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Simulation ballasted track behavior: numerical treatment and field application

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Abstract

The load deformation of ballasted rail tracks subjected to cyclic loading is investigated experimentally using a large-scale track process simulation apparatus and numerically through a combined discrete element-finite-difference approach. Laboratory tests were performed to examine the deformation and degradation of ballast subjected to cyclic loading at 15 Hz and a lateral confinement of 10 kPa. The laboratory results reveal that ballast undergoes significant deformation during the initial load cycles, followed by gradually increasing deformation attaining a steady value toward the end of testing. A numerical model based on a combined discrete element method (DEM) and finite-difference method (FDM) is introduced to study the load-deformation response of the ballast assembly while considering interaction between the ballast aggregates and the subgrade layer. In this coupled model, the discrete ballast grains are modeled by DEM, and the subgrade domain is modeled as a continuum by FDM. Interface elements are introduced to transmit the interacting forces and displacements between adjoining material domains in which the DEM transfers contact forces to the FDM, and then the FDM updates the displacements, which provides subsequent input into the DEM. This computational cycle continues with the increasing number of loading cycles. The numerical model is validated by comparing the predicted cyclic load-deformation response with the laboratory measurements. Contact force distributions and stress contours in the assembly are analyzed and presented graphically to interpret the behavior of the model track, and the effects that subgrade stiffness have on the axial strain and bond breakage of the ballast are investigated. This combined DEM-FDM analysis is also used to analyze the load deformation of an instrumented track in the town of Singleton, Australia, and the numerical predictions are compared with the field data.

Disciplines

Engineering | Science and Technology Studies

Publication Details

Ngo, N., Indraratna, B. & Rujikiatkamjorn, C. (2017). Simulation ballasted track behavior: numerical treatment and field application. *International Journal of Geomechanics*, 17 (6), 04016130-1-04016130-12.

Simulation of Ballasted Track Behaviour - A Numerical Treatment and Field Application

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Technical paper, Submitted to ASCE- International Journal of Geomechanics

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Simulation of Ballasted Track Behaviour - A Numerical Treatment and Field Application

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ABSTRACT

The load-deformation of ballasted rail tracks subjected to cyclic loading is investigated experimentally using a large-scale track process simulation apparatus, and numerically through a combined discrete element-finite difference approach. Laboratory tests were carried out to examine the deformation and degradation of ballast subjected to cyclic loading at 15 Hz and a lateral confinement of 10 kPa. The laboratory results reveal that ballast undergoes significant deformation during the initial load cycles, followed by gradually increasing deformation attaining a steady value towards the end of testing. A numerical model based on combined discrete element method (DEM) and finite difference method (FDM) is introduced to study the load-deformation response of the ballast assembly while considering interaction between the ballast aggregates and the subgrade layer. In this coupled model, the discrete ballast grains are modelled by DEM and the subgrade domain is modelled as a continuum by FDM. Interface elements are introduced to transmit the interacting forces and displacements between adjoining material domains whereby the DEM transfers contact forces to the FDM, and then the FDM updates the displacements which provide a subsequent input to the DEM; this computational cycle continues with the increasing number of loading cycles. The numerical model is validated by comparing the predicted cyclic load-deformation response with the laboratory measurements. Contact force distributions and stress contours in the assembly are analysed and presented graphically to interpret the behaviour of the

75 model track, and the effects that subgrade stiffness have on the axial strain and bond breakage of the
76 ballast are investigated. This combined DEM-FDM analysis is also used to analyse the load-
77 deformation of an instrumented track in Singleton and the numerical predictions are compared with
78 the field data.

79 **Introduction**

80 Ballast is an essential component of rail track substructure that is commonly used to distribute the
81 wheel load from sleepers to the underlying subgrade, maintain track alignment, and provide track
82 drainage (Selig and Waters 1994; Indraratna *et al.* 2013). It normally consists of medium to coarse
83 gravel sized particles (10-53 mm) and a small proportion of sand size grains. Upon repeated train
84 loading these aggregates degrade to smaller sizes that seriously decrease the shear strength and
85 impede the drainage capacity of the track substructure (Suiker *et al.* 2005; Indraratna *et al.* 2011a;
86 Huang and Tutumluer 2011). It is widely believed that the deformation of ballasted tracks can be
87 attributed to particle breakage followed by re-compaction and lateral spreading of aggregates (i.e.
88 parallel to sleepers), in the absence of sufficient lateral confinement (Suiker and Borst 2003;
89 Indraratna *et al.* 2011b; Anderson and Fair 2008). In an actual track, ballast settlement and lateral
90 spreading progressively increase with increased train speeds and impact loads associated with wheel
91 and rail irregularities. This results in excessive differential settlements which eventually alters the
92 track geometry and causes instability (Lackenby *et al.* 2007; Anderson and Fair 2008).

93 Continuum approaches (e.g. finite element or finite difference method) have been used to model
94 granular material assemblies, and various conventional continuum constitutive models have been
95 introduced to capture the stress-strain behaviour of granular materials (e.g. Indraratna and
96 Nimbalkar 2013). Due to the discrete nature of aggregates, the continuum approach cannot
97 accurately capture their micromechanical behaviour governed by fabric anisotropy, contact force
98 orientations, and localised strain. This is why the discrete element method (DEM) rather than
99 continuum-based methods are increasingly used to study granular materials. DEM was first
100 introduced by Cundall and Strack (1979) and has been widely applied to examine the mechanical
101 behaviour of ballasted tracks (McDowell *et al.* 2006; Cui and O'sullivan 2006; Lobo-Guerrero and
102 Vallejo 2010; Tutumluer *et al.* 2012; Ngo *et al.* 2014). DEM provides a deeper insight into the
103 micromechanical characteristics of granular materials, such as the contact force distribution, fabric
104 anisotropy, and particle breakage that are almost impossible or unlikely to be measured
105 experimentally (McDowell and Harireche 2002; Lobo-Guerrero and Vallejo 2006; Cui and
106 O'sullivan 2006; Bhandari and Han 2010; Ngo *et al.* 2016a).

DEM has recently been used to model the cyclic load-deformation behaviour of ballast subjected to monotonic and cyclic loads. Indraratna *et al.* (2010) used DEM to model fresh ballast subjected to 1,000 load cycles to study the shear stress-strain and particle degradation under various load frequencies, while Lu and McDowell (2010) introduced a DEM model to simulate ballast subjected to 100 load cycles and found it could still capture the stress-strain behaviour of ballast. In these studies the loading was limited to only a few hundred load cycles, and the role of angularity with regards to grains of various shapes and sizes could not be captured accurately. In addition, the high computational cost of simulating a granular assembly with an extremely large number of particles may relegate DEM to practical (large scale) problems. For instance, Indraratna *et al.* (2014a) simulated a typical laboratory test on fouled ballast (e.g. a conventional direct shear test) and the computation time was almost 500 hours (using a high performance workstation) for just one simulation. Given the excessive cost of solving a particular problem in DEM, and the numerical inability to accurately model a large geometrical problem with different particle assemblies and layering, there is a need to develop a coupled discrete-continuum approach that would fully utilise both approaches with acceptable computational effort and reasonable accuracy.

The coupled DEM-FDM model proposed here utilises each numerical scheme to provide a realistic solution to model an integrated and layered ballasted track, and a mathematical framework representing ballast-subgrade interaction while considering subgrade stiffness and particle breakage is also introduced. The cyclic load and deformation of ballast measured in the laboratory are used to calibrate and validate the proposed model.

Track Process Simulation Apparatus

A novel large-scale Track Process Simulation Apparatus (TPSA) with a dynamic actuator (specimen size: 800 mm × 600 mm × 600 mm) is used (Fig. 1) to test and monitor the deformation and degradation of ballast (Indraratna and Salim 2005; Indraratna *et al.* 2013). The ballast came from Bombo quarry, New South Wales, Australia, and was then cleaned and sieved according to Australian Standards (AS 2758.7, 1996); its average particle size is $d_{50}=35$ mm, which is similar to current Australian practices in Queensland and New South Wales. To represent field conditions, a 150 mm thick capping and subgrade layer made from a mixture of coarse sand and gravel was placed at the bottom of the apparatus and compacted to a bulk unit weight of 18.5 kN/m³. Ballast was then placed above the capping layer (i.e. subballast) and compacted in 50 mm thick sublayers to a field unit weight of approximately 15.5 kN/m³, until the ballast layer was 300 mm high.

To represent a typical track condition in NSW, Australia, a relatively low lateral confining pressure of $\sigma_{xx} = 10$ kPa was applied parallel to the sleepers to simulate the low confinement typically

provided by shoulder ballast. Meanwhile, lateral displacement in the direction of train passage was not allowed to simulate plane strain conditions, thus mimicking a straight track where ballast displacement in this direction is very small compared to lateral movement parallel to the sleepers (Indraratna *et al.* 2005; Ngo *et al.* 2016a). The four vertical walls of the apparatus were connected with a system of ball bearings and hinges which allowed them to displace laterally with minimum resistance. Cyclic loads were determined in accordance with Esvelde (2001) where a maximum induced cyclic stress of $\sigma_{yy} = 420$ kPa subjected to a frequency of $f = 15$ Hz was used to simulate a freight train with an axle load of 30 tonnes, travelling at approximately 90 km/h. All the tests continued to 500,000 load cycles. The results and analysis of these tests were presented earlier by Indraratna *et al.* (2013), and highlighted that the ballast settled rapidly within the first 100,000 load cycles, followed by a decreased rate of settlement up to 300,000 cycles, and then remained relatively unchanged towards the end of testing. Ballast breakage was also investigated in the laboratory by measuring the differences in the ballast particle size distribution curves before and after every test, using the ballast breakage index (*BBI*) proposed earlier by Indraratna and Salim (2005). The test results are used here to calibrate and validate the coupled DEM-FDM model.

Combined Discrete-Finite Difference Method

A schematic geometric model of the combined discrete-finite difference (DEM-FDM) is illustrated in Fig. 2a, where the dimensions represent the large-scale TPSA. Given that the ballast displaced along the direction of train passage is very small due to confinement by the sleepers (ties), a coupled DEM-FDM analysis was carried out while assuming an equivalent plane strain condition that usually represents long and straight tracks. A ballast layer was modelled in DEM using the Particle Flow Code, PFC2D (Itasca 2012). Ballast of different shapes and sizes are simulated by connecting and bonding many circular balls at appropriate sizes and positions to resemble the actual aggregates (Fig. 2b). Capping (subballast) and subgrade layers were modelled by the finite difference method (FDM), using the Fast Lagrangian Analysis of Continua, FLAC (Itasca 2010). Interaction between the ballast layer and subgrade was facilitated by a series of interface elements generated at the DEM and FDM boundary. Principally, coupling between DEM and FDM at the ballast-subballast interface can be done by: (i) using the forces acting on the discrete particles as force boundary conditions for the finite difference grids, and (ii) treating the finite difference nodal displacements as velocity boundary conditions for the discrete elements. A mathematical framework to help the coupled model transfer the forces and displacements between the two domains is presented in the following sections.

172 **Discrete Element Method**

173 The Discrete Element Method (DEM) has been often used to model ballast because it can capture
 174 the discrete nature of particulate materials (e.g. Oda and Iwashita 1999; O'sullivan 2011). DEM can
 175 examine the mechanical behaviour of a granular assembly consisting of a collection of arbitrarily
 176 shaped discrete particles subjected to quasi-static and dynamic conditions (Lu and McDowell 2010;
 177 O'sullivan and Cui 2009; Tutumluer *et al.* 2012). In DEM, the force-displacement law derives the
 178 contact force acting on two particles in contact to the relative displacement between them (Itasca
 179 2012). At a given time, the contact force vector $\vec{\mathbf{F}}$ that represents the interaction between the two
 180 particles (Fig. 3) is resolved into normal ($\vec{\mathbf{F}}_N$) and shear component ($\vec{\mathbf{F}}_T$) with respect to the contact
 181 plane:

$$182 \quad \vec{\mathbf{F}}_N = K_N U^n \quad (1)$$

$$183 \quad \delta \vec{\mathbf{F}}_T = -K_T \cdot \delta U^s \quad (2)$$

184 where, K_N and K_T are the normal and shear stiffnesses at the contact; U^n is the normal contact
 185 displacement; δU^s is the incremental shear displacement; and $\delta \vec{\mathbf{F}}_T$ is the incremental shear force.
 186 The new shear contact force is determined by summing the old shear force existing at the start of
 187 the time-step with the shear elastic force increment.

$$188 \quad \vec{\mathbf{F}}_T \leftarrow \vec{\mathbf{F}}_T + \delta \vec{\mathbf{F}}_T \leq \mu \vec{\mathbf{F}}_N \quad (3)$$

189 where, μ is the friction coefficient.

190 The average stress tensor, $\bar{\sigma}_{ij}$ in a given volume V of the ballast assembly is determined by in terms
 191 of the summation of discrete contact forces as:

$$192 \quad \bar{\sigma}_{ij} = \frac{1}{V} \sum_{N_c} \mathbf{F}_j^c x_i^c \quad (4)$$

193 where, $\mathbf{F}_i^c$ and x_i^c are the contact force and contact vector at contact locations; and N_c is the number
 194 of contacts in a given volume V . These forces include both forces resulting from contact at a point
 195 and parallel-bond forces. The contact location, x_i^c can be rewritten as:

$$196 \quad x_i^c = x_i^p + |x_i^c - x_i^p| n_i^{c,p} \quad (5)$$

197 where, x_i^p is the location of the particle centroid; $n_i^{c,p}$ is the unit-normal vector directed from the
 198 particle centroid to the contact location.

199 Substituting Eq. (5) to Eq. (4), and noting that $\sum_{N_c} \mathbf{F}_j^c = 0$, results in:

$$200 \quad \bar{\sigma}_{ij} = \frac{1}{V} \sum_{N_c} |x_i^c - x_i^p| n_i^{c,p} \mathbf{F}_j^c \quad (6)$$

201 Once the stress tensor, $\bar{\sigma}_{ij}$ is obtained, the vertical and shear stress contours developed across the
202 ballast assembly are plotted and analysed in the following sections.

203 ***Finite Difference Model***

204 Subballast and subgrade were modelled using Finite Difference method (FDM) as 100mm thick and
205 50mm thick homogeneous layers (i.e. similar to those in the laboratory), respectively. Given that
206 tracks are symmetrical, the left and right boundaries of the subgrade model were prevented from
207 lateral movement, but allowed to displace vertically. Indeed, the boundaries were considered to be
208 absorbent (viscous) to avoid any spurious reflection of cyclic waves, where the nodes at the bottom
209 boundary were modelled as a pinned supports (i.e. the lateral and vertical displacements were both
210 restrained). The subballast and subgrade were represented with a standard Mohr-Coulomb model
211 (linear elastic-perfectly plastic). In the current analysis, subballast with Young's modulus $E = 140$
212 MPa, Poisson's ratio $\nu = 0.35$, cohesion $c = 3.5 \text{ kPa}$, friction angle $\phi = 35^\circ$, and dilatancy angle
213 $\psi = 5^\circ$; and subgrade with $E = 60 \text{ MPa}$, $\nu = 0.33$, $c = 17 \text{ kPa}$, $\phi = 14^\circ$, and $\psi = 4^\circ$ were
214 selected based on data measured from compression tests and direct shear tests. Initially, a series of
215 walls is generated at the bottom boundary of ballast layer (Zone 1), such that each segment of wall
216 corresponds to a single segment of the surface at the top boundary of the subgrade (Zone 2), as
217 shown in Fig. 2a. Upon cyclic loading, Zone 2 deforms at large strain and the grid-point
218 displacements are then transferred to Zone 1. The resulting wall forces, due to particles interacting
219 with the walls in Zone 1, are transferred to Zone 2 as the grid-point forces are applied via interface
220 elements, as described in the Appendix.

221 It is noted in the Appendix that only the discrete particles that are in direct contact with the interface
222 elements can exert forces on the interface and the force calculation based on equivalent moment is
223 applied. If an element is moving away from the interface (i.e. discrete particles in the DEM zone),
224 then there are no forces acting on the interface. On the other hand, Equations 1-6 were used to
225 determine the forces and stresses induced for those discrete particles that were located away from
226 the interface. The developed subroutines in the coupled model automatically detect particles that are
227 in contact with the interface and implement the force calculation procedure (i.e. relationship among
228 d_1 , d_2 and L with respect to the radius of particle is also determined accordingly). It is also noted
229 that Equation 13 (i.e. determine the distribution of shear along the interface) was only applied for
230 those particles that are in direct contact with the interface. The inertia effect of the element was not

considered in the current analysis; as the coupled DEM-FDM model was simulated using an approximate quasi-static mode with relatively small time step of 1.9×10^{-5} per second, in order to avoid any unduly disturbance to the assembly (Indraratna *et al.* 2014a, Ngo *et al.* 2016b).

Modelling the TPSA

A coupled DEM-FDM model of the TPSA apparatus for ballast in a plane strain condition, where the ballast assembly is simulated by a 600 mm wide and 300 mm high layer placed over a 150 mm thick layer of subballast (100 mm) and subgrade (50 mm) is illustrated in Fig. 2a. A top wall platen is then forced to move vertically in a sinusoidal waveform to apply the desired cyclic load ($q_{max} = 420 \text{ kPa}$; $q_{min} = 50 \text{ kPa}$; frequency, $f = 15 \text{ Hz}$) while the two vertical walls are moved laterally by a servo controller (Itasca 2012) to maintain a lateral confining pressure of $\sigma_{xx} = 10 \text{ kPa}$. The simulated ballast particles (Fig. 2b) are placed at random locations within the specified wall boundaries without any overlapping, and then compacted to a bulk unit weight of 15.5 kN/m^3 to represent actual laboratory conditions (Indraratna *et al.* 2013). Micromechanical parameters are chosen by calibrating the shear stress-strain response of ballast obtained from DEM simulations with those measured in the laboratory (Table 1). Cyclic tests at a frequency of $f = 15 \text{ Hz}$ are simulated over a number of load cycles, $N=10,000$ where most ballast deformation and degradation occurred, as observed in the laboratory. During loading, displacement of the top plate and two vertical walls were recorded to determine the axial and lateral strains, while stress tensors were computed at specific load cycles to determine the corresponding stress contours.

Results and Discussion

Load-Deformation Response

Figure 4a illustrates the applied cyclic stress versus accumulated axial strain obtained from the coupled DEM-FDM model under $N=10,000$ load cycles. It is seen that the predicted axial strains increases remarkably up to around 2% within the first 1000 load cycles, followed by gradually increasing axial strains up to about 3% (i.e. within 5,000 cycles), and then remained relatively stable to the end (10,000 cycles). Indeed, the area confined by the cyclic (hysteresis) loops becomes increasingly smaller as the number of cycles increases, indicating that the ballast specimen through cyclic densification begins to respond more elastically with time. The hysteresis loops are also very similar to those obtained in laboratory tests and numerical studies (e.g. Indraratna *et al.* 2010). The predicted axial and lateral strains are compared with those measured by Indraratna *et al.* (2013), as shown in Figs. 4b-c. This indicates that the predicted strains agree with the experimental data, and show that most strains occurred after the first 100 cycles, after which permanent deformation decreased considerably. This indicates that the ballast sample experiences considerable

264 rearrangement and densification during the initial load cycles. However, after attaining a threshold
265 compression, any subsequent loading would resist further deformation and promote particle
266 crushing (Indraratna *et al.* 2011). This agrees with the results presented by Lobo-Guerrero and
267 Vallejo (2010) where they stated that ballast deformation increased significantly when particle
268 breakage is included in the analysis. The coupled analysis shows a noticeable discrepancy in the
269 strain curves compared to the experimental data, with a marked increase in strain at $N=5,000-8,000$
270 load cycles. This difference may be attributed to excessive particle degradation (i.e. contact bond
271 breakage) that could not be captured accurately in the coupled model, as well as the rigid loading
272 plate. Indeed, as the contact bonds broke, it would increase compression on the ballast assembly,
273 followed by a subsequent increase in lateral strain.

274 ***Stress Analysis***

275 The applied stress transmits to the discrete ballast aggregates in the form of contact force-chains
276 where the force-fabric pattern varies with the packing structure, and directly governs the
277 deformation and strength of the discrete (granular) assembly (Oda and Iwashita 1999). Fig. 5 shows
278 the inter-particle forces of the ballast assembly together with vertical stress contours at $N=10, 1000,$
279 and $N=5000$ cycles. Each contact force is represented at the contact point by a ‘black line’ oriented
280 in the direction of the force and with a thickness proportional to its intensity. Here, most of the
281 contact forces plot in a vertical direction, but upon repeated loading, large contact forces
282 concentrate beneath the loading plate and around edges of the wall, while a significant part of the
283 applied load is still being transmitted vertically to the underlying subballast and subgrade (Figs. 5b-
284 c). Moreover, the force distributed in the DEM region is always heterogeneous, where the
285 maximum contact forces change with the load cycles, and compressive stress (σ_{yy}) in the subgrade
286 is greater around the interface area that is in direct contact with the aggregates, and as expected, it
287 decreases with depth.

288 Fig. 6 presents the vertical (σ_{yy}) and shear (σ_{xy}) stress contours in the ballast assembly that are
289 subjected to a maximum cyclic load at $N = 100$ and 1000 . The stresses induced in the ballast
290 assembly are determined by the contact forces acting on it (Equation 6). Note that the stresses are
291 distributed non-uniformly across the assembly (i.e. for both σ_{yy} and σ_{xy}) and decrease with depth,
292 although maximum stresses tend to occur beneath the loading plate and the edges of the wall, which
293 is consistent with the contact force distributions described above. Locally, the vertical stress σ_{yy} has
294 a maximum value of about 410 kPa, while the peak shear stress, σ_{xy} is around 100 kPa. It is noted
295 that loads (i.e. vertical and shear stresses) applied on the top surface of ballast assemblies are
296 transmitted to ballast grains through an interconnected network of force chains at contact points
297 (Fig. 5). These forces create the deformation of the assembly (i.e. vertical and lateral displacements)

298 and the breakage of contact bonds (i.e. particle breakage). The stresses at the bottom of ballast
299 assembly, therefore, are much lower than those measured at the top surface (gravitational force is
300 considered in the DEM analysis). It is also noted that the applied loads are diminished with depth
301 and the influence of rigid boundary walls leading to unbalance forces the top and the bottom of the
302 ballast.

303 ***Particle Breakage and the Effect of Subgrade Stiffness***

304 In this current analysis the degradation of contact bonds within a cluster of particles represents
305 particle breakage. The effect the loading frequency has on the deformation of a ballast assembly
306 was investigated by conducting a DEM-FDM analysis at varying frequencies. Fig. 7a shows the
307 predicted evolution of bond breakage at various cyclic loadings subjected to different load
308 frequencies $f = 15 \text{ Hz}$ to 30 Hz . The ballast breakage index (*BBI*) measured in the laboratory is
309 also plotted for a comparison. Here, the evolution of broken bonds is similar to the changes of *BBI*
310 observed in the laboratory where bond breakage generally increases as the load frequency increases.
311 The predicted bond breakage increased rapidly and attained a relatively stable value after about
312 5,000 cycles. The amount of broken bonds in the range of $100 \leq N \leq 5,000$ increased markedly
313 with only a marginal increase in bond breakage in the subsequent loading. In comparison to 15-25
314 Hz, the predicted amount of broken still increased slightly, even after 8,000 cycles for $f = 30 \text{ Hz}$,
315 thus demonstrating continuing cyclic densification at high frequencies. Figs. 7b-d illustrate the
316 number and location of bond breakages at different stages of cyclic loading, varying from 500 to
317 10,000 cycles under a frequency of $f=15 \text{ Hz}$. For $N<1,000$ cycles, most bonds break just below the
318 loading plate due to induced high contact forces (Fig. 7b), so it is obvious that an increase in the
319 cyclic load will result in more bond breakage (Figs. 7c, 7d), and a re-arrangement of broken
320 particles (i.e. densification) will lead to a denser and more uniform distribution of contact force, as
321 shown earlier in Fig. 5.

322 The influence of subgrade stiffness on the deformation of the ballast assembly (excluding subgrade
323 deformation) was studied where the subgrade varied in stiffness from 15 MPa for soft subgrade to
324 100 MPa for relatively stiff subgrade at $f = 15 \text{ Hz}$. Fig. 8 presents the evolutions of axial strains
325 and the amount of broken bonds for different subgrade moduli. With weak subgrade (e.g. $E=15$
326 MPa) the ballast assembly exhibits the highest axial strain (Fig. 8a) while sustaining the lowest
327 amount of broken bonds (Fig. 8b), unlike the stiffer subgrade. Indeed, weak subgrade could act like
328 an energy absorbing layer that attenuates the transmission of impact induced cyclic loads and
329 substantially reduces ballast breakage.

330 **Application to a Case Study**

331 A DEM-FDM analysis was carried out to predict the load-deformation response of actual tracks in
332 the town of Singleton (located about 200 km from Sydney, NSW, Australia). Field data measured
333 by Indraratna *et al.* (2014b) were used to compare with the current numerical analysis. A field trial
334 was carried out along a section of instrumented track where two types of subgrade, i.e. relatively
335 soft general fill and hard rock (i.e. Section A and Section C, respectively) were examined. The track
336 substructure consisted of a 300 mm thick layer of ballast placed over 150 mm thick capping layer,
337 overlying structural fill (thickness varying from 500 to 900 mm).

338 The combined DEM-FDM model used to simulate the settlement S_v of the half- track is shown in
339 Fig. 9a, where the contact force chains and vertical stress contours were captured at $N=10,000$.
340 Note that the geometry, boundary conditions, and applied cyclic loads were modelled like those
341 carried out in the field (i.e. mean cyclic stress of 235 kPa; frequency of 15 Hz) to realistically
342 represent simulated freight trains with axle loads of 25-30 tonnes travelling at 80 km/h. The
343 simulation shows that ballast directly underneath the sleeper experiences denser contact forces than
344 ballast at the shoulder of the embankment, and this causes vertical stress (σ_{yy}) at the interface
345 between the ballast and capping to be distributed non-uniformly along the interface, where
346 increased stresses occur towards the centre of track; this supports the data measured in the field by
347 Indraratna *et al.* (2014b).

348 Comparisons of the settlement S_v of ballast for two types of subgrades obtained from the numerical
349 model and field measurements are presented in Fig. 9b, where settlement was measured beneath the
350 sleeper. It is evident that the model predicts the vertical deformation of ballast well in relation to the
351 observed field data using settlement pegs, where only a slight deviation from those measurements
352 was found. Where $N= 10,000$, the measured value of S_v for hard subgrade of 6.07 mm compares
353 well with the predicted value of 6.71 mm. The notable increase in settlement predicted for soft
354 subgrade ($S_v=10.21$ mm) compared to the field observation of $S_v = 8.72$ mm can be attributed to
355 inevitable discrepancies between the field conditions and the computational model that adopted
356 plane strain conditions.

357 **Conclusion**

358 Large scale simulation testing of the track process was carried out to study the load-deformation of
359 ballast subjected to cyclic loading. Laboratory results showed that ballast experienced a significant
360 axial strain during the initial load cycles, followed by a gradually increasing settlement that became
361 relatively stable towards the end. A coupled DEM-FDM model was introduced to study the load-
362 deformation responses of ballast in a plane strain, where the aggregates were modelled by the

363 discrete element method and the subgrade (continuum) by the finite difference method. The two
364 domains, DEM and FDM interacted with each other via interface elements, where DEM transferred
365 the forces as an input to FDM, which then updated the displacements back to DEM using
366 mathematical subroutines.

367 The computed values of axial and lateral strains compared to the experimental data, indicated that
368 the coupled DEM-FDM model proposed here could capture the correct load–displacement
369 behaviour under cyclic loading ($N=100-10,000$). The model indicated that the extent of bond
370 breakage in the ballast assembly increased with the frequency ($f=15-30$ Hz). The contact force
371 distributions and stress contours induced in the ballast at varying stages of cyclic loading were
372 analysed, and confirmed that the stresses would transfer non-uniformly across the ballast assembly
373 such that maximum stress would occur beneath the sleeper corresponding to greatest number of
374 broken bonds. The influence of subgrade stiffness on axial strain and bond breakage was also
375 analysed, and it showed that an increase in the subgrade stiffness resulted in an increased number of
376 broken bonds. The DEM-FDM analysis was also used to predict the load-deformation response of
377 an instrumented track in Singleton. The predicted settlement was comparable to the measured field
378 data indicating that the proposed DEM-FDM model is reliable.

379

380

381 **Appendix: Interface Elements**

382 Interface elements were created at the boundary of ballast and subgrade to help implement the
 383 coupling process where conditions of equilibrium and compatibility must be satisfied at the
 384 interface. Assuming an interface element (segment) is defined by end-point locations, X_1 , X_2 ; a
 385 length, L and a particle contact force, $\mathbf{F}_i^c = \mathbf{F}_1 + \mathbf{F}_2$, act at the particle centroid, X_p , as shown in
 386 Fig. 10. The contact forces acting at the segment end points, $\mathbf{F}_1$, and $\mathbf{F}_2$ can be described in terms
 387 of shear and normal component vectors with corresponding unit vectors, $\hat{t}$ and $\hat{n}$:

$$388 \quad \mathbf{F}_1 = F_{1x}\hat{t} + F_{1y}\hat{n} \quad (7)$$

$$389 \quad \mathbf{F}_2 = F_{2x}\hat{t} + F_{2y}\hat{n} \quad (8)$$

390 The force components F_{1x} , F_{1y} , F_{2x} , and F_{2y} were determined by satisfying a condition whereby
 391 the force acting at the segment end points should produce the same moment about X_1 as the particle
 392 contact force:

$$393 \quad d_1 \times \mathbf{F}_i^c = L\hat{t} \times \mathbf{F}_2 = L\hat{t} \times (F_{2x}\hat{t} + F_{2y}\hat{n}) \quad (9)$$

$$394 \quad d_1 \times \mathbf{F}_i^c = LF_{2x}(\hat{t} \times \hat{t}) + LF_{2y}(\hat{t} \times \hat{n}) \quad (10)$$

$$395 \quad \text{or, } (d_{1x}\hat{t} + d_{1y}\hat{n}) \times (F_x\hat{t} + F_y\hat{n}) = LF_{2y}(\hat{t} \times \hat{n}) \quad (11)$$

$$396 \quad \text{given that: } \hat{t} \times \hat{t} = 0; \hat{n} \times \hat{n} = 0; \text{ and } \hat{t} \times \hat{n} = \hat{k}$$

$$397 \quad \text{thus, } (d_{1x}F_y - d_{1y}F_x)\hat{k} = LF_{2y}\hat{k} , \text{ or}$$

$$398 \quad F_{2y} = \frac{(d_{1x}F_y - d_{1y}F_x)}{L} \quad (12)$$

399 Distributing the shear components based on the nearest of X_p to each end point to obtain F_{1x} , as
 400 given:

$$401 \quad F_{1x} = \left(\frac{|d_2|}{|d_1| + |d_2|} \right) (\mathbf{F}_i^c \cdot \hat{t}) \quad (13)$$

402 The forces acting at the segment end points should produce the same total force as the particle
 403 reaction force: $\mathbf{F}_i^c = \mathbf{F}_1 + \mathbf{F}_2 \quad (14)$

404 Expressing Eq. (14) in terms of the unit-tangent vector, which gives two equations:

$$405 \quad \mathbf{F}_x = (F_{1x} + F_{2x})\hat{t}_x - (F_{1y} + F_{2y})\hat{t}_y \quad (15)$$

$$F_y = (F_{1x} + F_{2x})t_y + (F_{1y} + F_{2y})t_x \quad (16)$$

Rearranging Eqs. (15) and (16), results in:

$$F_{1y} = [(F_y - F_{2y}t_x - F_{1x}t_y)t_x + (-F_x - F_{2y}t_y + F_{1x}t_x)t_y]/2 \quad (17)$$

$$F_{2x} = \begin{cases} \frac{F_x + (F_{1y} + F_{2y})t_y - F_{1x}t_x}{t_x}, & \text{or} \\ \frac{F_y - (F_{1y} + F_{2y})t_x - F_{1x}t_y}{t_y} \end{cases} \quad (18)$$

When the above equations are applied the segment forces acting in Zone 2 (F_{1x} , F_{1y} , F_{2x} , and F_{2y}) should produce the same total force and moment as the particle contact forces acting on Zone 1 at the interface (Fig. 2a), so both zones along the interface boundary experience the same loading condition. Subroutines written in FISH language were developed by the Authors so that all the aforementioned equations could be used to implement a fully coupled DEM-FDM analysis.

Acknowledgements

The authors are grateful to Professor Glenn McDowell who provided valuable discussions and comments during the preparation of the paper. The authors are grateful for the financial support provided by the Australian Research Council (ARC). Laboratory assistance from technicians, Alan Grant and Ian Bridge during the laboratory tests is also very much appreciated.

423 **Notation**

424 *The following symbols are used in this paper:*

425 BBI = ballast breakage index;

426 c = cohesion;

427 $d_{1,2}$ =distances from particle centroid to the segment end points;

428 d_{50} = medium value of the particle size distribution;

429 E = Young's modulus;

430 f = cyclic frequency;

431 $\vec{F}$ = contact force;

432 $\vec{F}_N$ = contact normal force;

433 $\vec{F}_T$ = contact shear force;

434 F_i^c = contact force at contact location, x_i^c ;

435 $F_{1,2}$ = contact forces acting at the segment end points;

436 $F_{1x}, F_{1y}, F_{2x},$ and F_{2y} = force components in horizontal and vertical directions;

437 $\delta\vec{F}_T$ = incremental shear force;

438 K_N = contact normal stiffness;

439 K_T = contact shear stiffness;

440 $\hat{k}$ = binormal vector to tangent and normal;

441 L = length of element

442 N = number of load cycles;

443 N_c = number of contacts;

444 $n_i^{c,p}$ = unit-normal vector directed from the particle centroid to the contact location;

445 $\hat{n}$ = unit normal vector;

446 q_{max} = maximum cyclic stress;

447 q_{min} = minimum cyclic stress;

448 S_v = vertical settlement;

449 x_i^c = contact location;

450 x_i^p = location of the particle centroid;

451 $\hat{t}$ = unit tangent vector;

452 U^n = normal contact displacement;

453 δU^s = incremental shear displacement;

454 μ = friction coefficient;

455 ν = Poison's ratio;

456 ε_x = lateral strain;

457 ε_y = vertical strain;
458 ϕ = friction angle;
459 ψ = dilatancy angle
460 σ_{xx} = horizontal stress;
461 σ_{yy} = vertical stress;
462 σ_{xy} = shear stress; and
463 $\bar{\sigma}_{ij}$ = stress tensor.
464

References

- Australian Standard: AS 2758.7 (1996). *Aggregates and rock for engineering purposes; Part 7: Railway ballast*, Sydney, NSW, Australia.
- Anderson, W. F. , and Fair, P. (2008). "Behavior of railroad ballast under monotonic and cyclic loading." *Journal of Geotechnical and Geoenvironmental Engineering*, 143(3), 316–327.
- Bhandari, A. , and Han, J. (2010). "Investigation of geotextile-soil interaction under a cyclic vertical load using the discrete element method". *Geotextiles and Geomembranes*, 28(1), 33-43.
- Cui, L. , and O'Sullivan, C. (2006). "Exploring the macro- and micro-scale response of an idealised granular material in the direct shear apparatus". *Géotechnique*, 56(7), 455-468.
- Cundall, P. A. , and Strack, O. D. L. (1979). "A discrete numerical model for granular assemblies". *Géotechnique*, 29(1), 47-65.
- Esveld, C. (2001). *Modern railway track*, MRT Press, The Netherlands.
- Itasca (2010). *FLAC - Fast Lagrangian Analysis of Continua, Version 6.0*, Itasca Consulting Group, Minneapolis, MN.
- Itasca. (2012). *Particle flow code in two and three dimensions*, Itasca Consulting Group, Inc., Minnesota.
- Indraratna, B. , and Salim, W. (2005). *Mechanics of ballasted rail tracks - A Geotechnical Perspective*, Taylor and Francis, Balkema, London, UK.
- Lackenby, J., Indraratna, B., McDowell, G. R. , and Christie, D. (2007). "Effect of confining pressure on ballast degradation and deformation under cyclic triaxial loading." *Géotechnique*, 57(6), 527–536.
- Indraratna, B., Thakur, P. K. , and Vinod, J. S. (2010). "Experimental and numerical study of railway ballast behavior under cyclic loading." *International Journal of Geomechanics, ASCE*, 10(4), 136-144.
- Indraratna, B., Salim, W. , and Rujikiatkamjorn, C. (2011a). *Advanced Rail Geotechnology - Ballasted Track*, CRC Press, Taylor & Francis Group, London, UK.
- Indraratna, B., Ngo, N.T. , and Rujikiatkamjorn, C. (2011b). "Behavior of geogrid-reinforced ballast under various levels of fouling." *Geotextiles and Geomembranes*, 29(3), 313-322.
- Indraratna, B., Ngo, N.T. , and Rujikiatkamjorn, C. (2013). "Studying the deformation of coal fouled ballast stabilised with geogrid under cyclic load". *Journal of Geotechnical and Geoenvironmental Engineering-ASCE*, 139(8), 1275–1289.
- Indraratna, B. , and Nimbalkar, S. (2013). "Stress-strain degradation response of railway ballast stabilized with geosynthetics." *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 139(5), 684-700.

499 Indraratna, B., Ngo, N.T, Rujikiatkamjorn, C. , and Vinod, J. (2014a). "Behaviour of fresh and
500 fouled railway ballast subjected to direct shear testing - a discrete element simulation." *International Journal of Geomechanics, ASCE*, 14(1), 34-44.

501

502 Indraratna, B., Nimbalkar, S. , and Rujikiatkamjorn, C. (2014b). "From theory to practice in track
503 geomechanics - Australian perspective for synthetics inclusions." *Transportation Geotechnics*,
504 1(2014), 171-187.

505 Lobo-Guerrero, S. , and Vallejo, L. E. (2006). "Discrete element method analysis of railtrack ballast
506 degradation during cyclic loading." *Granular Matter*,. 8(3-4), 195-204.

507 Lobo-Guerrero, S. , and Vallejo, L. E. (2010). "Crushing of particles under simulated static and
508 centrifuge forces." *GeoFlorida 2010: Advances in Analysis, Modeling & Design (Geotechnical
509 Special Publication, 199)*, ASCE, 550-559.

510 Lu, M. , and McDowell, G. R. (2010). "Discrete element modelling of railway ballast under
511 monotonic and cyclic triaxial loading." *Géotechnique*, 60(6), 459-467.

512 Huang, H. , and Tutumluer, E. (2011). "Discrete element modeling for fouled railroad ballast."
513 *Construction and Building Materials*, 25, 3306-3312.

514 McDowell, G. R. , and Harireche, O. (2002). "Discrete element modelling of soil particle fracture."
515 *Géotechnique*, 52(2), 131-135.

516 McDowell, G. R., Harireche, O., Konietzky, H., Brown, S. F. , and Thom, N. H. (2006). "Discrete
517 element modelling of geogrid-reinforced aggregates." *Proceedings of the ICE - Geotechnical
518 Engineering*, 159(1), 35-48.

519 Ngo, N.T., Indraratna, B. , and Rujikiatkamjorn, C. (2014). "DEM simulation of the behaviour of
520 geogrid stabilised ballast fouled with coal." *Computers and Geotechnics*, 55, 224-231.

521 Ngo, N.T., Indraratna, B., Rujikiatkamjorn, C. , and Biabani, M. (2016a). "Experimental and
522 discrete element modeling of geocell-stabilized subballast subjected to cyclic loading". *Journal
523 of Geotechnical and Geoenvironmental Engineering-ASCE*, 142(4), 04015100,1-14.

524 Ngo, N.T, Indraratna, B., and Rujikiatkamjorn, C. (2016b). "Micromechanics-based investigation of
525 fouled ballast using large-scale triaxial tests and discrete element modeling". *Journal of
526 Geotechnical and Geoenvironmental Engineering-ASCE*, DOI: 10.1061/(ASCE)GT.1943-
527 5606.0001587, pp:04016089, 1-16.

528 O'Sullivan, C. (2011). *Particulate Discrete Element Modelling: A Geomechanics Perspective*, Spon
529 press, London.

530 O'Sullivan, C. , and Cui, L. (2009). "Micromechanics of granular material response during load
531 reversals: Combined DEM and experimental study." *Powder Technology*, 193, 289–302.

532 Oda, M. , and Iwashita, K. (1999). *Mechanics of granular materials: An introduction*, Rotterdam:
533 A. A. Balkema.

534 Selig, E. T. , and Waters, J. M. (1994). *Track geotechnology and substructure management*,
535 Thomas Telford, London.

536 Suiker, A. S. J. , and Borst, R. (2003). "A numerical model for the cyclic deterioration of railway
537 tracks." *International Journal for Numerical methods in Engineering*, 57, 441–470.

538 Suiker, A. S. J., Selig, E. T. , and Frenkel, R. (2005). "Static and cyclic triaxial testing of ballast and
539 subballast." *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 131(6), 771–
540 782.

541 Tutumluer, E., Huang, H. , and Bian, X. (2012). "Geogrid-aggregate interlock mechanism
542 investigated through aggregate imaging-based discrete element modeling approach."
543 *International Journal of Geomechanics, ASCE*, 12(4), 391-398.

544

545 **List of Table**

546

547 **Table 1.** Micro-mechanical parameters of ballast particles and walls applied in DEM simulation

548

Micro-mechanical parameters	Values
Radius of particle (m)	$1.8 \times 10^{-3} - 16 \times 10^{-3}$
Inter-particle coefficient of friction	0.80
Particle normal and shear contact stiffness (N/m)	3.58×10^8
Normal and shear stiffness of wall (N/m)	3×10^7
Parallel bond normal and shear stiffness (N/m)	6.25×10^{10}
Parallel bond normal and shear strength (N/m ²)	5.78×10^6
Parallel bond radius multiplier	0.5
Particle unit weight (kN/m ³)	15.5

549

(a)

(b)

552
553 **Fig. 1.** Track Process Simulation Apparatus, TPSA : (a) perspective views of the apparatus; (b)
554 cross-section of the TPSA.

555

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559

560 **Fig. 2.** (a) Schematic diagram of a coupled DEM-FDM to model TPSA for ballast (dimensions are
561 in mm); (b) Irregular-shaped particles

562

Fig. 3. Notations used to describe contacts in DEM: (a) ball-ball contact; (b) ball-wall contact

Fig. 4. (a) applied cyclic stress versus axial strain; (b) axial strain versus number of load cycles; (c) lateral strains versus number of load cycles

568

569

570

571 **Fig. 5.** Vertical stress contour (σ_{yy}) in FDM region and contact force distribution in DEM region at
572 different load cycles; (a) 10 cycles; (b) 1000 cycle; and (c) 5000 cycles; (contour unit in Pascal)

573

575 **Fig. 6.** Vertical and shear stress contours developed in the ballast assembly at different load cycles;
576 (a) 100 cycles; (b) 1000 cycles; (contour unit in Pascal)

Fig. 7. (a) Comparison of the number of simulated broken bonds with actual ballast breakage index (BBI) from laboratory testing; (b)-(d): Snapshots of bond breakage at varying load cycles: (b) $N=500$; (c) $N=5,000$; (d) $N=10,000$

583

584

585 **Fig. 8.** Effect of subgrade modulus on: (a) axial strain; (b) number of bond breakage.

586

Fig. 9. Half- track model on the Singleton track: (a) contact force in ballast and vertical stress contour in subgrade (unit in Pascal); (b) comparisons of settlement between the combined model and field data.

592

593

Fig. 10. Interface element: mapping particle contact force to segment end points