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**VERTICAL DRAIN CONSOLIDATION ANALYSIS IN ONE, TWO AND THREE
DIMENSIONS**

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Submitted to: Computers and Geotechnics

First submitted Apr 11, 2011

Revised submission Jun 29, 2011

VERTICAL DRAIN CONSOLIDATION ANALYSIS IN ONE, TWO AND THREE DIMENSIONS

Rohan Walker¹

1 Abstract

A new method, the η or 'eta' method, for modeling consolidation by vertical and horizontal drains is presented. The approach is applicable in one, two and three dimensional as well as axisymmetric cases. Material and geometry properties are familiar from unit cell vertical drain analysis and are consistent across dimensions. An uncoupled finite element method (FEM) program is used to test the efficacy of the new approach. Because drains are not explicitly modeled in the finite element mesh, mesh complexity and computational time are greatly reduced. Unlike existing plane strain matching methods there is no special transformation of permeability or drain properties. The analyses conducted indicate that the η method provides an efficient and consistent means of modeling drains in any dimension.

Keywords: vertical drains; PVD; smear zone; finite element method; consolidation;

2 Introduction

Prefabricated vertical drains (PVD) are a ground improvement technique used to hasten the consolidation process in low permeability soils. Drains, typically 100 mm $\times$ 4 mm channeled rectangular plastic cores wrapped in a permeable geotextile filter, are installed

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in a 1 m to 3 m square or triangular pattern over the footprint of an embankment. Because PVD are small compared to the area being treated and so numerous, including each drain in an analysis can be computationally intensive. To reduce computational effort the full three dimensional multi drain situation is usually simplified. Single drain “unit cell” models are applicable beneath the center line of symmetrical embankments on symmetrical soil profiles. Two dimensional plane strain models are suitable for modeling long embankments with a uniform cross section. Three dimensional effects are important for circular and square embankments, embankment ends/corners and soil with complicated sloping stratigraphy. For the later cases where three dimensional effects are important, using a plane strain analysis can lead to significant errors in estimating lateral displacement and stability ([1][2][3]).

Unit cell modeling is commonly used as it is amenable to analytical solution, most notably Barron [4] and Hansbo [5]. The square/hexagonal influence area of each drain, the reduced permeability smear zone next to the drain caused by drain installation, and the rectangular drain itself are converted to equivalent circles for axisymmetric analysis. Vertical settlement and consolidation rate can then be obtained. For a multi drain plane strain analysis, lateral displacement and other full embankment behavior can be investigated. Typically using the finite element method (FEM), the plane strain properties of drain spacing, soil permeability or drain permeability are altered, alone or in combination, such as to match consolidation times with the axisymmetric unit cell approach. A number of different axisymmetric to plane strain matching procedures have been developed: [6] [7][8][9][10][11][12][13][14]. Indraratna et al.[3] developed a

separate matching procedure converting multiple drains in a circular region to the simpler axisymmetric case of drainage rings.

The details of each matching procedure mentioned above are not important for this paper other than that all methods modify the value of one or more properties and all except Chai et al. [12] still require drains in the finite element mesh. Because Chai et al. [12] alters gross vertical permeability and removes drains from the mesh it can be easily used in a three dimensional analyses. However, the equivalent vertical permeability approach has faster consolidation closer to a drainage boundary and slower consolidation far from drainage boundaries so an unrealistic pore pressure distribution develops. Also, if soil depth varies, as might happen in a 3D model, then permeability would need to vary for different soil thicknesses. Attempts have been made to study multiple drains in full 3D: [15][16][17][18]. Only simple 3D geometries such as single rows of drains or large cubes of soil have been modeled because the fundamental problem, which motivated the original plane strain matching procedures, of too many elements in a 3D FEM mesh with explicit drains still exists. This paper aims to remove the need for matching procedures and the need for explicit inclusion of drains by introducing a method for implicitly including drains, vertical or horizontal, in one, two and three dimensions.

3 The η or ‘eta’ method

The equal strain radial consolidation equation of Hansbo [5] can be re-derived with a non-zero pore pressure in the drain (w) to give the following relationship between strain rate, $\partial\epsilon/\partial t$, and average excess pore pressure $\bar{u}$:

$$\frac{2k_h}{r_e^2 \gamma_w \mu} (\bar{u} - w) = \frac{\partial \varepsilon}{\partial t} \quad (1)$$

Where, k_h is the undisturbed horizontal permeability; r_e is the vertical drain influence area radius; γ_w is the unit weight of water; and μ is a parameter dependent on geometry and permeability distribution discussed below. This paper proposes that drains can be modeled in one, two and three dimensions by incorporating Equation (1) into the relevant continuity equation as an additional strain term. A general 3D continuity equation, with drainage to both vertical and horizontal drains, is thus:

$$\left[-\frac{k_x}{\gamma_w} \frac{\partial^2 u}{\partial x^2} - \frac{k_y}{\gamma_w} \frac{\partial^2 u}{\partial y^2} - \frac{k_z}{\gamma_w} \frac{\partial^2 u}{\partial z^2} + \frac{k_x}{\gamma_w} \eta_x (u - w_x) + \frac{k_y}{\gamma_w} \eta_y (u - w_y) + \frac{k_z}{\gamma_w} \eta_z (u - w_z) \right] = \frac{\partial \varepsilon}{\partial t} \quad (2)$$

where, in the appropriate direction,

$$\eta = \frac{2}{r_e^2 \mu} \quad (3)$$

Because Equation (1) is based on average pore pressure $\bar{u}$, whereas Equation (2) employs only pore pressure u , any value of pore pressure obtained within a drain treated area using Equation (2) should be interpreted as pore pressure spatially averaged over the cross sectional influence area of a single drain. Pore pressure between drains can be found from the average value by equations discussed in Section 4 below.

Equation (2) is general and allows for a large variety of behavior to be modeled. Appropriate terms can be omitted from Equation (2) when dealing with less than three dimensions. The boundary between areas with and without drains is termed the influence

boundary. The influence boundary is the combined extreme extent of all the individual drain influence areas determined from a conventional unit cell analysis. As in unit cell analysis, where a single-drain influence area is approximated as a circle, reasonable results are obtained if the potentially complicated multi-drain influence boundary is approximated by smooth lines enclosing an area equivalent to the sum of all single drain influence areas. In areas without drains (outside the influence boundary or beyond the end of drains) η is simply set to zero and Equation (2) reverts to the conventional continuity equation. In areas with drains (inside the influence boundary) η might be set to zero initially to simulate a period before drain installation. The primary reason for including the non-zero drain pore pressure term w , is to allow negative vacuum pressures in a drain. The w term is also used in total pore pressure formulations in which it is set to the hydrostatic pressure in the drain at a particular depth. The parameters k , η , and w may vary over space and time. For example, permeability may decrease with effective stress, η may decrease with time to simulate drain clogging, and vacuum pressure may be switched off at some time. Any constitutive model can be substituted for $\partial\varepsilon/\partial t$. For convenience the method is referred to as the η or ‘eta’ method.

One dimensional variants of Equation (2) have been used to obtain analytical and numerical solutions for combined vertical and radial flow problems. The FEM software program ‘PVD-SD’ uses the approach [19]. Tang and Onituska [20] study a single layer under arbitrary load. Tang and Onituska [21] study double layered ground. Wang and Jiao [22] look at partially penetrating drains. Walker and Indraratna [23] and Walker et al. [24] present a solution for multi-layered ground with depth/time varying load. The

above authors justify their use of a one-dimensional formulation by assuming that flow in the vertical direction of a unit cell is based on the average hydraulic gradient at a particular depth. An interpretation when extended beyond the unit cell to multiple drains is that at any point in the drain-treated-area a sink term removes water from the system proportional to the pore pressure difference between that point and a specified pore pressure in the drain.

With large scale problems with multiple drains involving complex soil models and geometry (e.g. embankments), the analytical consolidation solutions become inadequate and recourse is made to numerical methods. By using Equation (2), the modeling of drains in two and three dimensions by numerical methods, particularly the finite element method, is greatly simplified over existing methods.

4 η and μ Parameters

Different μ parameters, used in Equations (1) and (2), can be derived based on different geometry and permeability assumptions in a unit cell. Hansbo[5] developed μ for a single smear zone with constant permeability or an ideal system with no smear. Walker and Indraratna[25] used a parabolic distribution of permeability in the smear zone. Walker and Indraratna[26] used a linear variation of smear to investigate overlapping smear zones. Basu et al.[27] developed a number of three-zone μ expressions with the two zones closest to the drain having either a constant or linear variation of permeability. As the smear zone configurations become more complicated so do the expressions for μ .

A more versatile approach to obtaining values of μ , which can yield numerically equivalent values to the above methods, is to divide the radial distribution of permeability into m zones each with constant horizontal permeability, k_{hi} , as in **Error! Reference source not found.**

Following the derivation of Hansbo[5], Walker[28] produced the following expression for μ based on piecewise constant permeability:

$$\mu = \frac{n^2}{n^2 - 1} \sum_{i=1}^m \kappa_i \left[\frac{s_i^2}{n^2} \ln \left(\frac{s_i}{s_{i-1}} \right) - \frac{1}{2} \left(\frac{s_i^2 - s_{i-1}^2}{n^2} \right) - \frac{(s_i^2 - s_{i-1}^2)^2}{4n^4} \right] + \psi_i \left(\frac{s_i^2 - s_{i-1}^2}{n^2} \right) \quad (4)$$

where,

$$\psi_i = \sum_{j=1}^{i-1} \kappa_j \left[\ln \left(\frac{s_j}{s_{j-1}} \right) - \frac{1}{2} \left(\frac{s_j^2 - s_{j-1}^2}{n^2} \right) \right] \quad (4a)$$

n is the ratio of influence radius to drain radius, r_m/r_0 ; s_i is the ratio of the i^{th} zone radius to drain radius, r_i/r_0 ; κ_i is the ratio of the m^{th} permeability to the i^{th} permeability, k_{hm}/k_{hi} . The r_0 , r_m and k_{hm} in index notation correspond respectively to the more conventional notation of drain radius r_w , influence radius r_e and undisturbed horizontal permeability k_h . Note that for Equation (4a) $\psi_1 = 0$. Equation (4) reduces to the μ expressions of Hansbo[5] for no smear and one smear zone when one and two zones are considered. If well resistance for a drain of drainage length l and discharge

capacity q_w is included in the derivation then at depth z an additional μ term is added to that of Equation (4):

$$\mu_w = \frac{k_m}{q_w} \pi z (2l - z) \left(1 - \frac{1}{n^2} \right) \quad (5)$$

An average value of μ_w for the whole soil layer can be obtained by integrating Equation (5) with respect to z between 0 and $2l$, yielding:

$$\mu_{w\text{average}} = \frac{2k_m l^2}{3q_w} \pi \left(1 - \frac{1}{n^2} \right) \quad (5a)$$

The pore pressure within the i^{th} zone, u_i , is described by:

$$u_i = \frac{\bar{u} - w}{\mu + \mu_w} \left[\kappa_i \left(\ln \left(\frac{r}{r_{i-1}} \right) - \frac{1}{2} \left(\frac{r^2}{n^2 r_0^2} \right) \right) + \psi_i + \mu_w \right] - w \quad (6)$$

Equations (4) through (6) describe a circular influence area draining to a circular drain. Corresponding expressions for a plane strain unit cell where a cell of half width B drains to a drainage wall of half width b_w are given in Equations (7) to (9) below. For the plane strain case **Error! Reference source not found.** is reinterpreted with radius coordinate r_i replaced with horizontal coordinate x and width b_i , and in Equations (1) and (3) influence radius r_e is replaced with influence width B .

$$\mu = \frac{n}{n-1} \sum_{i=1}^m \kappa_i \left[\frac{s_i}{n} \left(\frac{s_i - s_{i-1}}{n} \right) - \frac{1}{3} \frac{s_i}{n} \left(\frac{s_i^2 - 3s_{i-1}^2}{n^2} \right) + \frac{1}{3} \frac{s_{i-1}^2}{n^2} \left(3 - \frac{2s_i}{n} \right) \right] + \psi_i \left(\frac{s_i - s_{i-1}}{n} \right) \quad (7)$$

$$\psi_i = \sum_{j=1}^{i-1} \kappa_j \left[2 \left(\frac{s_j - s_{j-1}}{n} \right) - \left(\frac{s_j^2 - s_{j-1}^2}{n^2} \right) \right] \quad (7a)$$

$$\mu_w = \frac{2}{B} \frac{k_m}{q_w} z(2l - z) \left(1 - \frac{1}{n}\right) \quad (8)$$

$$\mu_{w\text{average}} = \frac{4l^2}{3B} \frac{k_m}{q_w} \left(1 - \frac{1}{n}\right) \quad (8a)$$

$$u_i = \frac{\bar{u} - w}{\mu + \mu_w} \left[\kappa_i \left(2 \left(\frac{x}{nb_0} - \frac{s_{i-1}}{n} \right) - \left(\frac{x^2}{n^2 b_0^2} - \frac{s_{i-1}^2}{n^2} \right) \right) + \psi_i + \mu_w \right] + w \quad (9)$$

To obtain the plane strain μ value when the drain is not explicitly modeled (i.e. drain thickness $b_w = 0$ and pore pressure is set equal to zero at $x = 0$), in Equations (7) and (9), s_i becomes b_i , n becomes B , $n/(n-1)$ becomes 1. The well resistance term in Equation (8) is obviously meaningless with a drainage wall of zero thickness.

Analytical expressions for η are based on axisymmetric approximations. Other arrangements are possible. Wang and Jiao[22] produced an expression for a polygonal drain draining a polygonal influence area. Laboratory studies ([29][30][31]) indicate that the shape of the smear zone for a rectangular drain/mandrel is elliptical. Cavity expansion analysis by Sathananthan et al.[32] and Ghandeharioon et al.[33] indicate that the size and properties of the smear zone may vary with material properties and insitu stresses, so η may vary with time and depth. It is possible to determine an η value by considering more realistic geometry (rectangular drain, elliptical smear zone, hexagonal influence area etc.) and permeability distributions. For an uncoupled analysis under instantaneous load with undisturbed coefficient of consolidation of unity and zero pore pressure on the drain boundary, the average pore pressure in a cross section through the drain, should decay almost exponentially such that η can be found by fitting the curve

$\bar{u} = \alpha \exp[-\eta t]$ (α should be very close to the initial value of pore pressure). Horizontal drains are occasionally employed, usually for dredging projects ([34][35]). Determining appropriate η values for horizontal drains may only be achievable with numerical methods as horizontal and vertical permeability may be different.

5 Implementation

To test the η method, Equation (2) was discretized with the finite element method and a variety of analyses were conducted. Movement of the soil particles was assumed to be vertical only, so strain was related to volume compressibility, m_v , and pore pressure change by:

$$\frac{\partial \varepsilon}{\partial t} = -m_v \frac{\partial u}{\partial t} \quad (10)$$

The analysis was thus uncoupled (i.e. no displacement degrees of freedom only pore pressure degrees of freedom). The program used for the finite element analyses was a modified version of program “p86” from the book by Smith and Griffiths[36]. Originally program “p86” was for general two (plane) or three dimensional analysis of the consolidation equation using implicit time integration with the “theta” method. The author modified “p86” source code to allow input of volume compressibility, axisymmetric analysis as well as element integrations involving η and w . FEM meshes were first prepared in ‘Gmsh’ by Geuzaine and Remacle[37], an open source 2D and 3D finite element mesh generation and post processing program. Meshes are output from Gmsh in a text file. The author wrote a Fortran program to take the mesh file generated

from Gmsh, apply material properties and boundary/initial conditions, and produce a new text file compatible with program 'p86'. Computer program source code is provided as supplementary material.

To demonstrate the validity of the η method, average pore water pressure for the entire mesh calculated with the η method is compared with the average pore pressure of the entire mesh calculated with the conventional finite element approach where drains are explicitly modeled in the mesh. Any mesh area without drains is included in the average calculations. Average pore pressure is calculated by numerically integrating the pore pressure across the volume of each element at a particular time, summing the contribution of each element and dividing by the total mesh area (element areas are also calculated by numerical integration). The analyses and associated results are presented below.

5.1 Unit cell - single drain, single soil layer

Walker and Indraratna [23] and Walker et al.[24] have already demonstrated that a one dimensional η formulation is excellent for modeling a unit cell. Unit cell examples are presented here and in section 5.2 to check the authors FEM program and investigate some nuances of the η method. **Error! Reference source not found.** shows an axisymmetric mesh with a single drain fully penetrating a single soil layer. Geometric and material properties, including the well resistance factor $D = q_w / k_h l^2 = 0.58$ ([38], l is the drainage

path), are shown in Table 1 (in Table 1 horizontal permeability has subscript x). A number of cases are considered:

- No well resistance, surcharge only – drain elements are removed; pore pressure on top and left hand side boundary set to zero; initial pore pressure is set to 100 kPa.
- No well resistance, surcharge and vacuum – drain elements removed; pore pressure on top and left hand side boundary set to -50 kPa; initial pore pressure set to 50 kPa.
- Well resistance, surcharge – drain elements included; pore pressure on top boundary set to zero; initial pore pressure set to 100 kPa.
- Well resistance, surcharge and vacuum – drain elements included; pore pressure on top boundary set to -50 kPa; initial pore pressure set to 50 kPa.

For the η analyses the same mesh is used but the drain and smear zone elements are given the same properties as the undisturbed soil and only pore pressure on the top boundary is fixed. For vacuum the variable w in the formulation is set to -50 kPa. For well resistance the μ and thus η variable in the formulation varies with depth according to Equation (5). Average pore pressure vs time plots for conventional FEM analyses and the η method are presented in **Error! Reference source not found.**. Also included are results using the approach of Walker and Indraratna[23] which is a semi analytical one dimensional form of the η method. Note that the analyses come in pairs, with or without well resistance. Well resistance cases are slower to consolidate and so are further to the right in **Error! Reference source not found.**. There is insignificant difference between the three types of analysis.

5.2 Unit cell – Single partially penetrating drain, three soil layers

Error! Reference source not found. shows an axisymmetric mesh with three soil layers and a single drain penetrating the top two layers. Geometric and material properties are shown in Table 1. The well resistance factor in each layer is calculated using the soil permeability of each layer and the total length of the drain. The mesh uses 564 eight node second order quadrilateral elements. The same cases as used in the single layer analyses are examined here. Results are presented in **Error! Reference source not found.**

The only case with any significant difference between the three methods is for the case with well resistance where the conventional FEM analysis exhibits slightly faster consolidation. Differences are explained by small amounts of horizontal flow in the bottom layer that are not captured by the η method. In the conventional analysis there is a small bulb of reduced pore pressure at the base of the drain extending slightly into the third layer. Depending on geometry and permeability the drainage path of pore fluid in the third layer can be diagonally towards the drain rather than vertical. For the η method if there are no drains in a layer then there is no horizontal flow so the effect is not captured. A similar effect might occur when using plane strain geometry matching if the transformed drain spacing is much greater than the actual drain spacing. To illustrate how horizontal permeability in the bottom layer affects consolidation, the surcharge only analysis with well resistance is repeated with different horizontal permeability in the bottom layer. **Error! Reference source not found.** shows the slightly increased rate of

pore pressure dissipation as the horizontal permeability is increased. It is seen that the reference case of zero horizontal permeability is still faster than the η analysis. This discrepancy is probably due to the fact that drain elements are included in the average pore pressure calculation whereas they have no effect in the η approach.

Thus far only comparisons of mesh averaged pore pressure have been made. Figure 6 shows the pore pressure profile with depth along the right hand side of the unit cell for the surcharge only case. For the η analysis the average pore pressure at a particular depth output from the FEM program was used with Equation (6) to calculate the pore pressure at $r=0.6$ m. There is insignificant difference between the profiles calculated with the conventional FEM method and the η method. A slight discontinuity occurs at the soil layer interfaces because different μ_w values are used on each side of the interface.

Also shown in Figure 6 is the pore pressure profile calculated with the matching procedure of Chai et al. [12]. Only vertical drainage is allowed and an equivalent vertical permeability, k_{ve} , in each layer is calculated according to:

$$k_{ve} = \left(1 + \frac{2.5l^2}{4\mu r_e^2} \frac{k_h}{k_v} \right) k_v \quad (11)$$

where μ uses the average well resistance term in Equation (5a). The derived properties are $\mu_1 = \mu_2 = 3.797$, $k_{ve1} = 25.19$ m/yr, $k_{ve2} = 50.38$ m/yr without well resistance and $\mu_1 = 6.051$, $\mu_2 = 8.311$, $k_{ve1} = 16.18$ m/yr, $k_{ve2} = 24.1$ m/yr with well resistance. The Chai et al.[12] method displays significant differences that are also reflected in the slower

dissipation of average pore pressure shown in Figure 7. With only vertical drainage there is no way for a one-way drainage condition to capture the lower pore pressure in the middle layer. Chai et al. [12] test their method with a two layer system where the top layer is more permeable than the second and obtain good average pore pressure match. This example shows that the Chai et al.[12] method may be unsuitable in some cases and that the proposed η yields an excellent pore pressure match at any depth.

Vertical drain treatment areas are finite in extent, so the interaction between the drain treated and untreated areas needs investigating and is done so in the next two examples.

5.3 *Horizontal flow to multiple drains in a square*

100 kPa of initial pore pressure dissipates in a 12 m $\times$ 12 m square to nine circular drains arranged in a 2.5 m square pattern. Geometric and material properties are shown in Table 1. The mesh consists of 5724 three node first order triangles shown in **Error! Reference source not found.** For the η analysis, drains and smear zones are removed, only the internal 7.5 m square has a non-zero η value and after re-meshing 670 elements were used. The internal square is simply half the drain-spacing-distance beyond the outermost drains. Results for the two analyses are presented in **Error! Reference source not found.** There is insignificant difference between the conventional analysis and the η method. Figure 10 shows the spatial distribution of pore pressure along a horizontal line through the middle of the mesh at time 1 yr and 4 yr. In the drain treated area the raw η method pore pressure is corrected with Equation (6) to give the proper pore pressure around the drains. The correction will always artificially produce a step change in pore

pressure at the boundary between drain and non drain treated areas (3.75m in Figure 10) because the corrected pore pressure at the edge of a drains influence area will always increase the pore pressure above the average which will be immediately adjacent to a non-corrected pore pressure in the non treated area. There are slight differences in the pore pressure profiles, especially at the earlier time but the η method still provides a good match to the conventional FEM approach both inside and outside of the drain treated area.

5.4 *Horizontal flow to multiple drains in a circle*

100 kPa of initial pore pressure dissipates in a circle of radius 7 m to 19 circular drains arranged in a 2.0 m triangle pattern. Geometric and material properties are shown in Table 1. The mesh consists of 10888 three node first order triangles shown in **Error! Reference source not found.** For the η analysis, after re-meshing, 708 elements were used. The choice of internal treated area is not as obvious as in **Error! Reference source not found.** because outer drains are not equidistant from a circular boundary. The treated circle was chosen with radius 4.58 m to give an area equal to 19 circular influence areas with radius 1.05 m. If the drains are removed from the mesh then the problem becomes symmetrical and an axisymmetric analysis can be performed. The results of such an analysis using a single row of 25 8-node second order quadrilateral axisymmetrical elements is shown along with the other analyses in **Error! Reference source not found.** There is insignificant difference between the conventional analysis, the η method or the axisymmetric- η analysis.

5.5 *Three drains in three dimensions*

The final example is a full three dimensional analysis. 100 kPa of initial pore pressure in a $2\text{ m} \times 9\text{ m}$ rectangle dissipates to a single line of three drains spaced at 2.0 m centers. The model is 1 m thick in the z -direction (into the page). Geometric and material properties are shown in Table 1. The three dimensional mesh consists of 109539 four node first order tetrahedral elements and is shown in **Error! Reference source not found.** (the drains are where the mesh lines are densest). The treated boundary is half the drain-spacing-distance beyond the outermost drains (9 m). So many elements are used because Gmsh automatically uses a very fine mesh to discretise the circular drains. For the η analysis, after re-meshing, 1080 8 node first order hexahedral elements were used. Without the drains the problem is a plane strain problem so an additional two dimensional plane strain η analysis is performed with 240 8-node second order quadrilateral elements.

The results, shown in **Error! Reference source not found.**, show slightly faster consolidation for the conventional analysis in the early stages than for the η analysis. This can be explained by two factors. The first is too coarse a mesh discretization in the z -direction for the conventional analysis. Unfortunately, using a finer mesh exceeds the ability of author's computer to solve the problem. The author could replicate this initial overshoot error by using a similar vertical mesh discretization in the 2D plane strain analysis. The second cause of the difference is that the η method is based on equal strain assumptions whereas the conventional method as modeled in this paper is essentially a free strain solution. Consider a unit cell. For the free strain case the initial

condition is 100 kPa everywhere. As soon as the analysis starts the pore pressure close to the soil-drain boundary drops rapidly so there is a corresponding rapid initial drop in average pore pressure. For the equal strain case the initial condition is an average pore pressure of 100 kPa. Inspection of Equation (9) shows that at time zero the equal strain case already has low pore pressure close to the drain and a value higher than 100 kPa at the cell periphery. There is no initial rapid drop in average pore pressure because the pore pressure distribution already has the shape that the free strain approach only develops after some time. The effect is only noticeable in this example and not the others because there is no smear zone in this example. When there is a smear zone the rapid drop in pore pressure in the free strain case is confined to the smear zone which is a small proportion of the mesh volume and so the average pore pressure is not greatly affected. In equal strain the smear zone forces the pore pressure in most of the unit cell to be very close to the average value and so the initial pore pressure distribution shape is much closer to that of the free strain case. It is the difference in shape of the pore pressure distributions just after the analysis starts, (calculated in the free strain case, implied in the equal strain case), that leads to more rapid initial consolidation of the free strain case. To show the effect without complications from three dimensions, **Error! Reference source not found.**, shows an equal strain vs free strain consolidation plot for a single drain with only horizontal flow (properties and geometry are the same as in the present example). The initial free strain drop is clearly visible. As the two solutions converge at larger times the choice of equal or free strain is somewhat irrelevant. The behavior in the field is expected to be somewhere between the two and practitioners are generally only concerned with later times. Also, even though the discrepancy is more pronounced for

drains with small influence areas (i.e. low $n = r_e/r_w$ values) the behavior at closer drain spacing is expected to be closer to equal strain conditions because it is easier for arching in fill materials to develop over a small distance and redistribute the load. Awareness of the free-strain equal-strain discrepancy is useful when validating numerical methods, which may use free strain, against easily calculated free strain solutions. Knowing beforehand that the solutions will be slightly different, considerable consternation can be avoided.

6 Discussion

6.1 Advantages

The most obvious advantage of using the η method is reduced FEM mesh complexity. Without explicit drains in the model the user is not forced to use tiny elements close to high hydraulic gradient areas next to the each drain. Lower numbers of small elements lead to easier selection of time step. Reducing the number of elements needed to analyze a problem has many beneficial consequences. It takes less time to define a mesh; program run time and file size is reduced. These coupled with not needing to convert permeability as in plane strain matching procedures lead to much shorter overall analysis time. Shorter total analysis time should not be underestimated. In the authors experience practitioners are hesitant to perform complicated FEM analyses, especially 3D models, simply because they take too long to set up and run.

In plane strain matching procedures the transformed variables tend to lose their physical meaning making it difficult to assess the plausibility or ‘correctness’ of a particular property without investigating the calculations performed to produce it. Beyond the normal conversions to obtain axisymmetric unit cell properties, and η itself, the η method requires no further manipulation of properties. The properties remain consistent across one, two and three dimensional analyses as well as in axisymmetric models. It is thus easy to scale up from simple analytical solutions to more complicated multi layer models.

Though not a dimensionless quantity the value of η provides a useful plausibility check to assess if the drain parameters are reasonable. A range of possible values for PVD without well resistance is between 3 m^{-2} ($r_w=0.025 \text{ m}$, no smear, 1 m triangular spacing, no smear) and 0.07 m^{-2} ($r_w=0.025 \text{ m}$, $s=5$, $\kappa=5$, 3 m square spacing). η also provides a simply means of comparing the accelerating effects on consolidation of drains across projects.

6.2 *Disadvantages*

In the spirit of Peck[39] quoting Terzaghi, “When you commit one of your ideas to print, emphasize every controversial aspect which you can perceive”, the disadvantages of the η method are discussed here.

One of the motivations behind the Chai et al.[12] plane strain matching procedure was to avoid the use of special drain elements (used in other matching procedures) that are not commonly available in FEM programs allowing the use of programs as is. Like the special drainage elements the η method cannot be used without first modifying a programs source code.

As identified in the three layer example with well resistance above the η method does not capture any horizontal fluid flow immediately beneath a partially penetrating drain meaning consolidation is slightly slower. This, as well as the free-strain equal-strain differences identified, are not expected to be important for small wick drains that yield high n ratios. However, the differences might be more pronounced if larger drainage inclusions such as stone columns or cement mixed columns, that produce low n ratios, are modeled. Further work is required in this area.

For the analyses presented no rigorous method was used in choosing the time step and mesh element size for each analysis. The stability properties of the Crank-Nicholson time stepping scheme[36] ($\theta=0.5$) and visual inspection of the pore pressure results were relied on to ensure each analysis was ‘well behaved’. Further work is needed to check if the η method requires different time stepping and mesh schemes than conventional consolidation analyses.

The expression allowing for well resistance in Equation (5) is an approximation as is the original expression of Hansbo[5] and the expressions used in the existing plane strain

matching procedures. It is an excellent approximation for homogenous soils but its use in multi layer problems is less well established. The rigorous treatment of well resistance is more complicated ([20]). Because no drains are explicitly modeled in the η method the effect of well resistance has to be prescribed by the user rather than developed as part of the solution process. For multilayered soil Chai et al. [12] recommend calculating well resistance terms using the full length of drain as the drainage path (l) or half the length if there is one way drainage. This appears reasonable; it achieved reasonable results in the three layer example above. However, multi layer systems with well resistance are very complicated. Different amounts of fluid enter the drain at different rates. PVD have reduced discharge with increased confining pressure ([40][41]) and may degrade/kink/clog over time. To date, no methods, including the η method account for these effects.

Based on equal strain unit cell consolidation, tested with an uncoupled finite program, there are a number of interesting areas where the η method has not or cannot be used. As no drains are modeled no reinforcement effect from the drains, however small, can be investigated. As there is no explicit horizontal movement of fluid to a drain then there can be no soil inhomogeneity between drains as investigated by Al-Tabbaa and Wood[42] and Pyrah et al.[43]. Fox et al.[44] models large strain effects in radial consolidation, Hansbo[45] and Ing and Xiaoyan[46] look at non-darcian flow. Both aspects are untested with the current implementation. Though vacuum pressure can be modeled it remains to be seen if the vacuum induced inward movement identified by Chai et al.[47] can be reproduced with the η approach. Finally, the method will need to be tested with a

coupled FEM formulation with complicated constitutive models as these are the situations that most interest geotechnical engineers.

7 Conclusions

A new method, the η or 'eta' method, for modeling drains in one, two, or three dimensions and in axisymmetric cases by modifying the continuity equation has been proposed. New expressions for determining η based on piecewise constant smear zone geometry and permeability are also presented. The η method was tested with an uncoupled finite element program. Drains are not explicitly modeled, greatly simplifying finite element meshes and reducing setup and analysis time. Comparison with conventional FEM analyses show that the η gives excellent simulation of pore pressure dissipation both inside and outside of drain-treated areas. Material and geometry properties are not modified as in existing plane strain matching procedures so properties maintain their physical meaning. The η method provides a simple, consistent and efficient way to model the effects of drains in analyses of any dimension.

8 Acknowledgements

The author wishes to thank the following people for their comments and suggestions in preparing this paper: Prof. Buddhima Indraratna and Dr. Cholachat Rujikiatkamjorn (University of Wollongong, Australia); Dr. Richard Kelly (Coffey Geotechnics, Australia).

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Property	Unit cell, 1 layer	Unit cell, 3 layers	Multi-drains in a square	Multi-drains in a circle	Three drains in 3D
h (m)	1.3	1.0, 1.3, 1.0	-	-	1
Drain spacing (m)	-	-	2.5 (sqr.)	2.0 (tri.)	2.0 (sqr.)
r_w (m)	0.025	0.025, 0.025, -	0.05	0.05	0.05
r_e (m)	0.6	0.6, 0.6, -	1.41	1.41	1.13
r_s (m)	0.1	0.1, 0.1	0.2	0.2	-
k_x (m/yr)	10	10, 20, 20	40	40	20
k_y (m/yr)	-	-	40	40	20
k_z (m/yr)	1	1, 2, 2	-	-	5
k_w (m/yr)	5000	25000, 25000, -	-	-	-
m_v (m ² /kN)	1	1, 1, 1	1	1	1
c_x (m ² /yr)	1	1, 2, 2	4	4	2
c_y (m ² /yr)	-	-	4	4	2
c_z (m ² /yr)	0.1	0.1, 0.2, 0.2	-	-	0.5
n	24	24	28.2	21.0	22.6
s	4	4	4	4	-
κ	2	2	2	2	-
μ	3.797	3.797, 3.797, -	3.96	3.658	2.37
η (m ⁻²)	1.463	1.463, 1.463	0.254	0.496	0.662
q_w (m ³ /yr)	9.8	49.1, 49.1, -	-	-	-
$D = q_w / k_h t^2$	0.58	0.93, 0.46, -	-	-	-
γ_w (kN/m ³)	10	10	10	10	10

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