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## A GENERALISED METHODOLOGY FOR EVALUATING VOLTAGE UNBALANCE INFLUENCE COEFFICIENTS

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### ABSTRACT

The recently released IEC Technical Report IEC/TR 61000-3-13 for voltage unbalance allocation requires quantitative measures of the propagation of voltage unbalance from one busbar to other neighbouring busbars of a sub-system in terms of influence coefficients. However, IEC/TR 61000-3-13 does not prescribe a method for their evaluation. This paper carries out preliminary studies in order to investigate the dependency of these influence coefficients on various load types/bases, and proposes a systematic method for estimating them. Proposed method is applied to a 3-bus MV test system and also to the IEEE 14-bus test system of which the results are verified using unbalanced load flow analysis. This new approach could be considered in future updates of IEC/TR 61000-3-13.

### INTRODUCTION

The recently released IEC Technical Report IEC/TR 61000-3-13 [1] which is based on the work that has been undertaken by the CIGRE/CIRED Joint Working Group C4.103 [2] provides guiding principles for determining requirements for the connection of unbalanced installations to MV, HV and EHV public power systems such that an adequate service quality to all connected customers is ensured. The report addresses the co-ordination of voltage unbalance between various voltage levels of a power system through the allocation of the system capacity to absorb voltage unbalance to individual customers.

Propagation of voltage unbalance from one busbar to another neighbouring busbar of a sub-system in terms of an influence coefficient is a key aspect considered in the IEC/TR 61000-3-13 allocation procedure. Influence coefficient  $k_{i-x}$  between busbars  $i$  and  $x$  is defined as the voltage unbalance arising at busbar  $x$  when 1pu of negative sequence voltage source is applied at busbar  $i$ , which can be expressed by (1). These influence coefficients are used in the allocation process in determining the total available apparent power of the entire sub-system as seen at a particular busbar to take into account the contributions from neighbouring busbars.

$$k_{i-x} = \left| \frac{V_{x1U_i}}{V_i} \right| \tag{1}$$

where,

$V_i$  – negative sequence voltage vector prevailing at busbar  $i$

$V_{x/U_i}$  – negative sequence voltage vector at busbar  $x$  which propagates from busbar  $i$

Objectives of this paper are to carry out preliminary studies in order to investigate the dependency of various load types/bases on influence coefficients, and eventually to develop a methodology for the evaluation of influence coefficients for interconnected network environments, an aspect that is not been covered in IEC/TR 61000-3-13.

### DEPENDANCY OF INFLUENCE COEFFICIENTS ON VARIOUS LOAD TYPES/BASES

Consider the radial MV–LV system shown in Figure 1 where the voltages at the sending end busbar (labelled ‘send’) of the line is taken as unbalanced. Purpose is to assess the voltage unbalance that propagates from the sending end busbar ( $V_{-send}$ ) to the receiving end busbar (labeled ‘rec’) of the MV sub-system or the influence coefficient  $k_{send-rec}$  between busbars *send* and *rec*. For this, the loads and the MV line (t) are considered as balanced.

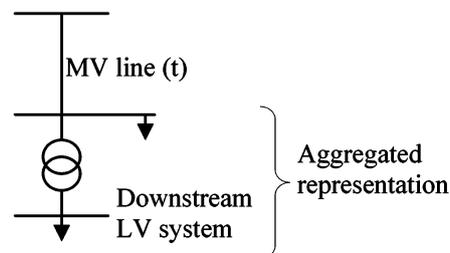


Figure 1: Radial MV sub-system

Negative sequence voltage vector  $V_{-rec/U_{send}}$  at the receiving end busbar that propagates from the sending end busbar can be written (ignoring zero sequence unbalance while replacing the negative sequence impedance  $Z_{-,-t}$  of the line with the positive sequence impedance  $Z_{+,+t}$ ) as:

$$\left| V_{-rec / U_{send}} \right| = \left| V_{-send} - Z_{+,+t} I_{-,-t / U_{send}} \right| \tag{2}$$

where,  $I_{-t/U_{send}}$  – negative sequence current vector in the line arising as a result of the unbalance at the sending end busbar.

Similar to the case of transfer coefficients [3, 4], the negative sequence voltage ratio or the influence coefficient  $k_{send-rec}$  can be expressed for the three basic types of passive loads (ie. constant impedance – Z, constant current – I and constant power – PQ) as:

$$\left| \frac{V_{-rec} / U_{send}}{V_{-send}} \right| = k_{send-rec} \approx \left| \frac{1}{1 + \frac{Z_{++t}}{Z_{++rec}}} \right| \approx (1 - VR_t)^\gamma \tag{3}$$

where,

$Z_{++rec}$  – downstream equivalent positive sequence impedance seen at the receiving end busbar

$\gamma = 1$  for Z loads, 0 for I loads and  $\beta$  for PQ loads

$\beta = -1$  and  $-2$  for low (eg. 0.9) and high (eg. 1) lagging power factor (pf) conditions respectively

$VR_t$  – voltage regulation of the line (calculated as the ratio between the positive sequence voltage drop and the sending end positive sequence voltage)

When the MV sub-system supplies three-phase induction motor (IM) loads (connected at LV), (2) can be rearranged to express the influence coefficient  $k_{send-rec}$  as:

$$k_{send-rec} = \left| \frac{1}{1 + \frac{Z_{++t}}{Z_{--rec}}} \right| \approx \frac{1}{1 + \left( \frac{VR_t}{1 - VR_t} \right) \left( \frac{1}{\frac{1}{k_s} + \frac{1}{k_{sc-lvagg}}} \right)} \tag{4}$$

where,

$Z_{--rec}$  – downstream negative sequence impedance seen at the receiving end busbar

$k_s$  – ratio between the positive and negative (which is inductive) sequence impedances of the aggregated motor load supplied by the LV busbar (typically,  $5 < k_s < 7$ )

$k_{sc-lvagg}$  – ratio between the short circuit capacity (in MVA) at the LV busbar (derived using the system impedance  $Z_{sys}$  that exists between the receiving end busbar and the downstream LV system) and the total MVA load supplied by the LV system

According to (3), the influence coefficient  $k_{send-rec}$  is equal to unity, smaller than unity by the factor  $(1 - VR_t)$  and greater than unity by the factor  $(1 - VR_t)^\beta$  for I, Z and PQ loads respectively. Considering most practical circumstances where  $VR_t < 10\%$ , the factor  $(1 - VR_t)^\gamma$  can

be approximated to unity (in other words  $I_{-t}/U_{send} \approx 0$ ) resulting a  $k_{send-rec} \approx 1$  for passive loads in general. However,  $k_{send-rec}$  for IM loads is considerably smaller than unity (eg. 0.6 for  $k_{sc-lvagg} = 20$ ,  $k_s = 6.7$  and  $VR_t = 10\%$ ) implying that the impact of the negative sequence current  $I_{-t}/U_{send}$  in the presence of IM loads on  $k_{send-rec}$  cannot be ignored as in the case of passive loads. Based on this,  $k_{send-rec}$  for a mix of passive (supplied at MV and/or LV) and motor (supplied at LV) loads can be expressed in a generalised form as:

$$k_{send-rec} \approx \frac{1}{1 + \left( \frac{VR_t}{1 - VR_t} \right) \left( \frac{k_{lv}}{\frac{1}{k_m k_s} + \frac{1}{k_{sc-lvagg}}} \right)} \tag{5}$$

where,

$k_m$  – ratio between the rated motor load (in MVA) and the total MVA load supplied by the LV system

$k_{lv}$  – fraction of LV loads supplied by the MV system

Figure 2 illustrates the variation of  $k_{send-rec}$  with  $k_m$  established using (5) compared with the unbalanced load flow results for a test case ( $VR_t = 8.5\%$ ,  $k_{sc-lvagg} = 19$ ,  $k_s = 6.7$ , passive load composition - equally shared constant impedance and constant power elements with 0.9 lagging pf) in relation to three cases where:

- $k_{lv} = 1$
- $k_{lv} = 0.5$
- $k_{lv} = 0$  (ie.  $k_m = 0$ )

These results confirm the above basis describing the behaviour of different load bases in relation to the propagation of voltage unbalance in a sub-system, also demonstrating the dependency of this propagation on the motor proportion.

### METHODOLOGY FOR EVALUATING INFLUENCE COEFFICIENTS

Consider an interconnected sub-system with  $n$  number of busbars. For the purpose of assessing the voltage unbalance that propagates from any busbar  $i$  to other busbars  $1, 2, \dots, n$  or the influence coefficients  $k_{i-1}, k_{i-2}, \dots, k_{i-n}$ , voltages at the upstream sub-system, all loads and lines in the network (including lines exist at lower voltage levels eg. MV lines for an HV sub-system) are considered to be balanced.

Extending the nodal equations  $[I] = [Y][V]$  to sequence domain, the nodal negative sequence currents  $I_{-x/U_i}$  at any busbars  $x$  ( $= 1, 2, \dots, n, x \neq i$ ) which arises as a result of the unbalance at busbar  $i$  can be expressed in terms of nodal negative sequence admittances  $Y_{-x,xy}$  ( $= Y_{++xy}$ , where  $x, y = 1, 2, \dots, n, x \neq i$ ), nodal negative sequence voltages  $V_{-x/U_i}$  at busbars  $x$  caused by the voltage unbalance at busbar  $i$  and the nodal negative sequence voltage  $V_{-i}$  at busbar  $i$  as given by (6). This can be decomposed and written in a concise form as given by (7):

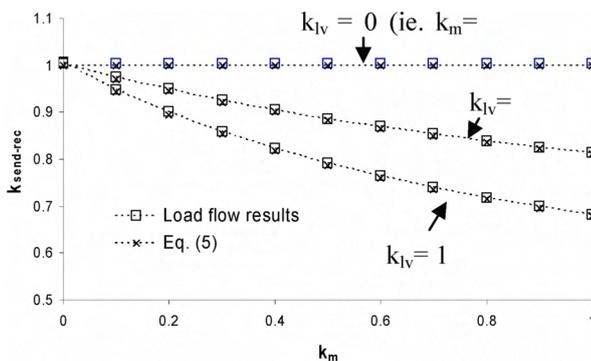


Figure 2: Variation of  $k_{send-rec}$  with  $k_m$

$$\begin{bmatrix} -I_{-U_i/1} \\ -I_{-U_i/2} \\ \vdots \\ -I_{-U_i/n} \end{bmatrix} = \begin{bmatrix} Y_{++:11} & Y_{++:12} & \dots & Y_{++:1i} & \dots & Y_{++:1n} \\ Y_{++:21} & Y_{++:22} & \dots & Y_{++:2i} & \dots & Y_{++:2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_{++:n1} & Y_{++:n2} & \dots & Y_{++:ni} & \dots & Y_{++:nn} \end{bmatrix} \begin{bmatrix} V_{-U_i/1} \\ V_{-U_i/2} \\ \vdots \\ V_{-i} \\ \vdots \\ V_{-U_i/n} \end{bmatrix} \quad (6)$$

$$- [I_{-:Ui/x}] = [Y_{++:xz}] [V_{-:Ui/x}] + [Y_{++:xi}] V_{-j} \quad (7)$$

where,  $x, z = 1, 2, \dots, n$  and  $x, z \neq i$ . As seen above, influence of the currents  $[I_{-:x/Ui}]$  on the voltages  $[V_{-:x/Ui}]$  should be taken into account in the presence of considerable proportions of IM loads. Thus, the current  $I_{-:x/Ui}$  at any busbar  $x$  can be generally written when the busbar supplies a mix of passive and motor (at LV) loads as:

$$I_{-:x/Ui} = Y_{--:x-im} V_{-:Ui/x} \quad (8)$$

where,  $Y_{--:x-im}$  – downstream negative sequence admittance seen at busbar  $x$  taking into account only induction motors supplied by downstream LV systems. This admittance is inductive in nature and dependant on system and load characteristics, system operating conditions and downstream load composition. As an example, taking the sub-system as an MV network,  $Y_{--:x-im}$  can be expressed as:

$$Y_{--:x-im} \approx -j \left( \frac{k_{lv:x}}{\frac{1}{k_{m:x} k_{s:x}} + \frac{1}{k_{sc-lv_{agg}:x}}} \right) \left( \frac{\sqrt{3} I_{+x}}{V_{n-mv}} \right) \quad (9)$$

where,

$k_{s:x}, k_{sc-lv_{agg}:x}, k_{m:x}, k_{lv:x}$  – as defined above where the additional subscripted  $x$  indicates the factors corresponding to busbar  $x$

$I_{+x}$  – nodal positive sequence current vector at busbar  $x$

$V_{n-mv}$  – nominal line-line voltage of the MV sub-system

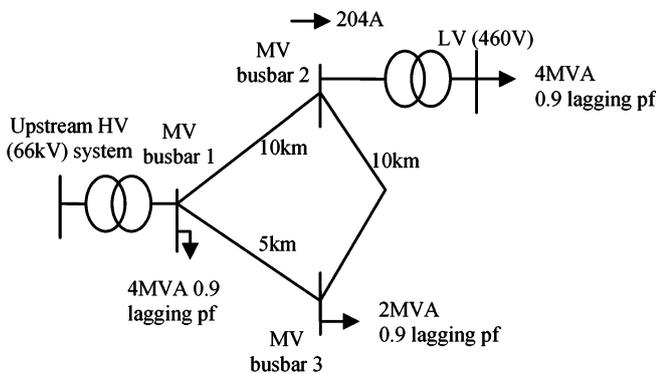
Substitution of (8) in (7) and rearrangement gives:

$$[k_{i-x}]_{(n-1) \times 1} \approx [Y'_{++:xz}]_{(n-1)(n-1)}^{-1} [Y_{++:xi}]_{(n-1) \times 1} \quad (10)$$

where,

$$Y'_{++:xz} = Y_{++:xz} + Y_{--:x-im} \text{ for } x = z$$

$$Y'_{++:xz} = Y_{++:xz} \text{ for } x \neq z$$



HV-MV coupling transformer – 12MVA, R = 1%, X = 10%, tap = 1.05pu  
 MV-LV coupling transformer – aggregated representation of fully loaded 1MVA transformers with R = 1%, X = 5% and tap = 1.05pu

Figure 3: Three-bus MV sub-system

Presence of positive sequence voltage controlled components such as PV generators and synchronous condensers in a system force the negative sequence voltage at the connected busbars to be zero disregarding the existence of sources of unbalance. Equation (10) does not consider the presence of such components and thus requires suitable adjustments such that the impact of zero voltage unbalance at given busbars on influence coefficients is accommodated. Matrix equation (10) can be modified incorporating the influence of a constraint  $V_{-j} = 0$  at a busbar  $j$  on the influence coefficients  $[k_{i-x}]$  (there are only  $(n - 2)$  number of influence coefficients to be determined as  $k_{i-j}$  is known to be zero) by:

- reducing the dimension of the matrix  $[Y'_{++:xz}]$  down to  $(n - 2) \times (n - 2)$  by removing both  $j^{th}$  row and column, and
- reducing the dimension of the matrix  $[Y_{++:ix}]$  down to  $(n-2) \times n$  by removing  $j^{th}$  row.

### VERIFICATION OF THE METHODOLOGY

Proposed methodology is applied to the three-bus MV network (60Hz, 12.47kV, three-wired) shown in Figure 3 for evaluating the influence coefficients  $k_{1-x}$  for  $x = 2, 3$ . Lengths of the lines which are taken as identical in construction and ideally transposed are shown alongside the lines. Positive sequence admittance per km of lines is  $(1.0098 - j2.0630) \text{ Skm}$ . Busbar 3 supplies an equal mix of Z and PQ elements at MV. That is,  $k_{m:3} = 0$  implying  $Y_{--:3-im} \approx 0$ . Busbar 2 supplies a mix of passive (equal mix of Z and PQ elements) and IM loads ( $k_s = 6.7$ ) at LV, which account for 40% of the total load supplied by the system. Note that  $Y_{--:2-im} \neq 0$  when  $k_{m:2} > 0$ .

Figure 4 illustrates the variation of  $k_{1-2}$  and  $k_{1-3}$  with  $k_{m:2}$  established using the methodology in comparison to the results obtained from unbalanced load flow analysis demonstrating the accuracy of the proposed technique. Further, these results reveal that motor loads help in reducing voltage unbalance that propagates from neighboring busbars compared to passive loads.

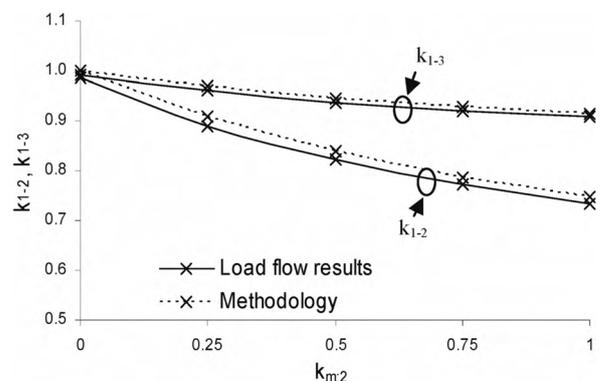


Figure 4: Variation of  $k_{1-2}$  and  $k_{1-3}$  with  $k_{m:2}$  for the 3-bus MV system

Methodology is applied also to the IEEE 14-bus test system shown in Figure 5 (which consists of positive sequence voltage controlled busbars ie. 1, 2, 3, 6 and 8), taking it as a 66kV, 60Hz and three-wired network supplying constant power loads at HV level itself. System (except line) data are as given in [5]. Lengths of the lines (which are taken as identical in construction and ideally transposed) with a positive sequence admittance per km =  $(0.2729+j1.0244) \times 10^2$  pu (on a 100MVA base) are given in Table 1. Figure 6 illustrates the influence coefficients  $k_{i-x}$  where  $i = 4$ ,  $x = 1 - 14$  and  $x \neq 4$  (ie. propagation of voltage unbalance from busbar 4 to other busbars) established using the methodology in comparison to the results obtained from unbalanced load flow where a good agreement is seen.

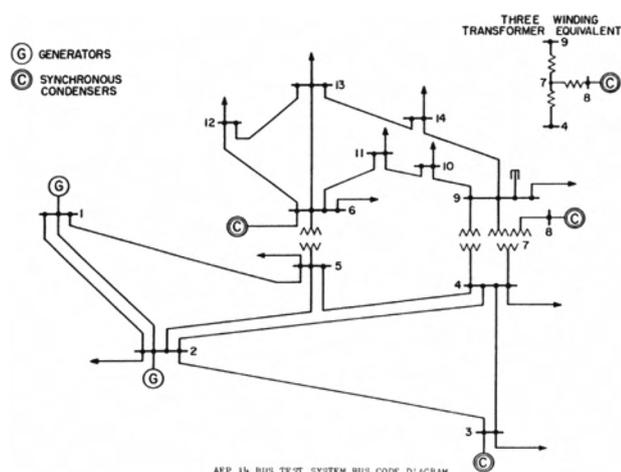


Figure 5: IEEE 14-bus test system

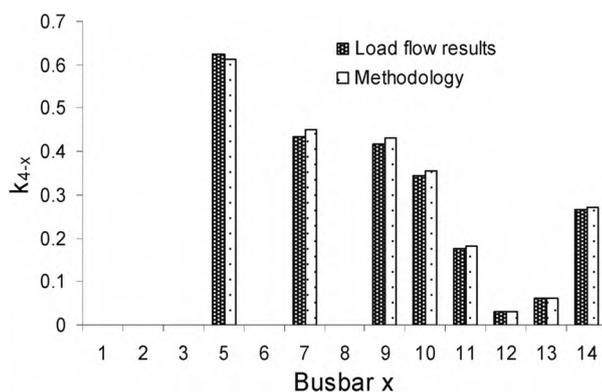


Figure 6: Influence coefficients  $k_{4-x}$  for the IEEE 14-bus test system

| Line | km    | Line | km    | Line  | km    |
|------|-------|------|-------|-------|-------|
| 1-2  | 6.56  | 3-4  | 19.34 | 9-10  | 9.51  |
| 1-5  | 24.17 | 4-5  | 4.65  | 9-14  | 31.46 |
| 2-3  | 21.43 | 6-11 | 23.21 | 10-11 | 22.00 |
| 2-4  | 19.55 | 6-12 | 29.89 | 12-13 | 31.37 |
| 2-5  | 19.27 | 6-13 | 15.39 | 13-14 | 40.83 |

Table 1: Lengths of lines of the IEEE 14-bus test system

## CONCLUSIONS

This paper has addressed the propagation of voltage unbalance from one busbar to other neighbouring busbars of a network, which is a key aspect employed in IEC/TR 61000-3-13.

Considering a simple two-bus radial sub-system, influence coefficients can be approximated to unity for passive loads in general. However, these influence coefficients can be considerably lower than unity when the network supplies a large proportion of induction motor loads. Major conclusion of this observation is that the negative sequence current which arises as a result of the unbalance that exists at a particular busbar introduces a considerable impact on influence coefficients in the presence of large proportions of motor loads although it is insignificant in the case of passive loads.

A systematic method for evaluating influence coefficients suitable for interconnected network environments has been developed. Accuracy of the new methodology has been verified employing a three-bus MV test system and the IEEE 14-bus test system where the results have been compared with those obtained using unbalanced load flow analysis.

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