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## STRESS-DILATANCY BEHAVIOR OF LOOSE SAND DURING DRAINED CYCLIC TORSIONAL SHEAR LOADING

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### INTRODUCTION

In nature, soil is frequently subjected to cyclic loading under drained or undrained conditions such as traffic loads and earthquakes. In order to investigate the behavior of soil during drained cyclic loading, an accurate volumetric change measurement is one of the key issues to obtain properly the stress-dilatancy relationship.

Several researchers such as Pradhan et al. (1989), Shahnazari and Towhata (2002), Nishimura (2002), De Silva (2008) among others, have conducted experimental studies to investigate the stress-dilatancy relationship of sands.

However, the effect of over-consolidation on dilatancy properties of loose sand is not well understood; therefore, in this study, investigations on the stress-dilatancy relationship were conducted by performing drained cyclic torsional shear tests.

### MATERIAL, APPARATUS AND TEST PROCEDURE

Toyoura sand was used as the test material. Its particles have an angular or sub-angular shape with the physical properties: specific gravity,  $G_s=2.656$ ; mean diameter,  $D_{50}=0.16\text{mm}$ ; fines content,  $F_c=0.1\%$ ; max. void ratio,  $e_{\max}=0.992$ ; min. void ratio,  $e_{\min}=0.632$ . Tests were conducted on hollow cylindrical specimens having the height of 30cm, the inner and outer diameters of 12cm and 20cm, respectively. Specimens were prepared at  $Dr=52\text{-}56\%$  by pluviation of air-dried sand particles into a mold. To obtain specimens with highly uniform density, the falling height was kept constant throughout the pluviation process. Due to the relatively large size of soil specimen, the double vacuum method was used to achieve a degree of saturation with the B value greater than 96%. During the tests, volumetric change of the specimen was measured by high sensitive electronic balance, with the accuracy of 1.0 mg, as illustrated in Fig. 1, which is free from the effect of surface tension or meniscus force of the water retained in the beaker. After consolidating the specimen isotropically, while applying the over-consolidation history or isotropic unloading and reloading cycles if necessary, drained cyclic torsional loading with single amplitude shear stress of 50 kPa was applied at constant shear strain rate of 2.0 %/min., while attempting to maintain the stress conditions of  $\sigma'_z = \sigma'_r = \sigma'_\theta = 100$  kPa.

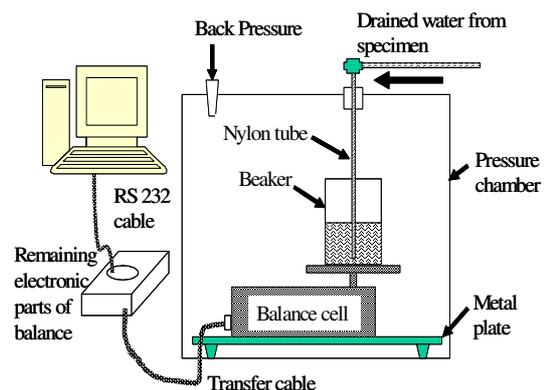


Fig. 1. High Sensitive Electronic Balance

## PROCEDURES FOR DATA ANALYSIS

The volumetric strain increment ( $d\epsilon_{vol}$ ) measured during cyclic loading consists of two components due to dilatancy ( $d\epsilon^d$ ) and consolidation/swelling ( $d\epsilon^c$ ), as shown in Eq.1:

$$d\epsilon_{vol} = d\epsilon^d + d\epsilon^c \quad (1)$$

The component due to dilatancy is perfectly plastic in nature, while the other component due to consolidation/swelling consists of both elastic and plastic deformations.

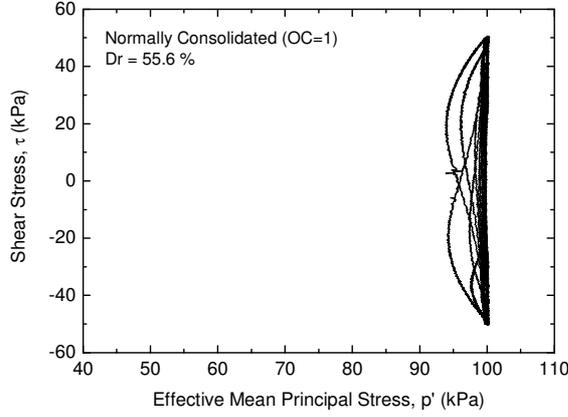


Fig. 2: Change in  $p'$  during drained cyclic torsional shear loading

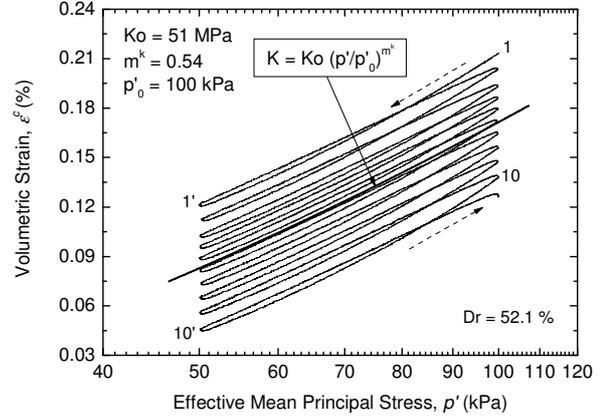


Fig. 3: Swelling curve during isotropic unloading and reloading cycles

Due to limited accuracy of machine control during cyclic loading, the axial stress could not be kept constant at  $\sigma_z' = 100$  kPa, resulting into a change in the effective mean principal stress,  $p'$ , in the range of 94 to 100 kPa, as typically shown in Fig. 2. Therefore, in order to correct for such a change in  $p'$ , the component of  $d\epsilon^c$  was evaluated by using Eq.(2) while referring to the swelling curve measured during isotropic unloading and reloading cycles in the range of  $p' = 50$  to 100 kPa, as shown in Fig. 3.

$$d\epsilon^c = \frac{dp'}{K_o \cdot (p'/p'_0)^{m^k}} \quad (2)$$

In Eq. (2), the bulk modulus,  $K$ , is assumed to depend on the current effective mean principal stress.

The total shear strain increment ( $d\gamma^t$ ) measured by potentiometer at the top cap was also decomposed into two components, plastic ( $d\gamma^p$ ) and elastic ( $d\gamma^e$ ) ones, as expressed in Eq.(3):

$$d\gamma^t = d\gamma^p + d\gamma^e \quad (3)$$

The elastic component ( $d\gamma^e$ ) was evaluated by Eq. 4 and Eq. 5, as formulated in the quasi-elastic constitutive model developed at the Institute of Industrial Science, University of Tokyo (Hong Nam, 2004).

$$d\gamma^e = d\tau / G \quad (4)$$

$$G = \{f(e)/f(e_o)\} \cdot G_o \cdot (\sigma_z' \cdot \sigma_r')^{n/2} / (\sigma_o')^n \quad (5)$$

where:  $G$  = shear modulus;  $f(e) = (2.17 - e)^2 / (1 + e)$ , (Hardin and Richart, 1963);  $f(e_o)$  = initial void ratio at  $\sigma_z' = \sigma_r' = \sigma_\theta' = 50$  kPa,  $G_o$  = initial shear modulus;  $\sigma_o'$  = reference isotropic stress state (=100 kPa);  $\sigma_z'$  and  $\sigma_r'$  = vertical and radial effective stress, respectively; and  $n$  = stress induced anisotropy material parameter. In this study,  $G_o$  and  $n$  were set equal to 85 MPa and 0.508, respectively.

There are many proposals among researchers to explain the stress-dilatancy relationship under different load conditions such as triaxial, plane strain, etc. In this study, stress-dilatancy behavior of sand is analyzed based on the relationship between the ratio of volumetric strain increment due to dilatancy and plastic shear strain increment ( $-d\epsilon^d/d\gamma^p$ ), and shear stress ratio ( $d\tau/p'$ ).

## TEST RESULT AND DISCUSSION

In order to investigate the effect of over-consolidation on the stress-dilatancy behavior of loose sand, two types of test with over-consolidation ratio of OC=1 (normally consolidated, NC) and OC=4 (over-consolidated, OC) were performed.

The relationships between the shear stress ratio ( $\tau/p'$ ) and the plastic shear strain ( $\gamma^p$ ) are shown in Figs. 4(a) and 5(a), respectively for NC and OC specimens. It may be seen that the hysteresis loops became smaller with cyclic loading. This phenomenon cannot be attributed totally to the increase in density with cyclic loading (i.e., after applying 10 cycles,  $Dr_{(N=10)} = 61.9\%$  for NC test and  $Dr_{(N=10)} = 63.7\%$  OC test), but it is probably due to the strain-hardening behavior.

The relationships between plastic volumetric strain due to dilatancy ( $\varepsilon^d$ ) and plastic shear strain ( $\gamma^p$ ) are shown in Figs. 4(b) and 5(b). The accumulation of positive volumetric strain with cyclic loading can be observed. This behavior is more significant for the NC specimen. During first cycle of loading (virgin loading), the NC specimen shows pure contractive behavior; on the other hand, the OC specimen initially behaves dilatative.

The numbers 1, 1', 2, 2'...10 in Fig. 4 and 5 correspond to the points where the loading direction was reversed. It can be seen that during the subsequent cyclic loading while approaching to the reversal points, the dilatancy characteristics change from negative to positive one; i.e., the specimen behaves dilatative. This behavior appears after the second cycle in the case of OC specimen and in the third cycle in the case of NC one.

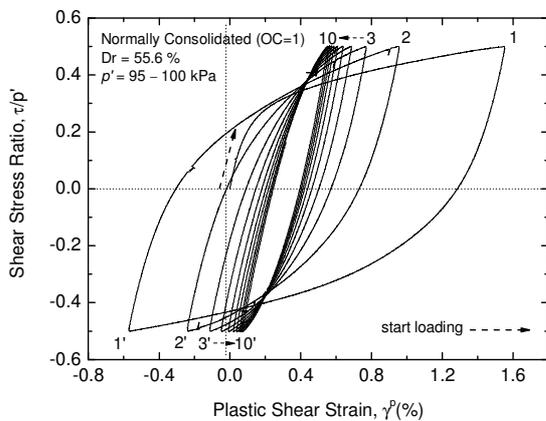


Fig. 4(a): Shear stress-strain relationship for normally consolidated sand

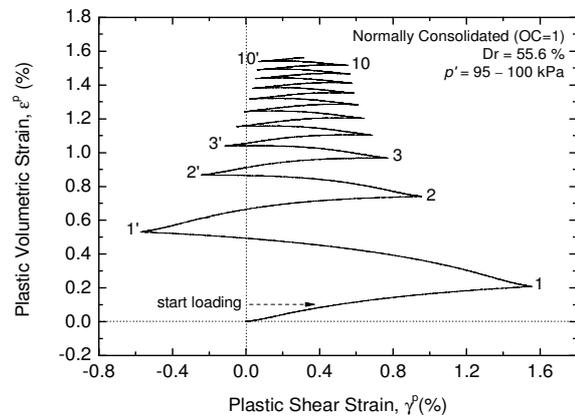


Fig. 4(b): Volumetric strain-shear strain relationship for normally consolidated sand

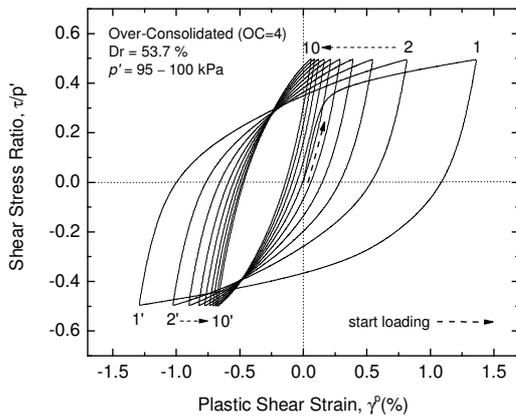


Fig. 5(a): Shear stress-strain relationship for Over-consolidated sand

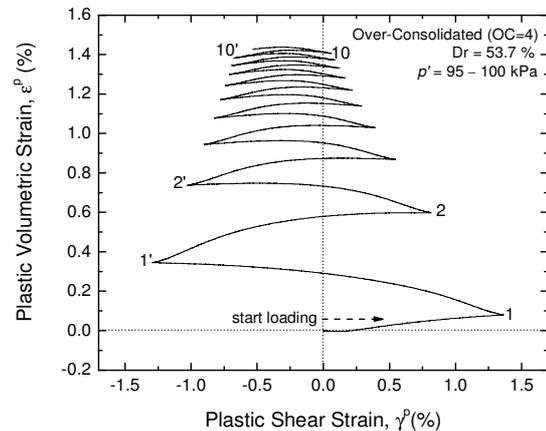


Fig.5(b): Volumetric strain-shear strain relationship for over-consolidated sand

The effects of over-consolidation on the dilatancy characteristics can be seen more clearly in the relation between the shear stress ratio ( $\tau/p'$ ) and the strain increment ratio ( $-d\varepsilon^d/d\gamma^p$ ) that are shown in Figs. 6(a) and 6(b). It should be noted that the line of  $\{-d\varepsilon^d/d\gamma^p\} = 0$  corresponds to the zero dilatancy state, which describes the change between the contractive and dilatative behaviors. Ishihara et al. (1975) called it as the phase transformation (PT)

It can be observed from these figures that the virgin loading curve is significantly affected by the over-consolidation history. NC sample shows a pure contractive behavior since the curve does not cross the zero

dilatancy state; on the contrary, the OC specimen clearly shows a change in dilatancy behavior from dilative to contractive.

The main effect of over-consolidation history on the subsequent cyclic loading consists of the shrinkage of loops with number of cycles. It may be seen that due to this shrinkage behavior, during reloading the dilatancy characteristics change from negative to positive one, repeatedly passing the zero dilatation state. However, it should be noted that, even after many cycles, during the unloading process sand exhibits a pure contractive behavior.

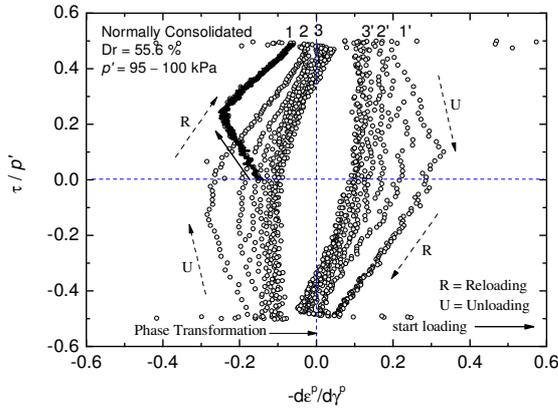


Fig. 6(a): Stress-Dilatancy relationship for normally consolidated sand

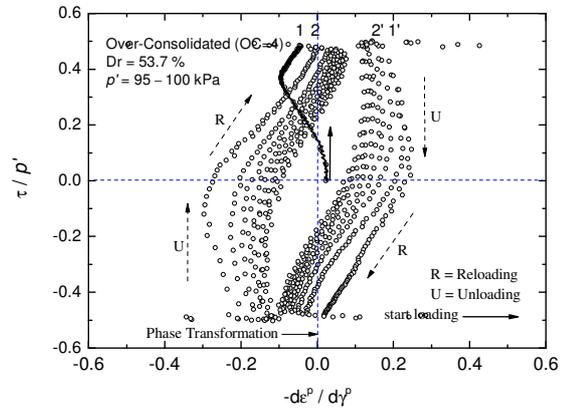


Fig. 6(b): Stress-Dilatancy relationship for over-consolidated sand

Two types of stress-dilatancy curves can be distinguished: (i) the ellipsoidal-shaped curve for NC specimen, which consists of two parallel segments of positive slopes (loading/reloading process) and two segments of negative slopes (unloading process); and (ii) the S-shaped for OC sample, which consists of two parallel segments of positive slopes (loading/reloading process) and two nearly vertical segments (unloading process). Further studies are required to investigate into such a difference to the shape of stress-dilatancy curves. The stress-dilatancy relationships of the subsequent cycles are different from that in the first cycle. For a large number of cycles these differences become remarkable.

## CONCLUSIONS

Results of drained cyclic torsional shear tests performed to investigate the effects of over-consolidation on the stress-dilatancy properties of loose Toyoura sand show that: (i) during the first cycle of loading (virgin loading) the NC sample shows a pure contractive behavior, while the OC specimen initially behaves dilative; (ii) during the subsequent cyclic loading, the hysteretic loops shrink progressively and within both cases of NC and OC the behavior of sand becomes more and more dilative.

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