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DEVELOPMENT OF EMPIRICAL RELATIONSHIPS FOR METALLURGICAL DESIGN OF HOT-ROLLED STEEL PRODUCTS

Tim Marchant*, Alysha Nickerson†, David Scott‡ and Steve Taylor§

Abstract

An empirical model is developed to predict the mechanical properties that steel inherits from the hot-rolling process and its chemical composition. In this process, slabs of steel are heated and rolled into thinner sheets which are either coiled or cut into plates. The mechanical properties of the coils and plates are sampled and must conform to national and international standards for steel products. The aim here is to use the statistical technique of multiple linear regression to develop relationships between the mechanical properties and the various processing temperatures and concentrations of chemical elements present in the steel. This analytical tool will allow better understanding of the steel making process and the ability to vary input parameters to improve the product. In particular the number of coils and plates which fail mechanical testing may be able to be reduced, with a subsequent fall in production costs.

1. Introduction

In hot rolling of steel, an essential procedure for steel mills, slabs of steel are heated and flattened by rolls to produce plates or coils of steel sheet. The process changes the metallurgical properties of the steel and must be monitored in order to produce steel that conforms to a number of national and international standards specifying mechanical properties.

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NZ Steel Ltd set the Study Group the task of developing a mathematical and statistical model that would allow them to predict mechanical properties from the steel's chemical composition and various rolling process temperatures. Such a model would allow the company to predict properties of products when changes are made to the chemistry or process parameters and thus it could prove to be a useful tool to improve the mechanical properties of existing products, reduce testing failure rates for the coil and plate products and for the development of new products.

The hot rolling process is illustrated in Figures 1, 2 and 3. First, a steel slab is heated in the Reheating Furnace. The hot slab then passes through the Reversing Roughing Mill, a coiling box and then the Finishing Mill for the final rolling. At this point the steel is either cut into plates or it is sent to the Downcoiler to be coiled up. At each step of this process, the temperature of the steel must be kept close to various aim temperatures, and actual temperature measurements are recorded. We describe these temperature variables in Section 3.

The other variables thought to have an effect are the concentrations of various chemical alloying and impurity elements, the gauge (thickness) and width of the steel strip and the coil mass. When coils are cut into smaller coils, the sampling position for mechanical testing changes, and the coil mass indicates if the coil has been cut up.

The study group's work involved a multiple linear regression analysis of the NZ Steel Ltd mill data. There was some discussion on whether or not a physical model could be developed. Indeed, such models have been developed and are discussed in the scientific literature: See Colas [2], Serajzadeh *et al* [6, 7], Zhou [8] and the references therein. These complex models have even led to computer packages that are commercially available. Such a package would require fine tuning for NZ Steel's hot rolling mill so there was some doubt that such a package would be any better than an empirical model obtained by multiple linear regression of the mill's data.

In Section 5 some preliminary use of the linear regression model was made, in order to illustrate its usefulness in reducing mechanical testing failure rates for a particular coil product.

2. Mechanical testing and properties

In this section we briefly discuss the mechanical tests that are used to measure the various properties of the steel that are of interest here. The tension test is a classical one in which a specimen is stretched in one direction until it breaks. The equipment for this type of test is commonly available in engineering laboratories worldwide and the tests have been standardised by professional bodies and international standards organisations. The description below is similar to that found in undergraduate text books such as Dowling [3] or Ashby and Jones [1].

The tensile test is a method for determining the behaviour of a metal under an actual stretch loading. This test provides the elastic limit, elongation, yield point, yield strength, tensile strength, and the reduction in area. When a metal is subjected to a stretch loading, its extension follows a curve similar to that shown in Figure 1. Extension depends linearly on load for smaller values of the load and in this region (the elastic region) the metal is able to return to its original shape when released. When the load reaches a certain point, the yield point, the change of extension relative to the increase in load begins to increase. At this point the metal begins to undergo a plastic deformation. If the metal is released after it moves beyond the yield point then it will not return to its original state.

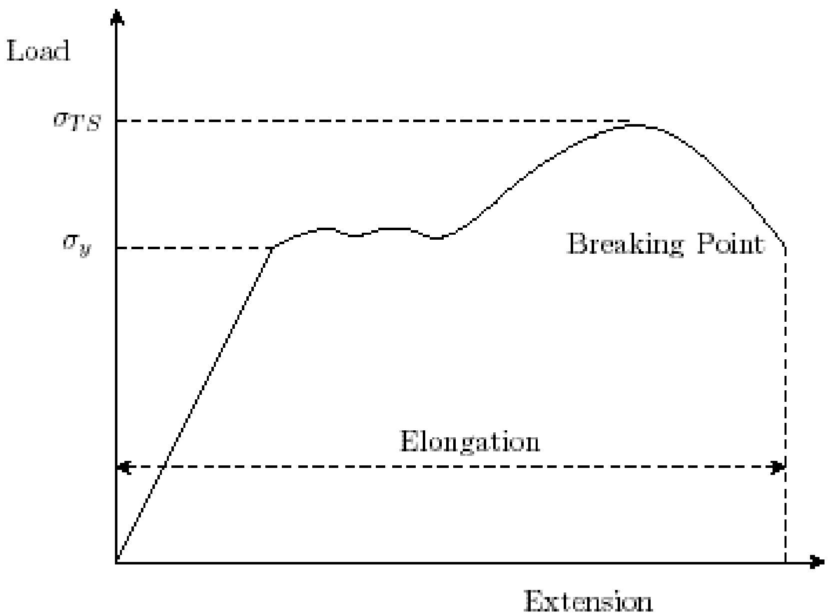

Figure 1. Schematic of a tensile test curve for steel. Yield Strength, YS = σ_y; Ultimate Tensile Strength, UTS = σ_{TS}

The Yield Strength, σ_y, is found by dividing the load at the yield point by the original cross-sectional area of the sample. The Proof Strength is another measure of yielding and it is the stress which produces a permanent strain equal to a certain percentage of the specimen length, eg $\sigma_{0.1}$ is the stress to stretch the sample by 0.1%. The Ultimate Tensile Strength, σ_{TS}, of the sample is obtained by dividing the maximum load by the original cross-sectional area of the sample. Both these quantities are measured in MegaPascals (MPa). The ductility of a metal is a measure of its ability to be stretched without breaking. The ductility is measured by the percentage of elongation, $100\epsilon_f$, of the metal when it is stretched to breaking point. The notation used for the mechanical properties is that of Ashby and Jones [1], see p.84. In the statistical analysis σ_y, σ_{TS}, and $100\epsilon_f$ were coded as YS, UTS, and Elong respectively.

3. Processing variables

The mechanical properties of hot-rolled steel depend on two classes of variables: those which can be measured directly from the final product, such as chemical composition and gauge; and variables associated with the hot-rolling process, such as temperature. The spreadsheet clipping in Figure 2 illustrates the presentation of the parameters.

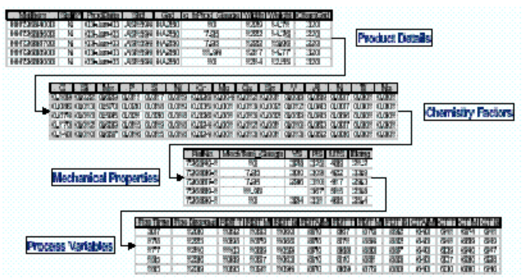

Figure 2. Format of the NZ steel data

NZ Steel Ltd provided data for 26 hot-rolled coil products, including between 100 and 4700+ sets of data for each product, in total around 20,600 rows of data, and 44 columns per row, each row of data representing a single coil or plate produced. Some variables are clustered about aim values (the input chemical concentrations and processing temperatures), while others are continuously distributed within a range (the output mechanical properties). There appears to be considerable scatter in mechanical property data and some processing variable data, and some data is missing.

The hot rolling process is illustrated in Figure 3. The slabs of steel first pass through the *Reheat Furnace*. There are two process variables associated with this. One is the *Residence Time* in minutes (total time spent in furnace), which is denoted Fce Time. The other is the *Dropout Temperature*, denoted Fce Dropout. This is a single value measured on slab discharge. It is affected by surface scale. The steel slab then passes through a hydraulic scale breaker, in which high-pressure water sprays are directed at the slab surface to remove the oxide scale formed in the furnace.

Next the slab passes through the *Roughing Mill*. The exit temperature, the *Roughing Delivery Temperature*, RDT, of slabs is measured in three places: The *front*, *middle* and *rear* of bar. These locations are sometimes called the "top", "middle" and "bottom" respectively, and the associated temperature measurements correspond to the three variables: RDT_T, RDT_M, RDT_B.

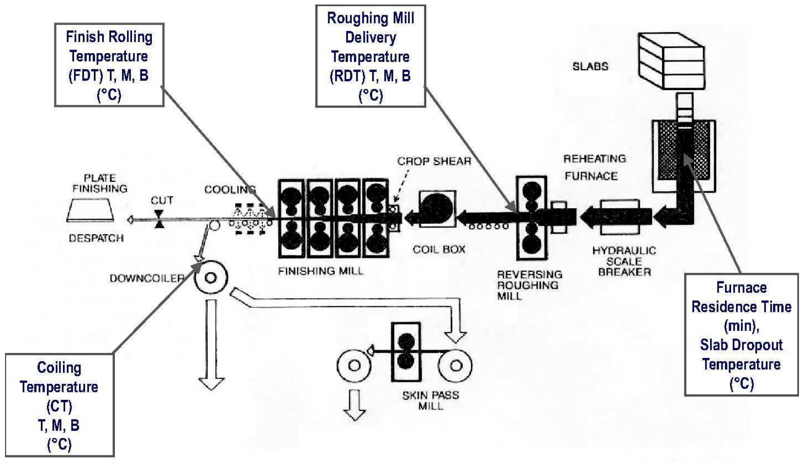

Figure 3. The hot-rolling process.

The steel strip is then coiled up in a coil box, which helps to retain heat in the strip and homogenise temperature prior to final rolling. The steel then passes through the *Finishing Mill* where the exit temperature, the *Finisher Dispatch Temperature*, FDT, is measured at the front, middle and rear of the bar again. The front and rear are interchanged relative to the RDT readings because of the coil box. The variables for these temperature readings are FDT_T, FDT_M, and FDT_B. The steel strip then passes under cooling water sprays on the Run-Out Table, which cool the strip from the finish rolling temperature to a temperature suitable for coiling. For coil products, the steel is finally coiled in the *Downcoiler*. The temperature of the steel is measured before it reaches the downcoiler, again at the front, middle and rear positions, giving variables CT_T, CT_M and CT_B.

We should note that there is another discrete variable associated with the process. The Split variable takes on the value "Y" or "N", depending on whether the steel coil is split up into smaller coils or not. The decision to split the steel depends on the order requirements, and applies only to strip thicknesses of 6mm or below, which are usually subsequently given a small cold-rolling reduction on the skinpass mill.

The significance of split coils is that the sample for mechanical testing is taken at the location of the split, in the body of the coil, rather than at the end. In addition, skinpassed coils which are not split are sampled for mechanical testing at the "top" or bore end of the coil, whereas non-skinpassed ($>$ 6mm thick) coils are sampled at the "bottom" end or outside end of the coil.

Note that some of the steel passes through the *Plate Line*. This is identical to the process described above, except for the fact that the steel is cut into a number of shorter flat plates instead of being coiled by the downcoiler. NZ Steel provided separate data for the plate line.

4. Statistical modelling

In this section the technique of multiple linear regression is used to analyse the data from NZ Steel Ltd. Most of the analysis was done using the package [5].

Before attempting to build a model, the data was examined to obtain some idea of distributions and relationships between variables, and to check for outliers or possible erroneous values. The relationships between the different mechanical properties were examined using a pairs plot which identified a very strong relationship between Yield and Proof Strength.

The predictor variables were also examined using pairs plots. Related groups of predictors were analysed in this way. Examination of the pairs plots for the concentration of the chemical trace elements (see Figure 4) reveals that within chemical grades, which are distinguished by having different carbon and manganese levels, concentrations of chemical elements appear to be largely independent of one another. Nickel is notable in that there are a number of outliers for this element. Roughing mill delivery temperatures at different locations were observed to be closely related according to a pairs plot of FDT_T, FDT_M and FDT_B. Likewise for the finisher delivery temperatures shown in Figure 5. This last plot is interesting in that for most of the data, there seems to be little difference between achieved temperatures even though aim temperatures are different. There is an isolated group in the finisher delivery temperature plots which was identified as a subgroup of the 120 grade steels. For the coil temperatures the pairs plot showed that the aim temperatures relate quite strongly to the achieved temperatures. This was observed in a multicolour version of Figure 6. For a fixed aim temperature the temperatures at the different locations were apparently independent as shown in Figure 7. In analysing the different groups of variables multicolour pairs plots are a very useful tool. With a data set of this size however they can require considerable amounts of computer memory and take time to display. Reproduction of them can produce very large files also so we have limited the number of plots in this document.

The Study Group analysed NZ Steel Ltd's data using multiple linear regression. Response and predictor variables were chosen after discussion with the problem proposers, bearing in mind the descriptive analysis. The responses chosen were σ_y, σ_{TS}, and $100\epsilon_f$. The predictors used were the chemical composition variables C, Si, P, S, Mn, Al, V, and N, finish rolling and coiling temperatures, the gauge, the aimed for finisher delivery temperature (FDTAIM), and aimed for coil temperature (CTAIM). Gauge, FDTAIM and CTAIM take discrete values and were included in the model as factors. The measured finish rolling and coiling temperatures used in the analysis were selected using a rule prescribed by NZ Steel Ltd: If the gauge was greater than 6 mm the bottom temperature was used, otherwise if the steel had been split the middle temperature was used, but if it had not been split the top temperature was used. Recall that steels of gauge up to 6 mm are skin passed.

A cautious approach was used by beginning with separate analyses for the different grades of steel which are distinguished principally by having different concentrations of carbon and manganese. It was initially thought that individual analyses might produce models with less residual variation and which thus might be more useful. In addition, model fitting on a subset of the data would be quicker and less complicated. In fact, the models obtained for the full data set produced residual standard errors which were quite similar to those obtained using separate models for different steel grades. Having one model for all steel grades is useful when it comes to using the model for controlling the steel making process. For these reasons it was decided to use a model based on the total data set.

In the modelling process no model fitting was attempted since eliminating variables using t-tests or F-tests is known to produce biases (see,

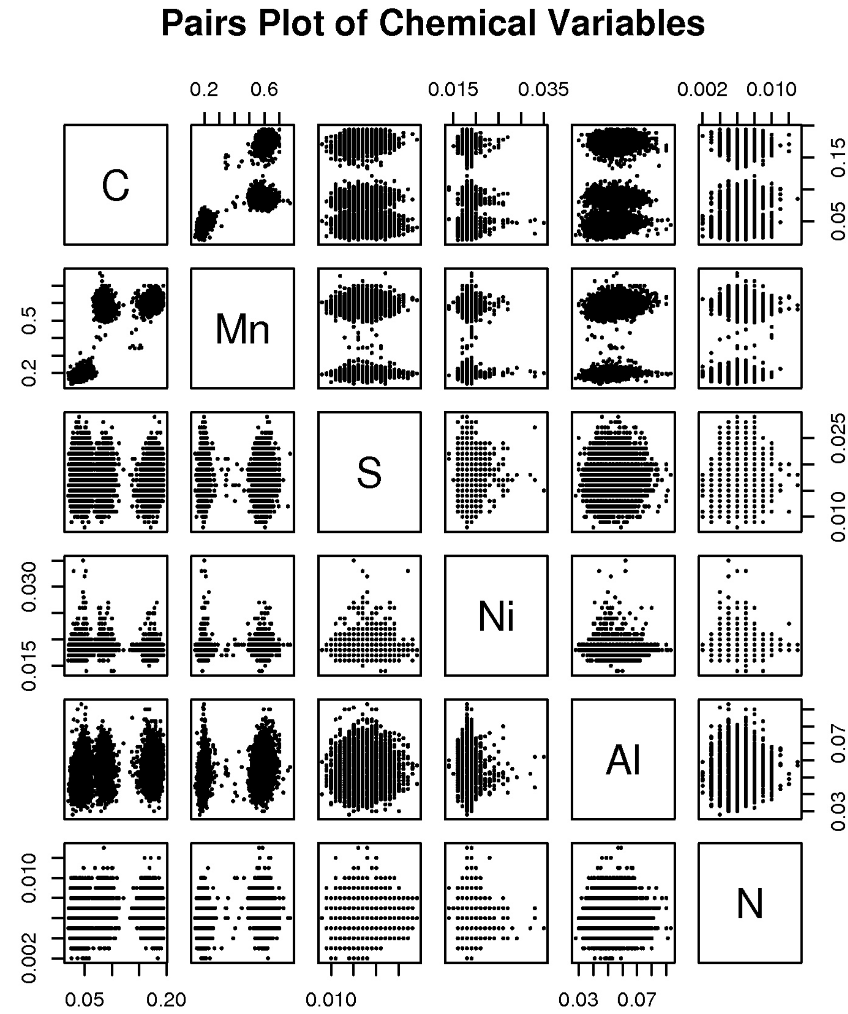

Figure 4. Chemical Properties

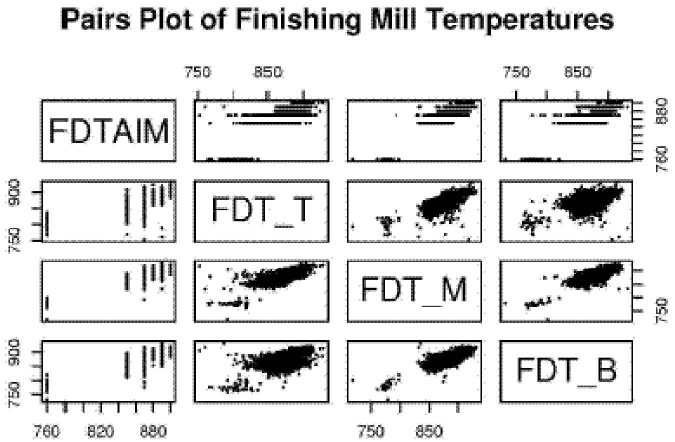

Figure 5. Finisher delivery temperatures

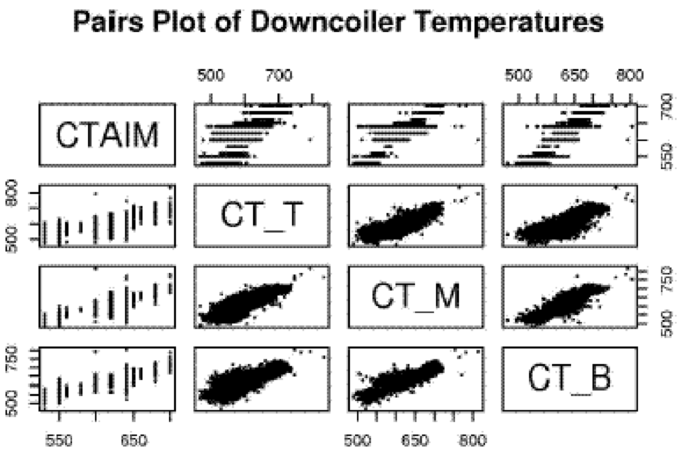

Figure 6. Downcoiler temperatures

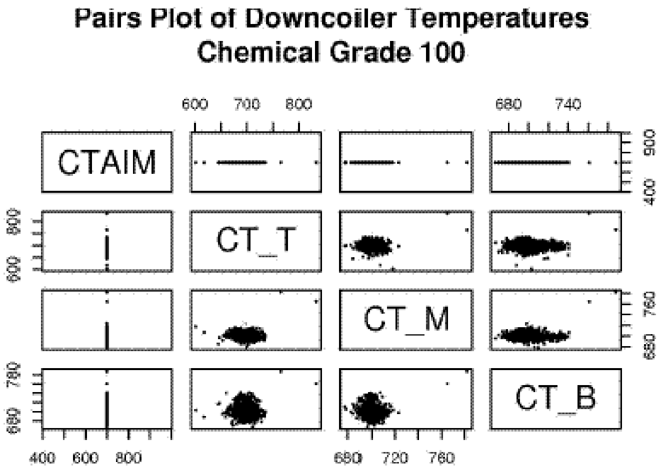

Figure 7. Downcoiler temperatures for 100 grade of steel

for example, Harrell [4]). Diagnostic checks were performed on the models obtained and showed that there were no serious violations of the regression assumptions. Since there were some fairly obvious outliers to be seen in the descriptive analysis, models were also fitted where the most egregious outliers had been removed. The models obtained differed little from those where all observations had been included. Since removal of observations needs to be justified and the models with outliers appeared to be satisfactory, no outliers were removed from the models which were eventually chosen as appropriate.

The code used for fitting the models and carrying out diagnostic checks is given in the Appendix. Full results from the Yield Strength fit are also given, for reference. The resulting models have many terms because of the fitting of factors for the Gauge, and aim temperatures. Ignoring these factors, which in any case cannot be varied in the manufacturing process, the multiple linear regression models for the mechanical properties are

$$\begin{aligned} \sigma_y &= 462\mathsf{C} - 84.9\mathsf{Si} + 434\mathsf{P} + 227\mathsf{S} + 49.7\mathsf{Mn} - 94.7\mathsf{Al} \\ &\quad +1136\mathsf{V} + 2451\mathsf{N} - 0.143\mathsf{FDT} - 0.115\mathsf{CT}, \\ \sigma_{TS} &= 766\mathsf{C} - 96.4\mathsf{Si} + 738\mathsf{P} - 206\mathsf{S} + 59.3\mathsf{Mn} - 114\mathsf{Al} \qquad (1) \\ &\quad +671\mathsf{V} + 2474\mathsf{N} - 4.69 \times 10^{-2}\mathsf{FDT} - 0.141\mathsf{CT}, \\ 100\epsilon_f &= -41.0\mathsf{C} - 5.14\mathsf{Si} - 27.1\mathsf{P} - 39.6\mathsf{S} - 3.83\mathsf{Mn} + 16.4\mathsf{Al} \end{aligned}$$

$$-41.4\mathsf{V} - 230\mathsf{N} - 3.80 \times 10^{-3}\mathsf{FDT} + 3.70 \times 10^{-2}\mathsf{CT},$$

where C, Si, P, S, Mn, Al, V, and N are the concentrations of Carbon, Silicon, Phosphorous, Sulphur, Manganese, Aluminium, Vanadium and Nitrogen respectively while FDT and CT are the Finish Rolling Temperature and the Coiling Temperature, respectively. Note also that the units of all chemical concentrations are % by weight and all temperatures are in degrees Celsius.

5. Reducing mechanical test coil and plate failures

After hot-rolling, the coil and plate products are tested for mechanical properties such as Yield Strength (σ_y), Ultimate Tensile Strength (σ_{TS}) (both in MPa) and the percentage of elongation ($100\epsilon_f$). Typically, to pass, test values of these properties must be above a certain minimum value. Up to three Retests on a new sample from the coil are performed if the initial test fails and the ultimate failure rate is 0.2%. However, the cost of failure is high as each coil is worth around US$12K. The analytical model (1) gives simple linear relationships between the mechanical properties and the chemical concentrations and processing temperatures. We wish to use these relationships to improve the mechanical properties of the hot-rolled product and to reduce testing failure rates.

As an example we consider the hot-rolled coil products made from the 110 grade of steel. There are three products, called HA250 (2m-3mm gauge only), MP1 and MP35CC. This example is chosen as the failure rate for the HA250 coils is higher than for other types of coils.

For the 110 grade of steel the three chemical elements have an allowed concentration range. They are $\mathrm{C} = (0.035, 0.05, 0.07)$, $\mathrm{Mn} = (0.14, 0.2, 0.25)$ and $\mathrm{Al} = (0.03, 0.05, 0.08)$, where the middle concentration in each range is the desired aim value. Table 1 shows the allowed ranges for the processing temperatures, CT and FDT, for the three hot-rolled coiled products made with this grade of steel. Also presented in this table are the minimum values that each test of the Yield Strength and the Ultimate Tensile Strength must exceed. The percentage of elongation $100\epsilon_f > 16$, for all three coil products.

NZ Steel Ltd supplied the data on over $20,600$ individual hot-rolled coils. Tables 2 and 3 contain the chemical and processing temperature (input) data and mechanical test (ouput) data for all of the coils manufactured with grade 110 steel. Presented are both the mean, μ and standard deviation σ of each parameter. The file contained data on 748, 683, and 770 coils of the HA250, MP1 and MP35CC products, respectively.

	CT	FDT	σ_y	σ_{TS}
HA250	500, 550, 590	810, 850, 890	250	350
MP1	500, 600, 650	830, 870, 910	207	311
MP35CC	500, 600, 650	830, 870, 910	207	331

Table 1. Allowable ranges for the processing temperatures and mechanical test minima for coil products made with 110 grade steel.

	C	Mn	Al	CT	FDT
HA250	0.049, 0.006	0.199, 0.014	0.049, 0.006	551, 7.33	858, 5.94
MP1	0.048, 0.005	0.199, 0.010	0.053, 0.008	601, 4.78	881, 6.54
MP35CC	0.049, 0.005	0.199, 0.009	0.054, 0.008	599, 10.18	880, 7.35

Table 2. Processing temperature and chemical data for NZ 110 grade steel products; mean, μ, and standard deviation, σ, are shown

It is noted that the chemical concentrations and processing temperatures are very close to their desired targets with small standard deviations. As these quantities represent inputs into the process it is not surprising that their standard deviations are so small; typically the upper and lower limits of the tolerance range are about four standard deviations away from the mean. For the mechanical properties, which represent outputs, the standard deviations are much larger. For the HA250 product the mean of σ_y is only about 1.7σ away the from test minimum, while the mean of σ_{TS} is about 2.15σ away from its test requirement. The percentage of elongation $100\epsilon_f$, is about 4.5σ away. For the MP1 and MP35CC products, the mean of σ_y is about 5σ above the test minimum. The reason for the higher failure rate of the HA250 coils is now clear. The mean values of σ_y and σ_{TS} are much closer to the test values than is the case for the other two coil products.

Hence the Yield Strength test for the HA250 product is by far the most likely one to fail. For a normal distribution about 5% of samples will be more than 1.7σ below the mean. Of course retesting reduces the failure rate below 5%, but increasing the mean of σ_y for the HA250

	σ_y	σ_{TS}	$100\epsilon_f$
HA250	284,20.0	378,13.5	30.2,3.10
MP1	285,17.3	380,13.4	29.4,2.39
MP35CC	283,19.2	377,11	29.1,2.74

Table 3. Mechanical properties data for 110 grade steel products; mean, μ, and standard deviation, σ.

coils will reduce the failure rate and eliminate the need for most of the retesting.

Ideally, we want to vary the input parameters to increase the mean of σ_y without decreasing the means of σ_{TS} or $100e_f$ and without moving the input parameters too close to the limits of their tolerance range. By using the analytical model (1), we find that varying the input parameters within the allowable bounds changes $100\epsilon_f$ by 2.5, about one standard deviation. Given that the mean of the percentage of elongation is about 4σ above the test requirement, we can safely ignore the effect of any change in elongation by varying the parameters. Next, we note that the expressions for σ_y and σ_{TS} are qualitatively similar in that the coefficients of each parameter have the same sign in each expression (except for S, which has no tolerance range anyway). This means that increasing σ_y, will also increase σ_{TS}.

Using Tables 1, 2 and 3 it can be seen that the means of the chemical concentrations range between 2.3σ and 5σ of their tolerance limits. For the HA250 product, the temperature processing means, CT and FDT, are between 5σ and 8σ of their tolerance limits. Hence, due to their wide tolerance band, there is some flexibility to adjust CT and FDT within their bands, without breaking any operational processing requirement.

Three standard deviations is taken to be a reasonable buffer between the mean and the edge of the tolerance band; the bands for the chemical concentrations are of this magnitude anyway. Also, for a normal distribution, only about 0.26% of samples will fall outside a tolerance band of the size $(\mu - 3\sigma, \mu + 3\sigma)$.

Considering the tables we can increase the target value of C by 0.0035 to 0.0525 as this is the midpoint of the tolerance band for the 110 grade steel. We can also decrease the target values of CT and FDT by about 30 for the HA250 product. This will increase the mean of σ_y by 9.4, hence $\mu = 293$, about 2.2σ above the test minimum. Hence, the initial failure rate on the Yield Strength test will decrease from about 5% to about 0.8%.

This provides a simple illustration of how the mechanical properties of coil products can be improved to minimise testing failure rates. For coil or plate products made with other grades of steel the task may be more complicated because adjusting the parameters to decrease the failure rate of one of the mechanical tests may increase the failure rate of another. Hence, in general an optimisation problem may need to be solved to achieve the optimal outcome.

6. Conclusions and recommendations

Multiple linear regression on the total data set supplied by NZ steel Ltd is shown to give acceptable results. Analyses were performed which showed that metallurgical properties do indeed depend linearly on the hot–rolling variables. Separate models were developed for each of these properties. For the popular 100–200–300 series of products, the model for Ultimate Tensile Strength had the largest R^2 value of 0.94, Yield Strength was next with a value of 0.78, and Elongation had a value of 0.57.

Commercial packages, based on physical models of this process, do exist but would need to be extensively calibrated to be useful to NZ Steel Ltd. The multiple linear regression model developed here is an excellent alternative to physical modelling. It is simple to develop, use and understand and should give accurate predictions, given it is based on NZ steel mill data.

The usefulness of the regression model was also illustrated, via a simple example. It was shown that the failure rate of the HA250 coil product could be substantially reduced, without adversely affecting other products made with the same grade of steel.

Acknowledgments

The project moderators are Tim Marchant, Alysha Nickerson (student moderator) and Steve Taylor. Tim and Steve wish to acknowledge the importance of Alysha's contribution to the project, which benefited from her dedication, knowledge of statistics and familiarity with the statistical package.

The moderators are very grateful to a small, dedicated, useful team of contributors: Ray Hoare, Lynne McArthur, David Scott and Lyndon Walker. David Scott deserves special attention because his work using formed the back–bone of the analysis.

We thank the New Zealand Steel representatives, Philip Bagshaw, Ben Garside and Mark Rouse, for bringing this project to MISG. Their knowledge and excellent preparation of materials and data for the project were vital for its success.

Appendix: R code and output for the linear regression analysis

```
# Linear models for steel data
#
# David Scott, 24/1/05
#
```

```
date()
options(width=80)
# Clean up after previous analyses
rm(list=ls(all=TRUE))

# Read data in
steel <- read.csv("SteelData.csv")
# Create date variable
steel$ProdDate <- as.Date(as.character(steel$ProdDate),"%d-%b-%y")

# Create gauge as a factor
gaugeBreaks <- c(2,2.5,3,4,5,6,7.99,9.99,11.99,16.05)
steel$GaugeCode <- cut(steel$Gauge,gaugeBreaks,right=FALSE,
                                       include.lowest=TRUE)
steel$FTemp <- ifelse(steel$Gauge>6,steel$FDT_B,
          ifelse(steel$Split=="N",steel$FDT_T,steel$FDT_M))
steel$CTemp <- ifelse(steel$Gauge>6,steel$CT_B,
          ifelse(steel$Split=="N",steel$CT_T,steel$CT_M))

# Consider selected grades
steelcomb <- steel[steel$ChemGrd%in%c(100,110,114,120,124,
                          210,211,217,220,221,223,230,
                          319,320,321,327)&
                         (!row.names(steel)==4705),]
steelcomb$GaugeCode <- factor(steelcomb$GaugeCode,exclude=NULL)


# Fit linear models for YS
YSfit <- lm(YS~GaugeCode+
            factor(FDTAIM)+factor(CTAIM)+
            C+Si+P+S+Mn+Al+V+N+FTemp+CTemp,data=steelcomb)
summary(YSfit)
pdf("Graphs/YScombfit.pdf",height=7,width=11,paper="a4",
                                        horizontal=TRUE)
plot(YSfit)

# Plots of residuals
YSfitRes <- stdres(YSfit)
index <- as.numeric(names(YSfitRes))
dates <- steel$ProdDate[index]

plot(dates,YSfitRes,xlab="Production Date",
```

```
     main="Residuals for fit of YS over time")
     lines(lowess(dates[!is.na(dates)],
     YSfitRes[!is.na(dates)]),col="red")
pred <-steel[,"GaugeCode"]
predictor <- pred[index]
print(summary(predictor))
plot(predictor,YSfitRes,xlab="Gauge",
     main="Residuals for fit of YS")


predVars <- c("C","Si","P",
              "S","Mn","Al","V",
              "N","FTemp","CTemp")
for (i in predVars){
  pred <-steel[,i]
  predictor <- pred[index]
  print(summary(predictor))
  plot(predictor,YSfitRes,xlab=i,
       main="Residuals for fit of YS")
  try(lines(lowess(predictor[!is.na(predictor)],
      YSfitRes[!is.na(predictor)]),col="red"))
}

dev.off()

# Fit linear models for UTS
UTSfit <- lm(UTS~GaugeCode+
   factor(FDTAIM)+factor(CTAIM)+
C+Si+P+S+Mn+Al+V+N+FTemp+CTemp,data=steelcomb)
summary(UTSfit)
pdf("Graphs/UTScombfit.pdf",height=7,width=11,
paper="a4",horizontal=TRUE)
plot(UTSfit)

UTSfitRes <- stdres(UTSfit)
index <- as.numeric(names(UTSfitRes))
dates <- steel$ProdDate[index]

plot(dates,UTSfitRes,xlab="Production Date",
     main="Residuals for fit of UTS over time")
     lines(lowess(dates[!is.na(dates)],
     UTSfitRes[!is.na(dates)]),col="red")
```

```
pred <-steel[,"GaugeCode"]
predictor <- pred[index]
print(summary(predictor))
plot(predictor,UTSfitRes,xlab="Gauge",
     main="Residuals for fit of UTS")

for (i in predVars){
  pred <-steel[,i]
  predictor <- pred[index]
  print(summary(predictor))
  plot(predictor,UTSfitRes,xlab=i,
       main="Residuals for fit of UTS")
  try(lines(lowess(predictor[!is.na(predictor)],
      UTSfitRes[!is.na(predictor)]),col="red"))
}

dev.off()

# Fit linear models for Elong
Elongfit <- lm(Elong~GaugeCode+
         factor(FDTAIM)+factor(CTAIM)+
  C+Si+P+S+Mn+Al+V+N+FTemp+CTemp,data=steelcomb)
summary(Elongfit)
pdf("Graphs/Elongcombfit.pdf",height=7,width=11,
paper="a4",horizontal=TRUE)
plot(Elongfit)

ElongfitRes <- stdres(Elongfit)
index <- as.numeric(names(ElongfitRes))
dates <- steel$ProdDate[index]

plot(dates,ElongfitRes,xlab="Production Date",
     main="Residuals for fit of Elong over time")
     lines(lowess(dates[!is.na(dates)],
     ElongfitRes[!is.na(dates)]),col="red")
pred <-steel[,"GaugeCode"]
predictor <- pred[index]
print(summary(predictor))
plot(predictor,ElongfitRes,xlab="Gauge",
     main="Residuals for fit of Elong")

for (i in predVars){
```

```
  pred <-steel[,i]
  predictor <- pred[index]
  print(summary(predictor))
  plot(predictor,ElongfitRes,xlab=i,
       main="Residuals for fit of Elong")
  try(lines(lowess(predictor[!is.na(predictor)],
     ElongfitRes[!is.na(predictor)]),col="red"))
}

dev.off()
```

The output for the Yield Strength model is presented below. For brevity, the output for Ultimate Tensile Strength and Elongation are not shown.

```
> # Fit linear models for YS
> YSfit <- lm(YS~GaugeCode+
+    factor(FDTAIM)+factor(CTAIM)+
+    C+Si+P+S+Mn+Al+V+N+FTemp+CTemp,data=steelcomb)
> summary(YSfit)

Call:
lm(formula = YS ~ GaugeCode + factor(FDTAIM)
           + factor(CTAIM) +
           C + Si + P + S + Mn + Al + V + N + FTemp
            + CTemp, data = steelcomb)

Residuals:
     Min       1Q   Median       3Q      Max
-89.5543 -12.5577  -0.4007  12.5511 118.6328

Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
(Intercept)           4.343e+02  1.851e+01  23.468  < 2e-16 ***
GaugeCode[2.5,3)     -4.584e+00  8.425e-01  -5.440 5.41e-08 ***
GaugeCode[3,4)       -4.848e+00  8.107e-01  -5.979 2.30e-09 ***
GaugeCode[4,5)       -1.453e+01  9.184e-01 -15.820  < 2e-16 ***
GaugeCode[5,6)       -2.374e+01  9.994e-01 -23.759  < 2e-16 ***
GaugeCode[6,7.99)    -3.095e+01  1.124e+00 -27.530  < 2e-16 ***
GaugeCode[7.99,9.99) -3.085e+01  1.111e+00 -27.760  < 2e-16 ***
GaugeCode[9.99,12)   -3.604e+01  1.250e+00 -28.820  < 2e-16 ***
GaugeCode[12,16.1]   -2.813e+01  1.804e+00 -15.590  < 2e-16 ***
factor(FDTAIM)850     1.206e+01  6.493e+00   1.857  0.06328 .
factor(FDTAIM)870     1.948e+01  6.542e+00   2.977  0.00292 **
```

```
factor(FDTAIM)880     1.390e+01  6.741e+00   2.062  0.03919 *
factor(FDTAIM)890     1.655e+01  6.749e+00   2.452  0.01422 *
factor(FDTAIM)900     1.383e+01  7.328e+00   1.887  0.05912 .
factor(CTAIM)550     -2.536e+01  1.034e+00 -24.518  < 2e-16 ***
factor(CTAIM)560     -2.710e+01  2.495e+00 -10.861  < 2e-16 ***
factor(CTAIM)580     -4.416e+01  4.558e+00  -9.689  < 2e-16 ***
factor(CTAIM)600     -3.340e+01  1.335e+00 -25.027  < 2e-16 ***
factor(CTAIM)620     -2.198e+01  1.622e+00 -13.551  < 2e-16 ***
factor(CTAIM)640     -3.190e+01  1.487e+00 -21.459  < 2e-16 ***
factor(CTAIM)650     -2.330e+01  2.297e+00 -10.143  < 2e-16 ***
factor(CTAIM)660     -2.356e+01  2.233e+00 -10.551  < 2e-16 ***
factor(CTAIM)680     -3.369e+01  2.097e+00 -16.065  < 2e-16 ***
factor(CTAIM)700     -1.480e+01  2.447e+00  -6.048 1.51e-09 ***
C                     4.622e+02  6.544e+00  70.618  < 2e-16 ***
Si                   -8.485e+01  5.062e+01  -1.676  0.09373 .
P                     4.342e+02  5.834e+01   7.443 1.05e-13 ***
S                     2.269e+02  6.304e+01   3.600  0.00032 ***
Mn                    4.968e+01  2.111e+00  23.536  < 2e-16 ***
Al                   -9.473e+01  2.227e+01  -4.254 2.11e-05 ***
V                     1.136e+03  8.192e+01  13.862  < 2e-16 ***
N                     2.451e+03  1.430e+02  17.145  < 2e-16 ***
FTemp                -1.430e-01  2.114e-02  -6.762 1.41e-11 ***
CTemp                -1.152e-01  9.631e-03 -11.963  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 19.3 on 13457 degrees of freedom
Multiple R-Squared: 0.7871,     Adjusted R-squared: 0.7865
F-statistic:  1507 on 33 and 13457 DF,  p-value: < 2.2e-16
```

References

[1] Ashby, M.F., Jones, R.H., *Engineering materials 1, 2nd Ed.*, (Butterworth-Heinemann, Oxford, 1986).

[2] Colas, R., “Mathematical modelling of hot rolling steel strip”, *Materials Science and Technology*, **14 (5)** (1998), 388–393.

[3] Dowling, N.E., *Mechanical behaviour of materials, 2nd Ed.*, (Prentice-Hall, New Jersey, 1999).

[4] Harrell, F.E., *Regression modeling strategies: with applications to linear models, logistic regression, and survival analysis*, (Springer, New York, 2001).

[5] R Development Core Team (2004). “*R*: A language and environment for statistical computing”. *R* Foundation for Statistical Computing, Vienna, Austria.

ISBN 3-900051-07-0, http://www.R-project.org.

[6] Serajzadeh, S., Mirbagheri and H., Taheri, A.K., "Modelling the temperature distribution and microstructural changes during hot rolling of a low carbon steel", *Journal of Materials Processing Technology*, **125–126**, (2002), 89–96.

[7] Serajzadeh, S.,"Prediction of temperature distribution and phase transformation on the run-out table in the process of hot strip rolling", *Applied Mathematical Modelling*, **27** (2003), 861–875.

[8] Zhou, S.X., "An integrated model for hot rolling of steel strips", *Journal of Materials Processing Technology*, **134** (2003), 338–351.