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# Radial consolidation equation with variable load, material and drain efficiency parameter

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Keywords: consolidation, vertical drains, analytical solution

## ABSTRACT

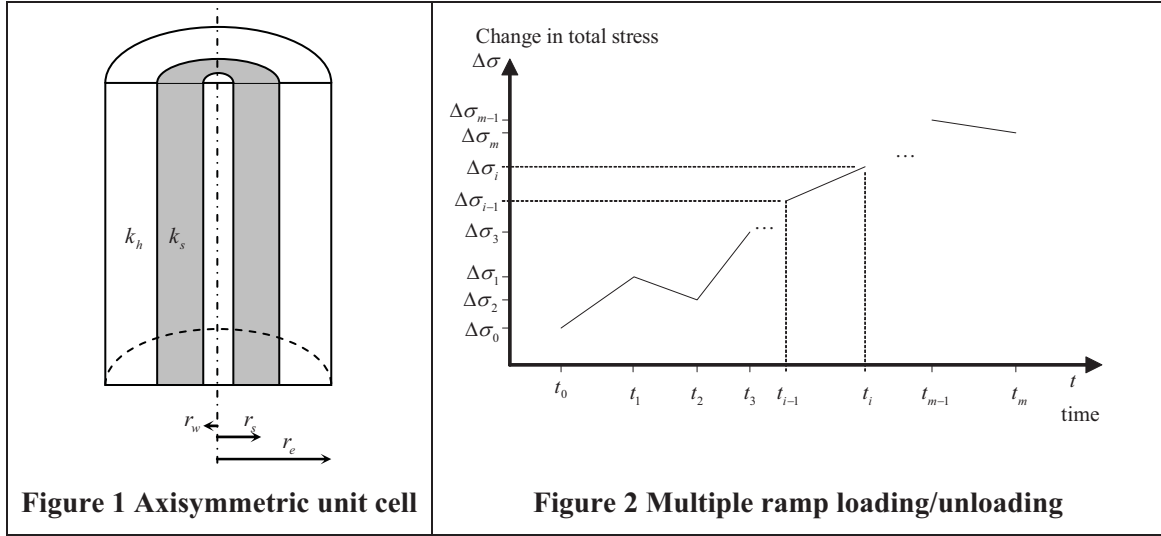
Numerous complex analytical solutions are available for analysis of consolidation problems involving radial drainage, yet the most common solution used for determining the rate of consolidation is the equal strain solution presented by Hansbo (1981). Maintaining the simplicity of Hansbo's (1981) solution this paper presents radial consolidation equations that capture the versatility of the more complex methods. Material and geometry properties vary piecewise-constant with time while loading/unloading varies piecewise-linear with time. A new drain efficiency parameter is introduced to model the detrimental effects of drain clogging and kinking. Drain clogging/kinking is simulated by specifying the excess pore pressure in the drain, normally set to zero, as a fraction of the average excess pore pressure in the soil. The new equations are easily implemented in a spreadsheet or computer program providing a simple yet versatile tool for analysis of consolidation problems dominated by radial drainage. Computer code for implementing the new approach in a Microsoft Excel is provided.

## 1 INTRODUCTION

Many analytical solutions to consolidation problems with radial drainage are available (Walker, 2006) but they are rarely used owing to their complexity and difficult implementation. In the authors experience the most common method used, especially for quick calculations, is the simplest, that of Hansbo (1981). Hansbo's (1981) method assumes constant soil properties and constant, instantaneously applied load, both of which are rarely encountered in the field. The radial consolidation equations presented in this paper build on the simplicity of Hansbo's (1981) approach but have the flexibility to account for varying load and material properties.

The equations presented can also roughly account for long term degradation of wick drain performance, sometimes associated with kinking or partial drain clogging. The only existing way to account for drain efficiency is to introduce a well resistance term related to the permeability and length of the drain. Modern wick drains usually have potential to become ineffective only after large deformations lead to kinking and partial blockage. Using a well resistance term like Hansbo (1981) tends to slow consolidation at the start of analysis not at the end. When drains are 100% efficient then the excess pore pressure in the drain is zero. If flow out of the drain is retarded then the excess pore pressure in the drain is non-zero. Exactly what the non-zero value is remains uncertain. As an approximation it may be appropriate (and mathematically convenient) to assume that the pore pressure in the drain is a fraction of the average pore pressure in the soil. In such a case pore pressure will still decay to zero but at a slower rate than if the drain pore pressure was always zero. The piecewise treatment of this drain efficiency parameter with time allows the clogging/kinking effects to be 'switched on' at a certain time. Indeed it is the piecewise treatment of all properties in the equations below that provide great flexibility. The equations are not 'hard wired' with a constitutive relationship and so it is up to the user to exercise judgment as to what period of time a soil or drain property is appropriate for. Thus the user can quickly assess many 'what if' scenarios: what if load gradually decreases due to submergence of fill? What if for a period the consolidation coefficient of a structured soil approaches zero? What if the smear effect decreases over time? What if I install more wick drains between existing drains?

## 2 DERIVATION



Hansbo (1981) presented the following radial consolidation equations for equal strain conditions in an axisymmetric unit cell (Figure 1):

$$\bar{u} = \bar{u}_0 \exp\left[\frac{-8T_h}{\mu}\right] \quad (1)$$

where,  $\bar{u}$  = average excess pore water pressure,  $\bar{u}_0$  = initial excess pore water pressure and  $T_h$  = horizontal time factor defined by:

$$T_h = \frac{c_h t}{4r_e^2} \quad (2)$$

$c_h$  = horizontal coefficient of consolidation,  $r_e$  = equivalent drain influence radius,  $t$  = time  
 $\mu$  = geometry and smear zone parameter. For a smear zone with constant permeability  $\mu$  is given by:

$$\mu = \frac{n^2}{n^2 - 1} \left( \ln \frac{n}{s} + \kappa \ln s - \frac{3}{4} \right) + \frac{s^2}{n^2 - 1} \left( 1 - \frac{s^2}{4n^2} \right) + \frac{\kappa}{n^2 - 1} \left( \frac{s^4 - 1}{4n^2} - s^2 + 1 \right) \quad (3)$$

$$n = r_e / r_w \quad (4)$$

$$s = r_s / r_w \quad (5)$$

$$\kappa = k_h / k_s \quad (6)$$

where,  $r_w$  = equivalent drain radius,  $r_s$  = smear zone radius,  $k_h$  = horizontal permeability in the undisturbed zone,  $k_s$  = horizontal permeability in the smeared zone. Other expressions for  $\mu$  can be found for non-constant smear zone permeability distributions Walker and Indraratna (2006), Walker and Indraratna (2007).

The penultimate step in the derivation of Equation (1) is the differential equation:

$$\bar{u} = \frac{4r_e^2 \mu}{8c_h} \left( \frac{\partial \Delta \sigma}{\partial t} - \frac{\partial \bar{u}}{\partial t} \right) + w \quad (7)$$

where,  $\Delta \sigma$  = change in load or total stress.  $w$  = excess pore water pressure in the drain. Equation (1) is produced when Equation (7) is solved with  $\partial \Delta \sigma / \partial t = 0$  (i.e. constant loading),  $w = 0$  (i.e. excess pore pressure in the drain is zero) and the initial condition of  $\bar{u} = \bar{u}_0$  at  $t = 0$  (i.e. instantaneous loading).

The somewhat limiting assumptions described above need not be followed. Consider the  $i$ th loading stage in a multiple ramp loading/unloading scenario as in **Figure 2**. The load varies in a linear fashion from  $\Delta\sigma_{i-1}$  at time  $t_{i-1}$  to  $\Delta\sigma$  at time  $t_i$ . Assume that between loading stages soil and drain properties may vary but are constant within each stage. Furthermore, assume that the excess pore pressure in the drain is not zero but a constant multiple of the excess pore water pressure in the soil (i.e.  $w = \bar{u}\theta$ ). With the modified assumptions Equation (7) can be written:

$$\bar{u}(1-\theta) = \Delta\dot{\sigma}_i - \frac{\partial\bar{u}}{\partial\tilde{T}}\eta_i \quad (8)$$

where

$$\Delta\dot{\sigma}_i = \frac{\Delta\sigma_i - \Delta\sigma_{i-1}}{(\tilde{T}_i - \tilde{T}_{i-1})\eta_i} \quad (9)$$

$\tilde{T}$  = a reference time factor given by:

$$\tilde{T} = 8\bar{c}_h t / 4\bar{r}_e^2 \bar{\mu} \quad (10)$$

The soil/drain parameters that make up  $\tilde{T}_h$  are any convenient reference values.

$\eta_i$  = the ratio of soil/drain properties in the  $i$ th loading stage to the reference values of properties in  $\tilde{T}_h$ . That is:

$$\eta_i = \frac{c_{hi}}{r_{ei}^2 \mu_i} \bigg/ \frac{\bar{c}_h}{\bar{r}_e^2 \bar{\mu}} \quad (11)$$

If soil properties remain constant throughout analysis then  $\eta$  will always be unity.  $c_h$  might change due to different loading and unloading stiffness.  $r_e$  might change if extra drains are installed between existing drains.  $\mu$  might change if the smear zone becomes less pronounced during consolidation. Using the initial condition  $\bar{u} = \bar{u}_{i-1}$  at  $t = t_{i-1}$  Equation (8) can be solved to give the excess pore pressure in the  $i$ th loading increment:

$$\bar{u}_i = \exp\left[-(\tilde{T}_{is} - \tilde{T}_{if})(1-\theta_i)\right] \left[ \bar{u}_{i-1} + \frac{\Delta\dot{\sigma}}{(1-\theta)} \left( \exp\left[(\tilde{T}_{is} - \tilde{T}_{if})(1-\theta_i)\right] - 1 \right) \right] \quad (12)$$

where

$$\tilde{T}_{is} = \min[\tilde{T}_{i-1}, \tilde{T}] \eta_i \quad (13)$$

$$\tilde{T}_{if} = \min[\tilde{T}_i, \tilde{T}] \eta_i \quad (14)$$

By taking the pore pressure at the end of a loading increment as the initial condition at the start of the following loading increment the pore pressure at any time is found to be:

$$\bar{u} = \sum_{i=1}^m \bar{u}_i \quad (15)$$

where

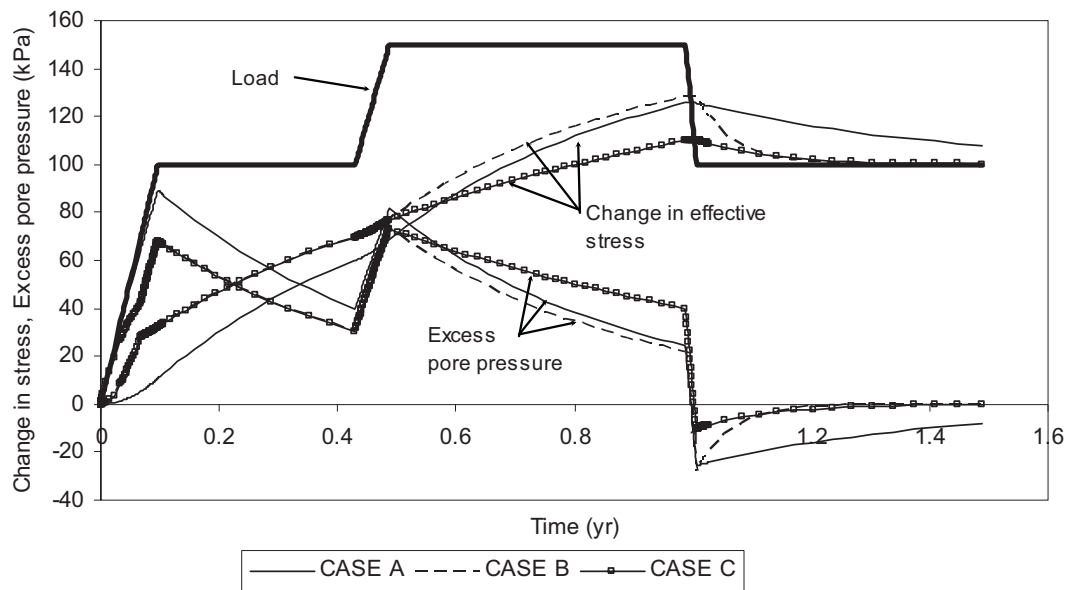
$$\bar{u}_0 = \Delta\sigma_0 \quad (16)$$

and  $u_i$  is from Equation (12).

The formulation of  $\tilde{T}_{is}$  and  $\tilde{T}_{if}$  prevents the need to determine which loading step the current time falls in. Contributions from future loading steps will evaluate to zero. Computer code for implementing the above equations in Microsoft Excel is given in APPENDIX A.

### 3 EXAMPLE

Consider a wick drain configuration where  $r_e=0.8$  m,  $n=32$ ,  $s=5$ ,  $\kappa=5$  (giving  $\mu=9.07$ ) under the loading scheme: load to 100 kPa over 5 weeks, wait for 4 months, load to 150 kPa over 3 weeks, wait for 6 months, unload to 100 kPa over 1 week, wait 6 months. For a constant  $c_h=7$  m<sup>2</sup>/yr the excess pore pressures and effective stress increase under the above loading scheme is shown in Figure 3 as CASE A. The effective stress increase is the excess pore pressure subtracted from the load. CASE B is similar to CASE A but it is assumed that until the effective stress increases by 30 kPa the soil is overconsolidated (i.e. preconsolidation stress is 30 kPa above the insitu effective stress) and so has a higher  $c_h$  of 49 m<sup>2</sup>/yr (i.e.  $\eta=7$ ). By trial and error, with the higher  $c_h$  30 kPa of excess pore pressure was dissipated in 3.5 weeks. Also, upon unloading the soil again becomes overconsolidated with  $\eta=7$ . CASE C is as per CASE B but when the load reaches 150 kPa the wick drains are assumed to become partially clogged and are 50% efficient ( $\theta=0.5$ ).



**Figure 3 Excess pore pressure and effective stress change vs time**

Effective stress increase is important when considering undrained shear strength gain and hence stability of embankments. CASE B and C have more rapid effective stress gain at early times and so will have faster strength gain than CASE A. The second loading stage may be able to be brought forward if this faster strength gain is sufficient. If the wick drains become clogged then the highest effective stress increase experienced, and thus strength gain, may be less than that predicted without clogging (CASE C vs CASE B) which may lead to stability issues. Finally if the  $c_h$  is not adjusted for unloading (CASE A vs CASE B) then the time for the end of primary consolidation is unrealistically long.

## 4 CONCLUSION

A flexible, easy to implement approach for investigating consolidation problems involving radial drainage has been presented (including computer code, see APPENDIX A). Material and geometry properties vary piecewise-constant with time while loading/unloading varies piecewise-linear with time. A new drain efficiency parameter was introduced to model the detrimental effects of drain clogging and kinking that may occur in the latter stages of consolidation. The consolidation equations build on the simplicity of Hansbo's (1981) approach, with the ability to more realistically assess of a wide range of radial consolidation problems.

## REFERENCES

- Hansbo, S. (1981). Consolidation of fine-grained soils by prefabricated drains. Proc. *10th International Conference Soil Mechanics and Foundation Engineering*, Stockholm, 677-682.
- Walker, R. (2006). "Analytical solutions for modeling soft soil consolidation by vertical drains." PhD thesis, University of Wollongong, NSW, Australia.
- Walker, R. & B. Indraratna (2007). *Vertical drain consolidation with overlapping smear zones. Geotechnique*, 57(5), 463-467.
- Walker, R. & B. Indraratna (2006). Vertical drain consolidation with parabolic distribution of permeability in smear zone. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 132(7), 937-941.

## APPENDIX A – COMPUTER CODE

The above equations can be implemented in Microsoft Excel, and then used like any other Excel function, by copy and pasting the code shown below into a VBA module (Tools...Macro...Visual Basic editor...Insert...Module). Note that the variables 'times' ( $t_0, t_{1, \dots}$ ), 'loads' ( $\Delta\sigma_0, \Delta\sigma_{1, \dots}$ ), 'etas' ( $\eta_0, \eta_{1, \dots}$ ) and 'ws' ( $\theta_0, \theta_{1, \dots}$ ) must be input as columns.

```
Function MultiRampConsol(ch, re, mu, t, times, loads, etas, ws) As Double
    Dim F As WorksheetFunction
    Set F = Application.WorksheetFunction
    Dim eta0, A1, u, term, The1, The2, I
    timesA = times: loadsA = loads: etasA = etas: wsA = ws
    eta0 = 8 * ch / (4 * re ^ 2 * mu)
    u = loadsA(1, 1)
    For I = LBound(timesA, 1) To UBound(timesA, 1) - 1
        A1 = (loadsA(I + 1, 1) - loadsA(I, 1)) / _
            (timesA(I + 1, 1) - timesA(I, 1)) / eta0 / etasA(I, 1)
        The1 = F.Min(timesA(I, 1), t) * eta0 * etasA(I, 1)
        The2 = F.Min(timesA(I + 1, 1), t) * eta0 * etasA(I, 1)
        term = A1 / (1 - wsA(I, 1)) * (Exp((The2 - The1) * (1 - wsA(I, 1))) - 1)
        u = (u + term) * Exp(-(The2 - The1) * (1 - wsA(I, 1)))
    Next I
    MultiRampConsol = u
End Function
```