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[1] This paper explores the underlying mechanism of flow resistance in a wide alluvial channel with bed forms. On the basis of published data, it is shown that the grain roughness can be taken as equal to 2 times the median diameter of the bed sediment. An empirical equation for the bed form roughness has been proposed, and it depends on the bed form height and bed form steepness. The influence of the bed form length and height on the total bed shear stress and energy slope is deliberated, and empirical expressions for the length of the separation zone behind the bed forms are also proposed. The study proposed an equation to compute the total bed shear stress as a function of the grain and bed form roughness as well as the important role of the bed form geometry in the overall flow resistance in alluvial channels. The model is tested and verified against 670 flume measurements and 1540 field observations. The computed and measured energy slopes are in good agreement with close to 71% of all data sets within the ±20% error band.

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1. Introduction and Background

[2] In an alluvial channel, the various regimes of bed forms are the results of complicated interactions between the overlying flow and the mobile bed sediments. The physics of bed form is complicated because the flow boundary is not fixed but changes dynamically according to the sediment characteristics, channel shape and flow strength, among other factors. The variable bed forms modify the flow resistance and therefore the stage-discharge relationship of the channel conveyance.

[3] The mobile bed resistance depends on many interrelated factors including the skin or grain resistance and form drag or bed form resistance. The former is dependent on the depth of flow and grain size at the boundary surface while the latter is the resistance associated with the eddy formations and secondary circulations set up by the flow over the bed form. Whereas the flow resistance for a given flow depth and velocity in a rigid boundary channel is approximately constant with time, it is not so for a mobile bed channel with bed forms. The flow resistance in the latter needs to consider the contribution of both the grain and bed form resistance. Generally, the equation for total shear stress acting on a sand bed is given by

$$\tau_o = \rho g R S \tag{1}$$

where τ_o is total bed shear stress, ρ is fluid density; R is hydraulic radius related to bed, g is gravitational acceleration, and S is energy slope.

[4] The current practice is to treat the total bed shear stress as the sum of two shear stress components corresponding to the grain and bed form resistance, i.e.,

$$\tau_o = \tau' + \tau'' \tag{2}$$

where τ' is shear stress due to grain resistance and τ'' is shear stress due to bed form resistance. These components are often assumed to be independent of one another and have been expressed in various forms by different researchers. For example, *Einstein and Barbarossa* [1952] assumed a constant $\rho g S$ on both sides of equation (2) and proposed that the bed hydraulic radius is the sum of two hydraulic radii corresponding to the grain resistance R' and bed form resistance R''. Equation (2) becomes

$$R = R' + R'' \tag{3}$$

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Chien and Wan [1999, p. 273] commented that equation (3) is useful only as a tool but the approach is not universally applicable because the grain and bed form resistance often affect one another. They argued that "…if bed forms form on the bed, the separation zone on the lee side of a bed form will cause a reduction of the direct contact between the flow and the bed, and correspondingly causes a reduction in the grain resistance…." Instead of using the two hydraulic radii, *Engelund* [1966] and *Smith and McLean* [1977] assumed a constant ρgR on both sides of equation (2) and introduced an alternative approach based on the direct summation of two energy slopes. Equation (2) becomes

$$S = S' + S'' \tag{4}$$

where S' is the energy slope due to grain resistance and S'' is the component due to the bed form resistance. The argument is that the additional energy loss associated with S'' is the result of the "sudden expansion" of flow at the lee side of the bed forms. *White et al.* [1981] compared the various proposed approaches and concluded that amongst the calculated overall bed resistance values which are within a factor of 2 of the measured values, *Einstein and Barbarossa* [1952] scored 21%, *Engelund* [1966] scored 83%, and *White et al.* [1981] scored 89%.

[5] Other researchers, such as *Karim* [1995], *Yu and Lim* [2003], *Wu and Wang* [1999], performed the best estimate of the bed shear stress by adjusting the Manning coefficient using either the regression techniques or dimensional analysis. *Wu and Wang*'s [1999] method gives less than 20% errors for 91% of the 811 data used; *Karim*'s [1995] empirical equation yielded results for which 74% of the measurements has less than 20% error; *Yu and Lim*'s [2003] formula showed less than 20% error for 86% of the 4824 sets of flume and field data used.

[6] The purpose of the present study is to investigate the mechanism of mobile bed shear stress or flow resistance caused by grain and bed form, or to express properly the bed shear stress with the presence of bed forms and to estimate the energy slope in alluvial channels.

2. Underlying Mechanism of Mobile Bed Resistance

[7] Consider a two-dimensional bed form as shown in Figure 1, in which, L = length of bed form; δ = bed form height; and h = flow depth. If the drop in the water level over a bed form is h_f over the distance L, then we can define the energy slope as $S = h_f/L$. Since the total head loss of a flow system is the direct summation of the component head losses, [*Daugherty et al.*, 1985, p. 248], it follows that one could introduce two component head losses for the case shown in Figure 1, i.e., h'_f corresponding to head loss due to grain resistance, and h''_f for head loss due to the bed form resistance. Thus the total energy loss over a bed form can be expressed as follows

$$h_f = h'_f + h''_f \tag{5}$$

We can further visualize that there are two characteristic energy slopes S' and S'', for which $S' = h'_f/L'$ and $S'' = h''_f/L''$. The physical interpretation $'$ and L'' is shown in Figure 1 in which L' is the characteristic length where the flow have direct contact with the bed, and L'' is the characteristic length of the separation zone behind the bed form, where because of the eddy formation, there is an obvious reduction of direct contact area between the flow and the bed that was realized by *Chien and Wan* [1999]. If we divide both sides of equation (5) by L, we obtained the energy slope as follows

$$S = \frac{h_f}{L} = \frac{h'_f}{L'}\frac{L'}{L} + \frac{h''_f}{L''}\frac{L''}{L} = S'\frac{L'}{L} + S''\frac{L''}{L} \tag{6}$$

Equation (6) provides a physical basis to Engelund's approach (equation (4)) and probably explains why his method performs best among the existing models as reported by *Bennett* [1995]. However, equation (6) implies that there is an implicit assumption such that $L' = L'' = L$ in Engelund's approach. This is not true as evidenced from Figure 1 where $L = L' + L''$ and for nonplane bed, L' and L'' are nonzero and will always be less than L. By multiplying ρgR to both sides of equation (6), one obtains

$$\tau_o = \tau'\frac{L'}{L} + \tau''\frac{L''}{L} \tag{7}$$

where $\tau' = \rho g R S'$ = shear stress caused by grain resistance; and $\tau'' = \rho g R S''$ = shear stress caused by bed form resistance. Comparing equation (7) with equation (2), it can be seen that the present model highlights the important role played by the bed form geometry. In other words, the total bed shear stress is a linear "weighted" sum of the component shear stresses, rather than the direct summation of the component shear stresses as shown in equation (2).

3. Grain Roughness

[8] A comprehensive review of the literature on grain roughness is given by *Bennett* [1995]. The grain roughness is associated with the skin drag that is generally defined as the force exerted by the flow on a portion of the bed several grain diameters in the length scale and includes both viscous and pressure drags that arise due to flow around individual particles on the bed [*McLean et al.*, 1999]. The grain shear velocity is widely expressed in the following form

$$u'_* = \sqrt{\tau'/\rho} = \sqrt{gRS'} = \frac{V}{2.5\ln\frac{11R}{k'_s}} \tag{8}$$

where V is cross-sectional mean flow velocity and k'_s is equivalent roughness related to grains. A wide range of k'_s values have been suggested in the literature; examples are $3d_{90}$ [*van Rijn*, 1984b], $2d_{90}$ [*Parker and Peterson*, 1980], $3.5d_{84}$ [*Hey*, 1989], $3d_{84}$ [*Whiting and Dietrich*, 1990], d_{84} [*Prestegaard*, 1983], d_{50} [*Griffiths*, 1989; *Millar*, 1999], $2.5d_{50}$ [*Engelund and Hansen*, 1967], d_{65} [*Einstein and Barbarossa*, 1952; *Wang and White*, 1993] and $2d_{50}$ [*Yang and Lim*, 2003], where d is diameter of sand grain and the subscript associated with d is the percentage finer of the particle size by weight. *van Rijn* [1982] cited several other publications and found k'_s to be within the range of $1.25d_{35} \le k'_s \le 5.1d_{84}$. *Millar* [1999] found that there was no significant difference between using either d_{35}, d_{50},

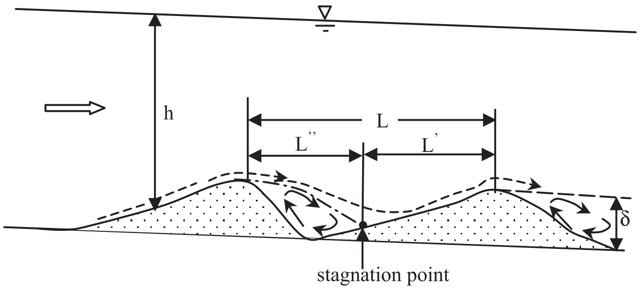

Figure 1. Definition sketch.

d_{84} or d_{90}. He attributed the wide variation of sediment size used to denote grain roughness to the presence of bed form roughness. With such a wide range of k_s' values, it is necessary to justify and derive an appropriate expression of k_s' for the present study. Generally, the grain roughness can be expressed as follows

$$k_s' = nd_{50} \tag{9}$$

where n is an empirical coefficient, typically determined from the best fit of the measured data. Here we will attempt to determine an appropriate n value based on published experiments on plane bed. In this case, there will be no bed form resistance and equation (6) is reduced to $S = S'$ since $L' = L$ and $L'' = 0$. We can then determine S' using equation (8) for any arbitrary n value in equation (9), then the inappropriate n will incur larger inherent error that is defined as follows

$$\text{Error} = \frac{\sum_{i=1}^{m} \left(S_i - S_i'\right)^2}{m} \tag{10}$$

where S and S' are the observed and calculated energy slope, respectively. Obviously, the energy slope can be calculated using equation (8) with the parameters of mean velocity (V), hydraulic radius/water depth (R) and n.

[9] *Guy et al.*'s [1966] data set was selected to test equation (10). This data set consists of 339 experiments conducted in two flumes of 60.96 cm and 243.84 cm width at the Colorado State University between 1956 and 1961. The experiments covered flow regimes ranging from a plane

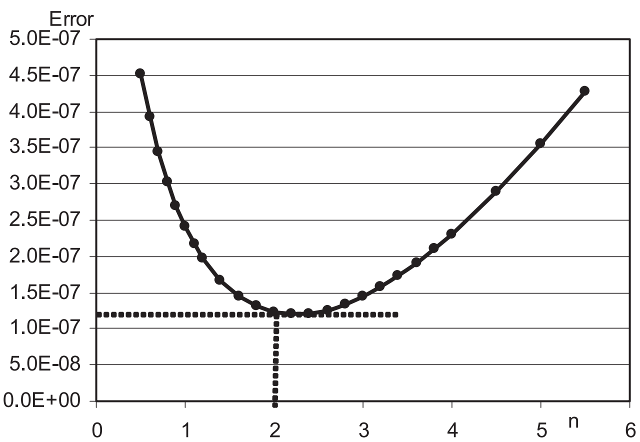

Figure 2. Relati between error and n.

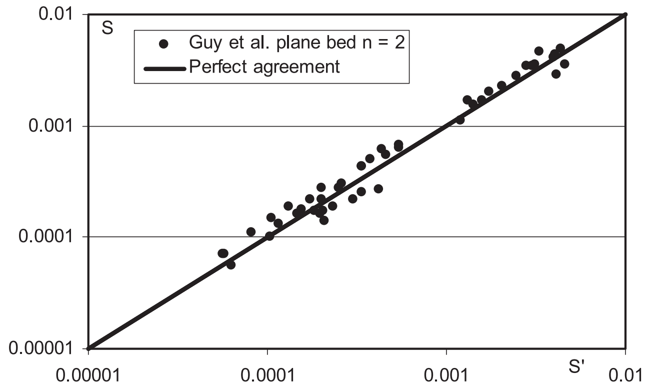

Figure 3. Relationship between S and S' in flat bed based on *Guy et al.*'s [1966] data.

fixed bed to antidunes. There were 46 plane bed experiments. Figure 2 shows the error in equation (10) incurred for various n values. Figure 2 shows that n = 2 give the minimum error. This observation is consistent with *Yang and Lim*'s [2003] conclusion that the grain roughness is best represented by $2d_{50}$. Figure 3 shows the good agreement between the measured S and calculated S' (based on $k_s' = 2d_{50}$) for Guy et al.'s plane bed data. Considering the difficulty and accuracy in the experimental measurement of the energy slope, the agreement depicted in Figure 3 is very good indeed.

4. Bed Form Roughness

[10] The bed form roughness is caused by the form drag created by the difference between the high pressure upstream and low pressure downstream of the bed form. This drag occurs when flow separation occurs behind the bed form and the length scale of the pressure variation over the bed forms is much larger than that of a single sediment particle.

[11] As the length of separation zone L'' behind the bed form is proportional to the bed form height, it can be expressed as follows

$$L'' = \alpha\delta \tag{11}$$

where α is a coefficient. Substituting equation (11) into (7), and noticing that $L' = L - L''$, one obtains

$$\tau_o = \tau' + (\tau'' - \tau')\frac{\alpha\delta}{L} \tag{12a}$$

$$\text{or } S = S' + (S'' - S')\frac{\alpha\delta}{L} \tag{12b}$$

Equation (12b) is different from equation (4) because the latter does not contain the bed form geometry. The relationship between the shear velocity and depth mean velocity is described by equation (8), which depends on the size of the roughness elements. Therefore the shear velocity related to the bed form may be approximated by

$$u_*'' = \sqrt{\tau''/\rho} = \sqrt{gRS''} = \frac{V}{2.5 \ln \frac{11R}{k_s''}} \tag{13}$$

Similar to equation (9), k_s'' for a channel bed with bed form is related to the bed form geometry, namely,

$$k_s'' = \delta f_{(\delta/L)} \tag{14}$$

where f is a function of bed form steepness, δ/L. *van Rijn* [1984c] analyzed the average bed form geometry and obtained an empirical equation for f as follows

$$f_{(\delta/L)} = 1.1\left[1 - \exp\left(-25\frac{\delta}{L}\right)\right] \tag{15}$$

By analyzing *Guy et al.*'s [1966] experimental data for both the lower and upper flow regimes, it is discovered that the equation

$$f_{(\delta/L)} = (\delta/L)^{0.1} \tag{16}$$

is suitable for the bed form roughness.

4.1. Bed Form Geometry: Length and Height of Bed Form

[12] There are many published empirical equations on bed form geometry. For example, the empirical equations on bed form height for the lower and upper regimes proposed by *Karim* [1999] are as follows:

$$\frac{\delta}{h} = \left[\frac{\left\{S - 0.0168\left(\frac{d_{50}}{h}\right)^{0.33}F_r^2\right\}\left(\frac{L}{h}\right)^{1.2}}{0.47F_r^2}\right]^{0.73} \tag{17}$$

for ripples, dunes and transition and

$$\frac{\delta}{h} = \left[\frac{\left\{S - 0.0168\left(\frac{d_{50}}{h}\right)^{0.33}F_r^2\right\}\left(\frac{L}{h}\right)^{1.2}}{0.085F_r^2}\right]^{0.73} \tag{18}$$

for antidunes or standing waves, where F_r is Froude number = $V/\sqrt{gh}$. There are also existing formulae for the length of the bed form L for both lower and upper regimes. For ripples, *Yalin*'s [1964] equation for ripple length is

$$L = 1000\ d_{50} \tag{19}$$

For dunes, *Julien and Klaassen* [1995] developed the relationship

$$L = 6.25\ h \tag{20}$$

Equation (20) is very similar to *van Rijn*'s [1984a] relationship of L = 7.3 h and *Yalin*'s [1964] L = 2π h.For antidunes or standing waves, *Kennedy*'s [1963] equation is as follows

$$L = 2\pi F_r^2 h \tag{21}$$

For transitional bed regime, *Karim* [1999] suggested the following equation

$$L = 7.37h\left\{0.00139\left[\frac{V}{\sqrt{g(\rho_s/\rho - 1)d_{50}^3}}\right]^{2.97}\left(\frac{u_*}{\omega}\right)^{1.47}\right\}^{0.295} \tag{22}$$

where ρ_s = sediment density, ρ = water density, and ω = particle settling velocity.

[13] *Karim* [1995] also proposed two limiting Froude numbers, F_t and F_u to demarcate the flow regimes associated with the various types of bed forms,

$$F_t = 2.716\left(\frac{h}{d_{50}}\right)^{-0.25} \tag{23}$$

$$F_u = 4.785\left(\frac{h}{d_{50}}\right)^{-0.27} \tag{24}$$

On the basis of equations (23) and (24), the different flow regimes may be determined from the flow Froude number, F_r as follows:

Lower regime (ripple, dunes)

$$F_r < F_t \tag{25}$$

Transition regime (washed out dunes)

$$F_t \le F_r \le F_u \tag{26}$$

Upper regime (plane bed, antidunes)

$$F_r > F_u \tag{27}$$

4.2. Length of Separation Zone Behind Bed Forms

[14] Ideally, the empirical expression for coefficient α in equations (11) and (12) should be developed using the direct measurements of the length of the separation zone behind the bed form. However, we are not aware of such data being available in the literature. Therefore an indirect empirical treatment would be necessary. With the bed form geometry known, one may determine S' and S'' using equations (8) and (13), respectively, and α may be determined from equation (12b) using the measured energy slope S. *Engel* [1981] found that for flow over dunes, α is virtually independent of the Froude number, but is correlated with the relative bed form height δ/h. The calculated α versus δ/h is plotted in Figure 4. It is clear that α decreases systematically with bed form development from the lower to upper regimes. This relationship is reasonable because the length of bed forms is a function of water depth as shown in equations (20), (21), and (22), and the length of separation zone is approximately proportional to the length of the bed form. Therefore δ/h is actually a function of δ/L''. For the lower regime, we fitted an empirical equation for α, namely,

$$\alpha = \frac{45}{1 + 5\delta/h} \tag{28a}$$

and for the upper regime, α can be expressed as

$$\alpha = \frac{8}{1 + 5\delta/h} \tag{28b}$$

Equations (28a) and (28b) are also plotted in Figure 4 for comparison.

5. Verification of Proposed Model

[15] To verify the proposed model, a large number of flume and field data sets selected from a database compiled

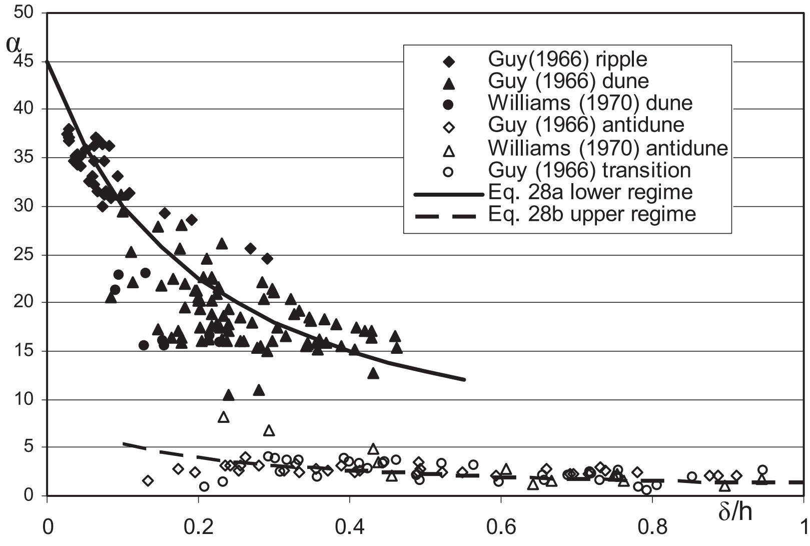

Figure 4. Variation of α with relative roughness of bed forms δ/h in the lower and upper regimes.

by *Brownlie* [1981] is used. A total of 670 sets of independent flume data and 1540 sets of field data were used to verify the validity of equation (12b). The database is comprehensive and covers all flow regimes in alluvial channels. Each data set includes complete records of flow discharge, channel width (b), water depth (h), energy slope (S), median sediment size (d_{50}), specific gravity of sediment ($\rho_s g$). The hydraulic parameters of the selected data sets are listed in Table 1.

[16] The energy slope S in equation (12b) involves calculation of a few parameters on the right-hand side of equation (12b), and the following equations are involved: (1) Calculate S' using equation (8) with $k_s' = 2\ d_{50}$. (2) Calculate S'' using equation (13) with k_s'' from equations

Table 1. Summary of Hydraulic Conditions of Measured Data for Verification

Source and Rivers	Runs	d_{50}, mm	S, ×1000	b, m	h, cm	Q, L/s	±10% Error	±20% Error	±30% Error
Chyn [1935]	33	0.59–0.84	1.1–2.47	0.61	4.7–10	12.2–35.9	0	45	100
Costello [1974]	20	0.51–0.79	0.45–1.01	0.915	14–16	42–60	15	50	70
Daves [1971]	72	0.15	0.11–2.51	1.37	7.6–30	25.5–322	55.6	80	90
Pakistan Water and Power Development Authority [1967]	13	0.44	0.2–3.45	1.219	17.8–24.3	62.3–170	23	92.3	100
Foley [1975]	12	0.29	3.74–10.6	0.267	2.95–4.7	3.7–7.5	58	100	
Franco [1968]	19	0.23–2.2	0.23–1.69	0.914	12.5–16	35.9–53	36.8	100	
Gibbs and Neill [1972]	9	4.37	2.9–5	1.219	17	158–198	100		
Gilbert [1914]	62	0.305	3.5–17.7	0.2–0.6	1.8–8.9	2.6–31.7	40	68	92
Stein [1965]	42	0.4	2–13	1.219	9–24.7	111–481	40	71	86
Barton and Lin [1955]	25	0.18	0.44–2.1	1.219	9–23.7	25.5–229	44	88	96
Mavis et al. [1937]	135	1.41–3.73	1.8–10	0.819	1.8–13.3	2.2–77.9	24	63	84
Guy [1966]	40	0.19	0.1–8.45	2.438	9–33.2	56.6–579	62.5	85	95
Guy [1966]	20	0.27	0.07–10	2.438	13.7–34	172–634	50	85	100
Guy [1966]	36	0.28	0.07–10	2.438	9–32.6	203–623	50	89	100
Guy [1966]	45	0.45	0.15–10	2.438	5.8–30.5	89.2–383	44	71	87
Guy [1966]	33	0.93	0.13–13.6	2.438	11.6–33.8	130–639	53	77	88
Guy [1966]	54	0.47	0.42–8.2	2.438	9.1–40	201–606	68.5	91	96
Acop canal	151	0.085–0.715	0.06–0.166	35.4–140.2	0.76–4.3	52131–486823	7.3	51.0	86.7
Red River	30	0.187–0.684	0.018–0.1336	542–1103	6.92–17.28	4247399–28825680	43.0	86.7	100
American Canal	11	0.096–7	0.058–0.302	3.2–15.118	0.8–2.6	1217–29420	18.0	45.5	54.5
Chop Canal	66	0.11–0.31	0.051–0.254	23.8–121	1.31–3.38	27523–427571	21.2	59.1	86.4
Rio Grande at Bernalillo	38	0.22–0.368	0.74–0.89	40.5–196.6	0.33–1.46	35111–285991	22.2	50.0	100
Mississippi River	165	0.163–1.129	0.0183–0.1336	455–1109	4.66–17.28	1512074–28825680	23.0	47.9	77.0
Colorado River	122	0.155–0.695	0.06–0.389	92.6–254.6	0.95–3.89	77531–500925	67.6	90.5	96.2
HII River	38	0.21–1.44	0.84–10.7	0.35–8	0.019–0.73	0.94–4851	42.1	60.5	81.6
Leopold River	55	0.14–0.814	0.037–0.346	88.7–152.4	0.96–4.1	83333–454301	63.0	87.0	94.0
Middle Loup River	38	0.267–0.429	0.928–1.496	37.49–46.63	0.292–0.376	9315–12855	39.5	100	
Snake and Cleanwater River	21	0.4–33	0.354–1.21	140.2–192	4.02–5.91	971238–3114759	52.4	95.2	95.2
Mountain Creek	100	0.899	1.37–3.15	3.923–4.334	0.046–0.177	64.4–1492	67	97	100
South American Canal	113	0.1–1.05	0.004–0.45	27–845	1.32–13.28	23999–14259996	27.7	50.0	67.0
Niobrara River	40	0.212–0.359	1.13–1.70	21	0.4–0.588	5861–16055	62.5	100	
Portugal River	219	2.204–2.6	0.62–0.94	69.7–188.9	0.457–2.44	28999–659980	6.4	90.4	100
Rio Grande Channel	27	0.17–0.24	0.45–0.8	16.76–22.86	0.39–1.51	3596–39077	37.0	48.1	70.4
Rio Grande River	2 3	0.173–10.954	0.69–2.46	14.02–121.9	0.158–1.88	498.4–285991	31.0	64.2	94.9

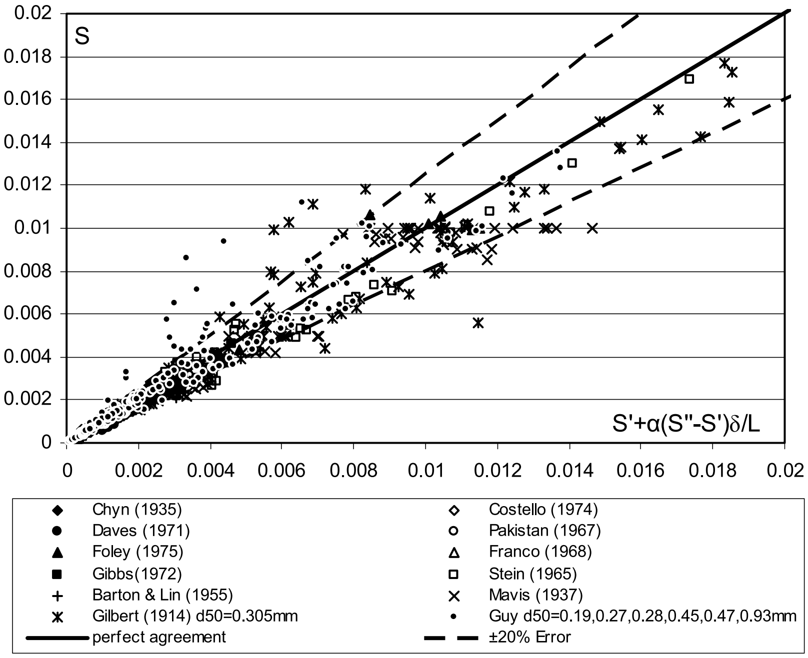

Figure 5. A comparison of measured and calculated energy slopes for laboratory data.

(14) and (16). (3) Calculate α using equations (28a) and (28b) for the lower and upper regimes, respectively. (4) Calculate the bed form steepness using equations (17)–(27), depending on the types of bed form regimes.

5.1. Laboratory Flume Data

[17] Figure 5 shows a comparison of the 670 sets of measured and calculated energy slopes using equation (12b). The agreement is generally good, and a closer look reveals that 42% of the data lies within the ±10% error band, 74% within the ±20% band and 91% within the ±30% band.

5.2. Field Data

[18] In natural rivers or canals, bed forms such as ripple, transition, standing wave and antidune are rare. Dune is the most commonly observed type of bed forms. A total of 1540 sets of field data [*Brownlie*, 1981] were used for the verification exercise. A brief description of these data sets is included in Appendix A. A comparison of the calculated

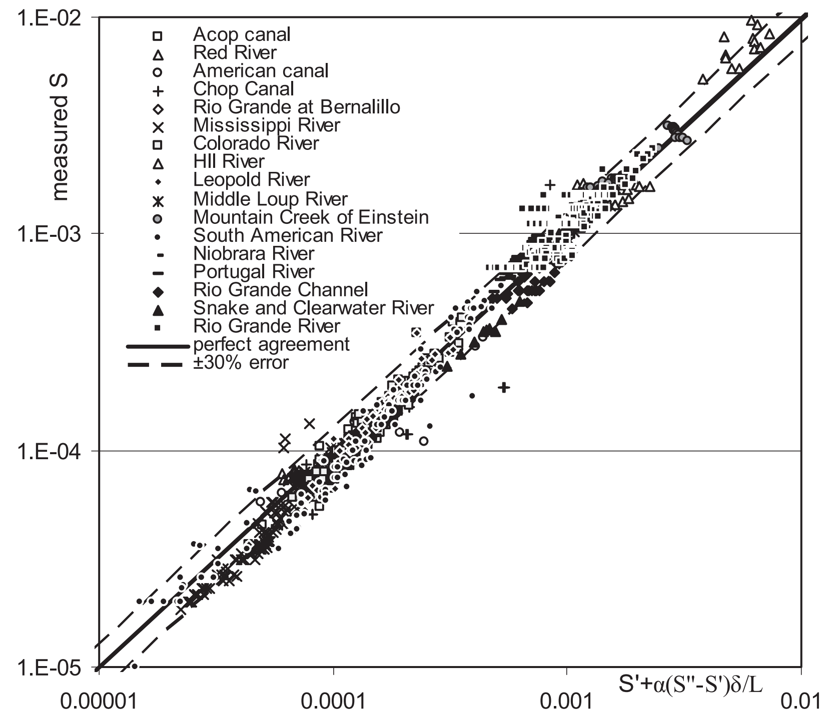

Figure comparison of measured and calculated energy slopes for field data.

and measured energy slope is shown in Figure 6. Among the field data, 32% lies within the ±10% band, 70% within ±20% and 90% within the ±30% discrepancy error band. The agreement is reasonably good considering the usually large degree of uncertainties in field measurements.

6. Conclusions

[19] The aim of the present study is to establish the relationship between the total bed shear stress and the grain and bed form shear stresses in alluvial channels. To this end, the study proposed an equation (equation (12b)) to compute the total shear and the formulation includes the important role of the bed form geometry vis-à-vis the overall flow resistance in the channel. For roughness related to grain, it is shown that the equivalent roughness is $2d_{50}$. For roughness related to the bed form, the equivalent roughness depends on the height and steepness of the bed form. Empirical expressions (equations (11), (28a), and (28b)) for the length of the separation zone behind the bed forms are also proposed. The validity of the proposed model (equation (12b)) has been tested with 670 flume measurements and 1540 field observations. The computed and measured energy slopes are in good agreement with 71% of all data sets falling within the ±20% error band.

Appendix A: A Brief Description of the Data Sets Used in the Verification Study

[20] Acop Canal data were recorded by Mahmood et al in 1979 at 17 reaches of five canals in Pakistan. American canal data were obtained by Simons in 1957 (12 canals in Colorado, Nebraska and Wyoming). Red river data were obtained by Toffaleti in 1968. Chop data were collected by Chaudry et al. in 1970 under the Canal and Headworks Observation Program of the West Pakistan Water and Power Development Authority, 1962–1964 from 9 canals. Toffaleti in 1968 measured the hydraulic conditions in Rio Grande near Bernalillo and Mississippi River. Colorado River data was measured by the U.S. Bureau of Reclamation in 1958. The location of measurements was at the Needles Bridge Station, Taylor's Ferry station, Palo Verde Weir Station and Adobe Ruins station. Hii River data (HII) were obtained by Shinohara and Tsubaki in 1959 at Igaya and Kurihara stations. Leopold River data was recorded by Pertson and Howells in 1969. Middle Loup River data were observed by Hubbell and Matejka in 1959 at Dunning, Nebraska, upstream from the confluence with the Dismal river. A turbulence flume was constructed at the bridge on State Route 2. Snake and Clearwater River data were collected by Seitz in 1976 in the vicinity of Lewiston, Idaho. In 1944, Einstein studied sediment discharge in Mountain Greek, a tributary of the Enoree River in Greenville County, South Carolina and West Goose Creek in Tallahachie River basin approximately four miles west of Oxford, Mississippi. A total of 81 records were collected in Mountain Creek and 19 records in West Goose Creek. South American River data were collected by Nedeco at 10 stations on the Rio Magdalena in Columbia, South America, and some records were made at 10 stations on the canal del Dique, also in Columbia in 1973. Niobrara River data were observed by Colby and Hambree near Cody in northern Nebraska from 13 July 1949 through 8 July 1953. The river has a natural co ed section, cut in bedrock, and almost rectangular in cross-section. The energy slope was not measured everyday, but was estimated from observations made on every other days, or the average of two observations if slope was measured shortly before and shortly after the other streamflow measurements. Portugal river data were measured by Da Cunha in 1969. Rio Grande Channel data were measured by Culbertson et al in 1976. Rio Grande river data were obtained by Nordin and Beverage in 1965 from 6 stations in New Mexico. The data were collected at the following stations: 18 records at Otowi bridge, near San Idelfson; 69 records at Cochiti; 69 records at san Felipe, 57 records near Bernalillo; 53 records at Albuquerque and 22 records near Belen.

Notation

- b width of channel.
- h flow depth.
- f function of bed form steepness.
- F_r Froude number.
- F_t limiting Froude number.
- F_u limiting Froude number.
- g gravitational acceleration.
- h_f energy loss.
- k'_s equivalent roughness related to grains.
- k''_s equivalent roughness related to bed form.
- L length of bed form.
- L'' length of separation zone behind the bed form.
- L' length dominating by grain friction.
- n empirical coefficient.
- R bed hydraulic radius.
- R' hydraulic radius related to grain.
- R'' hydraulic radius related to bed form.
- S energy slope.
- S' energy slope due to grain friction.
- S'' energy slope due to the bed form resistance.
- u local velocity.
- V cross-sectional mean velocity.
- y distance from bottom.
- α coefficient.
- τ_o bed shear stress.
- τ local shear stress.
- τ' $\rho gR'S$ or $\rho gRS'$.
- τ'' $\rho gR''S$ or $\rho gRS''$.
- ρ fluid density.
- ρ_s density of sand.
- δ bed form height.
- ω particle settling velocity.

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