2003

Drugs and Growth-Efficient Prevention: An Inter-temporal Portfolio of Machos, Narcos and Angels

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WP 03-09
August 2003
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Abstract
In many countries substance abuse is a social epidemic. This paper deals conceptually with some macroeconomic aspects of widespread substance abuse with special reference to narcotic drugs. The labour force is divided into non-using and therefore fully productive workers, a number of whom are employed by the government in drug-use prevention, and only partially productive drug users. An efficient management of the nation’s portfolio of workers is taken to be the trajectory of drug-prevention that maximises the present value of the stream of the disposable national incomes. 

(JEL I12, J21, J24, D90)

Keywords: Substance abuse, labour productivity, national income

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1. Introduction

Many studies have attempted to analyse conceptually the mechanics of either individual decision making of addicts or of the market for a particular addictive substance. This study attempts to analyse conceptually the effects of the spread of substance abuse on aggregate economic variables and growth within a dynamic optimisation framework and with special reference to narcotic drugs.

One hundred years ago almost all drugs were freely available in many countries. However, since that time, progressively tougher restrictions have been introduced on the traffic and consumption of many substances, and particularly mind altering drugs such as narcotics. These restrictions have on many instances been the outcome of significant international co-operation. For instance, the Hague Convention post WWI limited the use of opium, morphine and heroin to medicinal purposes only, with the Geneva Convention on 1925 adding cannabis to the list. In 1953 the World Health Organisation recommended that the use of heroin for medicinal purposes be banned. The Single Convention on narcotic drugs in 1961 consolidated the numerous earlier agreements, while the Convention on Psychotropic Substances of 1971 introduced controls on amphetamines, hallucinogens, barbiturates and some tranquillisers. Finally, the Convention Against Illicit Traffic in Narcotic Drugs and Psychotropic Substances was introduced in 1981. It prescribed measures against the drug trade that included the confiscation of assets of those so convicted. During this period individual countries have also introduced a myriad of domestic measures against trade and consumption of narcotics and other drugs.

Given this brief history one could justifiably ask what has been the return on this increasing worldwide investment in drug prohibition and control? Unfortunately and perhaps perversely, the international efforts against drugs and the worldwide level of drug use have escalated simultaneously. Arguably, the trade in and consumption of
illicit drugs is one of the most serious social problems confronting governments in many countries today. Perhaps in no other country is this most starkly the case than in Colombia. For instance, according to Thoumi (2002), “The drug industry has acted as a catalyst that accelerated a process of ‘delegitimation of the regime’, that has …produced a sharp decline in trust that increased transaction costs, contributed to increased violence and impunity, that has induced ‘free’ capital flight and larger security costs, promoted expectations of very fast wealth accumulation that produced highly speculative investments and increased bankruptcies, embezzlements and so forth… Increased criminality has (resulted in) lost growth (exceeding) two (percentage points) per year, without including its longer term effects on factor productivity and capital formation” (pp.110-111, parentheses ours). The trade and consumption of illicit drugs has had deleterious effects in other countries also such as Mexico (see Chabet 2002) and Nigeria (see Klein 1999).

A number of individual country studies have attempted to quantify these social costs with the estimates typically running to the hundreds of millions or billions of dollars annually. For instance, Cartwright (1999), reports on a study jointly published by the National Institute on Drug Abuse (NIDA) and the National Institute on Alcoholism and Alcohol Abuse (NIAAA) that estimates that the cost for the USA in 1995 as US$110 billion. Whilst this estimate includes the costs of alcohol abuse, it is clear that the illicit drug industry, even if the junior partner, nevertheless generates massive social costs in the USA. Xie et. al. (1998) investigate the economic costs of illicit drug use in Ontario, Canada, using a cost-of-illness approach and estimate that cost as C$489 million in 1992. Finally, Cleeland et. al. (1988) estimated the direct annual cost of drug law enforcement in Australia at A$123 million. This excludes the social cost of the illicit drug industry in that country. One gets an idea of the likely
magnitude of the industry worldwide from a 1998 United Nations report that concluded it to be worth 400 billion dollars annually, beaten into second place only by the international arms trade estimated to be worth US$800 billion annually (see Allen 1999, p.5). In some less developed countries, daily use of drugs (relatively mild in the case of Yemen) has been an integral part of the local culture and has considerably affected the households and aggregate time allocation, supply of labour, income, consumption and investment. Notable examples are Yemen, where the addiction to a relatively mild drug called Kat has been common, and Cambodia where there has been a widespread use of opium, and where the economy of, and political affairs in, a part of the country has been dominated by the production and supply of this drug.

Some researchers have concluded that the costs of drug use may actually have been increased over time by the increasingly tougher worldwide prohibition on narcotics and other drugs. For example, Miron (2001) argues that tougher drug law enforcement simply results in higher rates of violence and criminal activity that subsequently adds to the social costs to be borne by society. Kennally (2001) argues that the prohibition restricts entry, reduces consumer information and thus increases the market power of existing traders who use violence to enforce contracts and produce products of unknown quality. He thus advocates the nationalization of the industry so as to greatly reduce prices thereby eliminating the criminal element from the market. This would also, according to the author, “…discourage any legitimate firm from diversifying into this area. On the other hand the government could control the quality of drugs sold, regulate the age requirements for sale and easily implement education and rehabilitation programs” (p.80).

Clearly then, public policy towards the trade in and consumption of drugs such as narcotics is a difficult and contentious issue in most countries. On the one hand
few would advocate complete decriminalisation and a free market approach as this would likely result in a much larger cohort of addicts with associated social costs. On the other hand the current approach of a continuously tougher prohibition and law enforcement regime also appears to be very costly. Our paper seeks to make a contribution to this debate by constructing a dynamic macroeconomic model where government seeks to maximise the value of output in the presence of productive non-using workers and less productive drug users. Drug use is modelled as a diffusion process with user numbers being increasing in the existing user population but decreasing in the costly government prevention effort. The objective of government is to determine the prevention effort which maximises the net present value of the nation’s human resources. The conceptual framework developed in our paper is also applicable to the use of other substances. For instance, alcohol abuse in Russia is now so widespread that it could be described as a social epidemic. Therefore, whilst we refer in this paper to users of drugs such as narcotics, the approach is sufficiently general to make it applicable to the abuse of any potentially debilitating substance.

The remainder of this paper is organised as follows. Section 2 introduces the building blocks of our drug-control model where the nation’s labour force is divided into non-using and therefore fully productive workers and less productive drug users. Section 3 presents and interprets the rule for drug-control effort that maximises the present value of the stream of the net national incomes stemming from the country’s using and non-using workers. The properties of the steady states of the system comprising this rule and the drug-proliferation equation are analysed, displayed and simulated in section 4. The effects of changes in the model parameters on the steady state numbers of users, the steady state preventive effort, and the steady state level of disposable national income are simulated in section 5. The model is expanded in
section 6 to incorporate the social costs of disharmony between non-users and users. Section 7 concludes.

2. Building blocks of an economic drug-control model

Our model focuses on the relationship between the use of narcotic drugs, labour productivity, aggregate income and the government preventive effort. For tractability, we ignore capital, capital accumulation and technological changes and assume that labour is the sole factor of production. More specifically, our model is based on the following assumptions.

Assumption 1 (labour-force size and composition): The size of the working-age population is time invariant and equal to $L$, of which $N(t)$ people use narcotics, hereafter Narcos.

Assumption 2 (drugs and employment): $L_g$ members of the labour force are Anti Narcotic Guards and Enforcers of Law (Angels) employed by the government, whilst the remaining $L - L_g$ members are employed in the private sector. Narcos cannot be Angels. For tractability, the Angel force is taken to be homogeneous and having a single activity.$^1$

Assumption 3 (drugs and productivity): Drug use reduces productivity. If the instantaneous output of each of the privately employed $L - N(t) - L_g$ non-users, hereafter Machos, is $y$ then the instantaneous output of a Narco is

---

$^1$ A broader framework may consider multiple prevention and rehabilitation activities with diminishing marginal returns and, in turn, the division (or allocation) of the Angel force (or time) into activity groups (or slots). An extension of the model to two types of activities with varying degree of popularity was attempted. In accordance with public preferences, some of the Angles were engaged in prevention activity and the rest in rehabilitation. Public preferences were assumed to be responsive to the change in the population share of Narcos. However, the increased level of complexity (two state variables and two control variables) limited the analysis and reduced the overall clarity of the paper.
\[ y_n = \varepsilon y, \ 0 \leq \varepsilon < 1 \]  
(1)

where, \( y \) is taken for simplicity to be a positive, time-invariant scalar, and \( \varepsilon \) is the relative productivity of a *Narco* with \( \varepsilon = 0 \) indicating total incapacitation and \( 0 < \varepsilon < 1 \) partial incapacitation.

**Assumption 4 (drug-proliferation):** The net conversion of *Machos* and *Angels* to *Narcos* is given by the difference between a concave diffusion function \( F \) and a linear\(^2\) prevention-rehabilitation function \( \delta L_g \)

\[ \dot{N}(t) = F(N(t); L) - \delta L_g (t) \]  
(2)

where, \( F'' < 0 \) indicates a diminishing marginal diffusion of the use of narcotics within the population so that \( F' \) is positive up to a critical level \( N^* \) and thereafter negative,\(^3\) and \( \delta \) denotes the instantaneous marginal and average productivity of each *Angel* in terms of the number of people prevented and rehabilitated from using narcotics. Our specification of the diffusion function is based on the premise that the use of narcotics is socially contagious -- as the number of *Narcos* increases drug using becomes more socially acceptable. Due to innate intellectual, moral, cultural and physiological differences people have different degree of resistance to drugs. Drug

\(^2\) An alternative concave specification -- \( R(L_g) \), \( R' > 0, R'' < 0 \) -- reflecting diminishing marginal prevention requires \(-\lambda(t)R''(L_g(t)) < 2c\) for an interior solution to the maximisation problem described in section 3 to exist, where \( c \) is the overhead and social cost coefficient indicated in assumption 5 and \( \lambda \) is the co-state variable of the maximisation problem’s Hamiltonian. It is also possible that the marginal prevention depends upon the number of *Narcos* \( R'(L_g; N) \). However, the effect of \( N \) on the marginal prevention is not clear a-priori. On the one hand, the greater the number of *Narcos* the easier the “catch”. On the other hand, a larger number of *Narcos* might be associated with a greater resistance to prevention.

\(^3\) For instance, \( N^* = 0.5(1 - \beta)L \) in the case of a logistic diffusion function \( F(t) = \alpha N(t)(1 - N(t)/(1 - \beta)L) \), where \( 0 < \alpha < 1 \) denotes the intrinsic diffusion rate and \( 0 \leq \beta < 1 \) the share of the population absolutely unsusceptible to narcotics.
use spreads gradually and in diminishing increments from highly susceptible people to less susceptible people.

**Assumption 5 (prevention costs):** The instantaneous cost of prevention is an increasing and convex function of $L_g(t)$ comprising a linear part of forgone private output ($y$ for each Angel) and a quadratic part that comprises both the overhead costs of, and the social costs stemming from, prevention. Consistent with Miron (2001) and Kennally (2001), we assume that social costs increase rapidly in the government prevention effort as drug traders resort to more serious forms of criminal activity and violence and drug users face greater uncertainties about supply and drug quality. In formal terms, the instantaneous costs of prevention are depicted as follows:

$$C(t) = yL_g(t) + cL_g(t)^2$$

where $c$ is the positive coefficient of the marginal overhead and social costs associated with prevention.

**Assumption 6 (balanced budget and tax neutrality):** At every instance the government fully finances the prevention effort by collecting income tax that, for simplicity, is assumed to not adversely affect the supply of labour.

### 3. Growth-efficient prevention effort

Assumptions 1, 2, 3, 5 and 6 imply that the instantaneous disposable national income (DNI), that is the gross national income net of the government spending on prevention and the social cost of drug-control, is given by

$$DNI(t) = [L - (1 - \varepsilon)N(t) - L_g(t)]y - cL_g(t)^2.$$ (4)
A growth-efficient drug-control effort is taken to be the trajectory of the number of \textit{Angels} ($L_g^o$) that maximises the sum of the discounted instantaneous DNIs generated over an infinite planning horizon subject to the conversion equation of \textit{Machos} and \textit{Angels} to \textit{Narcos}. That is,

$$L_g^o = \arg \max \int_0^\infty e^{-\rho t} \left[ (L - (1 - \varepsilon)N(t) - L_g(t))y - cL_g(t)^2 \right] dt$$

subject to the motion Eq. (2) and where $\rho$ is a positive fixed rate of the social planner’s time preference.

The \textit{Hamiltonian} associated with this decision problem is concave in both the control variable ($L_g$) and the state variable ($N$) and hence there exists an interior solution. Along the growth-efficient drug-control path there is an equality between the marginal financial and social costs of \textit{Angels}, $e^{-\rho t} [y + 2cL_g(t)]$, and the value for the social planner of the people prevented from using drugs by an additional \textit{Angel}, $-\delta \lambda(t)$, where $-\delta \lambda(t)$ — the present-value shadow cost of \textit{Narcos} — diminishes in a rate which is equal to the sum of the marginal diffusion of drugs and the ratio of the marginal return (MR) on \textit{Angels} to the marginal costs ($C'$) of \textit{Angels}:

$$\frac{\dot{\lambda}(t)}{\lambda(t)} = -F'(N(t); L)\delta (1 - \varepsilon) y / (y + 2cL_g) .$$

The evolution of the number of \textit{Angels} along the growth-efficient path is given by the no-arbitrage rule:

$$\dot{L}_g^o(t) = \frac{UC(L_g^o)}{2c} \left[ \frac{C'(L_g^o)}{MR(L_g^o)} - \delta (1 - \varepsilon) \right] .$$
The first term in the numerator of the no-arbitrage rule is the instantaneously forgone gross national income stemming from an additional infinitesimal investment in prevention. It is equal to the product of the user cost (UC) of the prevention capital (namely, the fully productive people employed as Angels) and the financial and social costs of employing an additional unit of prevention capital (Angel). The user cost of the prevention capital (Angels) includes the government’s, or social planner’s, rate of time preference (presumably the forgone national interest on any dollar spent on prevention) but is reduced by the instantaneous “infection” of the Machos and Angels labour forces generated by an additional Narco, which is positive (negative) up to (beyond) the critical mass of $N^*$. The second term in the numerator of Eq. (7), $\delta (1-\epsilon) y$, indicates the marginal return on prevention capital (Angels). The employment of an Angel increases the number of the fully productive Machos by $\delta$ and, in turn, increases gross national income by $\delta (1-\epsilon) y$.

The no-arbitrage rule suggests that the government efficient employment of Angels changes during the planning horizon in accordance with the difference between the forgone gross national income stemming from an additional infinitesimal effort invested in prevention and the return, in terms of gross national income, on an additional unit of effort invested in this activity. If the loss of national income from employing an additional Angle is greater (smaller) than the return on an Angle, investment in prevention capital has to be accelerated (decelerated). The intertemporal change in the number of Angels is moderated by the coefficient (2c) of the associated marginal overhead and social cost. By adhering to this no-arbitrage rule the government facilitates the construction of a growth-efficient trajectory of the national
portfolio of privately employed inputs comprising a fully effective labour force of

\( L - N(t) - L_g(t) \) Machos and a less effective labour force of \( N(t) \) Narcos.

4. Steady states: nature and values

By setting \( \dot{L}_g = 0 \) in the no-arbitrage rule, the steady-state level of the effort

invested in prevention is

\[
L^*_g = C^*(1-\varepsilon)y/(\rho - F'(N_{ss};L))
\]  

(8)

Recalling Eq. (2), it can also be expressed as

\[
L^*_g = F(N_{ss};L)/\delta .
\]  

(9)

Thus, the stationary number of Narcos satisfies the following equality

\[
C^*(1-\varepsilon)y/(\rho - F'(N_{ss};L)) = F(N_{ss};L)/\delta .
\]  

(10)

The phase-plane diagram presented below is constructed for the case where

the planner’s rate of time preference (\( \rho \)) is larger than the intrinsic proliferation rate

of drugs (\( F'(0;L) \)) – the only case shown by numerical simulations to have

(computable) steady states with our choice of an explicit drug-diffusion function. (See

appendix B for a detailed and complete phase-plane analysis.) In this case, as well as

in the case of \( \rho < F'(0;L) \), the steady states are asymptotically unstable.

Nevertheless, two arms converge to the low-prevention saddle point, \( SS_2 \), from west-

north-west and from east-south-east, as displayed in Figure 1. The high-prevention

steady state, \( SS_1 \), is an asymptotically unstable spiral. That is, if initially the system is

off that steady state, diverging oscillations in both the number of Narcos and the

number of Angels characterise the path maximising the sum of the discounted

disposable national incomes. These oscillations reveal periods of high preventative

effort leading eventually to a decline in number of Narcos, followed by periods of
reduced effort subsequently leading to a rise in the number of *Narcos*. The directions of the efficient, diverging trajectories stemming from possible initial points in the vicinity of $SS_1$ and $SS_2$ are indicated by horizontal and vertical arrows.

![Phase-plane diagram with $\rho > F'(0, L)$](image)

To facilitate our numerical simulations, the drug-diffusion function ($F$) is set to be logistic:

$$F(t) = \alpha N(t)[1 - \frac{N(t)}{(1 - \beta)L}]$$  \hspace{1cm} (11)

where $0 < \alpha < 1$ indicates the intrinsic proliferation rate of narcotics within the population and $0 \leq \beta < 1$ the share of the population resistant to narcotic use. In this case, the aforementioned no-arbitrage rule and the state-equation are displayed by the following system of non-linear differential equations
\[ \dot{L}_g(t) = \{\rho - \alpha[1 - 2N(t)/(1 - \beta)L(t)]\}[(y + 2cL_g(t)) - \delta(1 - \varepsilon)y]/2c \]  \hspace{1cm} (12)

\[ \dot{N}(t) = \alpha N(t)[1 - N(t)/(1 - \beta)L(t)] - \delta L_g(t). \] \hspace{1cm} (13)

We substitute the steady-state condition \( \dot{L}_g = 0 = \dot{N} \) into this system and solve to obtain the following stationary values:

\[ L_g^{ss} = \frac{1}{2c} \left[ \frac{\delta(1 - \varepsilon)y}{\rho - \alpha + \frac{2\alpha N_{ss}}{(1 - \beta)L}} - y \right] \] \hspace{1cm} (14)

where \( N_{ss} \) satisfies the following polynomial

\[ \frac{2\alpha}{(1 - \beta)L} N_{ss}^3 + (\rho - 3\alpha) N_{ss}^2 - [(\delta y/c) + (\rho - \alpha)(1 - \beta)L] N_{ss} = \delta[\rho - \alpha - \delta(1 - \varepsilon)y(1 - \beta)L]/2\alpha \] \hspace{1cm} (15)

Consequently, the stationary level of gross national income (\( GNI_{ss} \)) is given by

\[ GNI_{ss} = [L - (1 - \varepsilon)N_{ss}]y \] \hspace{1cm} (16)

whilst the stationary disposable national income (\( DNI_{ss} \)) is given by

\[ DNI_{ss} = [L - (1 - \varepsilon)N_{ss} - L_g^{ss}]y - cL_g^{ss^2}. \] \hspace{1cm} (17)

As mentioned earlier, our numerical simulations reveal that steady states can be computed only for the case where the rate of time preference is larger than the intrinsic diffusion rate of drugs (\( \rho > \alpha \)). The numerically computed numbers of Angels and Narcos and the level of disposable net income in the steady states \( SS_1 \) and \( SS_2 \) are reported in Table 1 for an imaginary country with the following characteristics:

- a labour force (\( L \)) of 10,000,000,
- the relative productivity of a Narco (\( \varepsilon \)) being 0.5,
- an income of $22,500 per annum per Macho,
an intrinsic drug-proliferation rate ($\alpha$) of 0.02 per annum,

one half of the labour force with absolute resistance to drug use ($\beta = 0.5$),

a productivity per Angel ($\delta$) of 12 prevented people per annum,

a marginal overhead and social-cost coefficient ($c$) of $1,500$ per annum, and

an annual social rate of time preference ($\rho$) of 0.04.

Table 1: High and low prevention effort steady states

<table>
<thead>
<tr>
<th></th>
<th>High-effort steady state ($SS_1$)</th>
<th>Low-effort steady state ($SS_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{ss}$</td>
<td>1515</td>
<td>800</td>
</tr>
<tr>
<td>$N_{ss}$</td>
<td>1,194,350</td>
<td>4,461,770</td>
</tr>
<tr>
<td>$DNI_{ss}$</td>
<td>$208,090,000,000$</td>
<td>$173,830,000,000$</td>
</tr>
</tbody>
</table>

As mentioned above, SS1 (SS2) is the steady state with a relatively high (low) government preventative effort and hence a relatively low (high) number of users. If maximising disposable national income is the objective then the high prevention regime at SS1 is clearly preferable to the low prevention regime at SS2. The much greater cost of the preventative effort at SS1, with the number of Angels being almost twice that at SS2, is more than compensated for by the reduced user numbers and hence the greater aggregate labour force productivity. Of course shocks to the system (see analysis below), such as changes to the labour force (L) or a change in social attitudes to drug use that reduces the resistance of some to drug use ($\beta$), mean that the economy is likely to be off steady state most of the time. Beginning at SS1 we again note that the economy will follow a diverging spiral path that, depending on the speed of adjustment, implies policy oscillations from periods of higher preventative effort to period of lower preventative effort.
5. The effects of changes in the model parameters

Plausible changes in the parameters of the model, such as an increased labour force or an increased overhead and social cost of drug use, will affect the steady state numbers of Angels and Narcos and the steady state level of DNI and may also affect the trajectory from a steady state given some perturbation to the system. The directions of these changes cannot be assessed by total differentiation (see Appendix C for details). However, the number of Angels and Narcos can be determined by evaluating the direction of the shifts of the isoclines $\dot{L}_g = 0$ and $\dot{N} = 0$. Table 2 summarises the impact of an increase in each parameter in turn on the isoclines and thus on the steady state numbers of Angels and Narcos.

Table 2: The directions of the effects of increased parameter values

<table>
<thead>
<tr>
<th>Parameters →</th>
<th>$L$</th>
<th>$\rho$</th>
<th>$\delta$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$c$</th>
<th>$y$</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isocline $\dot{L}_g = 0$</td>
<td>up</td>
<td>down</td>
<td>up</td>
<td>down</td>
<td>down</td>
<td>up</td>
<td>up</td>
<td>down</td>
</tr>
<tr>
<td>Isocline $\dot{N} = 0$</td>
<td>up</td>
<td>---</td>
<td>down</td>
<td>up</td>
<td>down</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$L_g^{ss_1}$</td>
<td>up</td>
<td>down</td>
<td>up</td>
<td>up</td>
<td>down</td>
<td>up</td>
<td>up</td>
<td>down</td>
</tr>
<tr>
<td>$N^{ss_1}$</td>
<td>up</td>
<td>down</td>
<td>up</td>
<td>down</td>
<td>up</td>
<td>up</td>
<td>up</td>
<td>down</td>
</tr>
<tr>
<td>$L_g^{ss_2}$</td>
<td>down</td>
<td>down</td>
<td>up</td>
<td>down</td>
<td>up</td>
<td>up</td>
<td>up</td>
<td>down</td>
</tr>
<tr>
<td>$N^{ss_2}$</td>
<td>up</td>
<td>up</td>
<td>down</td>
<td>up</td>
<td>down</td>
<td>up</td>
<td>down</td>
<td>up</td>
</tr>
</tbody>
</table>

The direction of the isocline shifts alone, however, cannot reveal the direction and size of the impact on the stationary level of disposable national income. Hence numerical simulations based on the initial parameter set were used to assess the percentage change in the stationary number of Angels and Narcos and on the stationary level of DNI induced by a one percent increase in each of the parameters in turn. Recalling Eq. (17), the elasticities of DNI with respect to the model parameters are given by

$$\xi_{DNI} = -\left(y + 2cL_g^{ss}\bar{\xi}_L L_g^{ss} + (1-\epsilon)\bar{\xi}_N N^{ss}\right)/DNI^{ss}$$

(18)
where \( \xi_{L_g} \equiv (\Delta L / L_g) / (\Delta \gamma / \gamma) \) and \( \xi_{N} \equiv (\Delta N / N_{ss}) / (\Delta \gamma / \gamma) \) denote the elasticities of the number of *Angels* and *Narcos*, respectively, with respect to any of the model parameters (denoted by \( \gamma \)). The simulated elasticities of the stationary numbers of *Angels* and *Narcos* and of the stationary level of DNI with respect to the model parameters are reported in Table 3 for \( SS_1 \) and in Table 4 for \( SS_2 \).

Table 3: The elasticities of the numbers of *Angels* and *Narcos* and of DNI at \( SS_1 \)

<table>
<thead>
<tr>
<th>Parameters → Elasticities ↓</th>
<th>( L )</th>
<th>( \rho )</th>
<th>( \delta )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( c )</th>
<th>( y )</th>
<th>( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_{L_g} )</td>
<td>0.297</td>
<td>-0.878</td>
<td>0.290</td>
<td>0.581</td>
<td>-0.350</td>
<td>-0.640</td>
<td>0.654</td>
<td>-0.706</td>
</tr>
<tr>
<td>( \xi_{N} )</td>
<td>0.018</td>
<td>-1.217</td>
<td>2.133</td>
<td>-0.542</td>
<td>0.0003</td>
<td>-0.893</td>
<td>0.999</td>
<td>-0.986</td>
</tr>
<tr>
<td>( \xi_{DNI} )</td>
<td>-0.011</td>
<td>0.108</td>
<td>-0.147</td>
<td>0.016</td>
<td>0.012</td>
<td>0.079</td>
<td>-0.086</td>
<td>0.087</td>
</tr>
</tbody>
</table>

Table 4: The elasticities of the numbers of *Angels* and *Narcos* and of DNI at \( SS_2 \)

<table>
<thead>
<tr>
<th>Parameters → Elasticities ↓</th>
<th>( L )</th>
<th>( \rho )</th>
<th>( \delta )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( c )</th>
<th>( y )</th>
<th>( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_{L_g} )</td>
<td>-0.075</td>
<td>-0.725</td>
<td>1.263</td>
<td>-0.388</td>
<td>0.125</td>
<td>-0.988</td>
<td>1.113</td>
<td>-1.088</td>
</tr>
<tr>
<td>( \xi_{N} )</td>
<td>1.147</td>
<td>0.099</td>
<td>-0.340</td>
<td>0.170</td>
<td>-1.156</td>
<td>0.133</td>
<td>-0.155</td>
<td>0.148</td>
</tr>
<tr>
<td>( \xi_{DNI} )</td>
<td>-0.330</td>
<td>-0.021</td>
<td>0.084</td>
<td>-0.045</td>
<td>0.332</td>
<td>-0.027</td>
<td>0.032</td>
<td>-0.031</td>
</tr>
</tbody>
</table>

As the steady states are not asymptotically stable, the direction and size of the changes imposed by the model parameters on the model endogenous variables reported in Tables 2–4 should not be interpreted as the parameters’ impact effects, but as the changes required for a new steady state to prevail.

Table 3 reveals that in the vicinity of the high-preventative-effort steady state the highest growth rate, though modest, is generated by a rise in the rate of time preference (\( \rho \)). This is due to the substantial negative impact of such a change on both the number of *Angels* and the number of *Narcos* in a new steady state. The underlying rationale is as follows. The higher the social planner’s rate of time
preference the lower his, or her, inclination to recruit/employ Angles, who can be Machos in current private production activity. This, in turn, leads to an increase in the number of Narcos and to diverging oscillations. Steady state can only be retained with a smaller number of Narcos than the initial number. In contrast, a rise in the prevention-recovery rate (\( \delta \)) leads to the largest decline rate in DNI due, in particular, to the considerable increase in the number of Narcos required for retaining a steady state. It is also interesting to note that in the vicinity of the high-effort steady state a rise in full-capacity (Macho’s) income increases the control effort substantially but at the same time also generates almost one percent rise in the number of Narcos and hence a slight decline in DNI. Opposite outcomes are generated by a rise in either the Narcos’ degree of productivity or the cost-coefficient.

Table 4 displays a markedly different parameter effects in the vicinity of the low-preventative-effort steady state. The highest growth rate, three times larger than the counterpart in the vicinity of the high-effort steady state, is generated by an increase in the share of the population absolutely unsusceptible to drugs. This is due to the large decline in the population of Narcos which more than compensating for the extra costs of drug-control associated with the increased effort. An equally large but opposite results are generated by an increase in the labour-force size. It is interesting to note that a rise in either the recovery rate or full-capacity income leads to a large increase in drug-control effort, but the decline in the number of Narcos is small and does not compensate for the rise in the costs of the drug-control effort and hence generate a slight decline in DNI. In contrast, a one per cent rise in the control-cost coefficient reduces the drug-control effort by a similar per cent without causing a large increase in the number of Narcos (as the control effort has already been low).
and as the net cost saving exceeds the extra loss of production capacity, the DNI of a new steady should be slightly larger.

We note from these results that changes in two parameter values have unambiguous impacts on the stationary level of disposable net income. Firstly, population (or more correctly, labour force) growth \((L)\) is unambiguously bad for DNI with the positive effect on the number of *Narcos* being much greater in SS\(_2\) than in SS\(_1\). Similarly, an increase in the proportion of the population totally resistant to drug use \((\beta)\) is unambiguously good for DNI with the negative impact on the number of *Narcos* being much greater in SS\(_2\) than in SS\(_1\). This result may be interpreted as support for government interventions such as public education campaigns that warn of the dangers of drug use and thus reduce the propensity of some to experiment with drugs.

6. Extension - societal disharmony

If substance abuse is sufficiently widespread, tensions between users and non-users might arise.\(^4\) It is possible that the level of societal disharmony intensifies, and hence social costs increase, as the difference between the number of *Machos* and *Angels* and the number of *Narcos* diminishes.

*Assumption 7:* The relationship between costs of societal disharmony \((CSDH)\) and the share of the population that are *Narcos* conforms to an inverted U-shaped curve:

\[
CSDH(t) = CSDH_{\text{max}} - \mu [(N(t)/L) - 0.5]^2
\]

\(^4\) In the case of AIDS, there exists tension between infected and non-infected people and incidence of atrocities have taken place.
where $CSDH_{\text{max}}$ is the maximum societal cost of disharmony that accrues when the number of users is equal to the number of non-users, and $\mu$ is a positive scalar reflecting the moderation of the cost of societal disharmony by the quadratic distance from equal number.

Assumption 7 implies, in conjunction with the assumptions made earlier, that the instantaneous DNI, now the gross national income net of the financial and social costs of prevention and the costs of societal disharmony, is given by

$$DNI(t) = [L - (1 - \varepsilon)N(t) - L_g(t)]y - cL_g(t)^2 - [CSDH_{\text{max}} - \mu((N(t)/L) - 0.5)^2].$$

Consequently, the efficient number of Angels is now

$$\hat{L}_g = \arg\max_{\tilde{L}_g} \int_0^\infty e^{-pt} \left\{[L - (1 - \varepsilon)N(t) - L_g(t)]y - cL_g(t)^2 - [SDH_{\text{max}} - \mu((N(t)/L) - 0.5)^2]\right\} dt$$

subject to the motion Eq. (2).

The no-arbitrage rule associated with this modification is

$$\dot{\hat{L}}_g(t) = \frac{[p - F'(N(t);L)](y + 2c\hat{L}_g(t)) - \delta [(1 - \varepsilon)y + (2\mu/L)[(N(t)/L) - 0.5]]}{2c}.$$  

(See details in Appendix D.)

Since an extra infinitesimal effort in reducing the number of Narcos does not necessarily reduce the level of societal disharmony, $\dot{\hat{L}}_g < \tilde{L}_g^o$ as $\frac{N(t)}{L} < 0.5$. If the number of drug users initially exceeds the number of non-users, a rise in the preventative effort reduces the groups’ size-differential and thereby intensifies the social tension. In this case, a smaller rise in preventative effort is recommended by a socially aware planner than by a planner ignoring social disharmony. Conversely, if the number of non-users initially exceeds the number of drug-users, a rise in the
preventative effort increases the groups’ size-differential and hence reduces the level of social tension. In this case, a larger rise in preventative effort is advocated by a socially aware planner than by a planner disregarding social disharmony.

7. Conclusion

Drug use is in many countries a social epidemic that reduces the number of fully productive workers and thereby aggregate output. This paper presented a social-epidemic-control model — a hybrid of an epidemiological diffusion process and economic (and to a lesser extent, social) objectives — with a special reference to narcotics. The model is generic and may also be applicable to other epidemics such as alcohol abuse and AIDS.

The model divided the labour force into fully productive workers who do not use drugs and only partially productive users, and assumed that the use of drugs is socially contagious. In addition to forgone private output, costs are borne by government and society with the provision of a preventative effort. Efficient management of the nation’s portfolio of human resources was perceived as embarking on a path of drug-control effort that maximises the present value of the stream of the disposable national incomes. The efficient level of prevention varies during the planning horizon in accordance with the difference between the forgone gross national income stemming from an additional infinitesimal effort invested in this activity and the return, in terms of gross national income, on an additional unit of effort invested in this activity. The intertemporal change in the preventative effort is moderated by the coefficient of the associated marginal overhead and social costs. The forgone national income was taken as the product of the user cost of the typical non-user employed by the government and the marginal financial and social costs of the invested preventative effort. The user cost of a publicly employed Angel rises with the
government rate of time preference but is moderated by the instantaneous marginal “infection” of the labour force by drug users. The possible steady states of the system comprising this rule and the drug-proliferation equation were found to be asymptotically unstable and were numerically simulated. The effects of the model parameters on the long-run equilibrium numbers of Narcos and Angels and subsequently on the long-run equilibrium disposable national income were simulated. Finally, the economic drug-control model was expanded to incorporate tension between users and non-users.

REFERENCES


Appendix A: The necessary and sufficient conditions and the no-arbitrage rule

The (present value) Hamiltonian associated with Eq. (5) and Eq. (2) is

$$H(t) = e^{-ho t} \left\{ [L - (1 - \varepsilon)N(t) - L_g(t)]y - cL_g(t)^2 \right\} + \lambda(t)[F(N(t); L) - \delta L_g(t)].$$

(A1)

The necessary and sufficient conditions for maximum are:

$$\dot{\lambda}(t) = -\frac{\partial H(t)}{\partial N(t)} = e^{-\rho t}[(1 - \varepsilon)y - \lambda(t)F'(N(t); L)]$$

(A2)

$$\frac{\partial H(t)}{\partial L_g(t)} = -e^{-\rho t}[y + 2cL_g(t)] - \delta \lambda(t) = 0$$

(A3)

Eq. (2) and the transversality condition $\lim_{t \to \infty} \lambda(t)N(t) = 0$.

Eq. (6) is obtained by dividing both sides of Eq. (A2) by $\lambda$ and considering that by virtue of Eq. (A3) $\lambda(t) = -e^{\rho t}[y + 2cL_g(t)]/\delta$. The no-arbitrage rule, Eq. (7), is obtained by differentiating the optimality condition (A3) with respect to $t$ (singular control), substituting the information contained in conditions (A2) and (A3) for $\lambda$ and $\dot{\lambda}$, respectively, multiplying both sides by $e^{\rho t}/2c$ and rearranging terms. It can also be obtained by using Euler equation.
Appendix B: Phase-plane analysis

The system comprising the no-arbitrage rule (Eq. (7)) and the net-loss of fully productive workers (Eq. (2)) has multiple steady states. The number and nature of these steady states is identified by the following phase-plane analysis.

The slope of the isocline $\dot{N} = 0$ is

$$\frac{dL_g}{dN} \bigg|_{N=0} = L_g = \left[ \frac{F'(N;L)}{\delta} \right] \quad \text{as} \quad N = N^*$$

and hence this isocline is displayed by an inverted U-shaped curve in the phase plane.

Since $\frac{d\dot{N}}{dL_g} = -\delta < 0$, $\dot{N}$ is negative (positive) and depicted by leftward (rightward) pointed horizontal arrows, in the region above (below) this isocline.

The slope of the isocline $\dot{L}_g = 0$ is

$$\frac{dL_g}{dN} \bigg|_{L_g=0} = L_g = \left[ \frac{F^*(N;L)C'(t)}{[\rho - F'(N;L)]C^*(t)} \right]$$

as $F'(N;L) = \rho$, which in turn implies that $\frac{dL_g}{dN} \bigg|_{L_g=0} = 0$ for $N = F^*-1(\rho)$ where $F^*-1(\rho) < N^*$.

If the government’s rate of time preference ($\rho$) exceeds the intrinsic diffusion rate ($F'(0;L)$) the isocline $\dot{L}_g = 0$ is negatively sloped in the entire phase plane. There are two asymptotically unstable steady states and the system’s dynamics is displayed in Figure 1.

If the intrinsic diffusion rate of narcotics exceeds the government’s rate of time preference the isocline $\dot{L}_g = 0$ also has an inverted U-shape as displayed by Figure 2 or Figure 3. Also in this case the stationary points are asymptotically unstable. Figure 3 demonstrates three steady states with a convergent arm to $SS_5$. 
Figure 2: Phase-plane diagram with $\rho < F'(0, L)$

Figure 3: Phase-plane diagram with $\rho < F'(0, L)$

For further identification of the nature of the steady states, the eigenvalues of the state-transition matrix ($\alpha$) of the linearised (in the vicinity of steady state) system comprising the no-arbitrage rule (Eq. (7)) and the state equation (Eq. (2)) are computed as follows:
\[ \mu_{1,2} = 0.5 \left\{ \rho \pm \sqrt{\rho^2 - 4\left[ \rho - F'(N_{ss})\right] F'(N_{ss}) - \delta F''(N_{ss}) C'(t) \over \det(a)} \right\}. \]  

(B1)

Recalling that \( F'' < 0 \) and \( C' > 0 \), the second term in \( \det(a) \) is positive (namely, \( -\delta F''(N_{ss}) C'(t) > 0 \)) for all of the steady states illustrated above. In the cases of \( SS_1 \), \( SS_2 \), \( SS_4 \), \( SS_6 \), and \( SS_7 \), \( \rho > F'(N_{ss}) \) because \( N_{ss} > N^* \). In the case of \( SS_1 \), \( F' > 0 \) and hence the first term in \( \det(a) \) is positive. In this case, \( \det(a) > 0 \) and hence \( SS_1 \) is not a saddle point. In the cases of \( SS_2 \), \( SS_4 \), \( SS_6 \), and \( SS_7 \), \( F' < 0 \) and hence the first term in \( \det(a) \) is negative. In the cases of \( SS_3 \) and \( SS_5 \), the sign of \( \rho - F'(N_{ss}) \) and consequently the sign of the first term in \( \det(a) \) are not clear.
Appendix C: The effects of changes in the model parameters

The effects of changes in $L, \rho, \delta, \alpha, \beta, c, y, \text{ and } \varepsilon$ on $N_{ss}$ are obtained by differentiating Eq. (17):

\[
\frac{dN_{ss}}{dL} = \frac{2\alpha N_{ss}^3}{(1-\beta)L^2} + \frac{(\rho - \alpha)(1-\beta)N_{ss}}{2c\alpha} + \frac{\delta [\rho - \alpha - \delta (1-\varepsilon)]y(1-\beta)}{2c\alpha}
\]

\[\text{(C1)}\]

\[
\frac{dN_{ss}}{d\rho} = \frac{6\alpha N_{ss}^2}{(1-\beta)L} + \frac{2(\rho - 3\alpha)N_{ss} - (\delta y / c) - (\rho - \alpha)(1-\beta)L}{2c\alpha}
\]

\[\text{(C2)}\]

\[
\frac{dN_{ss}}{d\delta} = \frac{6\alpha N_{ss}^2}{(1-\beta)L} + \frac{2(\rho - 3\alpha)N_{ss} - (\delta y / c) - (\rho - \alpha)(1-\beta)L}{2c\alpha}
\]

\[\text{(C3)}\]

\[
\frac{dN_{ss}}{d\alpha} = \frac{3N_{ss}^2 - \left[2N_{ss}^3 / (1-\beta)L\right] + (1-\beta)L[N_{ss} + \delta (1-\beta) L(\gamma\alpha - ((\rho - \alpha - \delta (1-\varepsilon))y) / 2c\alpha^2)]}{2c\alpha}
\]

\[\text{(C4)}\]

\[
\frac{dN_{ss}}{d\beta} = \frac{\left(2\alpha N_{ss}^3 / (1-\beta)L^2 + (\rho - \alpha)LN_{ss} + \frac{\delta L[\rho - \alpha - \delta (1-\varepsilon)]y}{2c\alpha}\right)}{2c\alpha}
\]

\[\text{(C5)}\]

\[
\frac{dN_{ss}}{dc} = \frac{-\delta [\gamma N_{ss} + (\rho - \alpha - \delta (1-\varepsilon))y(1-\beta)L / 2c\alpha^2]}{2c\alpha}
\]

\[\text{(C6)}\]

\[
\frac{dN_{ss}}{dy} = \frac{(\delta / c)[N_{ss} + (\rho - \alpha - \delta (1-\varepsilon))(1-\beta)L / 2\alpha]}{2c\alpha}
\]

\[\text{(C7)}\]
\[
\frac{dN_{ss}}{d(1-\epsilon)} = \frac{-\delta^2 y(1-\beta)L/2c\alpha}{6\alpha N_{ss} - 2(\rho - 3\alpha)N_{ss} - (\delta y / c) - (\rho - \alpha)(1-\beta)L}.
\] (C8)
Appendix D: The no-arbitrage rule of Eq. (22)

The Hamiltonian associated with this decision problem is:

\[
H(t) = e^{-\rho t} \{ [L - (1 - \varepsilon)N(t)]y - C(L_g(t)) - [CSDH_{\text{max}} - \mu((N(t)/L) - 0.5)^2] + \lambda(t)[F(N(t);L) - \delta L_g(t)] \}. \tag{D1}
\]

Since \( H \) is concave in both the control variable and the state variable there exists an interior solution. The necessary conditions for maximum are

\[
\dot{\lambda}(t) = -\frac{\partial H(t)}{\partial N(t)} = e^{-\rho t} \{ (1 - \varepsilon)y + (2\mu/L)[((N(t))/L) - 0.5]\} - \lambda(t) F'(N(t);L) \tag{D2}
\]

\[
\frac{\partial H(t)}{\partial L_g(t)} = -e^{-\rho t} C'(L_g(t)) - \delta \lambda(t) = 0 \tag{D3}
\]

Eq. (2) and the transversality condition \( \lim_{t \to \infty} \lambda(t)N(t) = 0 \).

The no-arbitrage rule, Eq. (22), is obtained by differentiating the optimality condition (D3) with respect to \( t \) (singular control), substituting the information contained in conditions (D2) and (D3) for \( \dot{\lambda} \) and \( \lambda \), respectively, multiplying both sides by \( e^{\rho t}/C'(L_g(t)) \) and rearranging terms.