Linear System Identification using Pseudo Random Binary Signals

W Charlton

Recommended Citation
Linear System Identification using Pseudo Random Binary Signals
Linear System Identification using Pseudo Random Binary Signals.

W. CHARLTON.

March, 1968
LINEAR SYSTEM IDENTIFICATION USING PSEUDO RANDOM

BINARY SIGNALS.

W. CHARLTON.
SUMMARY

Recently there has been a good deal of interest in the application of correlation methods to the problem of process identification. Various classes of perturbing signal have been investigated and some experimental work has been described in the literature.

This report describes the circuits designed and preliminary tests made to initiate research into process identification using maximum length sequences as the perturbing signal.

The basis for using correlation functions with suitable test signals to obtain system weighting functions is reviewed. The properties of maximum length sequences and the methods of generating them and their delayed versions are outlined.

Experimental results are given for the identification of first and second order linear systems using varying clock pulse periods and code lengths. The effect of background noise on accuracy is also examined.
ACKNOWLEDGEMENTS

The author wishes to acknowledge the support given to this project by Professor C.A.M. Gray.

The circuits were built by Mr. J. Garcia Moll who also contributed many useful ideas and co-operated with the experimental work.
Consider the system shown in Fig. 1, where the input to and output from the system are \( F_i(t) \) and \( F_o(t) \) respectively.

\[ F_i(t) \quad \text{system} \quad F_o(t) \]

The autocorrelation function of the input \( F_i(t) \) is defined as

\[
\phi_{ii}(x) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} F_i(t)F_i(t+x) \, dt \quad \text{.................(1)}
\]

In a related manner, the cross correlation function between the system input and output is defined as

\[
\phi_{io}(x) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} F_i(t)F_o(t+x) \, dt \quad \text{.................(2)}
\]

Suppose the system of Fig. 1 is linear and has a weighting function \( w(t) \).
Then, from a superposition integral
\[ F_0(t) = \int_{-\infty}^{\infty} w(u) F_i(t-u) \, du \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (3) \]
\[ = w(t) * F_i(t) \] in convolution notation

Combining (2) and (3)
\[ \phi_{i0}(x) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} F_i(t) \int_{-\infty}^{\infty} w(u) F_i(t+x-u) \, dt \, du \]
changing the order of integration
\[ \phi_{i0}(x) = \int_{-\infty}^{\infty} w(u) du \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} F_i(t) F_i(t+x-u) \, dt \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (4) \]
But
\[ \phi_{ii}(x-u) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} F_i(t) F_i(t+x-u) \, dt \]
So (4) is
\[ \phi_{i0}(x) = \int_{-\infty}^{\infty} w(u) \phi_{ii}(x-u) \, du \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (5) \]
i.e. \( \phi_{i0}(x) \) is found by convolving the weighting function with the input auto correlation function.

The Fourier Transform of (5) is
\[ \Phi_{i0}(\omega) = \mathcal{F}(w) \Phi_{ii}(\omega) \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (6) \]
\[ \Phi_{ii}(\omega) = \mathcal{F}\{\phi_{ii}(x)\} \] is called the power density spectrum.

If \( \phi_{ii}(x) = K \delta(x) \), an impulse, then the power density spectrum is a constant, \( K \) and the input is termed "white noise" by analogy with white light.
If the input to the linear system is "white noise"
i.e. $\phi_{ii}(t) = K \delta(t)$, then

$$\phi_{io}(x) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} F_i(t) F_o(t-x) dt$$

$$= \int_{-\infty}^{\infty} \omega(u) \phi_{ii}(x-u) du$$

$$= \int_{-\infty}^{\infty} \omega(u) K(x-u) du$$

i.e.

$$\phi_{io}(x) = K \omega(x) \quad \text{(7)}$$

Equation (7) states that if white noise is used as an input to the system, a cross correlation between the input and output of the system will yield a function proportional to the system weighting function.

Next consider the situation shown in Fig. 2 where a noise $n(t)$ is added to the system input.
Suppose the noise enters the system at a point \((3)\) such that the weighting function between \((3)\) and point \((2)\) is \(W_n(t)\).

In this linear system the output \(F_o(t)\) is given by
\[
F_o(t) = \int_{-\infty}^{\infty} w(u)F_i(t-u)\,du + \int_{-\infty}^{\infty} w_n(u)n(t-u)\,du \quad \ldots\ldots(8)
\]

Hence,
\[
\phi_{io}(x) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} F_i(t) \left( \int_{-\infty}^{\infty} w(u)F_i(t+x-u)\,du + \int_{-\infty}^{\infty} w_n(u)n(t+x-u)\,du \right) dt
\]
\[
= \int_{-\infty}^{\infty} w(u)\phi_{ii}(x-u)\,du + \int_{-\infty}^{\infty} w_n(u)du \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} F_i(t)n(t+x-u)\,dt
\]
i.e.
\[
\phi_{io}(x) = \int_{-\infty}^{\infty} w(u)\phi_{ii}(x-u)\,du + \int_{-\infty}^{\infty} w_n(u)\phi_{in}(x-u)\,du \quad \ldots\ldots\ldots\ldots(9)
\]

If \(F_i(t), n(t)\) are independent, then
\[
\phi_{in}(x) = \frac{F_i(x)n(x)}{w(x)} \quad \text{and} \quad \phi_{io}(x) = \int_{-\infty}^{\infty} w(x)\phi_{ii}(x-u)\,du + F_i(t)n(t)\int_{-\infty}^{\infty} w_n(u)\,du \quad \ldots\ldots\ldots\ldots(10)
\]

The last integral term is a constant and if \(F_i(t)\) is white noise, the result is a simple expression.
\[
\phi_{io}(x) = Kw(x) + \text{constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)
\]
The constant will be zero if any of the three terms is zero. In particular, if the mean value of $F_i(t)$ or $n(t)$ is zero, the constant is zero.
SECTION 2. **PSEUDORandom Binary Sequences**

As shown by equation (11) and assuming the constant term is zero, if white noise is used as the perturbing signal, the impulse response (weighting function) could be obtained by cross-correlation.

In practice, white noise is not very satisfactory, principally because of the long averaging times needed to reduce statistical errors.

A desirable characteristic of a suitable test signal is that it have an autocorrelation function that approximates to a delta function. A class of deterministic signals having this property have been found to lead to more accurate results than that readily attainable with white noise. Such signals are said to be pseudorandom. In particular we are concerned with pseudorandom binary sequences, i.e. sequences that can assume one of two discrete amplitudes, usually called +1 and -1 states.

The two types of pseudorandom codes which have been commonly studied are those based on quadratic residues (q.r. codes) and those sequences generated by linear recurrence relationships. This latter type, known as maximal length or in-sequences, are the ones used in this investigation.

Maximum length chain codes may be generated by means of
linear, feedback shift registers. Since they are binary, cyclic codes they have several important attributes. The cross correlation integration is only necessary over a time interval corresponding to an integral number of code lengths. The binary nature of the code enables a delayed version of the signal to be produced relatively easily and relay cross correlation can be employed, thus simplifying this operation.

If the sequence of states in the cycle which make up the code is written as a vector $\mathbf{X}$ (all elements are either +1 or -1) then all other possible delayed versions of the code may also be expressed as vectors $D\mathbf{X}$, $D^2 \mathbf{X}$, ..., $D^n \mathbf{X}$ where $D$ is an operator to effect one unit of delay.

A characteristic of the codes considered is that the set of all delayed versions of the code from an "almost orthogonal" set of vectors.

Suppose $D^0 \mathbf{X} (= \mathbf{X})$, $D \mathbf{X}$, $D^2 \mathbf{X}$ etc. are $N$-vectors where $N = 2^n - 1$, a number whose significance will appear later. Form a matrix whose columns are the set $N$ of vectors i.e.

$$
D = \begin{bmatrix}
D^0 \mathbf{X} & D^1 \mathbf{X} & D^2 \mathbf{X} & \cdots & D^n \mathbf{X}
\end{bmatrix}
$$

The produced $D^\top D$ yields a symmetric matrix.
The "almost orthogonal" term comes from the inner product result
\[
\langle D^i \bar{X}, D^j \bar{X} \rangle = \begin{cases} 
N & i = j \\
-1 & i \neq j 
\end{cases}
\]
i.e. -1 rather than zero as it would be in the truly orthogonal case.

When this property is considered in relation to equation (1), the general form of the auto correlation function will be seen to have a "spike" of unit amplitude at the origin and be \(-1\) elsewhere. Thus these codes, as test signals, have an auto correlation function that approaches the desired "impulse at the origin" type.
SECTION 3  FEEDBACK SHIFT REGISTER

GENERATION OF BINARY SEQUENCES

In an n-stage shift register there will be a possible \(2^n\) different states that the register may assume. If all of these states could be generated in succession, the sequence from any given location in the register would have a period of \(2^n\) digits. But this would include the "all zero" state and for practical reasons this state should not be generated. Hence the largest possible period for a linear, n-stage shift register is \(N = 2^n - 1\).

If a cyclic code from an n-stage shift register has a period \(N = 2^n - 1\) units it is called a maximum length sequence or an m-sequence.

As a matter of interest, methods are known for producing \(2^n\) states by inserting and removing the "all zero" state, but as this only results in a gain of one state in a generally large number, the extra circuitry is not justified.

A feedback shift register connection to generate pseudorandom sequences is shown in Fig. 3.
Fig. 3

$G_1$, $G_2$, $G_3$, perform modulo-two additions and these have truth tables corresponding to the logical EXCLUSIVE-OR. Although Fig 3 shows three modulo-two gates, many m-sequences can be generated with only one gate. Similarly, some m-sequences will require more than three gates. As specific examples, code lengths of $2^n - 1$ with $n=10$, 9, 7 can be generated with one modulo-two gate, but $n=8$ requires three gates for an m-sequence.
The Theory of linear feedback shift registers

Consider a feedback shift register such as in Fig 3. Suppose that the state of the register at some time \( t_1 \) is expressed as an \( n \)-vector \( \overline{c}_1 \), where

\[
\overline{c}_1 = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_n \end{bmatrix}
\]

and the \( S_i \) are either +1 or -1.

Let \( m \) be a modulo-two summation

\[
m = a_1 S_1 (+) a_2 S_2 (+) \ldots a_n S_n
\]

The symbol (+) implies modulo-two addition

i.e.

\[
1 (+) 0 = 1 \quad 0 (+) 1 = 1 \\
1 (+) 1 = 0 \quad 0 (+) 0 = 0
\]

In the sum for \( m \) the \( a_i \) are either 1 or 0.

The next state succeeding \( \overline{c}_1 \) is

\[
\overline{c}_2 = \begin{bmatrix} m \\ S_1 \\ S_2 \\ \vdots \\ S_{n-1} \end{bmatrix}
\]
The relationship indicated can be written in vector-matrix notation,

\[
\begin{bmatrix}
  m \\
  S_1 \\
  S_2 \\
  \vdots \\
  S_{n-1}
\end{bmatrix}
= \begin{bmatrix}
  a_1 & a_2 & \cdots & a_n \\
  1 & 0 & \cdots & \cdots \\
  0 & 1 & \cdots & \cdots \\
  \vdots & \vdots & \ddots & \ddots \\
  0 & 0 & 1 & \cdots \\
  \vdots & \vdots & \vdots & \ddots
\end{bmatrix}
\begin{bmatrix}
  S_1 \\
  S_2 \\
  \vdots \\
  \vdots \\
  S_n
\end{bmatrix}
\]

or, more concisely as

\[
\overline{c}_2 = [T] \overline{c}_1 \quad \ldots \quad (12)
\]

Since the matrix \([T]\) is characteristic of the feedback shift register configuration and defines the relationship between one register state and the next following, it may be called a transition matrix.

Since \(\overline{c}_3 = [T] \overline{c}_2 = [T]^2 \overline{c}_1\),

it is evident that

\[
\overline{c}_k = [T]^{k-1} \overline{c}_1 \quad \ldots \quad (13)
\]

From an equation such as (12) it is seen that \(\overline{c}_1 = [T]^{-1} \overline{c}_2\) and this implies that a condition for each state to have a unique predecessor i.e. for a one-to-one mapping to exist between adjacent states, is that \([T]\) be non-singular.
When \( \det (T) \) is evaluated using modulo-two algebra, we get,
\[
\begin{vmatrix}
1 & 0 & 0 & \cdots & 0 \\
0 & 1 & \cdots & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & 1
\end{vmatrix} = a_n
\]
or \( a_n \neq 0 \) ensures that \( T \) is non-singular.

If \( [T]^p = [I] \) the unit matrix, then
\[
[T]^p \overline{c}_j = \overline{c}_j
\]
and a cycle of length "p" units exists. Further, if \( p = N = 2^n - 1 \), then the sequence is maximal length.

In general, if
\[
\overline{c}_m = [T]^{m-1} \overline{c}_i = \overline{c}_i
\]
i.e.
\[
[T]^k \overline{c}_i = \overline{c}_i \quad \cdots \quad (14)
\]
where \( (m-i) = k \) say, then the code is cyclic, with a period of "k" different states.

As indicated previously, the rules of modulo–two algebra are similar to ordinary algebra except for
\[
1 (+) 1 = 0 \text{ and } -1 = 1
\]
Rewriting equation (14), and remembering modulo-two algebra

\[ ([T]^k(+)[I])\overline{c}_i = 0 \quad \ldots \quad (15) \]

If \( \overline{c}_i \) is not the zero vector, then

\[ \det([T]^k(+)[I]) = 0 \]

i.e., a necessary condition for equation (15) to be satisfied (\( \overline{c}_i \neq \overline{0} \)) is that the determinant of \( ([T]^k(+)[I]) \) be zero. Similarly, a sufficient condition is that

\[ ([T]^k = [I]) \]

A useful aid in finding the possible values of \( k \) is the characteristic polynomial of \([T]\)

Direct evaluation will show that the characteristic polynomial \( p(y) \) is

\[ p(y) = \det([T] (+) y[I]) = y^n(a_1y^{n-1}(+)a_2y^{n-2} \ldots (+)a_n = 0 \]

and so, by the Cayley-Hamilton Theorem

\[ ([T]^n(+a_1[T]^{n-1}(+) \ldots a_n[I]) = [0] \]

and recall that the \( a_i \) are either 1 or 0.

As an example, consider the feedback shift register shown in Fig 4.
For this connection, $n = 4$ and

$$a_1 = a_4 = 1$$
$$a_2 = a_3 = 0$$

Hence, the characteristic polynomial in $[T]$ is

$$[T]^4 (+)[T]^3 (+)[I] = [0] \quad \text{.... (16)}$$

i.e. $$[T]^3 ([T](+) [I]) = [I] \quad \text{.... (17)}$$
Raise (17) to the 4th power
\[
\begin{bmatrix} T \end{bmatrix}^4 \begin{bmatrix} T \end{bmatrix}^4 \begin{bmatrix} I \end{bmatrix} = \begin{bmatrix} I \end{bmatrix} \quad \text{.... (18)}
\]

But from (16)
\[
\begin{bmatrix} T \end{bmatrix}^4 = \begin{bmatrix} T \end{bmatrix}^3 \begin{bmatrix} I \end{bmatrix} \quad \text{.... (19)}
\]

substituting (19) into (18) yields
\[
\begin{bmatrix} T \end{bmatrix}^4 \begin{bmatrix} T \end{bmatrix}^3 \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} I \end{bmatrix} = \begin{bmatrix} I \end{bmatrix}
\]
i.e. \[
\begin{bmatrix} T \end{bmatrix}^5 = \begin{bmatrix} I \end{bmatrix}
\]

For \( n=4 \), \( 2^n-1=15 \), so this defines an m-sequence.

As an extension to the example, assume that the feedback is taken from the second and fourth stages.

Then,
\[
a_1 = a_3 = 0
\]
\[
a_2 = a_4 = 1
\]

and the characteristic equation form is
\[
\begin{bmatrix} T \end{bmatrix}^4 \begin{bmatrix} T \end{bmatrix}^2 \begin{bmatrix} I \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \quad \text{.... (20)}
\]
i.e. \[
\begin{bmatrix} T \end{bmatrix}^2 \begin{bmatrix} T \end{bmatrix}^2 \begin{bmatrix} I \end{bmatrix} = \begin{bmatrix} I \end{bmatrix} \quad \text{.... (21)}
\]

From (20)
\[
\begin{bmatrix} T \end{bmatrix}^4 = \begin{bmatrix} T \end{bmatrix}^3 \begin{bmatrix} I \end{bmatrix} \quad \text{.... (22)}
\]

Putting (22) into (21)
\[
\begin{bmatrix} T \end{bmatrix}^6 = \begin{bmatrix} I \end{bmatrix}
\]
so the maximum length sequence is six for this feedback connection.
It will be found that
\[
\left| \begin{bmatrix} T \end{bmatrix}^3 (+) [I] \right| = 0
\]
and so there exists a possible minor cycle of 3 states.

To check this case, equation (15) is
\[
\left( \begin{bmatrix} T \end{bmatrix}^3 (+) [I] \right) \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix}
\]

Solving these equations for the \( S_i \), we find the possible solutions

\[
\begin{array}{cccc}
S_1 & S_2 & S_3 & S_4 \\
0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 \\
1 & 0 & 1 & 1 \\
\end{array}
\]

Excluding the null state, the three states of the minor cycle are
\[
(S_1 \ S_2 \ S_3 \ S_4) = (0110), (1101), (1011)
\]

This indicates that if the register initial state is (0110) or (1101) or (1011) it will cycle through these three states. But if the initial state is any one of (1111), (0111), (0011), (1001), (1100), (1110) it will cycle through these six states.

Tabulated details of maximum length codes may be found in several references listed in the bibliography. Some of these are repeated here in table 1.
### Table 1.

The existence of maximum length sequences may also be checked from a consideration of the properties of the polynomial in $D$ formed by description of the feedback connections used. This approach is described in the literature.
SECTION 4 CIRCUITS AND EXPERIMENTAL RESULTS

The equipment for identification studies was built to meet a number of restrictions and requirements.

The aim initially was to provide, with a minimum outlay, equipment suitable for laboratory investigations to gain experience with the method. For this reason the weighting function ordinates are obtained sequentially rather than concurrently. This greatly reduces correlation and delay circuitry and is satisfactory for laboratory experiments.

The equipment generally is limited to low frequency operation, but correspondingly has higher noise immunity. The reed relays used in the correlator have operating times of the order one millisecond.

Basically the changes necessary to make the method more suited to practical situations and higher frequencies are only those of adding additional but similar circuitry and replacing relays by electronic switches. Other schemes of correlation could of course be substituted.

The circuitry is solid state, discrete components. While integrated circuit elements are more economical of space and possibly in cost, for local reasons the former was chosen.
Figures 5, 6 and 7 show some of the basic circuits.

A block type diagram of the system is shown in Figure 8. Because weighting function ordinates are obtained in sequence, it is convenient to have a quick and easy method of setting any required delay. This is done by using a second identical code generator and advancing one code by impulses from a telephone dial.

Figure 9 is the circuit scheme for relay correlation and also shows provision for noise addition.
Fig. 5 BASIC BISTABLE
Fig. 6 EXCLUSIVE OR GATE
Fig. 7 OUTPUT STAGE
Fig. 8 CODE GENERATORS & CONTROL DIAGRAM
FIG. 9 CORRELATOR
Systems Tested

The experimental results shown on succeeding pages are concerned with obtaining weighting functions for a first order and a second order system. The work is restricted to single-input, single-output systems, although some tests were made to check interaction effects with multiple inputs derived from a common code generator.

To produce an unwanted signal at the correlator (background noise) a variable frequency sinusoid was used. Although inherently deterministic, the variable phase of this signal at code start introduces a random property to the "noise".

Further details of the graphical results are given on page 35.
FIG. 10

\[ f = 8 \, \text{Hz} \quad N = 63 \]

With preliminary code period

Without " " " "

--- O --- O ---

--- + --- + ---
FIG. 11

\[ f = 8 \text{ Hz} \quad N = 31 \]

With preliminary code period

Without

2 sec.
FIG. 12

$f = 8$ Hz  \hspace{0.5cm} N = 15
FIG. 13

\[ f = 16 \text{ Hz} \quad N = 31 \]
FIG. 15

\[ f = 20, \quad N = 63 \]
First order system with noise.

\[
\frac{\text{r.m.s. noise}}{\text{signal}} = 1 \text{ at correlator.}
\]
\[ f = 20, \quad N = 127 \]

First order system with noise.

\[ \frac{r.m.s. \text{ noise}}{\text{signal}} = 1 \text{ at correlator.} \]
First order system with noise.

\[ \text{r.m.s. noise signal} = 1 \text{ at correlator.} \]

**FIG. 17**

\[ f = 40, \quad N = 127 \]

Noise frequency - Hz.

Spread about mean value of \( w(t) \) ordinates.
FIG. 18

\( f = 40 \text{ Hz} \quad N = 63 \)
FIG. 19

\[ f = 60 \text{ Hz} \quad N = 127 \]
FIG. 20

\[ f = 20 \text{ Hz} \quad N = 127 \]

No preliminary code period.
FIG. 21

\[ f = 20 \quad N = 127 \]

With preliminary code period.
FIG. 22

$f = 30 \quad N = 31$
FIG. 23

\[ f = 40 \quad N = 31 \]
Experimental results - general comments

There are various sources of error in the experimental method.

(i) The basic binary sequence and any modulators which are assumed to change state by step function transitions and in between changes, to behave as zero-order hold devices. In practice, the step transitions take place over a finite time and this causes errors which become more significant as the clock frequency increases.

(ii) Practical circuits to start and stop the test code have initial and final condition errors. These are reduced with multiple code lengths.

(iii) Interference from other signals present may have some effects, depending on amplitude and frequency. Part of the experimental data is concerned with this effect.

(iv) Bias in the auto correlation function and the chain code. This error can be compensated.

(v) Using a delta function approximation to the auto-correlation function. This is related to the bandwidth limitations of the binary test signal and merits the further comment that it can be shown from the power spectrum of the test signal that the majority of the signal power is confined to the frequency range $f/N$ to $f$. 
With 'f' the clock frequency, \( \frac{N}{f} \) is the code period (T).

Provided,

(i) Period T is greater than the system settling time.

(ii) \( w(t) \) does not vary much from a value \( w(x) \) over the time interval 
(\( x - \frac{1}{f} \leq x \leq (x+\frac{1}{f}) \)),

then when the input signal is an m-sequence, equation (5) is 

\[
\phi_{io}(x) = \frac{N}{N-1} \cdot \frac{1}{f} \cdot w(x) - \frac{1}{N-1} \int_{0}^{T} w(x) dx
\]

for \( x \geq \frac{1}{f} \)

and

\[
\phi_{io}(x) = \frac{N}{N-1} \cdot \frac{1}{2f} \cdot w(0) - \frac{1}{N-1} \int_{0}^{T} w(x) dx \bigg|_{x=0}
\]
Comments on experimental results obtained

The results shown in Figures 10 to 14 are for a first order system having a time constant of $\frac{1}{2}$ second. The system is noise-free and the curves have been adjusted for bias.

Initial conditions errors can be greatly reduced by applying the test signal for one code length (period $T$) before correlation starts. The relative effects of initial condition errors are shown in Figures 10 and 11, where it is seen that running a signal period prior to correlation is still much more economical of time than extended periods to eliminate initial condition effects.

Provided reasonable values of $f$ and $N$ are chosen the correlator gives a very good estimate of $w(t)$ in the noise-free situation.

Figures 15 to 17 show the effects of sinusoidal background noise with a random phase angle at start of correlation. The spread in results is that obtained from approximately 20 tests and is roughly the same for all ordinates of $w(t)$. The spread decreases as $N$ is increased ($\propto$ constant). It also appears that a suitable clock frequency can be used to minimise the effects of dominant frequencies in noise. Using a
signal length of greater than one code period (i.e. 2 or more) will reduce errors due to random noise but it correspondingly increases the identification time.

The results of tests with a second order system are shown in Figures 18 to 23. These are all for the noise-free condition and are not adjusted for bias. For this system it was found that initial condition errors in the results were noticable unless the code period exceeded six seconds. As for the first order case, one code period before correlation practically eliminated this error and this practice was followed on all tests.

Figure 23 illustrates the effect when the code period is too short relative to the system settling time.

With multivariable systems it is convenient to be able to derive all required signals from the one generator. This is only feasible if all the required inputs and delayed signals are separated in time phase by at least the system settling time. Several tests - conducted with more than one signal from the p.r. generator as input to the plant. It was found, as predicted, that provided the required separation of phase could be maintained, there was no interference.
5. CONCLUSIONS

The research described here is the preliminary work in devising laboratory equipment for system identification using chain codes as probing signals.

It is evident that suitable combinations of frequency and code length will identify weighting functions well in the absence of noise. The effects of background noise can be reduced by using multiple periods or increasing N. Both of these measures increase the time for identification.

Extension of the single-input to the general multi-input system can be met in some cases by using the same code generator. Large variations in settling times of the component weighting functions could cause difficulties in this scheme. Alternative schemes are described in the literature.
References

A comprehensive bibliography on the topic of pseudo random binary noise has been compiled by Mr. C. A. Stapleton, School of Electrical Engineering, University of N.S.W. The following is a short list of references.

W.D.T. Davies - "Generation and properties of maximum-length sequences" (3 parts)

Control, June, July, August 1966.

P.E.K. Chow and A.C. Davies - "The synthesis of cyclic code generators"

Electronic Engineering, April 1964.

P. Briggs, P. Hammond, M. Hughes, G. Plumb - "Correlation analysis of process dynamics using pseudo random binary test perturbations"


A. Hazlerigg and A. Noton - "Application of cross-correlating equipment to linear system identification"


W. Peterson - "Error correcting codes"

P.A.N. BRIGGS and K.R. GODFREY

"Pseudorandom signals for the dynamic analysis of multivariable systems"


W.D.T. DAVIES

"Using the binary maximum length sequence for the identification of system dynamics"
