2011

Structure nonlinearity and response of laterally loaded piles

Wei Dong Guo
University of Wollongong, wdguo@uow.edu.au

B T. Zhu
Arup Pty Ltd, Furtitude Valley, Qld

https://ro.uow.edu.au/engpapers/4542

Publication Details
STRUCTURE NONLINEARITY AND RESPONSE OF LATERALLY LOADED PILES

Wei Dong Guo and B. T. Zhu

1School of Civil, Mining & Environmental Engineering, University of Wollongong
2Arup Pty Ltd, Level 4 108 Wickham Street, Fortitude Valley, Qld, 4006

ABSTRACT

In light of a generic limiting force profile (LFP), closed-form solutions for laterally loaded free- and fixed- head piles in elastic-plastic media have been developed, and implemented by the first author into a spreadsheet program called GASLFP. The solutions offer an expeditious and sufficiently accurate prediction of response of lateral piles. Conversely, they allow input parameters to be deduced using measured pile response, as has been conducted for over 70 test (elastic) piles to date. Nevertheless, structure nonlinearity of pile body is an important issue at a large deflection.

In this paper, a semi-empirical approach is established to capture pile response owing to structural nonlinearity. Expressions were provided for gaining cracking moment \( M_{cr} \), flexural rigidity of cracked cross section \( E_I k \), and ultimate bending moment \( M_{ult} \). Against measured response of two laterally loaded single piles, back-estimation indicates that (1) the parameters for elastic piles are quite consistent with the previous findings for piles in sand and clay; (2) The proposed variations of \( M_{cr} \), \( E_I k \) and \( M_{ult} \) for nonlinear piles provide good prediction of the pile response against measured data and (3) the modulus of rupture \( k \) of 16.7 (clay) and 33.0 (sand) are close to those adopted for structural beams, although a very high \( k \) of 62.7 (thus resulting in higher \( M_{ult} \)) for a pile in sand was deduced (shown elsewhere). The use of the \( k \) for beams would render pile deflections of the later pile to be significantly overestimated. The conclusions may be incorporated into design of laterally loaded piles.

1 INTRODUCTION

A number of approaches based on p-y concept were developed to capture the behaviour of piles subjected to lateral loads (McClelland and Focht 1958; Matlock 1970; Reese et al., 1974; Reese et al., 1975). The behaviour is generally dominated by the limiting force per unit length \( p_c \) mobilized between the pile and soil, especially at high load levels (Randolph et al.,1988; Guo 2006). The profile of \( p_c \) (also referred to as LFP) along the pile may be constructed using empirical or semi-empirical methods proposed previously (Brinch Hansen 1961; Broms 1964; Matlock 1970; Reese et al., 1974; Reese et al., 1975; Barton 1982; Guo 2006). In light of a generic LFP, elastic-plastic, closed-form (CF) solutions were developed for laterally loaded free- and fixed- head piles (Guo 2006; Guo 2009), respectively. The solutions allow pile response to be predicted in an effective and efficient manner. Conversely, they enable the LFP to be deduced against measured pile response. They, however, are limited to elastic piles, and structure nonlinearity of pile body is an important issue at a large deflection (Nakai and Kishida 1982; Reese 1997; Huang et al., 2001; Ng et al., 2001; Zhang 2003).

2 SOLUTIONS FOR LATERALLY LOADED PILES

The problem of a laterally loaded free-head pile is schematically shown in Figure 1(a), with a lateral load \( P \), applied at an eccentricity, \( e \) above ground level (GL). The pile-head is free to rotate and translate with no constraint imposed at the pile head and along the effective pile length except for soil resistance. The pile-soil interaction is captured by a physical spring-slider model presented in Figure 1(b) with the lateral load \( P \) and a moment, \( M_c (= P_c e) \) applied at the GL. The pile is described by deflection \( y \) (along the pile axis) at a depth \( x \) (below the origin at the GL). The interaction at each depth is modelled by a spring in series with a slider characterized by an idealized elastic-plastic p-y curve (see Figure 1c). The spring has a subgrade modulus, \( k \) (i.e. the slope of the p-y curve), and the slider provides a limiting force per unit length \( p_y \) for the pile-soil interaction. The soil resistance normally reaches the \( p_y \) (varying with depth) over a depth called slip depth, \( x_p \) that increases with loading. Above the depth \( x_p \), the pile-soil interaction is in plastic state; whereas below the \( x_p \), it is in elastic state. The elastic state is modelled by the independent springs linked by a fictitious membrane with a constant fictitious tension, \( N_p \) (i.e. incorporating the coupled effect among springs via a coupled load transfer model). In the plastic zone, however, the tension is neglected by taking \( N_p \) as zero (employing an uncoupled load transfer model).
Figure 1: Coupled load transfer analysis for a free-head pile (Guo, 2001a, 2006): (a) The problem addressed, (b) Coupled load transfer model and (c) Load transfer (p-y) curve.

In the elastic state \((x=x_p)\), the subgrade modulus \(k\) and the fictitious tension, \(N_p\), are obtained previously (Guo and Lee 2001). As shown in Table 1, they are mainly functions of Young’s modulus of an equivalent solid pile, \(E_p\) \([-E_p/\sqrt{\pi(\nu/64)}]\) and an equivalent soil shear modulus \(G_s^{\prime} = (1+0.75\nu_s)G_s\), in which \(E_p\) and \(d\) = flexural rigidity, and outside diameter of the pile, respectively; \(\nu\), and \(G_s\) = Poisson’s ratio and shear modulus of soil, respectively. In the plastic zone \((x>x_p)\), the limiting force per unit length \(p_u\) is well captured by (Guo 2001a)

\[
P_u = A_L (\alpha_o + x)^n
\]

where \(p_u\) = limiting force per unit length [FL\(^{-1}\)]; \(A_L = s_oN_d^{1/3}\) (clay), and \(\gamma_s N_d^{2/3}\) (sand), gradient of the LFP [FL\(^{-1}\)]; \(\alpha_o\) = a constant to include the force at the GL [L]; \(x\) = depth below the GL [L]; \(n\) = a power to the sum of \(\alpha_o\) and \(x\), \(N_d =\) limiting force factor; \(x_o =\) undrained shear strength of cohesive soil [FL\(^{-1}\)]; and \(\gamma_s\) = effective unit weight of the soil [FL\(^{-1}\)]. Guo (2006) shows

- \(G_s = (0.25-0.62)N\) (MPa) \((\bar{G}_s = 0.5N\) MPa, note the bar denotes average and \(N =\) blow counts of SPT), \(\alpha_o = 0\), \(n = 1.7\) and \(N_p = (0.4-2.5)K_p^2\) with respect to a uniform sand profile in which \(K_p = \tan^2 (45^\circ + \phi/2)\); and \(\phi_s =\) effective friction angle of the soil

- \(G_s = (25-315)s_s\) \((\bar{G}_s = 92.3s_s), n = 0.7, \alpha_o = 0.05-0.2 \text{ m} (\bar{G}_s = 0.11\) m), and \(N_p = 0.6-3.2(\bar{N}_g = 1.6)\) for a uniform clay profile.

- The construction of LFP and use of an average \(G_s\) for layered soil profiles.

The LFP described by Eq. (1) will be referred to as Guo LFP. It is provided in Table 1 along with Reese LFP for cohesionless soil, Matlock LFP for cohesive soil and Hansen LFP for any subsoil (using cohesion c and \(\phi_s\)).

The pile-soil interaction is governed by two differential equations that are dominated by the \(p_u\) in the plastic zone \((0\leq x\leq x_p)\) and by the \(k\) and \(N_p\) in elastic zone, respectively. They were resolved using boundary conditions and presented in closed-form expressions for pile deflection, slope, bending moment, shear force and soil resistance. The expressions are featured by reciprocal of characteristic length of the pile \(\lambda = [\sqrt[k]{4E_p}]\)\(^{0.35}\) with \(k/G_s = 2.4-3.92\), the slip depth \(x_p\) and the parameters \(A_L\) (or \(N_p\)), \(\alpha_o\), \(n\) (for the LFP). For instance, ignoring ground level resistance (by taking \(\alpha_o = 0\)) and using a uncoupled model \((N_p = 0\), the pile deflection at GL \(y_o\) is given by (Guo 2006)

\[
\frac{y_o}{A_L} = \frac{2}{3} \frac{\bar{x}^n}{3^n} \frac{2\bar{x}^2 + (2n+10)\bar{x} + n^2 + 9n + 20}{(\bar{x} + 1 + \bar{e})(n + 2)(n + 4)} + \frac{2\bar{x}^2 + 2\bar{x} + 1}{\bar{x} + 1 + \bar{e}} \frac{\bar{x}^n}{3^n} + \frac{2\bar{x}^4 + (n + 4)(\bar{x} + 1)[2\bar{x}^2 + (n + 1)\bar{x} + 1]}{(\bar{x} + 1 + \bar{e})(n + 1)(n + 4)} e^{x}\]

where \(\bar{x} = \lambda x_p, \bar{e} = \lambda e\), and \(P_L = p_p^{1/n}/A_L\). The solutions were entered into a spreadsheet program GASLFP. The solutions well captures the response of laterally loaded piles. Conversely, they allow the values of \(A_L\) (or \(N_p\), \(\alpha_o\), \(n\) and \(G_s\) to be deduced by matching the predicted with measured pile response of maximum bending moment \((M_{max})\), pile deflection \(y_o\) and so forth under each load \(P_L\). The solutions are developed for a
pile with a length L exceeding the critical value \( L_{cr} = 1.05d(E_p/G)^{0.25} \), beyond which any extra length of a pile will impose negligible influence on the pile response; otherwise the solutions for short rigid piles (Guo 2008) should be adopted. Note the \( G \) may be taken as the average \( \overline{G} \) over a depth of 10d. GASLFP is used to conduct current investigation.

### Table 1  Typical limiting force profiles

<table>
<thead>
<tr>
<th>Name</th>
<th>Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matlock LFP</td>
<td>( N_g = \gamma_i d \int s_u + 0.5, \alpha_o = 2d / N_g, n = 1 ) (Matlock 1970)</td>
</tr>
</tbody>
</table>
| Reese LFP (sand) (Reese et al., 1974) | \( p_u = \gamma_i x \left( f_1 x + \frac{\tan \beta}{\tan(\beta - \phi_\alpha)} - K_q \right) \) \( x < x_l \) \[
p_u = \gamma_i \left[ (K_q d (\tan \beta - 1) + K_q \tan \phi_i \tan^4 \beta) \right] \quad (x \geq x_l)
\]
|               | \( f_{r1} = \frac{\tan \beta}{\sin \phi_i} \left( \frac{K_q \tan \phi_i \cos \beta}{\cos \alpha} + \tan \beta \tan \alpha + K_q \tan \beta (\tan \phi_i \sin \beta - \tan \alpha) \right) \) \[
K_q \sin \phi_i \cos \beta = 0.5 \phi_i, \beta = 45^{\circ} + 0.5 \phi_i, K_q = \tan^2(45^{\circ} - 0.5 \phi_i), \quad \text{and} \quad x_l = \text{critical depth. A modification of the calculated} \ p_u \text{is used}
\]
|               | \( p_u = (K_q^D + K_q^C \phi_i) \) \[
K_q^D = \left( K_q^0 + \alpha_c K_q^\infty \frac{X}{d} \right) (1 + \alpha_q \frac{X}{d}) \quad K_q^C = \left( K_q^0 + \alpha_c K_q^\infty \frac{X}{d} \right) / (1 + \alpha_c \frac{X}{d}) \cdot
\]
| Hansen LFP (Hansen 1961) | \( \alpha_q = K_q^0 \tan(\phi_i) / [K_q^\infty - K_q^0 \tan(45^{\circ} + 0.5 \phi_i)], \alpha_c = 2 \sin(45^{\circ} + 0.5 \phi_i) K_q^0 / [K_q^\infty - K_q^0] \)
|               | \( K_q^0 = e^{(0.5 \pi + \phi_i) \tan \phi_i} \cos \phi_i \tan(45^{\circ} + 0.5 \phi_i) - e^{-0.5 \pi \phi_i \tan \phi_i} \cos \phi_i \tan(45^{\circ} - 0.5 \phi_i) \),
|               | \( K_q^\infty = [e^{(0.5 \pi + \phi_i) \tan \phi_i} \cos \phi_i \tan(45^{\circ} + 0.5 \phi_i) - 1] \tan \phi_i \)
|               | \( K_q^\infty = (1.58 + 4.09 \tan^4 \phi_i)[e^{0.5 \pi \phi_i \tan \phi_i} \tan(45^{\circ} + 0.5 \phi_i) - 1] \tan \phi_i \)
|               | \( K_q^\infty = K_q^\infty (1 - \sin \phi_i) \tan \phi_i \)
| Guo LFP or Eq. (1) (Guo 2006) | Clay: \( A_1 = \overline{s}_g N_d^{1/4n} \), with \( \alpha_o = 0.05 - 0.2 \) (m), \( n = 0.7 \) for clay, 1.5 for stiff, and \( N_g = 2 - 4 \)
|               | Sand: \( A_1 = \gamma_i N_d^{1/3n} \), with \( \alpha_o = 0, n = 1.7, \) and \( N_g = s_g k_p^2 \), in which \( s_g \) is an integral factor to cater for all sorts of influence; and \( K_q^0 = \tan(45^{\circ} + 0.5 \phi_i) / 2 \), the passive earth pressure coefficient.
|               | \( k = \frac{3 \pi}{2} \left[ 2 \gamma_i K_i(\gamma_i) - \gamma^2 \left( \frac{K_i(\gamma) - 1}{K_i(\gamma)} \right)^2 \right] \) \[
\text{and} \quad \frac{4N_g}{\pi d^2 G} = \left[ K_i(\gamma)/K_o(\gamma) \right]^2 - 1
\]

Note:

\[
k_i(\gamma) \text{ is modified Bessel function of second kind of } j^{th} \text{ order (} j = 0, 1 \text{) (Guo and Lee 2001)}
\]

where \( K_i(\gamma) \) is for \( L > L_{cr} + x_l \) and given by \( K_i(\gamma) = k_i (E_p / G)^{0.25} \); with \( k_i = 1.0 \) for a lateral load (e = 0) applied at point O (sliding level), and \( k_i = 2.0 \) for a pure moment. The governing equations were resolved (Guo 2001a; Guo 2006) for free-head case, and a pile length exceeding the sum of \( L_{cr} + x_l \).

### 3  MODELLING STRUCTURE NONLINEARITY

#### 3.1  CRACKING MOMENT MCR

Tensile strength is only a small fraction of compressive strength for a concrete pile. Tensile cracks may be developed at a large deflection, once the tensile stress at the extreme fibre arrives at the modulus of rupture \( f_c \). Section analysis of a beam allows the modulus \( f_c \) to be correlated with cracking moment, \( M_c \) (upon which cracks are initiated) by:

\[
M_c = f_c I_c = k_c \sqrt{f_c} I_c
\]

\[
M_c = k_r \sqrt{f_c} I_c
\]
where \( k_c = 19.7\text{--}31.5 \) for a normal weight concrete beam (ACI 1993), and 16.7\text{--}62.7 for lateral piles (shown elsewhere); \( y_c = \) distance from centroidal axis of gross section to extreme fibres in tension; \( I_g = \) moment of inertia of gross section about centroidal axis (neglecting reinforcements); \( f_c' = \) characteristic value of the compressive strength of concrete, in kPa. The \( f_c' \) and \( E_c \) may be empirically correlated by \( E_c = 151,000(f_c')^{0.5} \) (kPa) (ACI 1993).

### 3.2 VARIATION OF EFFECTIVE FLEXURAL RIGIDITY \( E_pI_p \) WITH \( M_{\text{MAX}} \)

Under a lateral load, cracks may be induced in a pile at the depth of \( M_{\text{MAX}} \), if the \( M_{\text{MAX}} \) exceeds the cracking moment \( M_{\text{cr}} \), although other regions may be still intact. This will render the flexural rigidity of the pile \( E_pI_p \) reduce from the elastic \( E_pI_g \) (rewritten as \( E_1 \)) to a new effective bending rigidity \( E_pI_{cr} \) (rewritten as \( E_{1,cr} \)) (ACI 1993) until a final (minimum) cracked rigidity \( (E_1)_{cr} (=E_pI_{cr}) \). This variation may be expressed by

\[
E_pI_{cr} = \left( \frac{M_{cr}}{M_{max}} \right)^3 EI + \left[ 1 - \left( \frac{M_{cr}}{M_{max}} \right)^3 \right] (EI)_{cr}
\]  

Equation (4) is used to deduce Young's modulus of a pile, \( E_p \) \([= E_1J/(\pi d^4/64), \) or \( E_1J/(bh^3/12)] \) from \( E_pI_{cr} \) of the entire pile, regardless of the position and scale of cracks. This should model the worst scenario as the cracks should generally be confined to a limited zone; otherwise much more complicated procedure is required (Choi et al., 2009). More importantly, replacing \( E_p \) in the elastic-pile solutions with this new \( E_{1,cr} \) nonlinear pile response is readily captured using the same solutions for elastic piles.

### 3.3 \( M_{\text{ULT}} \) AND \( I_{CR} \) FOR RECTANGULAR AND CIRCULAR CROSS SECTIONS

The ultimate bending moment \( M_{\text{ult}} \) and \( I_{cr} \) may be obtained by bending theory via moment-curvature method that involves stress-strain relationships of concrete and reinforcement (Hsu 1993). For instance, Reese (1997) provided the \( M_{\text{ult}} \) and \( I_{cr} \) for closely-spaced-cracks using a Hognestad parabolic stress-strain relationship for concrete and an elastic-perfectly plastic stress-strain relationship for steel. The cracks may initially be spaced at some distance and the concrete stress-strain relationship depends on strength, rate and duration of loading etc. A rational flexural theory for reinforced concrete is thus yet to be developed (Nilson et al., 2004). Pragmatically, limit state design underpinned by simplified rectangular stress block method (referred to as RSB hereafter) (Whitney 1937) has been widely adopted to calculate the \( M_{\text{ult}} \) and \( I_{cr} \) (BSI 1985; EC2 1992; ACI 1993).

A pile may generally fail by crushing of the concrete in the outmost compression fibre as tension failure of steel rebar is rare. The compression failure may occur at a strain \( \varepsilon_{\text{cr}} \) of 0.0035 (BSI 1985; EC2 1992; Nilson et al., 2004), or 0.003 (ACI 1993). The tension failure strain \( \varepsilon_{\text{tu}} \) varies from 10 to 40% (Lui 1997), and was taken as 0.015 (Reese 1997). The accuracy of \( \varepsilon_{\text{tu}} \) has limited impact on analysing lateral piles showing compressive failure.

Figure 2 shows typical cracked cross-sections, with four rows of rebar, and of a circular pile (Figure 2a), and a rectangular pile (Figure 2b), respectively. With a linear strain distribution (Figure 2c), the compressive stress in concrete is simplified as a rectangular stress block (Figure 2d) characterised by those highlighted in Table 2, including the intensity of the stress, \( \sigma_x \), the depth \( d_x \) of the stress block, and the stress induced in a rebar in the \( i \)th row, \( \sigma_{xi} \). The stresses allow axial force bending moment in either section to be gained as,

\[
\begin{align*}
\int_0^d \sigma_xA_x \, dx &= P_{xc} + P_{xs} = P_x \\
\int_0^d \sigma_x \, dx &= M_c + M_s = M_n
\end{align*}
\]  

where \( A_x = \) area of cross-section excluding the concrete in tension; \( \sigma_x = \) normal stress in concrete (\( \sigma_x \)) or rebar (\( \sigma_{xi} \)); \( P_{xc} = \sigma_xA_x \), axial load taken by the concrete; \( A_x = \) area of concrete in compression; \( P_{xs} = \sum \sigma_{xi}A_x \), axial load shared by the rebars; \( A_n = \) total area of rebar in the \( i \)th row; \( P_x = \) imposed axial load; \( x_i = \) distance from neutral axis; \( M_c \) and \( M_s = \) moments about the neutral axis induced by normal stress in the concrete and rebars, respectively; and \( M_n = \) nominal or calculated ultimate moment. Generally, the values of \( P_{xc} \), \( P_{xs} \) and \( M_c \) can be readily obtained. The \( M_c \) may be estimated using Equations (7) and (8) developed herein for a rectangular and a circular cross-section, respectively.

\[
M_c = f_{ce}ab(2c-a)/2
\]  

or

\[
M_c = 2r^2f_{ce} \left[ \frac{a}{3r} - \frac{2-a}{r} + \frac{1}{2} \left( 1 - \frac{c}{r} \right) \left( 1 - \frac{a}{r} \right) \sqrt{2ra-a^2 + \frac{r-c}{2} \left( \arcsin \left( \frac{r-a}{r} \right) - \frac{\pi}{2} \right)} \right]
\]  

where \( b = \) width of a rectangular pile.
Note \( f_c' = 0.85 f_c' \); \( \alpha_1 = 0.85; \beta_1 = 0.85 - 0.05(f_c' - 27.6)/6.9 \) and \( \beta_1 \geq 0.65; \theta = \text{curvature}; \ r = \text{cover thickness}; \ A_{1i}, \sigma_{li}, A_{i2}, \sigma_{l2}, A_{i3}, \sigma_{l3}, A_{i4}, \sigma_{l4} \) are axial forces of the first to fourth row reinforcing bars and the rows are numbered consecutively from the extreme tensile bars to the compression side, in which \( \sigma_{li} = \text{steel stress of a rebar in the } i\text{-th row and } A_{li} = \text{total area of rebars in the } i\text{-th row.} \)

Figure 2: Simplified rectangular stress block for ultimate bending moment calculation: a) Circular section, (b) Rectangular section (c) Strain, and (d) Stress.

### Table 2 Stress block for calculating \( M_{ult} \)

<table>
<thead>
<tr>
<th>Items</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_c )</td>
<td>The intensity of the stress, ( \sigma_c ) is ( \alpha_c f_c' ) with ( \alpha_1 = 0.85 ), and ( f_c' = 0.85 f_c' ).</td>
</tr>
<tr>
<td>( a )</td>
<td>The depth ( a ) of the stress block is given by ( a = \beta_1 c ), where ( \beta_1 = 0.85 - 0.05(f_c' - 27.6)/6.9 ) and ( \beta_1 \geq 0.65 ), and ( c = \text{distance from the outmost compression fibre to neutral axis} ). If concrete fails, otherwise ( c = 2r - t - d_r/2 = 0.015/\theta ) concerning the debond (tension) failure. Note: The diameter of ‘2( r )’ for a circular pile should be replaced with ( h ) for a rectangular pile; ( r = \text{radius of a circular pile, } \theta = \text{curvature at the limit state, } d_r = \text{diameter of rebars, and } t = \text{cover thickness (Figure 2b).} )</td>
</tr>
<tr>
<td>( \sigma_{li} )</td>
<td>The stress induced in a rebar in ( i\text{-th row}, \sigma_{li} ) is calculated according to level of rebar strain ( \varepsilon_{li} ): ( \sigma_{li} = \phi f_y ) (yield stress) if ( \varepsilon_{li} \leq \varepsilon_{iy} ) otherwise, ( \sigma_{li} = \varepsilon_{li} E_i ). Note: The ( i\text{-th row of rebars are counted from the farthest tensile side towards the compressive side (see Figure 2d); } \varepsilon_{iy} = \phi d_y/E_i, E_i = \text{Young’s modulus of reinforcement, generally taken as } 2 \times 10^5 \text{ kPa}; \text{ and } \phi = 0.9, \text{ reinforcement reduction factor for tension and flexure.} )</td>
</tr>
</tbody>
</table>

Computing \( M_{ult} \) and \( I_{cr} \) may follow the steps highlighted in Table 3. First, the nominal ultimate bending moment \( M_n \) of \( M_{ult} + M_t \) is calculated using Equation (6); Second, the ultimate bending moment \( M_{ult} \) is calculated as \( \phi M_n \) (\( \phi = \text{a reduction factor} \) (see Table 3); and finally the cracked flexural rigidity, \( E_i I_{cr} \), is obtained by

\[
E_i I_{cr} = M_{ult} / \theta
\]

The aforementioned calculation has been entered into a simple spreadsheet program that operates in EXCEL™. The program offers consistent results against the ACI, and the AS3600 methods (not shown herein), and was thus used in this study.
Table 3: Calculation of $M_{\text{max}}$ and $I_{\text{cr}}$ using RSB method

<table>
<thead>
<tr>
<th>Steps</th>
<th>Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Select an initial curvature $\theta$ (see Figure 2) of 0.0035/\tau;</td>
</tr>
<tr>
<td>2</td>
<td>Calculate position of neutral axis by evaluating $c$ and $a = \beta_1 c$;</td>
</tr>
<tr>
<td>3</td>
<td>Compute the axial forces in concrete $P_{xc}$ and rebars $P_{xr}$;</td>
</tr>
<tr>
<td>4</td>
<td>Find squash load capacity $P_{so}$ by: $P_{so} = A_e f_{c} A_e + E_s f_s \varepsilon_e$ where $A_e, A_s = \text{total areas of concrete and rebars, respectively}$;</td>
</tr>
<tr>
<td>5</td>
<td>If $P_{xc} + P_{xg} - P_{so} &gt; 10^4 P_{so}$, increase $\theta$ by a designated increment, say 0.0035/1000\tau. Otherwise, if $P_{xc} + P_{xg} - P_{so} &lt; 10^4 P_{so}$, reduce the $\theta$ by the said amount. Repeat steps (1)–(4) until the convergence is achieved.</td>
</tr>
<tr>
<td>6</td>
<td>Estimate the nominal ultimate bending moment of $M_{c} + M_{g}$ by Eq. (6); and finally</td>
</tr>
<tr>
<td>7</td>
<td>Compute the ultimate bending moment, $M_{ult}$, using $M_{ult} = \phi(M_{c} + M_{g} + M_{rf})$, in which $\phi = \text{ultimate moment reduction factor, catering for difference between actual and nominal pile dimension, rebar cage off-position, and assumptions and simplifications inherent in the analysis. The $\phi$ is given by (Nilson et al., 2004)}$</td>
</tr>
<tr>
<td></td>
<td>$\bullet \phi = 0.493 + 83.3 \varepsilon_{se}$, and $0.65 \leq \phi \leq 0.9$, for laterally tied rebar cage;</td>
</tr>
<tr>
<td></td>
<td>$\bullet \phi = 0.567 + 66.7 \varepsilon_{se}$, and $0.70 \leq \phi \leq 0.9$, for spirally reinforced rebar cage.</td>
</tr>
<tr>
<td>8</td>
<td>Compute the cracked flexural rigidity, $E_{cj}I_{cr}$ by $E_{cj}I_{cr} = M_{ult} / \theta$</td>
</tr>
</tbody>
</table>

Notes: As an example, the calculations for the pile in sand are as follows: $\beta = 0.65$, $\theta = 0.0164$ (m$^{-1}$), $c = 214$ mm, $c_{se} = 0.00586$, $a = 139$ mm, $P_{xc} = 2.8651$MN, $P_{xg} = -2.8654$MN, $M_{c} = 0.38$ MNm, $M_{g} = 1.72$ MNm, $\phi = 0.9$, $M_{ult} = 2.1$ MNm, and $M_{ult} = 1.89$ MNm

3.4 MODELLING STRUCTURE NONLINEARITY

As mentioned earlier, the analysis of piles with structural nonlinearity essentially employs the same solutions as those for a linear (elastic) pile, but with varying bending rigidity $E_{pl}$ beyond cracking load. The analysis for an elastic pile was elaborated previously (Guo, 2006), and recaptured in Table 4. As shown later, the values of $M_{\text{max}}$ gained are negligibly different between elastic pile ($E_{pl}$) and cracked pile, but for an extremely high EI (shown elsewhere). The new $E_{pl}$ for each $M_{\text{max}}$ (or load) may thus be calculated using Equation (4), together with $M_{c}$ from Equation (3), and $(EI)_{cr}$ (via Equation (9) from rectangular biaxial stress method (RBS)).

The $M_{cr}$ and $E_{pl}$ may be deduced by using measured pile response, as with the parameters $G$ and LFP for linear piles, using GASLFN. A measured $M_{cr}$ should correspond to initial point of deviation of the predicted $P_{c}-\gamma_{a}$ curve (using EI) from the measured curve. Beyond the $M_{cr}$, the $E_{pl}$ is adjusted to minimise the deviation between the two curves, until it reduces to the final cracked rigidity $(EI)_{cr}$ at maximum failure load. A 'measured' $k_{c}$ may be deduced using Equation (3) and the measured $M_{cr}$ (such as $k_{c} = 16.7$–62.7 mentioned earlier), and the obtained $E_{pl}$ for various $M_{max}$ together with $M_{c}$, and $(EI)_{cr}$ allow Equation (4) to be justified.

The impact of reduced rigidity (owing to cracking) on a fictitious pile response is illustrated using GSLP prediction. The pile with $d = 0.373$ m, $L = 15.2$ m, and $EI = 80.0$ MNm$^2$ was installed in the sand having $f_{s} = 35^\circ$, $k' = 9.9$ kN/m$^3$, $G = 11.2$ MPa, and $\nu = 0.3$. The LFP was described by $a_{se} = 0$, $n = 1.6$ and $N_{p} = 0.55 K_{p}$. Under a head load $P_{h}$ of 400 kN, the predicted profiles of bending moment $M_{cr}$, slope $\theta$, shear force $V$, and on-pile force per unit length $p$ are illustrated in Figure 3 for $EI = 80.0$ MNm$^2$ and $E_{pl} = 8.0$ MNm$^2$. The figures indicate the reduction in rigidity leads to (1) significant increase in the pile deflection and slope (all depth), and some increases in the local maximum $V$ and $p$, and in the slip depth $x_{p}$, however, (2) little alteration in the bending moment profile. The results legitimize the back-estimation of $E_{pl}$ using measured $y_{cr}$ and the $M_{\text{max}}$ predicted from elastic-pile in Equation (4).
Figure 3: Effect of concrete cracking on pile response: (a) LFPs, (b) deflection ($y$) profile, (c) bending moment ($M$) profile, (d) slope ($\theta$) profile, (e) shear force ($V$) profile and (f) soil reaction ($p$) profile.
Table 4: Procedure for analysis of nonlinear piles

<table>
<thead>
<tr>
<th>Steps</th>
<th>Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Input pile properties including: ( d, L, E_p, I_p, ) and ( e ), and calculate ( E_p ) by ( E_p = E_p'(\pi d^4/64) ) or ( = E_p/(\pi d^4/12) ).</td>
</tr>
<tr>
<td></td>
<td>Determine parameters ( \alpha_s, n ) and ( N_p ) for LFP based on Eq. (1). For sand, average values of ( \gamma_s, \phi ) are estimated over a depth of ( 5d ); and average shear modulus ( G_s ) over a depth of ( 10d ). For clay, an average value of ( s_c ) is estimated over a depth of ( 5d ), and shear modulus ( G_s ) are correlated to an average ( s_c ) over a depth of ( 10d ). For a layered soil profile with shear strength increases or decreases dramatically, ( n ) may be higher or lower than 0.7, respectively.</td>
</tr>
<tr>
<td>3</td>
<td>Calculate critical length ( L_c ) if the value of ( L_c \leq 5d ) for sand (or ( &lt; 10d ) for clay), the average shear modulus is reselected to repeat the calculation of 1 and 2.</td>
</tr>
<tr>
<td>4</td>
<td>Compute ( k ) and ( N_p ).</td>
</tr>
</tbody>
</table>

Nonlinear pile

1. Calculate the \( M_{max} \) and pile deflection \( \gamma_p \) at each load level for elastic pile (EI).

2. Determine \( M_a \) using Eq. (3).

3. Calculate the critical load \( P_{cr} \) by taking \( M_{max} \) in step 2 as the \( M_a \).

4. Compute the \( M_{ub} \) and \( (EI)_{ub} \) using the rectangular block stress method (RBS), in terms of pile size, rebar strength and its layout, and concrete strength.

5. Calculate bending rigidity \( E_pI_p \) for each load level above the \( P_{cr} \), by substituting values of \( EI \), \( (EI)_{ub} \), \( M_{ub} \) and a series of \( M_{max} \) into Eq. (4).

6. Compute pile deflection using new \( E_pI_p \), and program GASPFLP for each load.

4  ANALYSIS ON TWO NONLINEAR PILES

Investigation was conducted into four piles tested in sand and two piles in clay that manifested structure nonlinearity. Owing to space limitation, only two piles are presented herein, for which (1) shear modulus is taken as an average value over a depth of \( 10d \); (2) Poisson’s ratio is assumed to be 0.3; and (3) \( \gamma_s \) and \( \phi_s \) are taken as average values over a depth of \( 5d \).

4.1 A PILE TESTED IN SAND

A prestressed pile, P7 was instrumented with strain gauges and inclinometers, and tested individually under a lateral load applied near the GL (e = 0) (Huang et al., 2001). The soil profile at the site consisted of fine sand (SM) or silt (ML), and occasional silty clay. The ground water is located 1 m below the GL. Over a depth of 15 m, the average SPT blow count, \( N \) was 16.9; the friction angle \( \phi_s \) was 32.6° (Teng 1962), and the effective unit weight, \( \gamma_s \) was 10 kN/m³.

The pipe pile (P7) was infilled with concrete, reinforced with 19@19 rebars, 38@9 high strength steel wires that have a concrete cover of 30 mm. It was 34.0 m long, 0.8 m O.D., and 0.56 m I.D. The strength values are (1) \( f_y \) (outer pipe pile) = 1226 MPa, \( f_c \) (infilled material) = 471 MPa, \( f_c \) (prestressed concrete) = 78.5 MPa, and \( f_y \) (infilled concrete) = 20.6 MPa, respectively; and (2) An equivalent yield strength of the composite cross-section of 67.74 MPa (= \( E_p/(151000) \)), as \( E_p = 3.93 \times 10^8 \) kPa determined from \( EI = 0.79 \) GN·m². The parameters allow the following to be calculated: (1) \( M_{cr} = 277.4 \times 443.9 \) kNm using \( k_p = 19.1 - 31.5 \), \( \gamma_p = 0.4 \) m and Equation (3). And (2) \( M_{ub} = 1.89 \) MN-m, and \( (EI)_{ub}/EI = 0.146 \), using the RSB method (see Table 3).

The elastic-pile analysis was based on (1) \( n = 1.7, \alpha_s = 0 \) and \( N_p = 1.0K_s^2 = 11.13 \), as deduced from the DMT tests shown in Figure 4 (Huang et al., 2001), and (2) \( G_s = 10.8 \) MPa \( (= 0.64N_c \) (Guo 2006). The predicted deflection \( \gamma_p \) is illustrated in Figure 4(b), which agrees with the measured data until \( P = 30 \) kN, at which \( M_{cr} = 464.7 \) kNm. This prediction provides (1) A slightly higher \( k_p \) of 33.0; and (2) A \( M_{ub} \) of 1.82 - 2.0 MN-m (as indicated by sharp increase in the measured \( \gamma_p \) in Figure). 4(b) at \( P = 804 \) kN to 863 kN, which agrees well with 1.89 MN-m gained using the RSB method.
Table 5: Analysis of Pile P7 using elastic pile, and deduced $E_{pl}I_p$

<table>
<thead>
<tr>
<th>$P_r$ (kN)</th>
<th>Measured $y_0$ (mm)</th>
<th>Using elastic-pile $EI$</th>
<th>$E_{pl}I_p/EI$</th>
<th>Using cracked $EI_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$M_{max}$ (kNm)</td>
<td>$y_0$ (mm)</td>
<td>$x_{p/d}$</td>
</tr>
<tr>
<td>284</td>
<td>11.6</td>
<td>464.7</td>
<td>10.5</td>
<td>1.85</td>
</tr>
<tr>
<td>361</td>
<td>21.8</td>
<td>631.4</td>
<td>14.5</td>
<td>2.13</td>
</tr>
<tr>
<td>498</td>
<td>46.8</td>
<td>959.9</td>
<td>23.1</td>
<td>2.56</td>
</tr>
<tr>
<td>566</td>
<td>60.6</td>
<td>1138.0</td>
<td>28.0</td>
<td>2.75</td>
</tr>
<tr>
<td>666</td>
<td>88.9</td>
<td>1411.8</td>
<td>35.8</td>
<td>3.02</td>
</tr>
<tr>
<td>732</td>
<td>106.0</td>
<td>1603.5</td>
<td>41.4</td>
<td>3.18</td>
</tr>
<tr>
<td>804</td>
<td>171.8</td>
<td>1818.0</td>
<td>47.9</td>
<td>3.35</td>
</tr>
<tr>
<td>826.2</td>
<td>218.2</td>
<td>1886.0</td>
<td>50.0</td>
<td>3.40</td>
</tr>
</tbody>
</table>

Figure 4: Comparison between measured (Huang et al., 2001) and predicted response of pile P7: (a) LFPs, (b) $P_r-y_0$ curves, (3) $P_r-M_{max}$ curves, and (4) $E_{pl}I_p/EI \sim M_{max}$. 

Australian Geomechanics Vol 46 No 3 September 2011
With $(EI)_{cr}$ of 0.1152 GN·m² (as per RSB method), $M_{cr}$ of 464.7 kNm, and $M_{sh}$ of 1.89 MN-m (an average of 1.82–2.0 MN-m), the $E_p f_p$ was estimated using Equation (4) for each $M_{max}$ (gained using $EI$). This $E_p f_p$ allows new $y_o$ and $M_{max}$ to be estimated, which are presented in Figures 4(b) and 4(c), respectively. The measured deflection was overestimated by ~28%, indicating the accuracy of the estimated $(EI)_{cr}$ and Equation (4).

### 4.2 A PILE TESTED IN CLAY

Pile D was tested at a site (Nakai and Kishida 1982) with an undrained shear strength $s_u = 35 + 0.75x$ (kPa, $x$ in m < 20 m), and an average $\bar{s}_u$ over 15.48 m ($= 10d$) of 43 kPa. The lateral load was applied at 0.5 m above the GL. The pile had $L = 30$ m, $d = 1.548$ m, $EI = 16.68$ GN·m², $E_p = 5.92 \times 10^7$ kPa, and $f_p' = 153.7$ MPa [$= (E_p/15,000)^2$]. The $M_{cr}$ was estimated as 2.81 ~ 4.50 MN-m (using $k_r = 19.7$ ~ 31.5, $y_o = 0.774$ m and Equation (3), which exceeds 1.33 MNm by 2.0 ~ 3.4 times. The ultimate bending moment $M_{sh}$ was not estimated, without the reinforcement information.

![Graphs showing comparison between calculated and measured responses of Pile D](image)

Figure 5: Comparison between calculated and measured (Nakai & Kishida, 1982) response of Pile D: (a) LFPs, (b) $P_f$-$y_o$ curves, (c) $P_f$-$M_{max}$ curves, and (d) $E_p f_p$/$EI$ ~ $M_{max}$.  

Australian Geomechanics Vol 46 No 3 September 2011
The elastic pile reaction was simulated using \( n = 0.7, \alpha = 0.1 \) and \( N_p = 2.0 \) (see LFP in Figure 5a), and \( k = 3.13G \), with \( G = 5.62 \text{MPa} = 130.8G \) (Kishida and Nakai 1977). In particular, Figure 5(a) demonstrates that the LFP within 5d is close to Hansen’s LFP gained using \( c = 8 \text{kPa}, \varphi = 10^\circ \), but is far below the Matlock’s LFP. The predicted pile deflection compares well with the observed one until \( P, \text{of } 690 \text{kN} \) (see Figure 5b). The cracking \( M_{cr} \) (\( M_{max} \) at \( P = 690 \text{kN} \)) was deduced as 2.38 MN·m, and \( k \), as 16.7 using a higher \( E_c (= E_p) \) rather than the concrete modulus.

The \( E_{cr,p} \) was deduced using the measured deflection \( \gamma_d \) (see Figure 5b), which in turn offers the bending moment \( M_{max} \) as plotted in Figure 5(c). For instance, the maximum load \( P, \text{of } 1289 \text{kN} \) incurs \( E_{cr,p} = 5.0 \text{GN·m}^2 \) (= 0.3\( E_p \)), \( \gamma_d = 90.1 \text{mm} \), \( M_{max} = 5.57 \text{MN·m} \), and \( x_p = 6.09 \text{m} \) (= 3.93\( A \)). The evolution of cracking renders \( \gamma_d \) increase by 118.2%, \( M_{max} \) reduce by 1.24%, and \( x_p \) increase by 29.3%, which resemble those noted for piles in sand (not shown herein).

Using \( M_{max} \) and \( E_{cr,p} \) at \( P = 1289\text{kN}, \ (E_{cr})_p \) was calculated to be 4.01 \text{GN·m}^2 using Equation (4), and \( (E_{cr})_p E I = 0.241 \). Substituting \( (E_{cr})_p \) and \( M_{max} \) into Eq. (4), \( E_{cr,p} \) was calculated, which in turn allows the deflection \( \gamma_d \) to be computed. This deflection designated as \( E_{cr,p} \). Equation (4) compares well with measured one (see Figure 5b). Equation (4) is validated for this case.

5 CONCLUSIONS

A semi-empirical approach is established to capture pile response owing to structural nonlinearity. Expressions were provided for gaining cracking moment \( M_{cr} \), flexural rigidity of cracked cross section \( E_{cr,p} \), and ultimate bending moment \( M_{ub} \). Against measured response of two laterally loaded single piles, back-estimation indicates that (1) the parameters for elastic piles are quite consistent with the previous findings by Guo (2006). (2) The proposed \( M_{cr}, E_{cr,p} \) and \( M_{ub} \) provide good prediction of the pile response against measured data and (3) the modulus of rupture \( k, \text{of } 16.7 \text{clay} \) and 33.0 (sand) are close to those adopted for structural beams, although a very high \( k, \text{of } 62.7 \) (thus resulting in higher \( M_{cr} \)) for a pile in sand was deduced (shown elsewhere). The use of the \( k \) for beams would render pile deflections of the latter pile to be significantly overestimated. The conclusions may be incorporated into design of laterally loaded piles.

6 REFERENCES


Australian Geomechanics Vol. 46 No. 3 September 2011

51
APPENDIX 1 – NOTATION

The following symbols have been adopted in this paper.

\(a\)  height of stress block;
\(A_d\) gradient of the LFP [\(\text{FL}^{-1}\)];
\(b\) width of a rectangular cross section of a pile;
\(c\) distance from extreme compression fibre to neutral axis at a section in which \(M_d\) occurs;
\(d\) pile diameter;
\(e\) eccentricity of loading above groundline (GL);
\(E_s\) modulus of elasticity of concrete;
\(E_{ls}\) initial flexural rigidity of the pile;
\(E_p\) equivalent Young’s modulus of pile;
\(f_c'\) characteristic value of compressive strength of the concrete;
\(f_y'\) design value of concrete compressive strength;
\(f_y\) yield strength of reinforcement;
\(G_s\) equivalent shear modulus of soil;
\(G_{ls}\) shear modulus of soil;
\(h\) depth of a rectangular cross section of a pile;
\(I_p\) moment of inertia of gross section of the pile;
\(I_{cr}\) moment of inertia of cracked section;
\(k\) subgrade modulus of a spring between pile and soil;
\(k_t\) a constant of load transfer factor;
\(k_s\) a constant for concrete rupture;
\(L_{cr}\) critical length of a pile;
\(M\) cracking moment;
\(M'(M_{\text{max}})\) bending moment in the pile (maximum \(M\));
\(M_u\) nominal or calculated ultimate moment;
\(M'\) moment applied at the pile at the GL;
\(M_{cr}'\) and \(M_u\) moments about the neutral axis induced by normal stress in the rebars and concrete, respectively;
\(n\) power of the sum of \(a_0\) and \(x\);
\(N\) blow count of Standard Penetration Test (SPT) test;
\(N_{cr}\) a gradient correlated compressive strength with the limiting pile - soil pressure at ground level;
\(N_{cr}\) fictitious tension of a fictitious membrane linking the springs around the pile;
\(P_a\) lateral load applied at pile head;
\(P_a\) and \(P_u\) axial loads distributed on the rebars and concrete, respectively;
\(P_c\) total axial load;
\(P_{\text{lim}}\) limiting force per unit length [\(\text{FL}^{-1}\)];
\(r\) pile radius;
\(s_s\) undrained shear strength [\(\text{FL}^{-1}\)];
\(V\) shear force in a pile;
\(x\) depth below ground level [L];
\(x_s\) slip depth;
\(y(x)\) pile deflection (y at ground level);
\(y_0\) distance from the centroidal axis of gross section to the extreme fibre in tension;
\(a_0\) a constant to include the force at ground level [L];
\(a_0\) ratio of average concrete stress;
\(p_t\) ratio of stress block depth;
\(\phi\) ultimate moment reduction factor;
\(\phi\) angle of friction of soil;
\(\gamma\) load transfer factor;
\(\gamma_i(\gamma_s')\) unit weight of the soil (effective \(\gamma_s\));
\(\theta\) the curvature;
\(\lambda\) reciprocal of characteristic length of the pile;
\(\sigma_r, \sigma_c\) normal stress in the rebars and concrete, respectively;
\(v\) Poisson’s ratio of soil.