Channel-optimized vector trellis source coding for the AWGN channel

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Channel-Optimised Vector Trellis Source Coding for the AWGN Channel

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Summary: A channel-optimised (joint source and channel) trellis source coder is designed for the AWGN channel. The optimum decoder is a non-linear function of the real channel information. The extension to 2D vector alphabets coupled with modifications to the signal space are found to improve performance. Favourable comparisons are made against a trellis source coder/TCM system.

1 INTRODUCTION

Trellis source coding is well established as an efficient source coding technique. Channel-optimised (CO) (or joint source and channel) trellis coding consists of a jointly designed source encoder and decoder for a given noisy channel. The design of a CO trellis source coder to account for the discrete memoryless channel (DMC) was given by Ayanoglu and Gray [1]. In this work we report on the design and performance of a CO trellis source coder designed for the additive white Gaussian noise (AWGN) channel. The optimum trellis decoder and encoder for this channel is given. Extending the trellis codewords to 2D vectors permits signalling in two dimensions without increasing encoder complexity. Moreover, this allows freedom to move signal points in the I/Q plane. Coding of the Gauss Markov (GM) source \(\rho = 0.9\) is undertaken at 1 bit/sample. It is found that the extension to vector alphabets coupled with optimised signal constellations yields good performance and favourable comparisons are made against a trellis source coder/TCM system.

2 CO TRELLIS SOURCE CODING

A description of the trellis source coder is given in [2, ch. 15] while an advanced treatment can be found in [3, ch. 7].

A trellis source coder, appropriate for the DMC, consists of a finite state machine decoder and matched encoder. The usual decoder, and the one described herein, is a shift register of length \(K\) (the constraint length) that accepts discrete symbols from alphabet \(\mathbb{V}^K\) where \(\alpha\) is the number of bits per time interval, hence the bit rate is \(R = \alpha/\gamma\) bits/sample. The contents of the shift register are used to index into a codebook consisting of the reproduction codewords \(\hat{x}\). Specifically, the decoder groups the last \(K\) channel symbols into a vector \(v\) and forms a reproduction codeword \(\hat{x} = c(v)\). In the case where the codewords are actually vectors (as opposed to scalars) of dimension \(\gamma\), the coder is termed a vector trellis and the codewords are denoted by \(\hat{x} = c(v)\) with bit rate \(R = \alpha/\gamma\) bits/sample.

The encoder minimizes a per-letter additive cost function by searching an encoder trellis. The trellis is a convenient representation of the decoder finite state machine. Both are depicted at a single time instant in Figure 1. In this case the encoding rate is \(R = 1\) bit/sample \((\alpha = 1)\) and \(K = 3\). The Viterbi algorithm [2, ch. 15] provides an optimum search strategy. The algorithm operates with a cost function of the form \(d(x_n|u_e)\) whereby \(x_n\) is the source symbol and \(u_e\) is an encoder index formed from the last \(K\) transmitted symbols. For the squared error distortion

\[d(x_n|u_e) = (x_n - c(u_e))^2\]

Dunham and Gray [4] proposed that the encoder minimize the expected distortion across the channel. Thus the cost function is modified to

\[d(x_n|u_e) = E[(x_n - c(V))^2|u_e]\]

It was proved in [4] that with increasing constraint length a CO trellis source coder approaches the distortion-capacity bound.

3 AWGN CHANNEL OPERATION

To achieve good performance on the AWGN channel consideration must be given to using the real or unquantized information provided by the channel ('soft decision' information in channel coding literature). In [5] a number of systems that used various degrees of quantized real channel information were developed. If the constraint was
given that the decoder should operate at the same information rate as the encoder, it was shown that a detector with feedback could be incorporated into the joint coder design. This gave improved performance over hard decision detection at the cost of increased encoder complexity. Quantizing the channel information with two bits (4-ary decoding, eg. \( V = \{0, 1, 2, 3\} \)) achieved better performance without increased encoder complexity but with a greatly increased codebook memory requirement. It was also determined that the performance of this system was only marginally inferior to the following system which uses unquantized channel information [6].

We begin by giving the optimum decoder that minimizes the mean squared distortion between source vector \( X \) and reproduction vector \( \hat{X} \) given received channel vector \( Z \). The encoder is assumed to transmit index \( u \), consisting of \( K \) \( \alpha \)-bit words \( u_{\alpha} \) with each word mapped to a signal point from the constellation \( \{u_0, \ldots, u_M\} \). The coding rate is \( \frac{\alpha}{\gamma} \) bits/sample where \( \alpha = \log_2 M \) and \( \gamma \) is the source vector dimension. The receiver accepts \( Z_n = U_n + N_n \) where noise \( N_n \) is an independent Gaussian RV with variance \( \sigma_n^2 = N_0/2 \). The decoder forms reproduction vectors with the function \( \hat{X} = c(Z) \) such that \( E[\|X - c(Z)\|^2] \) is minimized. A well known result from estimation theory states that the optimum decoder is given by \( c(z) = E[X|z] \). By summing over the transmitted symbols this is equivalent to

\[
c(z) = \sum_{u_\alpha} E[X|z, u_\alpha] p(z|u_\alpha) P(u_\alpha) / p(z)
\]

The term \( E[X|z, u_\alpha] = E[X|u_\alpha] \) represents the centroid of those source vectors encoded by \( u_\alpha \), \( P(u_\alpha) \) is the probability of their occurrence and

\[
p(z) = \sum_{u_\alpha} p(z|u_\alpha) P(u_\alpha)
\]

Thus the optimum decoder is a non-linear (due to the Gaussian density) function of the received channel information. The decoded value is a scaled sum of centroids. Clearly this function provides a convenient way of avoiding the large codebooks that occur with finely quantized channel information. The downside is the rather high complexity compared to simple lookup tables. Equivalent formulations for the vector quantizer were given by Vaishampayan and Farvardin [7] and Liu et. al. [8]. In [7] the optimum decoder was abandoned in favour of an analytically tractable linear decoder.

The optimum encoder for the given decoder is one that utilizes the Viterbi algorithm to search the encoder trellis with the per-vector cost function

\[
d(x_n|u_\alpha) = E[\|x_n - c(Z)\|^2|u_\alpha] = \|x_n\|^2 - 2x_n^T m_1(u_\alpha) + m_2(u_\alpha)
\]

where

\[
m_1(u_\alpha) = \int_{\mathbb{R}^\alpha} c(z) p(z|u_\alpha) dz
\]

\[
m_2(u_\alpha) = \int_{\mathbb{R}^\alpha} \|c(z)\|^2 p(z|u_\alpha) dz
\]

A major constraint in coder design is the numerical integration over \( \alpha K \) dimensional real space.

In addition to the encoder and decoder design a joint system entails optimisation of the signal space. A target bit rate of 1 bit/sample was set with \( \alpha = \gamma = 2 \); thus the coder signals with \( M = 4 \) signal points. These are optimised under a constant and average energy constraint and compared to QPSK signalling. The high complexity of the decoder function prohibits an analytical solution to the constellation design problem. For the constant energy constellation, the procedure is to determine a locally optimum encoder and decoder (based on a training set) and then to optimise the phase of the signal points with a non-derivative search technique. The process is repeated until convergence. For the average energy constel-
lution design, the phase optimisation is alternately substituted with an amplitude optimisation step which includes a penalty function to ensure the average signal point energy is constrained [5].

4 RESULTS

Performance of the proposed system was evaluated for the GM source. Training and test sequences were of length $10^4 K$ and 16384 samples respectively. A sufficient number of tests were performed to ensure the standard deviation was less than 0.1 dB. The coders were designed for $K=2, 3$ and noise densities of $N_0=0, 0.25, 0.5, 0.75, 1.0, 1.25$ with the average energy per bit $E_b=1.0$.

Numerical results are given in Table 1. The table includes the performance of the CO scalar trellis with antipodal signalling (BPSK). At $K=4,6$ this coder operates with complexity equivalent to the $K=2,3$ vector trellis. The extension to vector alphabets (QPSK) is seen to yield a significant improvement. The vector trellises with constant (4PSK) and average (4QAM) energy optimised constellations provide further improvements. Different gains are achieved with different constraint lengths. However, irrespective of the constraint length, at high noise level designs an overall gain of slightly less than 1 dB was made in going from the scalar trellis with BPSK signalling to the vector trellis with average energy optimised constellation. The distortion-capacity bound is determined by evaluating the distortion-rate function for the GM source [10, eqns 4.5.28, 4.5.29] at the capacity of the Gaussian channel [10, eqn 5.2.39].

Figure 2 illustrates the constant and average energy constellations determined for the $K=2$ coder. For the constant energy case, a departure away from QPSK occurs as the designed noise level increases. For average energy designs, the more frequently used signal vectors $u_0, u_3$ have a reduced energy in contrast to the increased energy of $u_1, u_2$.

Figure 3 illustrates the performance of $K=2, 3$ CO vector trellis coders with estimator decoding and average energy optimised signal constellation. The coder is designed for a channel SNR (10 log10 $E_b/N_0$) of 3 dB. In doing so, the coder sacrifices performance at high SNRs but maintains good performance at low SNRs. Also included is a scalar trellis source coder/TCM combination reported by Soleymani [9] and the optimum PAM bound [10, sec 5.2]. Of course the decoders of the respective systems are quite different with the TCM system requiring a delayed decision Viterbi search while the estimator decoder is without delay. The results indicate that the performance of the low complexity coder (4QAM, $K=2$) is superior to the trellis source coder/TCM ($K=4$) combination, even though the encoder is designed for an SNR of 3 dB. The high complexity coder (4QAM, $K=3$) is slightly inferior to the trellis source coder/TCM combination at high SNRs but again becomes superior as the noise level increases. Both TCM systems breakdown as the noise level increases while the proposed systems degrade gracefully, somewhat in parallel with the optimum PAM curve.

In conclusion, a CO vector trellis source coding system is developed for the AWGN channel. The optimum decoder and encoder is given and, with optimised signal constellations, a favourable comparison is made against a trellis source coder/TCM combination. The scheme described achieves improved performance over the scalar trellis due to at least three reasons. Firstly, the vector trellis achieves a gain by dealing with vectors rather than scalars. The higher dimensional advantage is well known in VQ terms and is applicable to the vector trellis. Secondly, the vector trellis uses twice the number of waveforms thus making better use of channel capacity. Finally, modifications to the signal space permit a relationship between distortion and the distance between codewords.

References

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<td>3</td>
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Table 1: Performance of CO trellis coders with estimator decoders with BPSK (scalar trellis), QPSK, optimised constant energy (4PSK) and optimised average energy (4QAM) signal sets (2D vector trellises); GM source ($\rho = 0.9$), $R = 1$ bit/sample, $E_b = 1.0$.

Figure 2: Constant (4PSK) and average (4QAM) energy constellations; $K = 2$ (drawn to scale)

Figure 3: Robustness of CO $K = 2, 3$ vector trellis coder (4QAM) designed for SNR = 3 dB compared to trellis/TCM [9]; GM source ($\rho = 0.9$), $R = 1$ bit/sample.