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AN ELASTO-PLASTIC MODEL FOR LIQUEFIABLE SANDS SUBJECTED TO TORSIONAL SHEAR LOADINGS

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Summary. This paper presents a modeling procedure for simulating the monotonic undrained torsional shear behavior of sands, including stress-strain relationship, and excess pore water pressure generation, while considering the void ratio and stress level dependence of stress-strain-dilatancy behavior of sand. A unique set of soil parameters is required by the model to satisfactorily predict the undrained behavior of loose and dense Toyoura sand over a wide range of initial void ratios and confining pressures, as proven by simulating experimental data produced by the authors and found in the literature.

1 INTROCUCTION

It is well known that sand behaves differently under different density states and confining pressures [1–5]. During shearing, loose sand exhibits a contractive behavior with strain-softening. Instead, dense sand dilates while showing strain-hardening.

Although there have been a number of comprehensive model developed in geomechanics, to predict in a very reliable manner the complex response of sands undergoing monotonic loading conditions for a large range of initial void ratios and confining pressures without the need to change the soil parameters remains to be a major challenge.

In models based on generalized plasticity [6, 7], complex mathematical formulations are used. In addition, the same sand is usually considered as different material depending on density and stress level, so that a large number of soil parameters is required. Alternatively, one of advantages of critical-state constitutive models [8, 9] is their ability to predict soil behavior over a range of densities and confining pressures by using a single set of soil parameters. [10] proposed a unified model based on critical state and generalized plasticity. Nevertheless, there may be debates over the uniqueness of critical state line. In addition the majority of such predictive model has been validated only for the case of triaxial loadings.

As an alternative, [11] used a generalized hyperbolic equation (GHE) to simulate the stress-strain behavior of Toyoura sand under drained plane strain compression loadings. Later, the GHE approach was successfully used by [12, 13] for simulating the drained/undrained monotonic/cyclic torsional shear behavior of Toyoura sand for the case of loose and dense Toyoura sands (considered as two different materials).

In this paper, a simple (i.e. use of non-complex mathematical formulations and of small number of soil parameters) elasto-plastic model able to predict all the characteristics of Toyoura sand behavior in monotonic undrained torsional shear tests over a wide range of void ratios and confining pressures using a unique set soil parameters under the GHE approach is proposed.

2 PROPOSED MODEL

2.1 Modeling of stress-strain behavior of sand

In this current study, the GHE approach was used to simulate the highly non-linear undrained stress-strain response of Toyoura sand subjected to torsional shear loadings. GHE has been proposed by [11] in the form:

$$y = \frac{x}{1/C_{1(x)} + x/C_{2(x)}} \tag{1}$$

where x and y represents a normalized plastic shear strain function and a normalized shear stress ratio function, respectively. Note that, in Eq. (1), for normally consolidated uncemented sand, similar to Toyoura sand, the elastic strain component (see Appendix) may be neglected.

$C_{1(x)}$ and $C_{2(x)}$ are two strain-dependent parameters, which for the case of torsional shear loadings can be formulated as follows:

$$C_{1(x)} = \frac{C_{1(0)} + C_{1(\infty)}}{2} + \frac{C_{1(0)} - C_{1(\infty)}}{2} \cos\left\{\frac{\pi}{(\alpha/x)^a + 1}\right\} \tag{2}$$

$$C_{2(x)} = \frac{C_{2(0)} + C_{2(\infty)}}{2} + \frac{C_{2(0)} - C_{2(\infty)}}{2} \cos\left\{\frac{\pi}{(\beta/x)^b + 1}\right\} \tag{3}$$

All the coefficients in Eqs. (2) and (3) can be determined by fitting the experimental data plotted in terms of y/x vs. x relationship as explained in details in [11].

To characterize the GHE while accounting for the void ratio and confining stress level dependence of sand behavior, the same x and y functions employed by [14] were adopted:

$$y = \frac{(\tau / p')}{(\tau / p')_{\max}} \tag{4}$$

$$x = \gamma^p / \gamma_R\,, \qquad \gamma_R = \frac{(\tau / p')_{\max}}{(G_0 / p_0')} \tag{5}$$

where γ^p is the plastic shear strain; τ is the shear stress; p' and p_0' are the current and initial effective mean principal stress, respectively; $(\tau/p')_{max}$ is the peak shear stress in the plot τ/p' vs. γ^p; and G_0 is the initial shear modulus.

A number of formulations have been reported in literature to express the shear modulus as a function of void ratio and confining pressure [2, 9, 10, 14]. In here, for the case of undrained torsional shear loadings, the following formulation is proposed for G_0:

$$G_0 = G_{R0} \frac{f(e_0)}{f(e_{R0})} \left(\frac{p_0'}{p_{R0}'} \right)^n = 7828 \frac{(2.17 - e_0)^2}{1 + e_0} (p_0')^{0.508} \tag{6}$$

where e_0 is the initial void ratio at the corresponding p_0'; $f(e_0) = (2.17 - e_0)^2/(1 + e_0)$ as defined by [15]; and n is a material parameter. G_{R0} is a reference initial shear modulus at the corresponding void ratio (e_{R0}) and confining pressure p_{R0}'. For Toyoura sand subjected to undrained torsional shearing, [13] reported the following values: G_{R0}= 80000 kPa at $f(e_{R0=0.828})$ = 0.985 and p_{R0}'= 100 kPa; and n = 0.508.

Referring to results of torsional simple shear tests, [16] evaluated the friction angle at failure (φ) over a wide range of void ratios and confining pressures. They observed that the pressure-dependence of φ is very small, while the density-dependence is predominant. In a similar manner to [16], in this study it was found that the angle of shear resistance at failure $\{\varphi' = \tan^{-1} (\tau/p')_{max}\}$ varies significantly with the void ratio, while the pressure–dependence can be neglected. By fitting the test results, the void ratio dependence of shear strength, $(\tau/p')_{max}$, was formulated as follows:

$$(\tau / p')_{max} = 0.67 \frac{(2.17 - e_0)^2}{1 + e_0} \tag{7}$$

2.2 Stress-dilatancy relations of sand under torsional shear

Volume change in drained shear can be considered as the mirror image of pore water pressure build-up during undrained shear. Change of volumetric strain in different stage of loading can be described by the stress-dilatancy relationship, which relates the dilatancy ratio $(-d\varepsilon^d_{vol}/d\gamma^p_{vol})$ to the stress ratio (τ/p') [17, 18]. For the case of torsional shear [19] proposed the following empirical linear stress-dilatancy relationship:

$$\frac{\tau}{p'} = N_d \left(-\frac{d\varepsilon^d{}_{vol}}{d\gamma^p} \right) + C_d \tag{8}$$

where N_d and C_d are the gradient and the intercept in the plot $(-d\varepsilon^d_{vol}/d\gamma^p_{vol})$ vs. (τ/p'), respectively. Note that, when $(\tau/p') < C_d$ soils behaves contractive, while when $(\tau/p') > C_d$ soils behaves dilative. Alternatively, $(\tau/p') = C_d$ corresponds to zero dilatancy state (i.e. phase transformation state [20]).

[5] reported that, for a given material, the mobilized angle of shear resistance at zero dilatancy, $\varphi'_{PTL} = \tan^{-1}(\tau/p')_{PTL}$, is independent of stress level and density. As a consequence, $C_d = (\tau/p')_{PTL}$ can be regarded as a constant, and in this current study, C_d was chosen as 0.6 (i.e. for Toyoura sand, $\varphi'_{PTL} = 31°$ [21]).

In the proposed model, by fitting test results reported in [12, 13], the void ratio dependence of N_d was formulated as follows:

$$N_d = 5.793 - 5e_0 \tag{9}$$

2.3 Modeling of pore water pressure generation

In modeling the undrained cyclic shear behavior it was assumed that the total volumetric strain increment ($d\varepsilon_{vol}$) during the undrained loading, which consists of dilatancy ($d\varepsilon^d_{vol}$) and consolidation/swelling ($d\varepsilon^c_{vol}$) components, is equal to zero. Therefore the following equation is valid during undrained loading:

$$d\varepsilon_{vol} = d\varepsilon^c{}_{vol} + d\varepsilon^d{}_{vol} = 0 \tag{10}$$

Experimental evidences suggest that the bulk modulus K (= $dp'/d\varepsilon^c_{vol}$) can be expressed as a unique function of p':

$$K = \frac{dp'}{d\varepsilon^c{}_{vol}} = K_0 \frac{f(e)}{f(e_0)}\left(\frac{p'}{p_0'}\right)^{m_k} \tag{11}$$

where K_0 is the bulk modulus at p_0'; $f(e)$ and $f(e_0)$ are the void ratio function at current and reference stress state, respectively; and m_k is a coefficient to model the stress-state dependency of K.

Considering that $f(e) = f(e_0)$ in undrained tests, and that the volumetric strain component due to consolidation/swelling is the mirror image of the one due to dilatancy ($d\varepsilon^c_{vol} = -d\varepsilon^d_{vol}$), the generation of pore water pressure during undrained shearing was evaluated as follows:

$$dp' = K_0 (p'/p_0')^{m_k}\left(-d\varepsilon^d{}_{vol}\right) = K_0 (p'/p_0')^{m_k}\left(\frac{\tau}{p'} - C_d\right)\frac{d\gamma^p}{N_d} \tag{12}$$

An empirical formulation, Eq. (13), was adopted to account for density and pressure dependence of the bulk modulus, similarly to the case of G_0 in Eq. (7). For Toyoura sand subjected to undrained torsional shearing, [14] reported the following values: K_{R0}= 47000 kPa at $f(e_{R0=0.828}) = 0.985$ and p_{R0}'= 100 kPa; and m_k = 0.5. Therefore, substituting those values in Eq. (12) yields:

$$K_0 = K_{R0}\frac{f(e_0)}{f(e_{R0})}\left(\frac{p_0'}{p_{R0}'}\right)^{m_k} = 4772\frac{(2.17-e_0)^2}{1+e_0}\left(p_0'\right)^{0.5} \tag{13}$$

3 MODEL CALIBRATION AND VALIDATION

A single set of soil parameters (i.e. GHE parameters), as listed in Table 1, was used to simulate the behavior of loose and dense sand consolidated at different confining pressure level and sheared under undrained torsional shear conditions.

The soil parameters were determined by fitting the results of a monotonic undrained torsional shear tests conducted on a loose Toyoura sand specimen (e_0 = 0.828, *D*r= 45.5%), which was isotropically consolidated at p_0'=100 kPa. Note that, tested Toyoura sand specimens refer to batch I, with e_{max}= 0.992, e_{min} = 0.632 and G_S = 2.65.

Table 1. GHE parameters for Toyoura sand

$C_{1(0)}$	$C_{1(\infty)}$	$C_{2(0)}$	$C_{2(\infty)}$	α	β	a	b
4	0.123	0.102	1.2	0.01073	0.85012	0.2	0.2

Fig. 1 compares predicted and observed (by the authors) behavior of Toyoura sand consolidated to p_0' = 100 kPa at two different void ratios of e_0 = 0.691 (dense, *D*r = 84%) and 0.859 (loose, *D*r = 37%). Fig. 1 shows that, the overall soil behavior of loose (contractive with strain-softening) and dense (dilative with strain-hardening) Toyoura sand specimens consolidated at the same confining pressure was well simulated by the model.

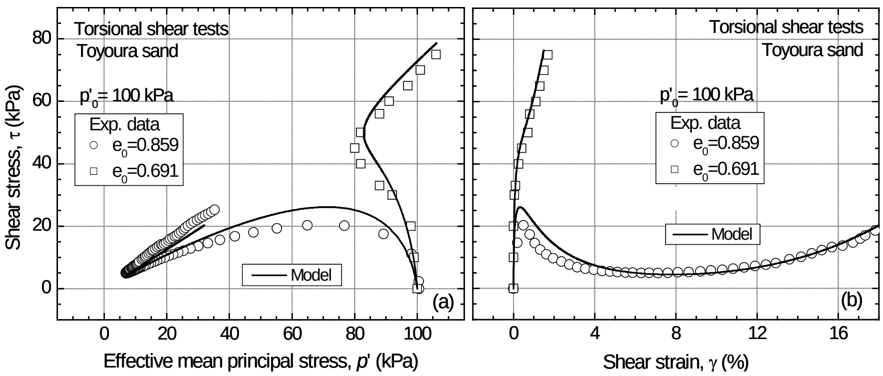

Fig. 1. Undrained monotonic torsional shear behavior of Toyoura sand consolidated to p_0' = 100 kPa: (a) effective stress path; and (b) stress-strain relationship.

To confirm the model performance, two test results presented by [4] for very loose Toyoura sand specimens (e_0 = 0.872-0.876) consolidated to p_0' = 50 kPa and 100 kPa were simulated. Note that, the Toyoura sand used by [4] was of different batch from the one used by the authors, thus the index properties were slightly different (i.e. e_{max}= 0.977 and e_{min} = 0.597). In Fig. 2, the model predictions are compared with the laboratory test results. It can be seen that, despite the differences in index soil properties of Toyoura sand, the model was able to satisfactorily predict the behavior of very loose sand until the offset of liquefaction.

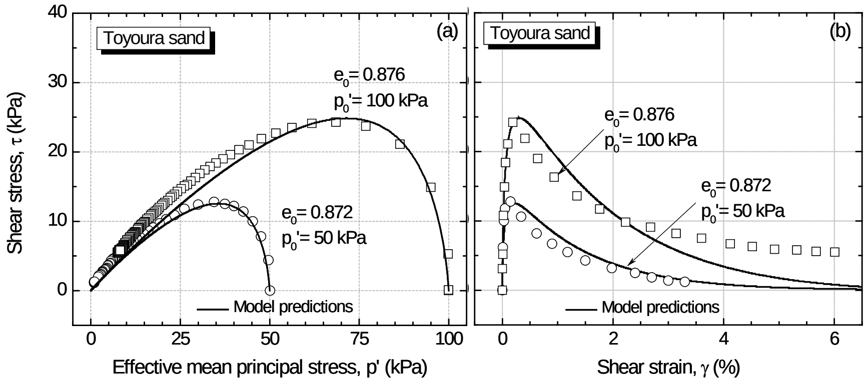

Fig. 2. Undrained monotonic torsional shear behavior of Toyoura sand consolidated to a void ratio e_0 = 0.872-0.876 (from [4]): (a) effective stress path; and (b) stress-strain relationship.

4 CONCLUSIONS

Under shearing, loose sand exhibits a contractive behavior with strain-softening, while dense sand shows dilative behavior with strain-hardening. To predict in a very consistent manner the complex response of sands subject to shearing while accounting for void ratios and stress level dependency is not an easy task.

In this paper, based on GHE approach, an elasto-plastic model to predict the monotonic undrained torsional shear behavior of sand was proposed. The model employs only a unique set of soil parameters to satisfactorily predict the behavior of Toyoura sand over a wide range of initial void ratios and confining pressures, including contractive behavior with strain-softening (loose sand), dilative behavior with strain-hardening (dense sand) and liquefaction occurrence (very loose sand).

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6 APPENDIX

For normally consolidated uncemented sand, the elastic shear strain (γ^e) is very small. In this study, γ^e (= $\Sigma d\gamma^e$) was found to be in the order of 0.03% for loose Toyoura sand (e_0 = 0.828; p_0' = 100 kPa) at the peak stress state, and of 0.11% for dense sand Toyoura sand (e_0 = 0.691; p_0' = 100 kPa) subjected to a stress state of τ = 100 kPa.

The elastic component was calculated by Eqs. (14) and (15), as formulated in the recently developed quasi-elastic constitutive model proposed by [22]:

$$d\gamma^{e} = d\tau / G \tag{14}$$

$$G = G_0 \frac{f(e)}{f(e_0)} \left(\frac{\sqrt{\sigma_z ' \sigma_r '}}{\sigma_0 '} \right)^{n} \approx G_0 \left(\frac{p'}{p_0 '} \right)^{0.508} \tag{15}$$

where G is the actual shear modulus; $f(e_0)$ and f(e) are the initial and current void ratio functions, note that $f(e) = f(e_0)$ in undrained tests; G_0 is the initial shear modulus at $f(e_0)$ and σ_0'; σ_z' and σ_r' are the vertical and radial effective stress, respectively; and n is a material parameter.

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