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Keywords

wires, coils, modelling, temperature, superconducting, high

Disciplines

Physical Sciences and Mathematics

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MODELLING OF HIGH TEMPERATURE SUPERCONDUCTING WIRES AND COILS

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Abstract

This paper discusses modelling techniques for high temperature superconducting (HTSC) wires and coils. Models are developed based on the behaviour of Bi-2223/Ag wires and coils. For many power-engineering applications it is necessary to form HTSC wires into coils. PSpice models for HTSC coils will be presented in this paper. The aim is to develop models for HTSC wires and coils so that they can be treated as circuit elements in PSpice. The critical current of a coil will in general be different from the wire it is manufactured from and this change in current for a HTSC coil will be modelled, as well as the AC losses in the coil. This model has also been verified experimentally and it will be shown that the models agree well with the measurement results.

1. INTRODUCTION

This paper discusses behaviour modelling techniques for HTSC wires and coils. Models are developed based on behaviour of Bi-2223/Ag wires and coils. The models were developed using the PSpice software package [1]. The aim is to develop models for HTSC wires and coils so that they can be treated as circuit elements in PSpice.

Firstly, the development of a comprehensive model of HTSC wire using Pspice will be presented in Section 2. Then in Section 3 PSpice models for HTSC coils will be presented. These models cover the DC critical current and the AC loss of pancake coils. Verification of the models is presented in Section 4.

2. BEHAVIOUR MODEL OF HTSC WIRE

The analog behavioural modelling capability of PSpice provides an easy way to include function blocks in a circuit without having to create a circuit that implements the function. Simple math functions like addition, subtraction, multiplication, and division of waveforms can be implemented as well as more complicated functions including log, sin, exp, gain, and absolute value. This enables the simulation of more complicated circuits by reducing the component count.

In the previous work [2] the model of HTSC wires has been described. The model of the bending properties of HTSC wires is constructed as the following. First measurements of the critical currents of three samples are made as a function of bending diameters. The plots are then reconstructed to provide the critical currents of the samples as functions of the bending diameters. These are then fitted using fourth order polynomial expressions. The polynomial fit of the sample having the average critical current is then recorded in the PSpice model using a polynomial VCVS (voltage controlled voltage source) and a voltage source. The input of the model is the bending diameter, D_b and the output is the altered critical current, $I_c(D_b)$ as can be seen in Fig. 1.

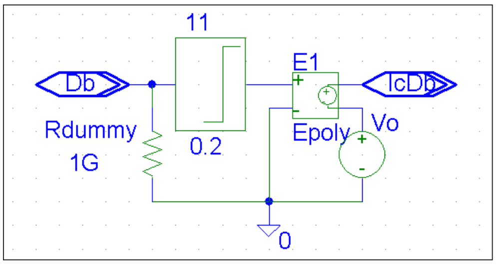

Figure 1. Model of bending properties of HTSC wire.

The model for the field properties of HTSC wires is built as follows: The dependency of magnetic fields on the critical current of the sample is experimentally measured. Different field magnitudes are exposed perpendicularly to the sample and the critical currents are recorded. The normalised critical current, $I_c(B)/I_c(0)$ is then fitted using the Kim [3] approximation and the value of B_0 is obtained from the curve fitting by observation. For any fields exposed to the sample in any direction α the perpendicular components are obtained by multiplying their strengths with the sine of the directions. The PSpice model can then be constructed using the magnitude and orientation of the fields as inputs and the output is the normalised critical current as shown in Fig. 2.

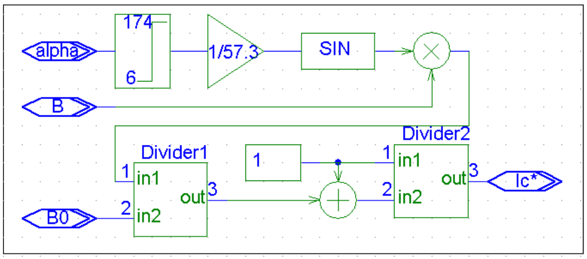

Figure 2. PSpice model of the field properties of a HTSC wire.

In Fig. 2 the inputs of the model are `B`, `alpha` and `B0`, where `B` and `alpha` are the magnitude and orientation of the field respectively and `B0` is the field parameter of the wire. The gain of `1/57.3` in the block diagram is used to convert the orientation of the field from degrees to radians. The input `alpha` is fed into a limiter because the disorientation of the grains in the HTSC wire is about 6° and therefore the value of `alpha` must be limited to the region of 6° to 174° [4]. The perpendicular component of the field is obtained by multiplying its magnitude with the sine of the field's orientation. The new value of normalised critical current I_c* altered by the field appears as the output `Ic*`.

The model description to provide the *I-V* characteristic is shown in Fig. 3 with the current *I* with a given parameter $I_c(D_b,B)$ as the input and the output is the voltage *V* across the wire.

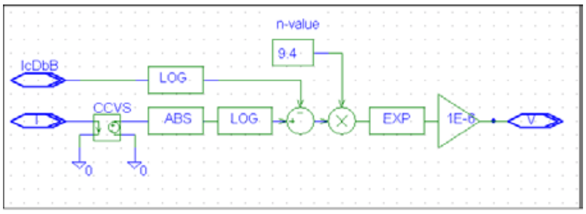

Figure 3. PSpice model for *I-V* characteristics of HTSC wire.

A model of the wire incorporating both bending properties and field properties to provide the *I-V* characteristic of the wire is given in Fig. 4. The block `IIcDb2V` is the model description to provide the *I-V* characteristic as shown in Fig. 3. The block `Db2IcDb` is the model description to incorporate the bending properties as in Fig. 1 and the block `B2Ic*` represents the field properties of the wire as in Fig. 2. The multiplier is used to get the critical current of the wire altered by the bending and field, as $I_c(D_b,B)=I_c(D_b)\times I_c{}^*$ with $I_c{}^*=I_c(B)/I_c(0)$. The current sources `I1` to `I4` and the resistors `R1` to `R4` are used in conjunction with the values of `@Db`, `@alpha`, `@B` and `@B0` to model the input parameters D_b, α, $|B|$ and B_0 respectively.

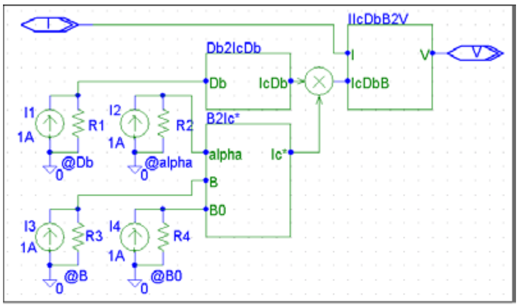

Figure 4. PSpice model of HTSC wire to provide *I-V* characteristic.

The PSpice model in Fig. 4 can be simplified as a block with an input *I* and an output *V* with input parameters `Db`, `alpha`, `B` and `B0` as can be seen in Fig. 5. The block `HTSC Wire` represents a model of a HTSC wire. A current source named `Source` is used as the input of the simulation and the output can be read from the terminal `out`. Parametric simulation can be done in PSpice and therefore the parameter `{Db}`, `{B}` and `{alpha}` can be varied. Their default values are set at 10 cm, 10 mT and 90 degrees respectively. The parameter of the wire B_0 is set at 20 mT.

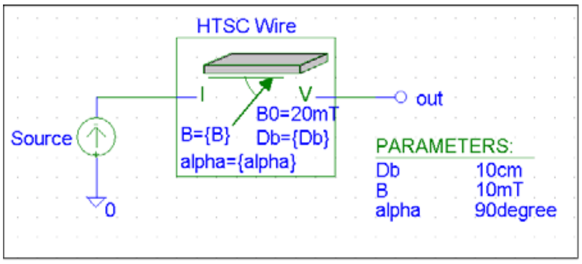

Figure 5. PSpice model for I-V characteristics of a HTSC wire incorporating the effects of bending and the magnetic fields.

3. BEHAVIOUR MODEL OF HTSC COILS

This section deals with the PSpice modelling of HTSC coils. Firstly the critical current estimation of a HTSC coil will be modelled and then the AC losses in the coil will be modelled. A block diagram representing the model of the coil is shown in Fig. 6.

As seen from Fig. 6 the coil model consists of a critical current and AC loss sub models. The critical current model has an input *z* (where *z* is the position across the width of the wire) and the output is $I_c(B_P,z)$. This output is then integrated over the width of the wire $z=-d$ to $z=+d$ to obtain the net critical current of the coil. Input parameters from the wire forming the coil $I_c(0)$ and B_0 as well as coil parameters such as N, R_{out} and R_{in} which represent the number of turns in the coil and the outer and inner radii of the coil are required.

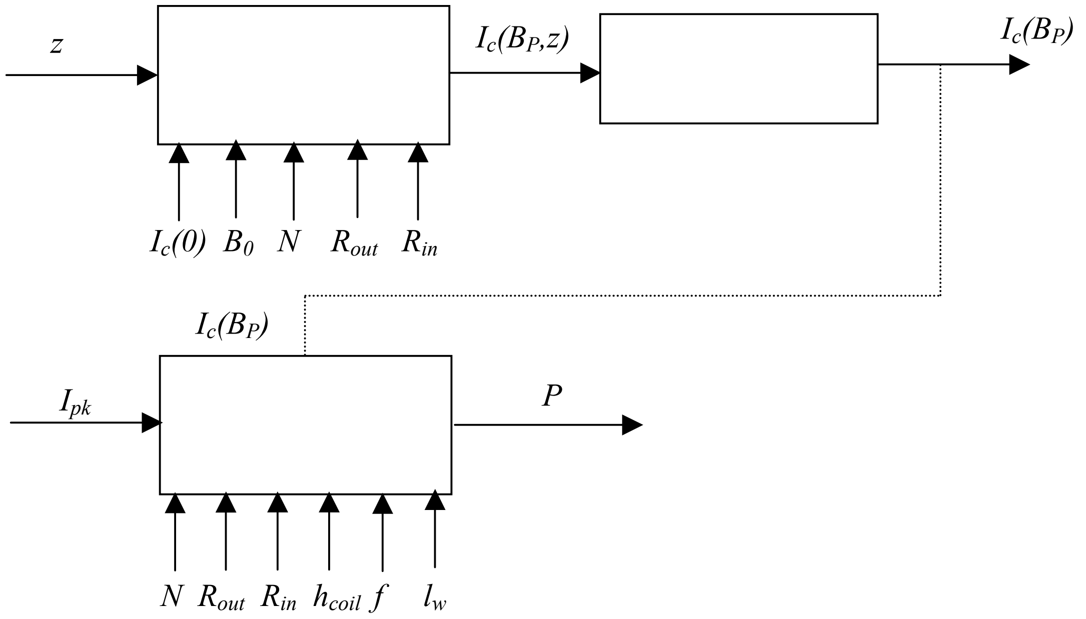

Figure 6. Block diagram of HTSC coil to give critical current and AC loss.

The AC loss model uses the peak value of the AC transport current (I_{pk}) as an input and the output is the total loss in the coil (P). The critical current obtained from the previous model, ie. from the critical current model ($I_c(B_P)$) will be used as an input parameter. In addition other parameters such as the number of turns in the coil, the radii and height of the coil (N, R_{out}, R_{in} and h_{coil}), the total length of the wire forming the coil (l_w) and the AC frequency (f) are also needed.

3.1. Model for the Critical Current of HTSC Coils

The behaviour of the coil is investigated based on the characterisation of pancake coils as discussed previously [5]. The critical current in a HTSC pancake coil at z position, $I_c(B_P,z)$, was calculated using the following equation:

$$I_c(B_P,z) = \frac{-1 \pm \sqrt{1-4uw}}{2u} \tag{1}$$

where

$$u = \frac{B^*_{rad}\,\frac{z}{d}}{B_0}$$

and

$$w = -I_c(0)\frac{\sqrt{1-\frac{z^2}{d^2}}}{\int_{-d}^{d}\sqrt{1-\frac{z^2}{d^2}}\,dz}\,.$$

So that the PSpice model can be more easily used it is advantageous to estimate the value of $B_{rad}{}^*$ by applying the Biot-Savart equation [6] instead of using finite element methods as described previously in [5]. When both diameters of a pancake coil are large enough it can be assumed as a straight slab with an infinite length.

Based on the sheet current analysis the magnitude of the radial field of a pancake coil with N turns carrying current of I per turn, B_{rad} is found to be: [6,7]

$$B_{rad} = \mu_0\frac{K}{2} = \mu_0\frac{NI}{2t_{coil}} \tag{2}$$

where K is the sheet current density, N is the number of turns in the pancake coil, I is the transport current through the wire, t_{coil} is the thickness of the coil ($t_{coil} = R_{out}-R_{in}$, where R_{out} and R_{in} are outside and inside coil radii) and μ_0 is the permeability of free space.

The normalised expression of the radial field $B_{rad}{}^*$ is:

$$B_{rad}{}^* = \frac{\mu_0}{2}\frac{N}{(R_{out}-R_{in})} \tag{3}$$

The coil can thus be represented as a block with an input z (distance across the wire) and given parameters: $I_c(0)$, B_0, N, R_{out}, and R_{in}. The output of the block is the current distribution per unit width in the wire of the coil in the presence of its self-field, $I_c(B_P,z)$. This is then integrated over z to get the critical current value, $I_c(B_P)$.

To model the critical current of the coil using PSpice Eq. (1) can be translated into the block diagram as shown in Fig. 7. The inputs are u and w and the output is $I_c(B_P,z)$. The `constant`, `sqrt`, `divider`, `subs` and `gains` blocks are used to represent the equation.

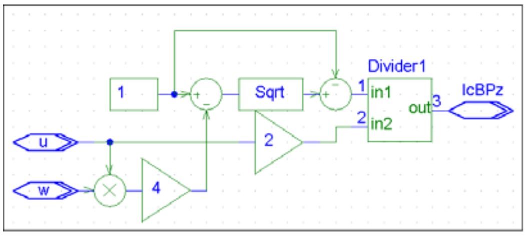

Figure 7. Block diagram to obtain the solution given by Eq. (1).

The values of u and w which are used in Fig. 7 are obtained using the block diagram shown in Fig. 8. The inputs are N, R_{out}, R_{in}, $I_c(0)$, B_0 and z and the outputs are u and w as shown in the figure. Again, the `constant`, `sqrt`, `divider`, `subs` and `gains` blocks are used to represent Eq. (1).

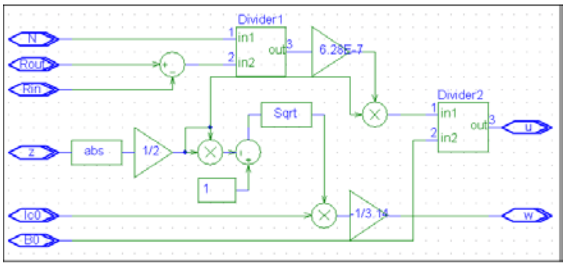

Figure 8. Block diagram for calculating u and w.

The value of $I_c(B_P,z)$ is obtained by connecting the block diagram presented in Fig. 7 with that presented in Fig. 8 which are shown in Fig. 9 as `uw2IcBPz` and `pars2uw` respectively. The input z can be entered into terminal `z` and the output $I_c(B_P,z)$ can be obtained from terminal `IcBPz`. Current sources (`I1-I5`) and resistors (`R1-R5`) with their values of `@B0`, `@Ic0`, `@N`, `@Rin` and `@Rout` are used to represent the input parameters B_0, $I_c(0)$, N, R_{in} and R_{out} respectively.

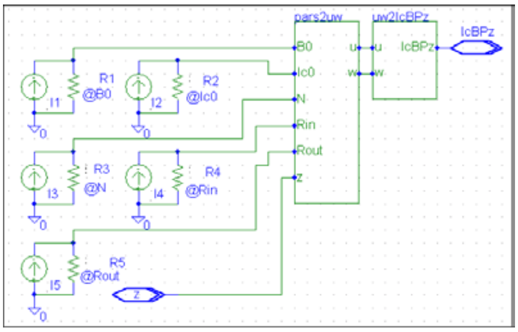

Figure 9. Block diagram for calculating $I_c(B_P,z)$.

The model of the critical current of the coil can then be simplified as a block named `CoilCriticalCurrent` with an input z and an output $I_c(B_P,z)$. The input z is represented as a current source `I_z` together with a resistor `R_z` of `@{z}`. The parameters which must be provided in the model are B_0, $I_c(0)$, N, R_{in} and R_{out}. These have default values as shown in Fig. 10. The output of the model is a current distribution per unit width in the wire of the coil in the presence of the coil's self-field.

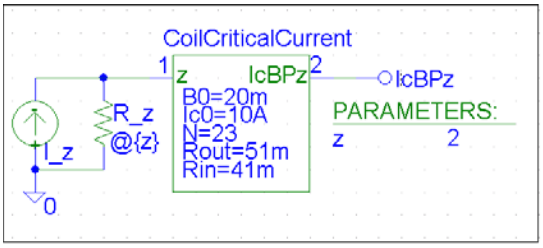

Figure 10. PSpice model for calculating $I_c(B_P,z)$ as a function of z.

The value of the critical current of the coil is obtained by integrating the output of Fig. 10. However, in PSpice this can be done while analysing the output using "Probe". This is done with a statement `s(V(Ic(BP,z)))` where `s` is the notation for the integral and `V(Ic(BP,z))` is the output of the model.

The model development for the critical current of a HTSC coil can be summarised as follows. Firstly the HTSC coil is characterised.. The critical currents of short samples from both ends of the coils are measured. Then the radial components of the magnetic fields generated in the coils are approximated using the Biot-Savart law with the coil's parameter as the inputs. The critical current distribution along the width of the sample is calculated using Eq. (1) with the critical current value of the worst sample and the approximated magnetic field value as the input parameters. This is then translated into PSpice using the available math functions. The critical current of the coil can then be obtained by integrating the current distribution along the width of the wire.

3.2. Model for the AC Losses of HTSC Coils

In this section a model development of the AC loss in a pancake coil will be presented. It has been shown that a pancake coil with a relatively large diameter can be assumed as straight wires arranged in a face-to-face stack as shown previously in [5]. It has been shown in that section that when the spacing D reaches its limit ($D \rightarrow 2a$) the loss per unit length of the wire can be simplified to that of an infinite slab of width $2d$.

The total loss in the coil (P) can be expressed as a function of I_{pk} by entering the dimensions and other parameters of the coil into the equation.

$$P = \frac{\mu_0}{6} f \frac{h_{coil}}{\frac{R_{out}-R_{in}}{N}} l_w \frac{1}{I_c(B_P)} I_{pk}^3 \qquad (4)$$

where f and I_{pk} is the frequency and the peak value of AC transport current respectively, $I_c(B_P)$ is the critical current of the coil, h_{coil}, R_{out}, and R_{in} are the height, outer and inner radii of the coil, N is the number of turns in the coil, l_w is the total length of the wire forming the coil and μ_o is the permeability of vacuum.

The coil can thus be represented as a block with an input I_{pk} and given parameters: $I_c(B_P)$, f, l_w, N, R_{out}, R_{in} and h_{coil}. The output of the block is the total AC loss in the coil. The critical current of the coil $I_c(B_P)$ is provided by the previous model described in Section 4.1.

To model the AC loss of the coil Eq. (4) can be translated into a block diagram as shown in Fig. 11. The input is I_{pk} and the output is P. Parameters such as $I_c(0)$, N, R_{out}, R_{in}, h_{coil}, f and l_w are required by the model. These parameters are defined using current sources `I1` to `I7` together with resistors `R1` to `R7` with their respective values as shown in the figure. The `constants`, `divider`, `subs` and `mults` blocks are used to represent the equation.

The AC loss model of the coil can then be simplified as a block named `CoilACLoss` with an input I_{pk} and an output P. The input I_{pk} is represented as a current source `I_Ipk` together with a resistor `R_Ipk` of `@{Ipk}`. The parameters which must be provided in the model are $I_c(0)$, N, R_{out}, R_{in}, h_{coil}, f and l_w with their respective default values as shown in Fig. 12. The output of the model is the total AC loss of the coil P which can be obtained from terminal `P`.

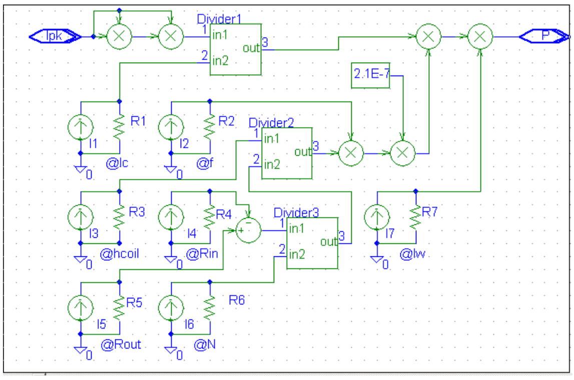

Figure 11. Model of AC loss of HTSC coil.

The construction of the model for the AC loss of the HTSC coil can be summarised briefly as follows: A theoretical approximation which assumes that the coil is constructed using face-to-face stacks is used as described in Eq. (4). The total loss of the coil is then obtained by using the coil diameters as the input parameters. Other parameters include the AC frequency and the critical current of the coil obtained from the model described in Section 4.1. The loss is plotted against the peak AC transport current. Then the PSpice model is constructed using the above equation and parameters.

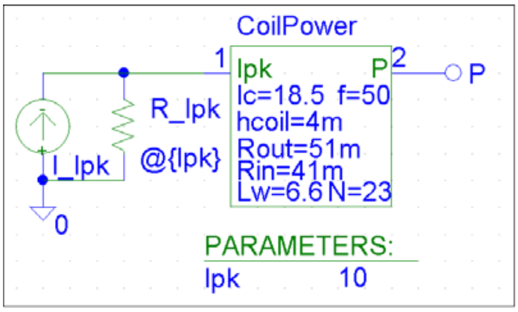

Figure 12. PSpice model of AC loss in a HTSC pancake coil.

4. VERIFICATION OF THE MODEL

This section verifies the modelling techniques by comparing the simulation results with the measurement results. Section 4.1 will present verification of the critical current model of HTSC coils. Results for the AC loss model of the HTSC pancake coils will be given in Section 4.2.

4.1. Verification of the Critical Current Model

Table 5.2 summarises the critical current results using the model. Here the worst, average and the best critical current values are used as input parameters. The calculation results using the finite element methods are also given in this table.

As seen from Table 5.1 the value of the critical current of the coil simulated using the PSpice model using the worst sample for Pancake 1 is 19.4 A. The discrepancy with the measured result (18.5 A) is 0.9 A. It is noted that the discrepancy between the finite element methods and measured results was 1.3 A. The PSpice calculation using the worst sample for Pancake 2 is 3.9 A with a discrepancy of 1.1 A compared with the measured value of 5 A. It is noted that the discrepancy using finite element analysis was 1.2 A. It can be concluded here that the net critical current of these coils as predicted using the PSpice model compares well with the calculation results using finite element analysis.

It is also seen from Table 5.1 that when the thickness of the coils (R_{out}-R_{in}) is large enough, eg. for Pancake 2, the calculation of the critical current of the pancake coils using a field value derived from the

Biot-Savart law is very close to the calculation using a field value derived from finite element simulations.

Table 5.1. Critical current result from PSpice model of pancake coils.

COIL	$I_C(0)$ (A)	B_{RAD}^* (MT/A)	$I_C(B_P)$ (PSPICE) (A)	$I_C(B_P)$ (FEM) (A)
Pancake1 I_c=18.5 A	22 (worst)	1.4	19.4	19.8
	29.1 (aver.)	1.4	24.9	25.5
	35 (best)	1.4	29.2	30.1
Pancake2 I_c=5 A	4 (worst)	2.0	3.9	3.8
	13.5 (aver.)	2.0	12.1	12.1
	21 (best)	2.0	17.9	18.0

4.2. Verification of the Model for the AC Losses

The PSpice model for the AC loss of HTSC coils is run based on the given parameters for Pancakes 1 and 2 and using the critical current value obtained from the previous model, ie. the model for the critical current of HTSC coils developed in Section 3.1. Fig. 13 presents the simulation of the PSpice model for AC loss of Pancakes 1 and 2.

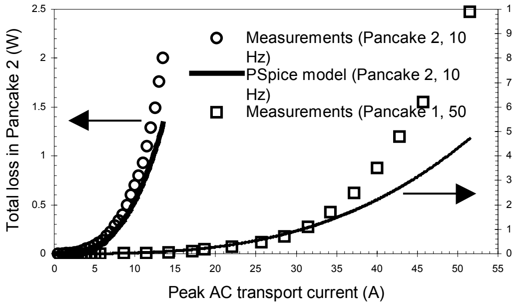

Figure 13. PSpice result of AC loss of Pancake coils.

In Fig. 13 the AC losses of two pancake coils, Pancake 1 and Pancake 2 are given as functions of the peak AC transport current. The frequency of AC transport current for Pancake 1 is set at 50 Hz while that for Pancake 2 is 10 Hz. The losses of Pancake 1 are about 0.4 W for a peak current of 18.5 A and the losses of Pancake 2 are about 0.1 W for a peak current of 5 A. It is seen in the figure that the models fit well with the measurement results for the peak current close to the critical current of the coils, ie. 18.5 A for Pancake 1 and 5 A for Pancake 1. Beyond that the measured losses are greater than the simulated values.

5. CONCLUSIONS

For many power-engineering applications it is necessary to form HTSC wires into coils. The development of PSpice models of pancake coils has been described. The characterisation of the pancake coils has been described. This requires various coil parameters and a field value derived from the Biot-Savart law with no need for a finite element analysis. The model has been tested to accurately predict the critical current of two pancake coils.

Then a model of AC loss for HTSC coils was developed. The model requires an input of the peak AC transport current and the output is the total loss in the coil. Parameters such as the critical current of the coil, dimensions of the coil, frequency of the transport current and the total length of the wire forming the coil are required by the model. The AC loss model of the coil has been verified using the measurement results. It is shown that the models agree well with the measurement results for a peak current close to the critical current of the coils, ie. 18.5 A for Pancake 1 and 5 A for Pancake 1. Beyond these currents the measured losses are greater than the simulated values since the coils become non superconducting above their critical current value.

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