the IP through the negotiations with the TSPs. $IP_{UT}$ is given in Eq. (6) where $n_k$ is the total number of successfully negotiated services with the TSPs.

$$IP_{UT} = \sum_{i=1}^{n_k} U_i$$  \hspace{1cm} (6)

The second component is the average utility for each service, $IP_{UA}$. It indicates the profit for each successfully negotiated service. $IP_{UA}$ is determined in Eq. (7), where $n$ is the total number of successfully negotiated services.

$$IP_{UA} = \frac{1}{n} \sum_{i}^{n} U_i$$ \hspace{1cm} (7)

Overall journey time is another important indication of service quality from the viewpoints of the customers. The second evaluation index, $EJT_{\theta}$, measures the average deviation in journey time of a train service operated by a TSP of type $\theta$ (i.e. freight, regional, intercity, etc.) from its desired schedule. For a TSP operating a set of $n_i$ services, $EJT_{\theta}$ is defined by Eq. (8), where $\tilde{t}_{ij}$ and $\hat{t}_{ij}$ (in min) are the actual and expected inter-station runtimes of train $i$ between stopping stations $j$ and $j+1$ respectively.

$$EJT_{\theta} = \frac{1}{n_{\theta}} \sum_{i=1}^{n_{\theta}} \sum_{j=2}^{n_i} \max(\tilde{t}_{ij} - \hat{t}_{ij}, 0)$$ \hspace{1cm} (8)
When $EJT_0 = 0$, the timetable contains no extension because all trains arrive at the stations no later than the time requested. When $EJT_0$ takes a value other than zero, the timetable is ‘extended’, in which one or more of the services suffers from extension in journey time.

Trains are preferred to arrive at a station at equally spaced time intervals. Any deviation, either earlier or later, may lead to overcrowding at platforms and trains. The third evaluation index, $DFR_\theta$, is the mean deviation from regularity of TSP $\theta$ at all stopping stations and defined by Eq. (9). $\hat{n}_\theta$ is the expected number of trains in an one-hour operation, $n_\theta$ is the actual number of trains in service, $t_i^j$ (in min) is the arrival time of the $i$-th train at station $j$, and $t_{\theta+1}^j = t_i^j + 60$, which assumes the timetable repeats in the subsequent hour.

$$DFR_\theta = \frac{1}{n_\theta} \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} \left| t_{i+1}^j - t_i^j - 60 / \hat{n}_\theta \right|$$

(9)

When $DFR_\theta = 0$, the timetable is referred as ‘periodic’ because the TSP operates trains with equally spaced time intervals at all stations.

4.1.2 Simulation and analysis

In order to facilitate the negotiations between IP and TSPs, a multi-agent system for open railway access market (MAS-ORAM) developed in a previous study (Tsang, 2007) has been adopted. The MAS-ORAM is built on the popular middleware, JADE (Java Agent DEvelopment Framework), which provides the essential software components for agent development. In this model, each stakeholder in the railway open market is considered as a self-interested entity, or agent, that is capable of interacting with other agents in the system through an iterative process of bid-offer submission.

Each TSP service bid contains numerous parameters and there are a huge number of combinations of values for these parameters, evaluation on individual cases may not necessarily reflect the overall situations and any conclusions drawn from the results are only valid to the
specific set of input values. In order to generalise the findings on the resulting timetables, the average results over repetitive simulations are employed to make deduction.

In this study, the parameters in the services bids are made random variables with known probability density functions (pdfs) and instances are drawn from the respective pdfs. The MAS-ORAM provides the simulator tool to enable the negotiations, to produce the resulting timetables and hence the evaluation indices, $IPU_T$, $IPUA$, $EJT_\theta$ and $DFR_\theta$. The timetables are compared against those produced by a commonly adopted sequential timetable generation approach of IP-TSP negotiations, first-come-first-served (FCFS). A total of 155 timetables (i.e. 765 train services if all negotiations are successful) are generated from random samples of the parameters.

The commencement times of the five services in the case studies are limited within 60 minutes. The probability density functions of the service bid parameters for the statistical analysis are given in Table 2. $U(a_1, a_2, \ldots, a_n)$ denotes a uniform distribution among feasible discrete values of $a_1, a_2, \ldots, a_n$. $N(\mu, \sigma^2)$ specifies a normal distribution with population mean $\mu$ and variance $\sigma^2$. $P(a, \lambda, t)$ represents a right-shifted Poisson distribution by $a$ units with decay constant $\lambda$ and time interval $t$. The flex level is set the same for all TSPs. From Table 2, the intercity service is more willing to pay higher track access charge while the freight service tends to pay lower track access charge, the train is slower and it spends more time at stations.

As PSO involves a number of parameters in its operations and there is no generic set of preferred values for these parameters, a series of preliminary studies has been carried out to calibrate the parameters to ensure reasonable computation time and optimisation performance. The set of parameters adopted for PSO is given in Table 3.

4.2 Results and Discussions

Table 4 shows the numbers of successfully negotiated services from the two timetabling approaches and the corresponding percentages of the 155 tests. The comparison results on $IPU_T$,
IPU_A, EJT_θ and DFR_θ between combinatorial (PSO) and sequential (FCFS) timetable generations are given in Tables 5-9. As there is just one intercity and freight service, the regularity, DFR, only applies to regional services.

It should be reiterated that the PSO approach attempts to consider all service bids at the same time in the formulation of the timetable while FCFS deals with the service bids one by one in a specific order. Even though it is a small example, there are still a total of 7 services within an hour and the variation of the services depicts realistic operational conditions of a mainline railway. The results serve the purpose of demonstrating the feasibility of the PSO approach on timetable generations.

From the results, the PSO approach provides timetables with service quality comparable to those attained from FCFS in most evaluation indices. While the intercity service manages the lowest successful rate, it enjoys a better service quality in shorter service extension time provided that the track access is secured (Table 6). However, the regional services, which demand more frequent operations, have the worst extension time (Table 7).

Evidently, the PSO approach achieves lower successful rate on securing the services during negotiations. When all service bids have to be considered simultaneously in combinatorial timetable generation, it is more likely that the conflicts among the service bids cannot be resolved. The successful rate is adversely affected as a result. On the other hand, FCFS allows each service bids to be dealt with in turn and thus it is easier to avoid conflict.

The low successful rate of PSO approach inevitably leads to slightly inferior performance in IP revenue and regularity. While the PSO approach considers all service bids at the same time, the difference in the service bids are better reflected. As the intercity service is willing to pay higher track access cost, it is given higher priority to secure the access rights. Therefore, its EJT is significantly reduced with PSO (Table 6) while the regional and freight services suffer in EJT and/or DFR (Tables 7-9).
5. Conclusions

In a railway open market, timetabling is a complicated resource allocation problem with self-interested stakeholders. Negotiations with bargaining and concession are necessary in the process of formulating the timetables. In order to facilitate train service timetabling with the consideration of the negotiation contents and behaviours of all stakeholders, a particle swarm optimisation approach has been employed.

The number of train service bid parameters from the TSPs defines the dimensions of the optimisation problem in timetabling. In order to maintain a manageable scale of the problem, the optimisation is achieved by a number of iterations, in which the services with track access rights successfully negotiated are taken out from the subsequent iterations. This approach also allows the service bids from all TSPs to be considered simultaneously, i.e. combinatorial timetable generations.

The performance of PSO is evaluated by comparing the service quality of the resulting timetables with that attained from a sequential timetable generation approach. It has been shown that PSO is capable of producing the train service timetables while meeting the requirements of the stakeholders as far as possible. However, PSO manages a lower successful rate of striking feasible timetables through negotiations, which drags down the quality of the resulting timetables on average. Further studies have to be carried out to enhance the successful rates of stakeholder negotiations. Intelligence behaviour modelling for stakeholder negotiations is one of the potential areas (Wong & Ho, 2010).

While many countries around the world are contemplating the introduction of open railway market, each market has its own unique characteristics in traffic demands, service provisions, stakeholder compositions, business objectives, operational conditions and market regulations. It is essential to conduct various feasibility studies to ensure the viability of the market and to encourage participation of service providers. This research work enables further investigations on
track access rights negotiations among the major stakeholders and the service quality in railway open markets.

Acknowledgements

The authors gratefully acknowledge the generous financial support provided by the Research Grants Council of the Hong Kong Special Administrative Region Government (PolyU 5174/06E) and the Hong Kong Polytechnic University.

References


Fig. 1  Timetable production process in the UK

Fig. 2  Timetable generations by IP in multiple negotiations

(a) Combinatorial generation

(b) Sequential generation
Optimisation of 1 service
- $k = 1 \Rightarrow U(k=1)$
- $k = 2 \Rightarrow U(k=2)$
- $k = 3 \Rightarrow U(k=3)$
- ...
- $k = n \Rightarrow U(k=n)$

Optimisation of 2 services
- $k = 1 & k = 2 \Rightarrow U(k=1, k=2)$
- $k = 1 & k = 3 \Rightarrow U(k=1, k=3)$
- ...
- $k = 1 & k = n \Rightarrow U(k=1, k=n)$
- $k = 2 & k = 3 \Rightarrow U(k=2, k=3)$
- $k = 2 & k = 4 \Rightarrow U(k=2, k=4)$
- ...
- $k = 2 & k = n \Rightarrow U(k=2, k=n)$
- ...
- $k = n-1 & k = n \Rightarrow U(k=n-1, k=n)$

Optimisation of 3 services

Optimisation of n services

For example,
$U^* = U(k=2, k=4)$

⇒ Propose offers to TSP 2 and TSP 4
⇒ Ask TSP 1, TSP 3, TSP 5, TSP 6, … to relax

Fig. 3 Optimisation of all combinations of services
Collects bids from \( n \) TSPs

Create \( S = \sum_{r=1}^{n} \frac{n!}{r!(n-r)!} \)
sub-optimisation problems

Set up each sub-optimisation problem

Generate the constraint sets

Perform a PSO optimisation

Obtain \( U^* \) among sub-optimisations

Derive actions from \( U^* \) and reply to TSPs

Fig. 4  Flowchart for each round of timetable generation
Input:
1. PSO settings
2. TSP's constraints

Initiation:
1. Set number of iterations, \( t \)
2. Set number of particles, \( p \)
3. Distribute particles to the solution space randomly

Return Gbest as the solution

\( t \) reach max.?

All particles processed?

Update "velocities" of particles

Update "locations" of particles

Calculate utility values of particles

Is it the best solution so far?

Set the utility value = 0

Utility value exceed the TSP constraint?

Update the global best solution

Fig. 5  PSO process for each sub-optimisation of timetable generation
Table 1  Track configuration

<table>
<thead>
<tr>
<th>Origin</th>
<th>Destination</th>
<th>Track Length (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>30</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
<td>15</td>
</tr>
<tr>
<td>D</td>
<td>E</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 2  Service bid parameter pdfs

<table>
<thead>
<tr>
<th>Number of service</th>
<th>Intercity</th>
<th>Regional</th>
<th>Freight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intercity</td>
<td>Regional</td>
<td>Freight</td>
</tr>
<tr>
<td></td>
<td>Intercity</td>
<td>Regional</td>
<td>Freight</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>TAC</td>
<td>N(1600, 625)</td>
<td>N(1500, 625)</td>
</tr>
<tr>
<td>Commencement time</td>
<td>U(0:59)</td>
<td>U(0:19)</td>
<td>U(0:59)</td>
</tr>
<tr>
<td>Dwell Time at A</td>
<td>N(5, 0.25)</td>
<td>P(1, 0.2, 1)</td>
<td>N(15, 1)</td>
</tr>
<tr>
<td>Dwell Time at B</td>
<td>-</td>
<td>P(1, 0.2, 1)</td>
<td>N(15, 1)</td>
</tr>
<tr>
<td>Dwell Time at C</td>
<td>-</td>
<td>P(1, 0.2, 1)</td>
<td>-</td>
</tr>
<tr>
<td>Dwell Time at D</td>
<td>-</td>
<td>P(1, 0.2, 1)</td>
<td>N(15, 1)</td>
</tr>
<tr>
<td>Dwell Time at E</td>
<td>N(5, 0.25)</td>
<td>P(1, 0.2, 1)</td>
<td>N(15, 1)</td>
</tr>
<tr>
<td>Runtime at AB</td>
<td>P(11, 0.3, 1)</td>
<td>P(15, 0.5, 1)</td>
<td>P(24, 0.7, 1)</td>
</tr>
<tr>
<td>Runtime at BC</td>
<td>P(16, 0.3, 1)</td>
<td>P(24, 0.5, 1)</td>
<td>P(35, 0.7, 1)</td>
</tr>
<tr>
<td>Runtime at CD</td>
<td>P(9, 0.3, 1)</td>
<td>P(14, 0.5, 1)</td>
<td>P(23, 0.7, 1)</td>
</tr>
<tr>
<td>Runtime at DE</td>
<td>P(11, 0.3, 1)</td>
<td>P(15, 0.5, 1)</td>
<td>P(24, 0.7, 1)</td>
</tr>
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</table>
### Table 3  PSO parameters

<table>
<thead>
<tr>
<th>Number of particles</th>
<th>Number of iterations</th>
<th>Inertia factor</th>
<th>Self recognition</th>
<th>Social component</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>6</td>
<td>0.8</td>
<td>2</td>
<td>2</td>
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</table>

### Table 4  Successfully negotiated services

<table>
<thead>
<tr>
<th>Timetabling approaches</th>
<th>FCFS</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of service</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercity</td>
<td>150</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td>97%</td>
<td>50%</td>
</tr>
<tr>
<td>Regional</td>
<td>461</td>
<td>399</td>
</tr>
<tr>
<td></td>
<td>99%</td>
<td>86%</td>
</tr>
<tr>
<td>Freight</td>
<td>142</td>
<td>188</td>
</tr>
<tr>
<td></td>
<td>92%</td>
<td>76%</td>
</tr>
<tr>
<td>Total</td>
<td>753</td>
<td>595</td>
</tr>
<tr>
<td></td>
<td>97%</td>
<td>79%</td>
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</table>

### Table 5a  $I_{PU_T}$

<table>
<thead>
<tr>
<th>Timetabling approaches</th>
<th>FCFS</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ($/service)</td>
<td>6791</td>
<td>5171</td>
</tr>
<tr>
<td>Standard deviation ($/service)</td>
<td>528</td>
<td>652.2</td>
</tr>
</tbody>
</table>

### Table 5b  $I_{PU_A}$

<table>
<thead>
<tr>
<th>Timetabling approaches</th>
<th>FCFS</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ($/service)</td>
<td>1394</td>
<td>1370</td>
</tr>
<tr>
<td>Standard deviation ($/service)</td>
<td>352.4</td>
<td>344.8</td>
</tr>
</tbody>
</table>

### Table 6  Intercity service - extension time $EJT$

<table>
<thead>
<tr>
<th>Timetabling approaches</th>
<th>FCFS</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (mins/service)</td>
<td>10.59</td>
<td>7.45</td>
</tr>
<tr>
<td>Standard deviation (mins/service)</td>
<td>2.79</td>
<td>2.31</td>
</tr>
</tbody>
</table>

### Table 7  Regional service - extension time $EJT$

<table>
<thead>
<tr>
<th>Timetabling approaches</th>
<th>FCFS</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (mins/service)</td>
<td>4.44</td>
<td>10.27</td>
</tr>
<tr>
<td>Standard deviation (mins/service)</td>
<td>2.81</td>
<td>3.02</td>
</tr>
</tbody>
</table>

### Table 8  Freight service - extension time $EJT$

<table>
<thead>
<tr>
<th>Timetabling approaches</th>
<th>FCFS</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (mins/service)</td>
<td>3.56</td>
<td>5.13</td>
</tr>
<tr>
<td>Standard deviation (mins/service)</td>
<td>2.81</td>
<td>3.02</td>
</tr>
</tbody>
</table>

### Table 9  Regional service - deviation from regularity $DFR$

<table>
<thead>
<tr>
<th>Timetabling approaches</th>
<th>FCFS</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (mins/service)</td>
<td>13.5</td>
<td>101.7</td>
</tr>
<tr>
<td>Standard deviation (mins/service)</td>
<td>2.41</td>
<td>6.53</td>
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