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Modelling of influence of matric suction induced by native vegetation on sub-soil improvement

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CHAPTER FOUR

4 NUMERICAL ANALYSIS OF COUPLED FLOW AND DEFORMATION EQUATIONS OF UNSATURATED SOILS CONSIDERING ROOT BASED SUCTION

4.1 GENERAL

In most semi-arid regions, the engineering properties of soils do not conform to conventional soil mechanics rules and patterns because they are not fully saturated. Besides unsaturated soil problems that include consolidation, flow, and shear strength, there are some phenomena not encountered in fully saturated soils such as heave from desiccated clays, swelling and additional settlement due to the grain structure collapsing when wet.

Achieving an exact solution of highly non-linear differential equations for coupled flow and deformation differential equations is often very difficult, so, proposing approximate solutions for the governing equations is the most appropriate method. The finite element method was the first numerical procedure used to solve multi-phase flow problems using approximation methods. The finite element method used in this study provides a systematic procedure for deriving the approximation functions over sub-regions of the domain. This method is introduced as a variational based technique using the philosophy of constructing piecewise approximation of solutions to problems described by the governing differential equations. Full details of the basic steps for this method are given by Zienkiewicz and Taylor (1989) and Reddy (1993).

The main objective of this chapter is to describe the fully coupled flow-deformation model for unsaturated soils within a three dimensional framework that embraces the proposed root water uptake model. The governing equations are developed using the deformation and flow models. The deformation model is based on the continuum mechanics theories of elasticity and plasticity and also the effective stress concept of unsaturated soils. The flow model is based on Darcy's law, Henry's law and the conservation of mass. The deformation and flow models are coupled through the effective stress parameters and the ABAQUS finite element code used to solve the flow and deformation equations considering the root water uptake model is presented.

4.2 EFFECTIVE STRESS EQUATION OF UNSATURATED SOILS

The effective stress principle converts a multi-stress state porous medium to a mechanically equivalent single-stress state continuum, allowing the application of the principles of continuum solid mechanics to deformable porous media filled with fluid. Terzaghi's effective stress principle may be stated as two propositions, (i) changes in volume and shearing strength of a soil are due exclusively to changes in effective stress and (ii) the effective stress σ'_{ij}, in soil is defined as the excess of the total applied stress σ_{ij}, over the pore pressure u. The first proposition is based on the assumption that water is incompressible and cannot sustain shearing stress (i.e. zero shear strength). Figure 4.1 schematically illustrates soil as a porous medium with water, air, and solid phases.

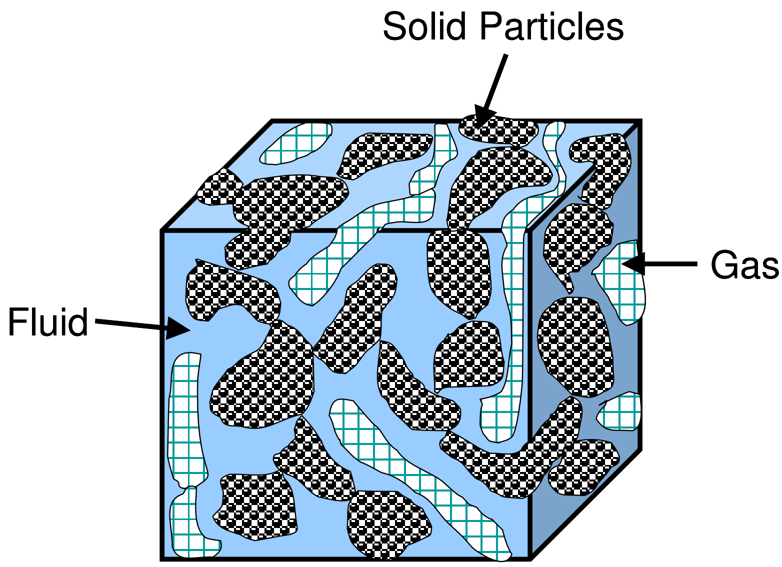

Figure 4.1 An illustration of three phases of porous media

The basic effective stress concept adopted in this study to analyse unsaturated soil behaviour was inspired by Bishop (1959):

$$\sigma'_{ij} = \sigma_{ij} - u_a\delta_{ij} + \chi(u_a - u_w)\delta_{ij} \quad (4.1)$$

where, σ'_{ij} is the effective stress of a point on a solid skeleton, σ_{ij} is the total stress in the porous medium at the point, u_a is the pore air pressure, u_w is the pore water pressure, δ_{ij} is Kronecker's delta ($\delta_{ij} = 1$ when $i = j$ and $\delta_{ij} = 0$ when $i \neq j$), and χ is the effective stress parameter attaining a value of unity for saturated soils and zero for dry soils. In unsaturated soil mechanics, the term $(u_a - u_w)$ is usually called the matric suction. The quantity $\lfloor \chi u_w\delta_{ij} + u_a\delta_{ij} - \chi u_a\delta_{ij} \rfloor$ may be considered as an equivalent pore pressure, which is a portion of the effective stress in a soil resulting from fluid pressure in the pores.

Bishop and Donald (1961) performed triaxial tests on partly saturated silt to verify the validity of Equation (4.1). In this test, the cell pressure (σ_3), the pore-water pressure (u_w), and the pore-air pressure (u_a) were varied during the shearing process such that both ($\sigma_3 - u_a$) and ($u_a - u_w$) remained constant throughout. These variations had no effect on the stress/strain curve. The validity of Bishop's effective stress concept for predicting the shear strength and change in volume of unsaturated soils has recently been confirmed by Khalili et al. (2004) and Lu and Griffiths (2004). The two main arguments, which are often cited in literature, are that the effective stress theory in unsaturated soils cannot explain the collapse that occurs in unsaturated soils and that there is no unique relationship between χ and the degree of saturation. Loret and Khalili (2000) and Khalili and Loret (2001) have shown that plastic deformation such as collapse can be readily described within the effective stress framework by defining the yield surface as a function of suction. As discussed by Khalili et al. (2004), collapse and dilation even in saturated soils cannot be explained in terms of effective stress alone without invoking an appropriate plasticity model. Although much criticism of Equation (4.1) has been made because of the uncertainty of the value of χ, which depends on a number of factors such as degree of saturation, soil type and hysteresis effects, Khalili and Khabbaz (1998) showed that by plotting the value of χ against a more appropriate parameter such as ratio of matric suction over the air entry (suction ratio), a unique relationship may be obtained for most soils, which have been used in this study.

$$\chi = \begin{cases} \left(\dfrac{\psi_e}{\psi}\right)^{0.55} & for \quad \psi \geq \psi_e \\ 1 & for \quad \psi < \psi_e \end{cases} \tag{4.2}$$

where, ψ is the matric suction and ψ_e is the matric suction value marking the transition between saturation and unsaturated states. For wetting, ψ_e is equal to the air expulsion, ψ_{ex}, whereas for drying, ψ_e is equal to the air entry, ψ_{ae}. The air expulsion and the air entry related to the soil structure can be determined using the soil water characteristic curve (see Figure 4.2).

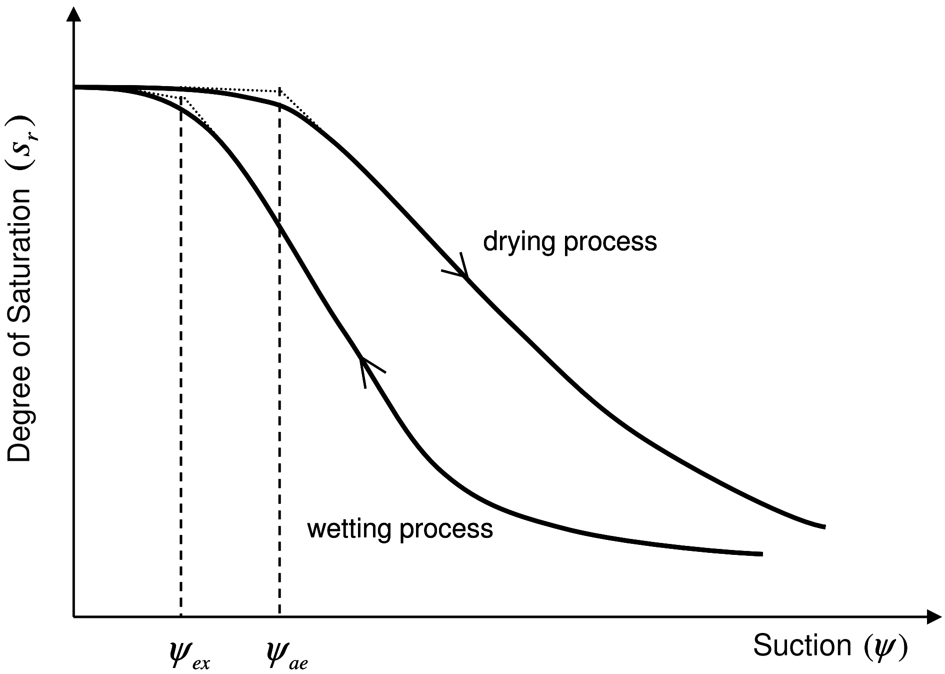

Figure 4.2 Schematic soil water characteristic curve

The principle of effective stress for partly saturated soils is similar to the effective stress theory of saturated soils and takes the form of two propositions, (i) all the measurable effects of a change of stress, such as compression, distortion, and a change in shearing resistance of the soil are exclusively due to changes in effective stress, and (ii) the effective stress (σ'_{ij}) in a partly saturated soil is defined as the excess of total applied stress (σ_{ij}) over the equivalent pore pressure $\lfloor \chi u_w \delta_{ij} - (1-\chi) u_a \delta_{ij} \rfloor$.

Fredlund and Morgenstern (1977) proposed a constitutive model based on the concept of independent stress state variables in which, the total stress tensor and air and water and pressure are considered as independent features. The state of stress model requires a considerable amount of time consuming and costly laboratory testing to identify the parameters. Also, different plasticity models for saturated and unsaturated soils need to be used in the state of stress theory. Consequently, in this study, the effective stress theory (Equation (4.1)) in conjunction with Equation (4.2) suggested by Khalili and Khabbaz (1998) for the value of effective stress parameter is used.

4.3 COUPLED FLOW AND DEFORMATION GOVERNING EQUATIONS

4.3.1 Deformation model

Based on solid mechanics for a non-rotational quasi-static soil element subjected to the body force and total stresses tensor applied on the boundaries, the simple equations of equilibrium can be expressed as,

$$\frac{\partial \sigma_{ij}}{\partial x_j} + F_i = 0 \qquad \text{i,j=1,2,3} \tag{4.3}$$

$$\sigma_{i,j} = \sigma_{j,i} \qquad \text{i,j=1,2,3} \tag{4.4}$$

where, F_i is the body force per unit volume and x_1, x_2, and x_3 are the three orthogonal coordinate directions.

Alternatively the total deformation of the element for the elasto-plastic behaviour of soil skeletons is decomposed into elastic and plastic components by superposition. Therefore,

$$\varepsilon_{ij} = \varepsilon_{ij}{}^{e} + \varepsilon_{ij}{}^{p} \tag{4.5}$$

where, ε_{ij} is the total deformation of the soil skeleton, $\varepsilon_{ij}{}^{e}$ and $\varepsilon_{ij}{}^{p}$ are the elastic and plastic components of the soil skeleton deformation, respectively. In this study, it has been assumed that the creep strain ($\varepsilon_{ij}{}^{cr}$) is negligible.

The constitutive law, which relates the stress to strain components of the soil in the elastic part, can be written based on Lame's constants G and λ, as follows,

$$\sigma'_{ij} = 2G\varepsilon_{ij}{}^{e} + \lambda\varepsilon_{kk}{}^{e}\delta_{ij} \tag{4.6}$$

where, G is the shear modulus, $G = \dfrac{E}{2(1+\upsilon)}$ and $\lambda = \dfrac{E\upsilon}{(1+\upsilon)(1-2\upsilon)}$ and E represents the drained Young's modulus and υ is the drained Poisson's ratio of the soil structure.

Assuming small or infinitesimal strain, strain and displacement is integrated through the six relations identified as,

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{x_j} + \frac{\partial u_j}{x_i}\right) \tag{4.7}$$

Substituting Equations (4.1), (4.6) and (4.7) into Equation (4.3) results in:

$$G\frac{\partial^2 u_i{}^{e}}{\partial x_j \partial x_j} + (\lambda + G)\frac{\partial^2 u_j{}^{e}}{\partial x_i \partial x_j} + \chi\left(\frac{\partial u_w}{\partial x_i} - \frac{\partial u_a}{\partial x_i}\right) + \frac{\partial u_a}{\partial x_i} + F_i = 0 \tag{4.8}$$

which is the differential equation governing the elastic deformation phenomenon in unsaturated soils with incompressible solid grains.

A basic notation in plasticity is the yield surface, which bounds the elastic domain. In fact, if the yield surface is defined it means that the boundary between elastic and plastic behaviour has been defined. The theory of plasticity is based on three basic relationships, yield surface, the hardening or softening rule and the flow rule. The yield criteria can be described as:

$$f(\sigma'_{ij}, \varepsilon^{p}, h) = 0 \tag{4.9}$$

where, h is the hardening variable, which controls the amount of hardening or softening. For example, the modified Cam-Clay yield function can be presented as follows,

$$f(p', q, p'_c) = \frac{q^2}{M^2 p'} + p' - p'_c = 0 \tag{4.10}$$

in which, $p' = -\sum_{i=1}^{3} \frac{\sigma'_{ii}}{3}$ is the mean effective stress, q is the second invariant of stress deviator, and M is the slope of critical state line, and p_c is pre-consolidation pressure, fixing the size of the yield surface.

In the Cam-Clay model, p'_c is the parameter determining and controlling the size of the yield surface. When effective stress increases and goes beyond the point of initial pre-consolidation pressure (p'_{c0}), the yield surface expands and the value of p_c increases. For example, when the effective stress of soil increases because of root based suction and the generated matric suction passes beyond the initial pre-consolidation pressure, the value of p'_c keeps increasing, and therefore, the soil gets stiffer as consolidation settlement is induced by root suction.

Based on the plasticity flow rule, the plastic strain ($\varepsilon_{ij}{}^{p}$) is obtained from:

$$\varepsilon_{ij}{}^{p} = \phi \frac{\partial g}{\partial \sigma'_{ij}} \tag{4.11}$$

where, ϕ is the plastic multiplier, which determines the plastic straining and g is the plastic potential. For example, in the modified Cam-Clay the flow rule can be written as:

$$de = -\lambda d(\ln p') \tag{4.12}$$

in which, λ is the slope of the consolidation curve (logarithmic scale).

4.3.2 Liquid Flow

Liquid flow based on the continuity equation for incompressible fluid is presented earlier in Equation (3.14). By rearranging the equation based on pore water pressure and liquid volume, and using the developed root water uptake model, the liquid flow equation can be written as:

$$\frac{1}{\gamma_w}\nabla.(k\nabla u_w)-\frac{\partial k_z}{\partial z}-G(\beta)F(T_p)f(\psi)=\frac{1}{V}\frac{\partial V_w}{\partial t} \tag{4.13}$$

in which, V is a representative volume of soil element and V_w is the volume of water. As discussed in Chapter 3, the most appropriate function for $f(\psi)$ is Equation (3.41) suggested by Feddes et al. (1978), which is used in our study. Also, the following two equations are suggested (see Chapter 3) for the root density and the potential transpiration factors, respectively;

$$G(\beta)=\frac{\tanh(k_3\beta_{\max}e^{-k_1|z-z_0|-k_2|r-r_0|})}{\int_{V(t)}\tanh(k_3\beta_{\max}e^{-k_1|z-z_0|-k_2|r-r_0|})dV} \tag{4.14}$$

$$F(T_P)=\overline{T}_p\frac{(1+k_4z_{\max}-k_4z)}{\int_{V(t)}G(\beta)(1+k_4z_{\max}-k_4z)dV}\times\frac{e^{-d_g(t/t_f-c_g)^2}-e^{-d_gc_g{}^2}}{1-e^{-d_gc_g{}^2}} \tag{4.15}$$

Influence of growth rate of tree on $z_{\max}$, $r_{\max}$, $\beta_{\max}$, k_1, k_2, z_0 and r_0 is included in the above equations by using Equations (3.23)-(3.25), (3.28), (3.29), (3.30), (3.32) and (3.33), respectively. Meanwhile, the flow equation considering root water uptake for compressible wetting liquid can be presented as:

$$\frac{1}{\gamma_w}\nabla.(k\nabla u_w)-\frac{\partial k_z}{\partial z}-G(\beta)F(T_p)f(\psi)=\frac{1}{V}\frac{\partial V_w}{\partial t}+\phi_w c_f\frac{\partial u_w}{\partial t} \tag{4.16}$$

where, ϕ_w is the volumetric water content and c_f is the compressibility coefficient of fluid. More detailed information on compressible liquid flow equation can be found in Khabbaz (1997).

4.3.3 Gas Flow

Unsaturated soils can be categorised into two main types depending on the degree of saturation and air phase, (I) soil with occluded air bubbles and (II) soil with continuous air phase. The form of air flow may vary for each of these cases. When the degree of saturation is relatively high, air is in an occluded form and its flow takes place as diffusion through the soil. Fick's law can be used to describe this process. At lower degrees of saturation, the air phase is predominantly continuous. This occurs when the soil is at the optimum water content or at the dry side of the optimum water content. In this case, movement of air through the soil matrix will be determined by resistance along pore space boundaries.

For the case of occluded air using Fick's law, the rate of mass transfer of diffusing air across a unit area may be written as (Fredlund and Raharajo, 1993):

$$J_{ai} = D_i \frac{\partial C_a}{\partial x_i} \tag{4.17}$$

where, J_{ai} is the mass rate of air diffusing across a unit area, D_i is the coefficient of diffusion, and C_a represents the concentration of the diffusing air in terms of mass per unit volume of the soil.

The concentration of air with respect to unit volume of the soil V, can be explained as:

$$C_a = \frac{m_a}{V} \tag{4.18}$$

where, m_a is the mass of air in the soil element. Using the definition of air volume in the soil element, it can be written

$$C_a = \frac{m_a}{V_a}(1 - S_r)n \tag{4.19}$$

in which, S_r represents the degree of saturation and n is porosity. Equation (4.19) can be written in terms of air density (ρ_a),

$$C_a = \rho_a(1 - S_r)n \tag{4.20}$$

For isothermal conditions, the density of air can be calculated using the following equation,

$$\rho_a = \frac{\zeta P}{RT} \tag{4.21}$$

where, ζ is the molecular weight of the air mass ($\zeta = 29 kg/kmol$), R represents the universal gas constant ($R = 8.3143 J/mol.K$), $P = u_a + u_{atm}$ is the absolute pressure, in which, u_{atm} is the atmospheric pressure ($u_{atm} = 101.3 kPa$), and T is the absolute temperature in Kelvin.

Substituting Equations (4.20) and (4.21) into Equation (4.17) results in a modified form of Fick's law as suggested by Blight (1971):

$$J_{ai} = -D_i^* \frac{\partial u_a}{\partial x_i} \tag{4.22}$$

in the above, D_i^* is the transmission coefficient for the air phase defined by the following relationship,

$$D_i^* = D_i \frac{\zeta}{RT}(1 - S_r)n \tag{4.23}$$

Using the definition of J_{ai}, an expression for velocity of air flow through soils (v_{ai})

can be written as follows:

$$v_{ai} = -\frac{D_i^*}{\rho_a}\frac{\partial u_a}{\partial x_i} \tag{4.24}$$

For the case of soils with continuous air, the air flow maybe defined as:

$$v_{ai} = -\frac{k_{ai}}{\gamma_w}\frac{\partial u_a}{\partial x_i} \tag{4.25}$$

where, k_{ai} is the permeability coefficient for air. A comparison of Equations (4.24) and (4.25) indicates that for both occluded and continuous air, the air flow through the soil matrix may be described using Darcy's law, but except in the case of occluded air for which, the coefficient of permeability is defined as:

$$k_{ai} = \frac{D_i^* \gamma_w}{\rho_a} \tag{4.26}$$

Now satisfying the conservation of air mass and following the same procedure as for liquid continuity, it can be written:

$$\frac{1}{\gamma_a}\nabla.(k_a \nabla u_a) - \frac{\partial k_a}{\partial z} = \frac{1}{V}\frac{\partial V_a}{\partial t} + \frac{\phi_a}{P}\frac{\partial u_a}{\partial t} \tag{4.27}$$

Henry's law is used to find the mass of dissolved air in water. The volume of air dissolved in volume $nS_r\Delta V$, of water is given by $HnS_r\Delta V$. Therefore, the volumetric air content, or air porosity (ϕ_a) in the soil element can be explained as:

$$\phi_a = (1 - S_r)n + HnS_r \tag{4.28}$$

where, H is Henry's constant ($H \cong 0.02$)

Equations (4.13) and (4.27) are the governing differential equations describing the flow of liquid and gas through a porous medium. As can be seen, there are two equations and four unknowns (i.e. u_w, u_a, V_w, V_a). Therefore, two additional equations are required. The additional equations can be obtained by relating $\frac{\partial V_w}{\partial t}$ and $\frac{\partial V_a}{\partial t}$ to the primary field variables of Equation (4.8), u_w, u_a and u_i.

According to Khabbaz (1997) and Khalili and Valliappan (1996), application of the Betti's reciprocal law yields:

$$\frac{\partial V_w}{V \partial t} = (1 - \chi - \phi_a) c_m \left(\frac{\partial u_w}{\partial t} - \frac{\partial u_a}{\partial t} \right) - \chi \frac{\partial}{\partial t} (\nabla u_i) \tag{4.29}$$

$$\frac{\partial V_a}{V \partial t} = (1 - \chi - \phi_a) c_m \left(\frac{\partial u_a}{\partial t} - \frac{\partial u_w}{\partial t} \right) - (1 - \chi) \frac{\partial}{\partial t} (\nabla u_i) \tag{4.30}$$

where, c_m is the compressibility coefficient of soil structure with respect to the matric suction. Therefore Equations (4.27) and (4.13), incorporating Equations (4.29) and (4.30) as flow equations of liquid and gas in conjunction with deformation equation (Equation (4.8)), form the general set of differential equations governing flow and deformation phenomena in unsaturated porous media. Therefore the following differential equations present the coupled flow and deformation equations considering the root water uptake model using the effective stress theory in the elastic region.

$$G \frac{\partial^2 u_i^{\ e}}{\partial x_j \partial x_j} + (\lambda + G) \frac{\partial^2 u_j^{\ e}}{\partial x_i \partial x_j} + \chi \left(\frac{\partial u_w}{\partial x_i} - \frac{\partial u_a}{\partial x_i} \right) + \frac{\partial u_a}{\partial x_i} + F_i = 0 \tag{4.31}$$

$$\begin{aligned} &\frac{1}{\gamma_w} \nabla . (k \nabla u_w) - \frac{\partial k_z}{\partial z} - G(\beta) F(T_p) f(\psi) = \\ &(1 - \chi - \phi_a) c_m \left(\frac{\partial u_w}{\partial t} - \frac{\partial u_a}{\partial t} \right) - \chi \frac{\partial}{\partial t} (\nabla u_i) \end{aligned} \tag{4.32}$$

$$\frac{1}{\gamma_a}\nabla.(k_a\nabla u_a)-\frac{\partial k_a}{\partial z}=\frac{\phi_a}{P}\frac{\partial u_a}{\partial t}+$$
$$(1-\chi-\phi_a)c_m\left(\frac{\partial u_a}{\partial t}-\frac{\partial u_w}{\partial t}\right)-(1-\chi)\frac{\partial}{\partial t}(\nabla u_i) \tag{4.33}$$

If the soil behaviour is in the plastic region, Equation (4.31) will be replaced by the hardening and flow rule of plasticity (Equation (4.12)) to calculate the plastic strains on the yield surface.

4.4 APPLICATION OF ABAQUS FINITE ELEMENT CODE

There are two common approaches for solving coupled equations. One approach is to solve two sets of equations first and then use the results to solve the last equation. These results in turn are fed back into the first two sets of equations to see what changes happen in the solution. This process continues until succeeding iterations produce negligible changes in the solution obtained. This is the so called staggered approach to the solution of coupled systems of equations. The second approach is to solve the coupled system directly. This direct approach has rapid convergence even in highly non-linear cases.

Although the choice of available finite element or finite difference software used in the field of geotechnical such as FLAC (2D or 3D) and PLAXIS had been considered, for the numerical analysis of the coupled flow deformation equations in this study, ABAQUS 6.5 has been used because it was designed as a flexible tool for multi-purpose finite element analysis and unsaturated soil mechanics. Softwares such as FLAC and PLAXIS are used for the simulation of fully saturated soil behaviour and cannot be used for the modelling of partially saturated soil with the high matric suction values generated by the tree roots. ABAQUS is a general purpose finite element programme for analysing non-linear engineering problems and the coupled pore fluid-stress of partially saturated soil. ABAQUS is using a direct approach to solve differential equations that are highly non-linear. After introducing the time integration operator, the basic equations form the iterative solutions of a time step that are non-symmetric. This lack of symmetry is due to changes in the geometry, dependency of permeability on void ratio, changes in saturation, dependency of the root water uptake model on pore water pressure and time, and the inclusion of fluid gravity load terms in total pore pressure analysis.

An important aspect of ABAQUS is the ability to write subroutines to modify the existing models. User subroutines increase the functionality of several ABAQUS capabilities for which, the usual data input methods alone may be too restrictive. The user subroutines are typically written in FORTRAN and can be included in a model for executing the analysis. As mentioned above, this analysis is highly non-linear mainly because the unsaturated permeability changes with the pore pressure and void ratio, and also the root water uptake is a function of pore pressure.

In this study the coefficient of soil permeability (k) described by Brooks and Corey (1964) is used,

$$k = k_s(e) s_e^{\frac{2+3\lambda}{\lambda}} \tag{4.34}$$

$$S_e = \left[\frac{S_r - (S_r)_{res}}{1 - (S_r)_{res}} \right] \tag{4.35}$$

where, $k_s(e)$ is the saturated coefficient of permeability estimated based on the Kozeny-Carman equation, S_e is the effective degree of saturation, S_r is the degree of saturation, $(S_r)_{res}$ is the residual degree of saturation, and λ is the slope of the soil water characteristic curve on a log-log plot $(i.e. \Delta \log S_e / \Delta \log \psi)$. Kozeny (1927, 1928) and Carman (1938, 1956) developed the following semi-empirical, semi-theoretical formula for predicting the saturated permeability of porous media,

$$k_s(e) = \frac{\gamma}{C_{k-c} \mu S_0{}^2} \frac{e^3}{1+e} \tag{4.36}$$

where, γ is the unit weight of the fluid, μ is viscosity of the fluid, C_{k-c} is Koezy-Carman empirical coefficient, S_0 is specific surface area per unit volume of particles and e is the void ratio of the porous media. The above permeability equations in conjunction with the soil water characteristic curve are included in the ABAQUS input file.

The proposed root water uptake model has been included in the coupled analysis by introducing the sink term ($S(x, y, z, t)$) as a FORTRAN subroutine in the finite element analysis. The main subroutine includes the rate of root water uptake as a moisture flux boundary applied to the top side of all elements within the root zone. In other words, Equation (3.16)) incorporating Equations (4.14), (4.15) and (3.41) has been implemented in the numerical model as boundary flux, which can determine the rate of root water uptake within the root zone at each increment of time.

The stiffness matrix on an element (K) relates the values of the nodal field variables to the load vectors at its nodes. The general information for the construction of the element stiffness matrix of an isoparametric element is a 16×16 matrix in the form of,

$$[K]^e{}_{16\times16} = \begin{bmatrix} [E] & \chi[C] & (1-\chi)[C] \\ \chi[C]^T & -\theta\Delta t[H_1] - a_{11}[M] & a_{12}[M] \\ (1-\chi)[C]^T & a_{21}[M] & -\theta\Delta t[H_2] - a_{22}[M] \end{bmatrix} \tag{4.37}$$

in which, $[E]_{8\times8}$ is the conventional stiffness matrix relating stress to strain problems, θ is the time weight factor changing between 0 and 1, which depends on the method of iteration, the matrices $[H_1]_{4\times4}$ and $[H_2]_{4\times4}$ represent material properties used in study steady state flow equation, which is function of root water uptake rate, $[C]_{8\times4}$ provides coupling between flow and deformation as a function of root water uptake rate, $[M]_{4\times4}$ is the conventional mass matrix, a_{11} and a_{22} are the apparent compressibility of water and air, respectively, and a_{12} and a_{21} are coupling factors relating microscopic pore water and pore air volumetric deformations calculated from Equations (4.30) and (4.31). If the liquid and grains are assumed incompressible, apparent compressibility and coupling factors can be estimated as follows,

$$a_{11} = a_{12} = a_{21} = c_m(1-\phi_a) - \chi c_m \tag{4.38}$$

$$a_{22} = \frac{\phi_a}{P} + (1-\chi-\phi_a)c_m \tag{4.39}$$

4.5 FLOWCHART OF NUMERICAL ANALYSIS

AQAQUS has the ability to conduct the analysis of porous media in two stages;

- *Geostatic*- for checking and modifying the initial conditions defined for the steady-state equilibrium of ground under geostatic loading: This stage is used to ensure that analysis begins from a state of equilibrium under geostatic loading.
- *Consolidation*- for transient response analysis of partially saturated soil under transpiration: To avoid non-physical oscillations and also convergence problems caused by non-linearities; this stage includes a time-dependent analysis using time intervals for the analysis period with continuous root water uptake.

The Integration procedure used in ABAQUS for consolidation analysis introduces a relationship between the minimum useable time increment and the element size as follows,

$$\Delta t > \frac{\gamma_w e_0 (1+\beta v_w)}{(1+e_0)6k_s k} \frac{ds_r}{du_w} (\Delta l)^2 \tag{4.40}$$

where, v_w is the magnitude of the velocity of the pore fluid and Δl is a typical element dimension. If time increments smaller than these values are used, spurious oscillations may appear in the solution. If the problem requires analysis in smaller time increments, a finer mesh is required. Generally, there is an upper limit on the time step except accuracy, since the integration procedure is unconditionally stable unless non-linearities cause convergence problems.

Figure 4.3 shows the flow chart to solve the coupled flow-deformation governing equations used in this study considering the developed root water uptake model. In this study, the porous media is modelled by attaching the finite element mesh to the solid phase and then fluid can flow through this mesh. The FORTRAN root water uptake subroutine employed in one of the case study analysis has been presented in Appendix A.

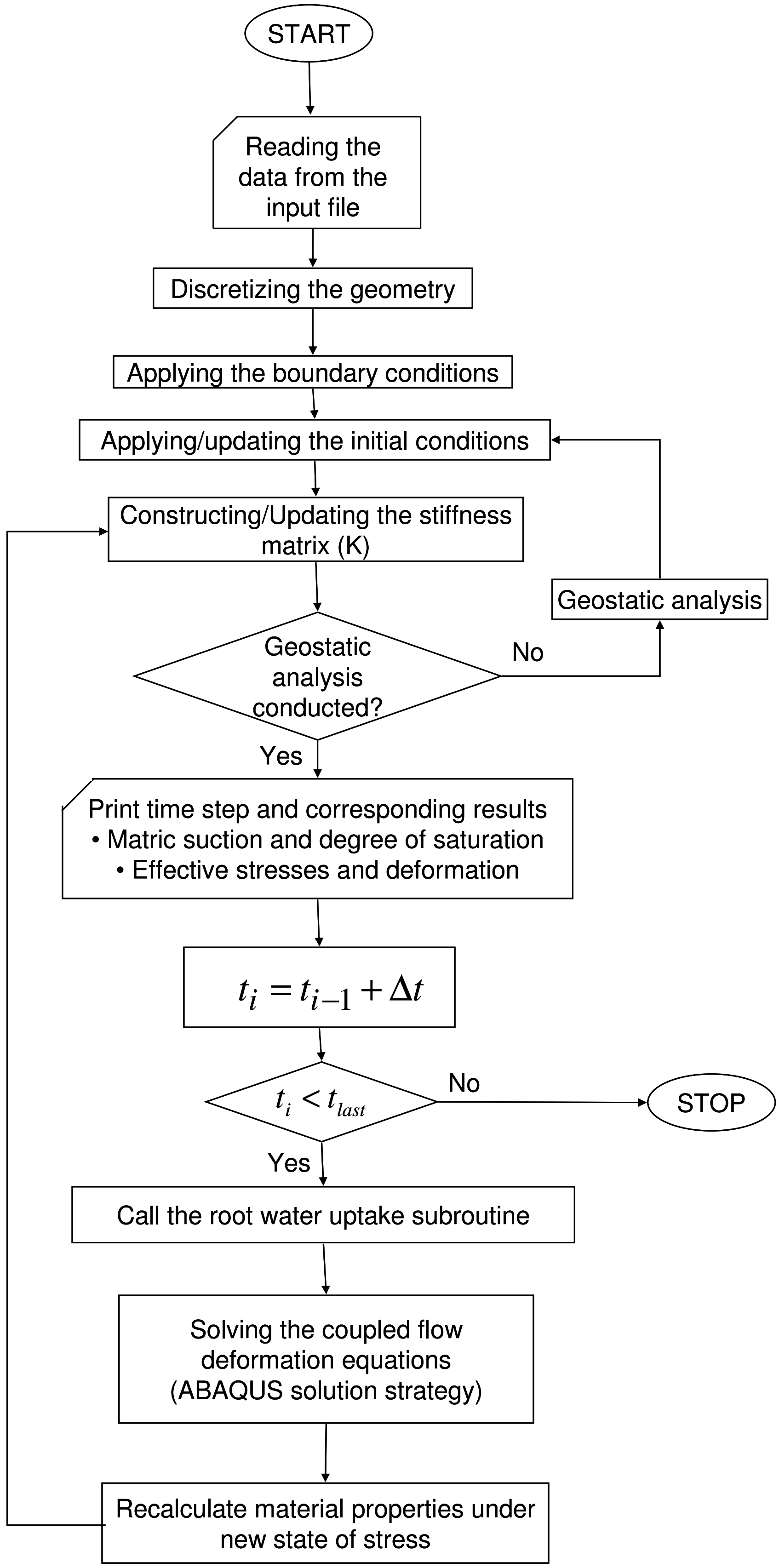

Figure 4.3 Flow chart of approximate solution of coupled-flow deformation governing equation

4.6 SUMMARY

In this chapter, a more rigorous treatment of the theory of coupled flow and deformation has been presented for unsaturated soils based on the effective stress concept considering root based suction. Furthermore, Darcy's law and conservation of fluid mass to define fluid flow in water and air phases in conjunction with the conservation of solid mass to couple flow and deformation have been used. Then the ABAQUS finite element code used to solve the governing equation has been described. The effects of root density distribution, soil suction, and potential transpiration have been incorporated in the flow equation using a sink term (see Equation (4.32)). The root water uptake model has been considered in the finite element analysis using the flow subroutines. These flow subroutines allow you to apply the sink term on any nodes or elements of the model to simulate the discharge of moisture from the soil medium due to evapotranspiration. It has been shown that by including the root water uptake model in the analysis, the stiffness matrices of elements and the whole model change. These changes stem from a variation of flow and deformation and coupling parameters as functions of the root based suction. By conducting finite element analysis at any time step, displacement, pore pressure, and boundary reactions are obtained. As the soil properties depend on their state of stress, the stiffness matrix is recalculated at the end of each time step with new material properties. An application of the theory of elasticity to define deformation matrix has been presented in this chapter in detail.