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Seiches in Jervis Bay, N.S.W.; theory vs. experiment
SEICHES IN JERVIS BAY, N.S.W.;

THEORY VS. EXPERIMENT

By
D. J. Clarke

NOVEMBER, 1971
Jervis Bay (Fig. 1) is situated on the New South Wales coastline some 100 miles south of Sydney. Recently this Bay has become the subject of environmental studies because of the three proposals (i) the building of a Nuclear Reactor south of Bowen Island to be cooled by Bay water, (ii) the building of a Steelworks near Hockinson with the possible waste discharge, and (iii) the discharge of treated effluent from nearby housing developments as

SEICHES IN JERVIS BAY, N.S.W.;

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The motion to determine the practicability of the above proposals, in terms of preserving the natural ecology of the area, is by D.J. CLARKE. Water movement in the Bay are (i) direct exchange of Bay water with the outside ocean through tidal flow, (ii) indirect exchange of Bay water due to circulation effects of the tidal flow, particularly around Bowen Island, and (iii) forced oscillations in the Bay caused by wind and/or the existence of particular frequencies of oscillation over the local continental shelf region.

The Department of Mathematics at Wollongong University College had a long wave recorder installed on the N.A.B. wharf at Jervis Bay Naval College for several months in 1968. Comparisons of predicted tide heights and those measured by the recorder gave excellent agreement in general. This would infer that the direct exchange of Bay water with the outside ocean through tidal
Jervis Bay (Fig. 1) is situated on the New South Wales coastline some 100 miles south of Sydney. Recently this Bay has become the subject of environmental studies because of the three proposals (i) the building of a Nuclear Reactor south of Bowen Island to be cooled by Bay water, (ii) the building of a Steelworks near Huskisson with the possible waste discharge, and (iii) the discharge of treated effluent from nearby housing developments as a means of sewerage disposal.

The motion of the Bay water will determine the practicability of the above proposals in terms of preserving the natural ecology of the area. Types of water movement in the Bay are (i) direct exchange of Bay water with the outside ocean through tidal flow, (ii) indirect exchange of Bay water due to circulation effects of the tidal flow, particularly around Bowen Island, and (iii) forced oscillations in the Bay caused by wind and/or the existence of particular frequencies of oscillation over the local continental shelf region.

The Department of Mathematics at Wollongong University College had a long wave recorder installed on the S.A.R. wharf at Jervis Bay Naval College for several months in 1968. Comparison of predicted tide heights and those measured by the recorder gave excellent agreement in general. This would infer that the direct exchange of Bay water with the outside ocean through tidal
flow is predictable. Also circulation effects are being studied over a long term by the Atomic Energy Commission particularly near Bowen Island. However the forced oscillations in the Bay have not been considered and it is the purpose of this article to report the first stages of such a study.

A previous report on short period oscillations or surging in the Bay was given by Clarke, Keane, and O'Halloran (1968).

2. ANALYSIS OF WAVE HEIGHT DATA

Examination of the long wave records obtained in 1968 revealed two facts. The first is that the Bay does not tend to oscillate in the presence of strong westerly winds, i.e., winds transverse to the major axis. The second is that the Bay does seiche under the action of strong southerly winds. Two such records of seiching were obtained on the 19th April and 15th May, and these records were used to form two power spectra.

The data to be analyzed were read manually at 3 minute intervals from a 5 hour length of record of wave heights for the 19th April and a 9 hour length for 15th May. A Fourier series spectrum was computed for each set of data by the Time Series Analysis Program FSAPS of the User Group G6 CSIR, (Clarke, 1965), and are shown in Figure 2.

The excitation mechanism of the wave motion on both days was a strong southerly wind. This wind is usually very
gusty on the N.S.W. coastline and hence it is not expected that the two spectra should be identical. However it is expected that many of the natural oscillations of the Bay should be excited and thus many of the energy peaks of the two spectra should correspond. In figure 2 the peaks have been identified by the time in minutes of the period of oscillation and these are seen to be in close agreement for most of the two spectra. One outstanding difference is the large energy peak at 33.3 minutes on the spectrum for 19th April. This peak has swamped the region corresponding to the period 24.0 minutes, where a peak appears on 15th May spectrum. The other region of disparity is that between 15.8 minutes and 17.4 minutes. Clearly further experimental data is required to resolve these areas.

3. SEICHES IN A ONE-DIMENSIONAL FLOW MODEL

There are several methods of analyzing the longitudinal oscillations in a lake or bay and these are mostly summarized by Defant (1961). A variational method was described by Clarke (1968) who applied the Galerkin method (Kantorovich and Krylov, 1964) to the boundary value problem of the longitudinal oscillations in a lake or bay. He showed that it is an easy-to-apply technique which is valid over a wide range of basin shapes provided the flow in the transverse direction could be neglected. This method is now applied to Jervis Bay.

The Bay is approximately oval in shape being about 9
miles long and 4½ miles wide. Jeffreys (1924) has shown that the frequency of the fundamental mode of oscillation in an elliptic basin of uniform depth varies only by 2.5% as the ellipse varies from a canal to a circle. Within this order of error it is assumed that the transverse flow is negligible in Jervis Bay. With regard to the tidal flow through the entrance the time cycle is of the order of 12 hours. Since the seiches in the Bay are of the order of 35 minutes and less (Clarke, Keane and O’Halloran, 1968), the tidal flow is neglected. For longitudinal seiching in the Bay it can be seen from figure 1 that very little energy will be lost through the entrance. Hence the Bay is assumed to be closed from Bowen Island to Longnose Point. The excitation mechanism being southerly winds it is also reasonable to seek the longitudinal oscillations only.

The boundary value problem to be solved is defined by the differential equation

$$\frac{d^2a}{dx^2} + \frac{\sigma^2}{g \ f(x)} \ a = 0,$$

where

$$f(x) = \lambda(x) b(x),$$

and the boundary conditions

$$a = 0, \ x = 0 \ and \ x = x_L,$$

where \(a\) is the time-reduced water transport over a cross-section
having an area \( A(x) \), and a breadth \( b(x) \) in the surface, \( x \) is the surface area variable defined by

\[
x = \int_0^x b(x') dx',
\]

\( x \) being the distance along the main axis of the Bay measured from one end, \( \sigma \) is the frequency of the oscillation, and \( g \) is the acceleration due to gravity. The Bay has a length \( l \) along the main axis and a total surface area \( x^l \).

The approximate solution of this boundary value problem is based on the Galerkin method. The transport, \( \alpha \), is approximated by a polynomial satisfying the boundary conditions, i.e.,

\[
\alpha = \alpha_n(x) = x(x^l - x)(a_1 + a_2x + a_3x^2 + \ldots + a_nx^{n-1})
\]

\[
= \sum_{i=1}^{n} a_i \phi_i.
\]

The functions \( \phi_i \) are to satisfy orthogonality conditions

\[
\left( \sum_{i=1}^{n} a_i \phi_i \right) \left( \int_0^{x^l} \frac{d^2 \alpha_n}{dx^2} + \frac{\sigma^2}{g f(x)} \alpha_n \right) \phi_j \, dx = 0,
\]

\( j = 1, 2, \ldots, n. \)

The co-efficients of the \( a_i \) are computed for each \( j \) value and are of the form \( -\mu_{ij} + \lambda \nu_{ij} \), where \( \lambda = \sigma^2/g \). Thus if \( A = (\mu_{ij}) \) and \( B = (\nu_{ij}) \) then the integrated system of orthogonality conditions becomes \( (A - \lambda B)x = 0 \), where \( x \) is the column vector of the \( a_i \). Hence \( \lambda \) and \( x \) are the eigen-values and eigen-vectors.
For Jervis Bay the origin was chosen at the southern end and the Bay was divided into 44 stations along the main axis. For each station the area of cross-section, \( A(x) \), and the breadth in the surface, \( b(x) \), were calculated. The integrations were carried out using Simpson's Rule on the original \( x \) variable. The first six eigen-values and their associated eigen-vectors were calculated from \( B^{-1}A \), sufficient accuracy being obtained for an \( n \) value of 8, i.e., the transport was approximated by a polynomial of degree 10. The periods of the oscillations were calculated from the eigen-values and these, together with the transport co-efficients, \( a_1 \), are given in Table 1. Wave heights, \( \zeta \), are then computed from the relation

\[
\zeta = -\frac{da}{dx},
\]

and are illustrated in Figure 3.

4. **THEORY VERSUS EXPERIMENT**

The two spectra have been compared earlier and it was pointed out that close agreement existed between them except for missing peaks on the 19th April with periods of 17.4, 20.0, and 24.0 minutes. For this reason the theoretical periods will be compared with the spectrum of 15th May.

It seems reasonable to try and match the theoretical values with the spectrum values in the following way
33.6 20.0 14.2 10.8 8.7 7.2
33.75 20.0 13.8 10.1 8.3 6.6

The non-matched spectrum values of 24.0, 17.4, 11.7, 9.0, and 7.3 each lie between the theoretical values, and it could be conjectured that they correspond to oscillations through the entrance. Further investigation would be necessary to resolve these points. The first two matched pairs above agree within the resolution factor of the spectrum and the other pairs show an advance in the theoretical periods of some 0.6 minutes.

5. CONCLUSION

The first stage in examining the behaviour of Jervis Bay to forced oscillations has been completed in that the first six longitudinal free oscillations have been determined for a one-dimensional flow model. The periods obtained were 33.6, 20.0, 14.2, 10.8, 8.7, and 7.2 minutes. Two power spectra obtained from wave records for 19th April and 15th May after strong southerly winds show close agreement in the periods of energy peaks. Every second peak is in reasonable agreement with the theoretical periods although some conjecture is required in identifying corresponding peaks. The absence of oscillations under the action of strong westerly winds strengthens the conjecture.

The next stage would be to determine the nature of the resonant oscillations through the entrance and then to consider the behaviour of the Bay under forced oscillations.
REFERENCES

CLARKE D.J.


CLARKE D.J., KEANE A., and O'HALLORAN P.J.


DEFANT ALBERT


JEFFREYS HAROLD


KANTOROVICH L.V., and KRYLOV V.I.

<table>
<thead>
<tr>
<th>HARMONIC PERIOD(MINS)</th>
<th>TRANSPORT CO-EFFICIENTS</th>
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<tbody>
<tr>
<td></td>
<td>$a_1$</td>
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<tr>
<td>1</td>
<td>33.6</td>
</tr>
<tr>
<td>2</td>
<td>20.0</td>
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<tr>
<td>3</td>
<td>14.2</td>
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<td>4</td>
<td>10.8</td>
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<td>5</td>
<td>8.7</td>
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<td>6</td>
<td>7.2</td>
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TABLE 1. The periods and the co-efficients, $a_1$, of the approximate transport, eqn. (7), for the first six longitudinal harmonics of Jervis Bay.
FIGURE 1: JERVIS BAY. DOTTED LINES INDICATE 10 AND 6 FATHOM LINES.
FIGURE 2: FOURIER SERIES ENERGY SPECTRUM FOR JERVIS BAY WITH MAXIMUM ENERGY PEAKS MARKED TO SHOW THE PERIOD IN MINUTES
FIGURE 3: THEORETICAL WAVE HEIGHTS IN JERVIS BAY NORMALISED TO -1 AT SOUTHERN END