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Abstract

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Keywords

thermal, on-site, system, hvac, residential, energy, control, storage, hybrid, model, predictive, generation

Disciplines

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Hybrid model predictive control of a residential HVAC system with on-site thermal energy generation and storage

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Abstract: This paper describes the development, implementation and experimental investigation of a Hybrid Model Predictive Control (HMPC) strategy to control solar-assisted heating, ventilation and air-conditioning (HVAC) systems with on-site thermal energy generation and storage. A comprehensive approach to the thermal energy management of a residential building is presented to optimise the scheduling of the available thermal energy resources to meet a comfort objective. The system has a hybrid nature with both continuous variables and discrete, logic-driven operating modes. The proposed control strategy is organized in two hierarchical levels. At the high-level, an HMPC controller with a 24-hour prediction horizon and a 1-hour control step is used to select the operating mode of the HVAC system. At the low-level, each operating mode is optimised using a 1-hour rolling prediction horizon with a 5-minute control step. The proposed control strategy has been practically implemented on the Building Management and Control System (BMCS) of a Net Zero-Energy Solar Decathlon house. This house features a sophisticated HVAC system comprising of an air-based photovoltaic thermal (PVT) collector and a phase change material (PCM) thermal storage integrated with the air-handling unit (AHU) of a ducted reverse-cycle heat pump system. The simulation and experimental results demonstrated the high performance achievable using an HMPC approach to optimizing complex multimode HVAC

26 systems in residential buildings, illustrating efficient selection of the appropriate operating
27 modes to optimally manage thermal energy of the house.
28 Keywords: Hybrid model predictive control; Photovoltaic-thermal collectors; Phase change
29 material thermal storage; Solar-assisted HVAC; System identification.

30

31 **Nomenclature**

32 A_i = equivalent area of internal solar gains [m^2]

33 A_e = equivalent area of wall solar gains [m^2]

34 c_p = specific heat of air [J/kg K]

35 C_i = indoor space equivalent capacitance [kWh/K]

36 C_e = effective thermal capacitance of walls [kWh/K]

37 C_{pcm} = effective capacitance of PCM unit [kWh/K]

38 \overline{COP} = heat pump average coefficient of performance

39 $\delta_{m..}$ = discrete Boolean variables for operating modes

40 $\delta_{m..}$ = discrete Boolean variables for natural ventilation activation

41 δ_{hp} = discrete Boolean variable for heat pump activation

42 ε = cost associated to thermal comfort constraint softening [$\text{kW}/^\circ\text{C}$]

43 η_{HP} = identifiable efficiency of the heat pump

44 Φ_{ig} = internal loads [kW]

45 Φ_{hp} = heat pump thermal generation [kW]

46 Φ_{PVT} = PVT system thermal generation [kW]

47 Φ_{PCM} = PCM unit thermal generation [kW]

48 Ψ = solar gains on building lumped capacitance surfaces [kW/m^2]

49 ρ = air density [kg/m^3]

50 R_{pcm} = equivalent PCM unit thermal resistance [K/kW]

- 51 R_w = equivalent half-wall resistance [K/kW]
 52 R_v = equivalent infiltration resistance, operable windows closed [K/kW]
 53 R_{vo} = equivalent infiltration resistance, operable windows open [K/kW]
 54 T_a = ambient temperature [°C]
 55 $T_{pcm,i}$ = PCM unit inlet air temperature [°C]
 56 $T_{pcm,o}$ = PCM unit outlet air temperature [°C]
 57 T_i = average indoor temperature [°C]
 58 \bar{T}_i = indoor temperature upper boundary [°C]
 59 \underline{T}_i = indoor temperature lower boundary [°C]
 60 $T_{pvt,o}$ = PVT outlet temperature [°C]
 61 $T_{melt,b}$ = PCM melting temperature, lower limit [°C]
 62 $T_{melt,t}$ = PCM melting temperature, upper limit [°C]
 63 \dot{V} = air volume flow rate [m³/s]

64

65 **1. Introduction**

66 Increasing energy efficiency in buildings has been recognized as one of the fastest and most
 67 cost-effective means to save energy and reduce greenhouse gas emissions, as the building
 68 sector accounts for approximately 40% of the world's energy demand [1]. A central piece in
 69 improving building energy efficiency and sustainability is the installation, commissioning
 70 and fine-tuning of efficient Heating, Ventilation and Air Conditioning (HVAC) systems and
 71 Building Management and Control Systems (BMCSs) [2].

72 The concept of Net-Zero Energy Buildings (NZEBs), which capture the high-level of energy
 73 usage performance required from buildings, has received growing attention as a research and
 74 technology driver, and has been extensively discussed in many energy policy forums (e.g.
 75 [1]). An instrumental methodology in achieving the goal of a NZEB is the embedding of

76 optimal control strategies in building HVAC systems and BMCSs. These optimal control
77 strategies can help match the building energy demand in face of weather-dependent energy
78 generation and disturbances by efficiently integrating energy storage systems, which are now
79 recognized as a proven demand management technology in building energy management.

80 However, the practical design and implementation of optimal control strategies that can
81 efficiently integrate on-site energy generation and storage with a complex communication
82 network between buildings, and to a smart grid, largely remains as a challenging problem [3].
83 Current industrial HVAC practice [4,5] is predominantly limited to the implementation of
84 Rule-Based Control (RBC) and Proportional-Integral-Derivative (PID) control strategies,
85 which have traditionally been reliable in the operation of building systems. However, such
86 relatively simple classical control approaches cannot in general deal with the high
87 performance requirements of modern buildings.

88 A proven advanced control strategy that can deal with such complex real-time optimal
89 integration problem is Model Predictive Control (MPC). MPC is a well-established
90 methodology in the control of complex interacting dynamic systems in the process industry,
91 and has been receiving wide attention from the building control research community. In
92 building systems, MPC has been applied to control the zone temperatures and building
93 thermal mass [6–9], with numerous encouraging experimental results reported [10]. The
94 formulation of dynamic models, a critical component of the MPC strategy, has been
95 undertaken in a number of building studies [11,12], based on identifiable models and the use
96 of model parameter identification techniques [13–16].

97 MPC has also been used to solve on-site generation and storage scheduling problems [17,18]
98 with continuous variables and discrete, logic-driven operating modes, generally leading to a
99 mixed–integer programming problem to be solved [19,20]. This is commonly referred as
100 Hybrid MPC (HMPC) [21–23].

101 In the area of residential buildings, implementation of MPC on a conventional heating system
102 showed a significant increase in the system energy efficiency [24]. A comprehensive
103 framework based on an MPC algorithm was presented by Di Giorgio and Liberati [25] to
104 optimally schedule electrical generation, storage and controllable demand in a house.

105 Many modern residential buildings, which are often aiming to achieve the Net Zero-Energy
106 target, implement renewable technologies on their HVAC systems, where the coordination of
107 solar thermal generation, thermal energy storage, conventional air conditioning systems and
108 operable windows must be optimally controlled to achieve efficiency and thermal comfort
109 objectives.

110 While quite extensive work has been undertaken in optimising temperature control or
111 scheduling electrical resources in buildings using the MPC framework, to the best of the
112 authors knowledge, there is a gap in current research in developing a comprehensive MPC
113 framework for this typology of HVAC systems at a residential scale, as well as a lack of
114 experimental demonstrations of their operation. Since these thermal energy resources are
115 highly weather-dependent, it is believed that these systems could strongly benefit from the
116 application of an MPC approach, through optimising the building thermal dynamics with the
117 management of the renewable and conventional thermal energy resources. As the
118 optimisation problem to be solved features a combination of continuous and discrete
119 dynamics, this will lead to the formulation of an HMPC controller. A preliminary
120 investigation on the application of an MPC strategy on the system studied in this paper
121 showed some promising results [26].

122 In this paper, an HMPC strategy is developed for optimising the operation of a solar-assisted
123 HVAC system, which includes Photovoltaic-thermal (PVT) collectors, an active PCM
124 thermal storage unit, a conventional reverse cycle air conditioning unit and operable windows
125 implemented in the ‘Illawarra Flame’ house, a net zero-energy retrofit house that won the

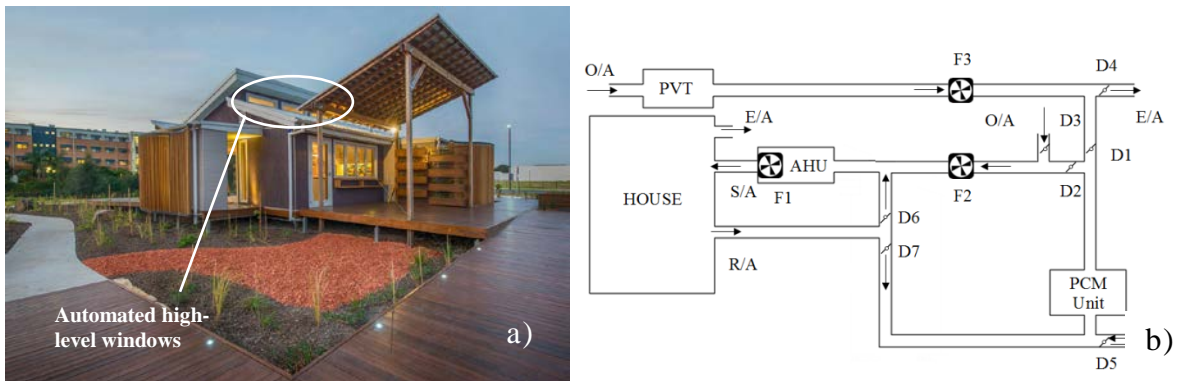
126 Solar Decathlon China 2013 Competition. The system was designed for operation during
127 both winter and summer, using daytime solar radiation and night sky radiative cooling to
128 increase overall energy efficiency. To compute a near-optimal solution for this type of system
129 [27], the problem formulated as a Mixed Logical Dynamical (MLD) model can capture both
130 continuous and discrete dynamics, for which an HMPC controller is then developed.

131 In this study, an R-C model was utilised to represent the house dynamics and a system
132 identification of the key parameters was performed using the experimental data collected
133 from the house operation. A hierarchical HMPC control structure was implemented in Matlab
134 using the Hybrid Toolbox for Matlab [28]. A key objective of this study was to
135 experimentally demonstrate the implementation of this HMPC controller. A live
136 communication link between the Matlab script and the BMCS of the ‘Illawarra Flame’ house
137 allowed the deployment of the real-time controller and its testing on the real system. Section
138 2 describes the case study HVAC system and its control system, followed by the description
139 of the grey-box building model and its identification in Section 3. The description of the
140 HVAC system models is provided in Section 4 and the MPC formulation is presented in
141 Section 5. The experimental results are reported in Section 6 and the conclusion is provided
142 in Section 7.

143 **2. Description of the case study HVAC System and Operating Modes**

144 The Team UOW Solar Decathlon house, its automated windows, and the schematic of its
145 HVAC system are shown in Figure 1. This HVAC system integrates a number of energy
146 components including a PVT system, a PCM thermal storage and a conventional air
147 conditioning system with an outdoor condenser unit and an indoor air-handling unit (AHU).
148 The PVT is used to generate the electricity and the low-grade thermal energy. The PCM
149 storage is used to store the low-grade thermal energy, which may then be used later for space
150 heating or cooling. The thermal storage is ‘charged’ by warm or cool air generated from the

151 PVT either by winter daytime solar radiation or summer night-time sky radiative cooling,
152 respectively.



153
154 Figure 1: a) Team UOW Solar Decathlon house and its automated high-level windows and b)
155 Schematic of the solar-assisted HVAC system where the symbols represent the following: S/A supply
156 air, O/A outside air, R/A return air, E/A exhaust air, F fan and D damper, respectively
157

158 There are five operating modes in this HVAC system, including three conditioning modes
159 and two PVT modes, which are controlled through the BMCS specifically designed for the
160 house.

161 Conditioning modes

162 Depending on the indoor conditions, the system can either work in the ‘natural ventilation’
163 mode through automatically controlling the opening of the high-level windows, or work in
164 the mechanical heating mode or mechanical cooling mode. Selection of the three
165 conditioning modes is determined based on the measured indoor and outdoor conditions.

166 In the mechanical heating and cooling modes, the system can operate in three different sub-
167 modes, i.e. *Direct Photovoltaic-Thermal Supply*, *Supply Air Preconditioned through Phase*
168 *Change Material*, and *Normal Heating and Cooling Mode*.

169 PVT modes

170 If there is no interfering operating mode activated, the system can operate in two other
171 modes, including *PCM Charging and PVT Exhaust*. A detailed description of the operating

172 modes for the HVAC system can be found in [26]. The mode of *PVT Exhaust* is not
173 considered in this study.

174 **2.1 BMCS Description and System Integration**

175 The control system of the Illawarra Flame house was designed to accommodate the
176 objectives and constraints of the overall project, capable of controlling lighting systems,
177 operable windows and the complex HVAC system. The BMCS system designed is also to be
178 capable of monitoring temperatures, energy consumption, electricity generation and flow
179 rates, and can effectively report this information to the user via a graphical interface. For
180 sourcing and project sponsoring reasons, the Australian manufacturer and supplier Clipsal,
181 and its proprietary building management and control system C-Bus were used in the project.

182 A supervisory BMCS (Tridium Niagara JACE) was integrated with the C-Bus system
183 because of its capability to utilize various communication languages and the availability of
184 drivers for common C-Bus units. This allowed a successful integration of the system
185 components on for example Modbus and oBIX [30] networks. The JACE controller acts as a
186 supervisor, informing C-Bus on which mode the system should operate and controlling the
187 associated variables.

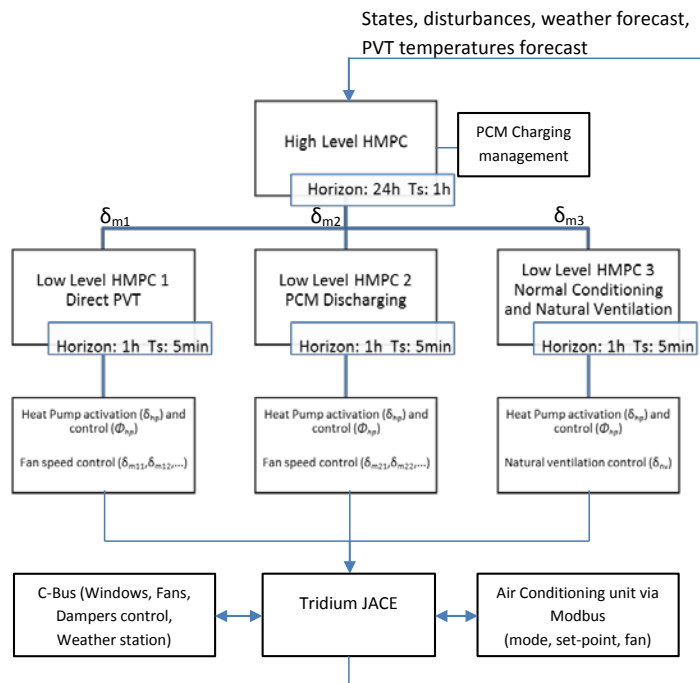
188 The JACE controller is also given the capability of overriding the local controller of the
189 reverse-cycle air conditioning unit, utilising a Modbus gateway. This allows the control
190 system to dynamically change the operation mode of the HVAC system, temperature set-
191 point and fan speed.

192 In order to easily implement the Hybrid MPC strategy, a machine running the Hybrid MPC
193 Matlab script is necessary, with the possibility to real-time override the JACE Niagara data
194 points. The Matlab script was interfaced to the JACE controller using an oBIX network
195 interface developed by the researchers from the Newcastle Energy Centre, Commonwealth
196 Scientific & Industrial Research Organisation (CSIRO). This facilitates the implementation

197 of virtually any desired control strategies in the house. The solver utilised for the optimisation
 198 was Gurobi [31]. The HMPC controller was implemented on a commercial laptop computer
 199 with an Intel® Core™ i5 Processor (2.7GHz).

200 2.2 Control Chain Structure

201 Given the complexity of the problem to be solved, the HMPC control strategy was divided
 202 into a high-level HMPC controller and multiple low-level HMPC controllers. The
 203 information collected from the weather station is used for the weather forecast and the
 204 prediction of the future PVT outlet temperature and its potential useful thermal generation
 205 together with the system states, which are then used to compute the optimal control action
 206 generated by the HMPC. The first set of actions from the optimal control sequence is then
 207 sent to the BMCS to control the individual components and HVAC equipment.
 208 The objective of the high-level HMPC is to compute the optimal sequence of the operating
 209 modes, using a control step of 1 hour and a prediction horizon of 24 hours.



210

211 Figure 2: Control chain structure: the high-level controller selects the Conditioning mode and
 212 manages the PCM Charging mode and; the low-level controllers optimise the Conditioning mode and
 213 control the system.

214 Once the high-level HMPC has selected the operating mode for the next hour, the
215 corresponding low-level HMPC controller is then activated (Figure 2). The objective of each
216 low-level HMPC controller is to optimise the selected operating mode for the time period of
217 concern, considering the range of the fan speeds that can be utilized for the PVT and PCM
218 storage, for example. The low-level HMPC computes the optimal solution with a control step
219 of 5 minutes and a prediction horizon of 1 hour. Once the optimal sequence of control actions
220 has been computed, the first set of the actions is then sent to the JACE controller and applied
221 to control the operation of the HVAC system (Figure 2).

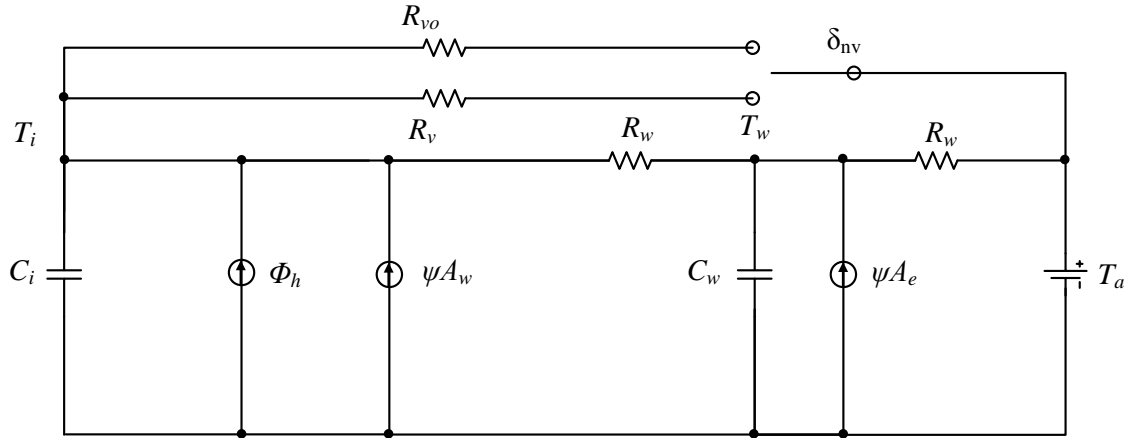
222 **2.3 Measurement Instrumentation**

223 The weather conditions, including ambient temperature, solar radiation, and wind speed and
224 direction, were monitored using a Davis Vantage Pro II Weather station and integrated
225 through an RS232 connection to the C-Bus and JACE control units. Air flow rate was
226 monitored using Siemens QVM62.1 air velocity sensors, located at one-third of the duct
227 diameter from the duct centreline. These sensors had a stated accuracy of ± 0.2 m/s + 3% of
228 the measured values. The temperatures in the ductwork were measured using Clipsal C-Bus
229 Digital Temperature Sensor Units 5104DTSI with a stated accuracy of $\pm 0.5^{\circ}\text{C}$ in the
230 temperature range of -10 - 80°C .

231 **3. Building Modelling and System Identification**

232 The HMPC control strategy is developed based on a grey-box state-space model. The model,
233 based on an R-C analogy, was developed in a similar manner to identifiable models available
234 in the literature [13]. The parameters of this model were estimated using the datasets of the
235 system in its various configurations, forcing the building with intermittent heating or cooling,
236 and using the average temperature (from five temperature sensors) of the building as the
237 thermal response. The building was therefore treated as a single thermal zone. The estimation
238 of the building parameters was performed using a nonlinear least-square fitting method.

239 The building was originally set to operate in two main modes, i.e. mechanical ventilation
 240 when the high-level windows were completely closed and natural ventilation when the high-
 241 level windows were fully open. In order to keep the system linear and easily identifiable,
 242 natural ventilation was simply modelled with different building infiltration rates, i.e. R_v when
 243 the windows are closed and R_{vo} when the windows are open. The R-C model of the building
 244 is presented in Figure 3.



245
 246 Figure 3: R-C representation of the building thermal system.

247 In this model, the state vector is $x = [T_i, T_w]^T$, and the input vector is $u = [T_a, \Psi, \Phi_h]^T$. The
 248 Boolean switch δ_{nv} represents the opening of the windows.

249 The parameters subject to identification are the equivalent half wall resistance R_w , the
 250 infiltration equivalent resistance with the windows closed R_v , the infiltration equivalent
 251 resistance with windows open R_{vo} , the equivalent internal capacitance C_i , the equivalent wall
 252 capacitance C_w , the equivalent windows area A_w , and the equivalent external wall area A_e .

253 The system dynamics are described by the state-space equations of the R-C model illustrated
 254 in Figure 3.

255 3.1 Identification of the House Parameters

256 The model parameters, which represent the physical characteristics of the building, were
 257 identified using a number of datasets obtained from the real operation of the building, where
 258 the indoor temperature was forced to intermittently use the PVT system or the heat pump

259 system. When the PVT system is active, the heating introduced to the building is calculated
 260 as

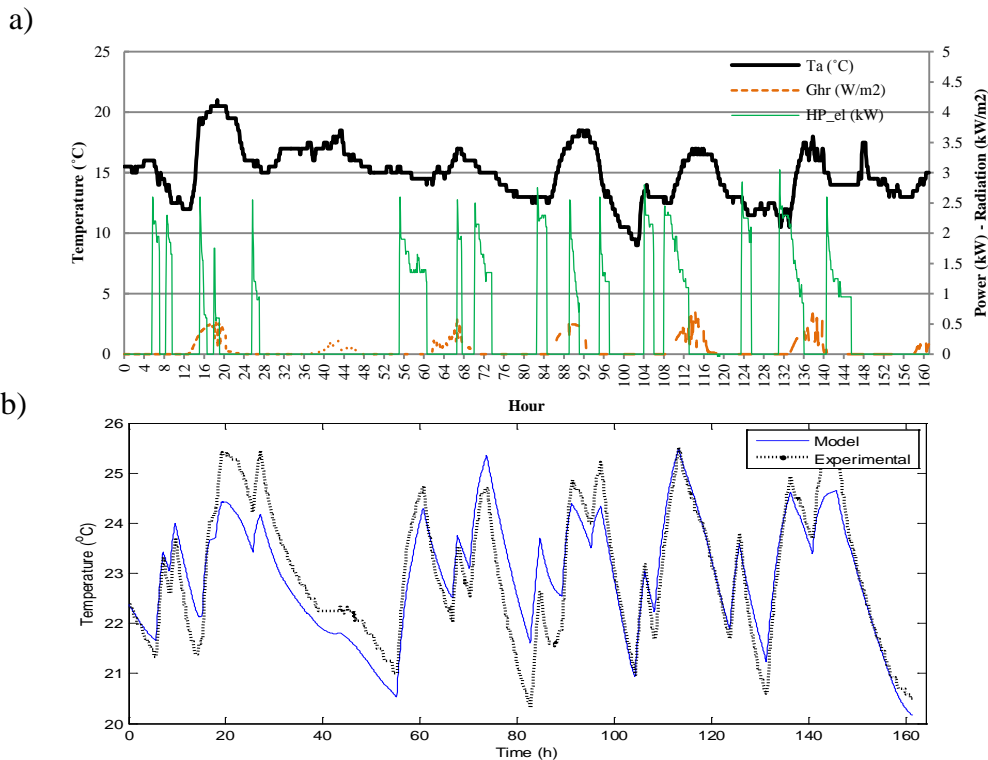
$$261 \quad \Phi_{h_PVT} = \rho \dot{V} c_p (T_{pvt,o} - T_i) \quad (1)$$

262 When the heat pump system is active, the heating or cooling input of the heat pump, which is
 263 not directly measurable in the house setup, is considered to be proportional to the electrical
 264 consumption of the air conditioning unit ($\Phi_{c,el}$), following the relationship

$$265 \quad \Phi_{h_HP} = \eta_{HP} \overline{COP} \Phi_{c,el} \quad (2)$$

266 In this study, a constant COP was considered to simplify the control problem, as the main
 267 objective of the controller is to optimise the operation of the whole system.

268 Three input datasets were used for this identification, including the data with intermittent
 269 heating of the building with the heat pump, intermittent heating with the PVT system and
 270 cooling with the heat pump. An example is presented in Figure 4a, where the data set used
 271 and the responses of the building are presented, in the case of the heating with the heat-pump
 272 test.



273

274

275 Figure 4: a) Identification input data heating with heat pump where HP_el is the electrical
 276 consumption and b) comparison between the data from the identified model and experimental data.

277 As the proposed model is linear and time-invariant, Matlab’s System Identification Toolbox
 278 in-built linear grey-box model estimation function [32] was used to fit the model parameters
 279 using the experimental data.

280 To ensure that the values of the parameters identified are representative to the physical
 281 lumped resistances and capacitances, the ‘first guess’ and boundary conditions were chosen
 282 based on the design values of the Illawarra Flame house and commonly used minimum and
 283 maximum design values for each parameter. The values used are summarized in Table 1.

284 Table 1: Building parameters identification

Parameter	First Guess	Lower Boundary	Upper Boundary	Identified Value
R_w (K/kW)	7.90	2	20	6.54
C_i (kWh/K)	0.76	0.06	5	3.39
C_w (kWh/K)	22	1	40	6.72
A_e (m ²)	20	0.50	50	2.91
A_w (m ²)	1	0.01	5	1.15
R_v (K/kW)	7	1	20	7.62
η_{HP}	0.90	0.60	1	0.61

285
 286 The comparison between the measured response and the predicted response of the identified
 287 system under the three operation conditions is shown in Figure 4b.

288 The goodness of fit between the measured and the predicted outputs, widely used metric,
 289 defined in the Matlab System Identification Toolbox and [33], was 48.38, 54.15 and 57.48%
 290 for the three different operation conditions, respectively.

291 The procedure was repeated for the natural ventilation mode to identify infiltration equivalent
 292 resistance with the windows open R_{vo} . To quantify this parameter, a test was conducted
 293 where the operable windows were kept fully open for a week, leaving the house free-running.
 294 The boundaries for the parameter R_{vo} were the same as for R_v in the identification with the
 295 windows closed. The identified infiltration equivalent resistance was 3.28, which is less than
 296 half of the infiltration resistance with the windows closed.

297 The goodness of fit between the both sets of the data was 47.09%. The results of these
 298 identifications can therefore be considered acceptable [11], considering that natural

299 ventilation was also included, the limitations of the model (e.g. no influence of wind speed
300 and direction on R_{vo}), the fact that multiple sources of thermal energy were used, the
301 relatively limited inputs available from the measurements available to the building model,
302 and the uncertainty associated with the experimental tests. While the statistical goodness of
303 fit of the identified model in the open loop is around 50%, due to measurement noise and
304 disturbances, it is evident from Figure 4b that the model captures the dominant system
305 dynamics, which is acceptable for control purposes. This observation is further supported by
306 the closed-loop performance of the system, presented in Sections 6.1 and 6.2.

307 **4. Modelling of Solar-PVT Assisted HVAC System**

308 **4.1 PVT System and PVT Direct Supply**

309 The PVT collector studied in this paper consists of a number of thin-film PV panels mounted
310 on a steel sheet flashing that is fixed onto the top of an existing sheet metal roof profile. This
311 system creates a cavity underneath the steel flashing through which the air exchanges heat
312 with the PVT panels.

313 To describe the heat transfer in the PVT, a steady-state model was employed to derive an
314 analytical solution to determine the PVT output [29]. The calculated outlet temperature of the
315 PVT is then used to calculate the heating that the PVT system can supply to the building
316 using Eq. (1).

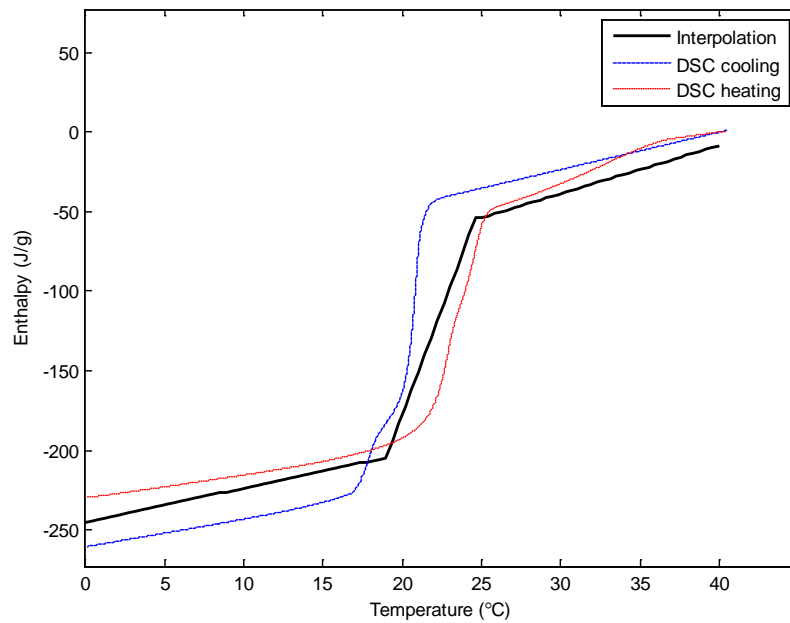
317 **4.2 PCM Thermal Storage unit and PCM Discharging**

318 The PCM thermal storage unit in the Solar Decathlon house was designed to exchange the
319 heat between the PCM and the air. It consists of PlusIce PCM bricks [34], which were placed
320 vertically and spaced to create the channels for the air to flow through. The phase change
321 temperature of the material used is 22°C. The relationship used to describe the heat exchange
322 in this PCM unit was presented in Ref. [29].

323 The heating that the PCM storage unit can supply to the building is determined by the
324 equation

$$325 \quad \Phi_{h_PCM} = \rho \dot{V} c_p (T_{pcm,o} - T_i) \quad (3)$$

326 The equivalent thermal capacitance of the PCM unit was calculated through interpolating the
327 differential scanning calorimeter test data measured at the Fraunhofer - Institut Für Solare
328 Energiesysteme in Freiburg. The equivalent thermal capacitance was divided into three
329 ranges, including the solid range ($C_{pcm,s}$), the phase change range ($C_{pcm,c}$), and the liquid range
330 ($C_{pcm,l}$).



331
332 Figure 5: PCM equivalent thermal capacitance tested under a cooling and heating rate of
333 0.01K/min and a temperature range of 0-40°C.

334 The equivalent capacitances of the PCM were 0.40, 5.10 and 0.57kWh/°C for the solid range,
335 the phase change range and the liquid range, respectively.

336 The curve used for the implementation in the HMPC system is represented in Figure 5.

337 4.3 PCM Charging with PVT

338 In the PCM charging with PVT, the total heat stored in the PCM unit can be determined by
339 the equation

$$340 \quad \Phi_{h_charged} = \rho \dot{V} c_p (T_{pvt,o} - T_{pcm,o}) \quad (4)$$

341 **4.4 Energy Consumption of the System Components**

342 The energy consumption of the system components in each operating mode is needed to
 343 compute the optimal solution of the controller to allocate a cost for each energy source.

344 Identification of the electrical consumption versus the air flow characteristics was the key
 345 aspect of the optimisation of the operating modes involving the PVT system or the PCM unit.

346 The fit used was a 3rd order polynomial of the form

$$347 \quad P_{fan} = \gamma_D \dot{V}^3 \quad (5)$$

348 where the coefficient γ_D is related to the system pressure losses and is calculated for each
 349 operating mode. The identification of these curves was presented in [35]. In this study, the
 350 consumption-airflow characteristic was considered at the discrete fan speed levels.

351 The energy used by the heat pump is instantaneously measured and represented by $\Phi_{c,el}$. The
 352 energy used in the natural ventilation mode is considered negligible and an infinitesimally
 353 small energy cost was associated to its operation.

354 **5 Hybrid Model Predictive Control Development**

355 The HVAC system presented in this study can operate in three different conditioning modes
 356 and one PVT mode. The switching between these operating modes would indicate a system
 357 model with a time-varying structure. The switching is also constrained, since some of these
 358 modes cannot operate simultaneously (e.g. PCM Charging and PCM Discharging modes) due
 359 to the physical constraints. Furthermore, the relationships between the PVT outlet
 360 temperature and the airflow rate are intrinsically nonlinear.

361 The switching and nonlinearity issues described above may be tackled by approaching the
 362 MPC problem formulation as an HMPC problem. HMPC can deal with both continuous and
 363 discrete dynamics, seeking optimal solutions by solving a mixed-integer programming
 364 problem. In particular, it is possible to describe this problem as a Mixed Logical Dynamical

365 (MLD) system, in which the logic, dynamics and constraints are integrated as described in
 366 [27,28].

367 Taking into account the future samples of the reference vector and measured disturbance, the
 368 prediction model has to be augmented with an additional linear model and treat the vector of
 369 the future references and measured disturbances as the additional states [36].

370 Writing the problem as an MLD system with the discrete fan speeds removes the difficulty of
 371 the aforementioned problems. Indeed, this results in a number of linear systems that can be
 372 activated by appropriate Boolean variables. For each of these linear systems, the cost is fixed
 373 at the energy used by the fan with the selected discrete speed step with the addition of the
 374 consumption of the heat pump.

375 Given the number of Boolean variables and the requirement to compute the solution in a
 376 relatively short time period (e.g. 5 minutes), the controller was divided into two levels: a
 377 high-level controller that selects the operating mode every hour, and three low-level
 378 controllers that optimize the operation of each operating mode every five minutes.

379 Each controller will have to minimise a cost function of the form

$$380 \min_{\{u,d,z\}_0^{N-1}} J(\{u, d, z\}_0^{N-1}, x(t)) \triangleq \sum_{k=1}^{N-1} \|Q_x(x(k) - x_r)\|_p + \sum_{k=1}^{N-1} \|Q_u(u(k) -$$

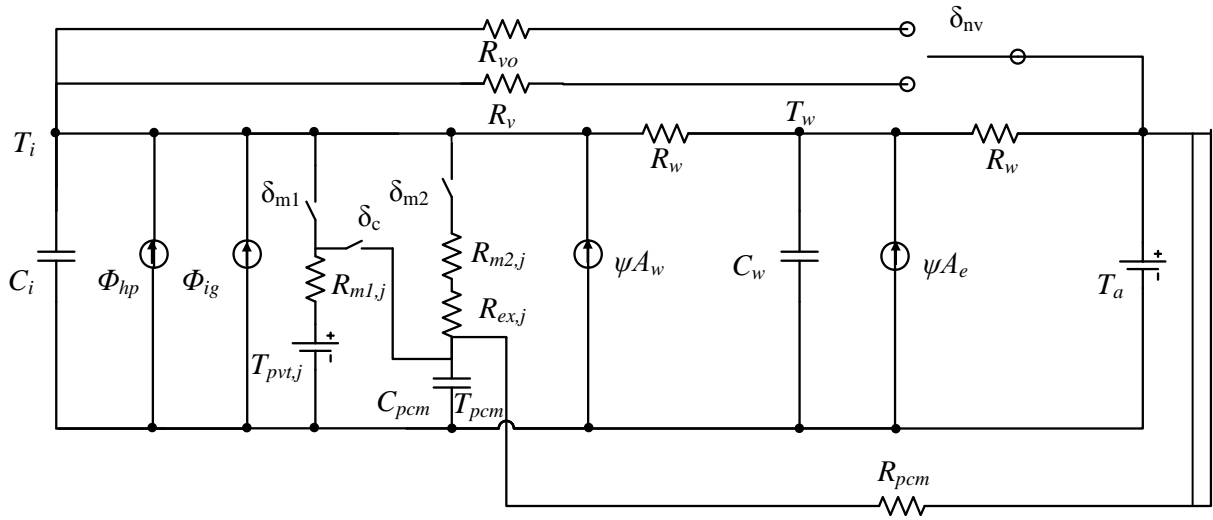
$$381 u_r)\|_p + \sum_{k=1}^{N-1} \|Q_z(z(k) - z_r)\|_p + \sum_{k=1}^{N-1} \|Q_y(y(k) - y_r)\|_p + \|Q_{xN}(x(N|t) - x_r)\|_p \quad (6)$$

382 The values and the parameters included in the cost function vary from controller to controller.

383 For this study, the infinity norm was used, with $p = \infty$.

384 **5.1 High-Level Controller**

385 The main purpose of the high-level controller (Figure 6) is to determine the system operating
 386 mode under a given working condition. To maintain the problem relatively simple and
 387 minimise the number of Boolean optimisation variables, each mode was considered to
 388 operate at its nominal conditions (e.g. the fan speed for PVT Direct Supply and PCM
 389 Charging mode is considered to be at 80% of the VSD range, corresponding to a flow rate of
 390 approximately 300 L/s).



391

392 Figure 6: Schematic of High-level Controller system.

393 The states of the high-level controller, the measured disturbances and the controlled variables
 394 are summarised in Table 2. The resistances R_{m1j} and R_{m2j} represent the delivered heating and
 395 cooling to the building using the nominal air mass flow rates in PVT Direct Supply and PCM
 396 Discharging modes, respectively. $R_{ex,j}$ represents the heat exchange rate of the PCM unit at
 397 the nominal air flow rate. The equivalent PCM capacitance C_{pcm} at each time step k is defined
 398 as

$$399 \quad C_{pcm}(k) = \begin{cases} C_{pcm,s} & \text{if } T_{pcm} < T_{melt,b} \\ C_{pcm,c} & \text{if } T_{melt,b} \leq T_{pcm} \leq T_{melt,t} \\ C_{pcm,l} & \text{if } T_{pcm} > T_{melt,t} \end{cases} \quad (7)$$

400 Once the high-level controller has selected the operating mode for the next hour, one of the
 401 low-level controllers will be activated. The objective of this controller is to track the
 402 reference temperature given by the high-level controller, using the next computed state of the
 403 average indoor temperature T_i . The states of the low-level controllers, the measured
 404 disturbances and the controlled variables are also summarised in Table 2.

405 Table 2: HMPC states, measured disturbances and controlled variables.

Items	Parameters	Description	High-level	Low-level 1	Low-level 2	Low-level 3
<i>States</i>	T_i	average inside temperature (measured)	✓	✓	✓	✓
	T_w	equivalent wall temperature (estimated)	✓	✓	✓	✓
	T_{pcm}	average PCM temperature (measured)	✓	✗	✓	✗
Measured disturbance	T_a	ambient temperature	✓	✓	✓	✓
	Ψ	global horizontal radiation	✓	✓	✓	✓
	Φ_{ig}	Internal loads	✓	✓	✓	✓
	$T_{pvt,j}$	PVT calculated outlet temperature at fan speed j	✓	✓	✗	✗
Controlled variable	ε	constraint softening on indoor temperature	✓	✗	✗	✗
	Φ_{hp}	thermal input to the building provided by the heat pump	✓	✓	✓	✓
	δ_{m1j}	PVT Direct Supply (at fan speed j in low-level controllers)	✓	✓	✗	✗
	δ_{m2j}	PCM Discharge Supply (at fan speed j in low-level controllers)	✓	✗	✓	✗
	δ_c	PCM Charging mode	✓	✗	✗	✗
	δ_{nv}	Natural ventilation mode	✓	✗	✗	✓
	δ_{hp}	Heat pump active (minimum heat delivery)	✗	✓	✓	✓

406

407 The set of the constraints associated with the high-level controller are as follows:

- 408
- 409 ■ Two conflicting modes are not activated at the same time;
 - 410 ■ The heat pump thermal input respects the physical limits of the real unit; and
 - 411 ■ The comfort conditions are maintained, keeping the average indoor temperature in the defined comfort range.

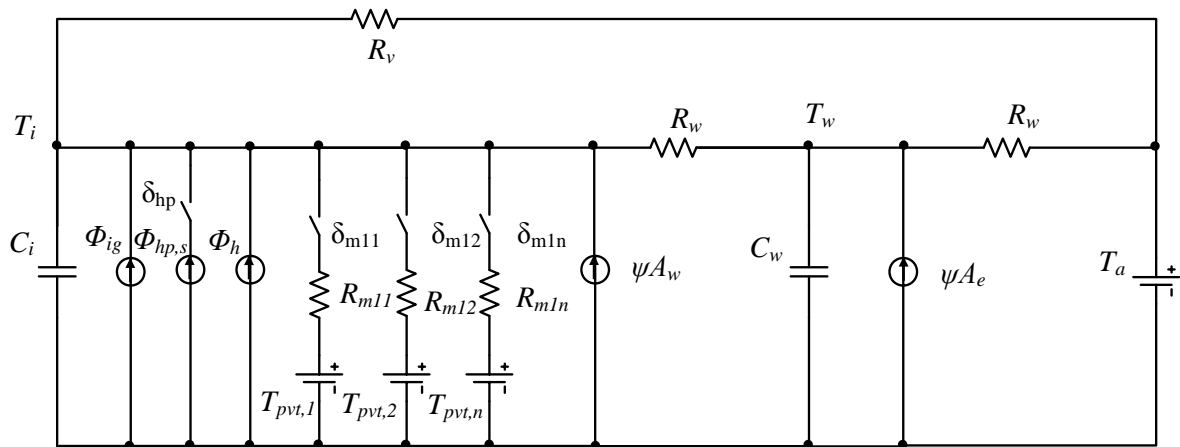
412 The limit of the air conditioning thermal capacity (Φ_{hp}) is represented by $[\overline{\Phi_{hp}}, \underline{\Phi_{hp}}]$, and the
 413 limits of the allowance for the temperature to go outside the comfort band (ε) is represented
 414 by $[\overline{\varepsilon}, \underline{\varepsilon}]$. The logical conditions are:

$$415 \sim(\delta_{m2} \& \delta_{m1}), \sim(\delta_{m2} \& \delta_{nv}), \sim(\delta_{m1} \& \delta_{nv}), \sim(\delta_{m2} \& \delta_c), \sim(\delta_{m1} \& \delta_c), T_i \leq \overline{T_i} + \varepsilon, T_i \geq \underline{T_i} - \varepsilon.$$

416 Where \sim is the “not” condition and $\&$ is the “and” condition.

417 5.2 Low-Level Controller 1 – Direct PVT and Normal Conditioning

418 Low-level controller 1 (Figure 7) is selected when the Boolean variable δ_{m1} of the high-level
 419 controller is active. This controller sets the system in mechanical ventilation and operates the
 420 PVT Direct Mode in conjunction with the operation of the heat pump. This controller can
 421 select various Boolean variables corresponding to the discrete fan speed levels.



422

423 Figure 7: System schematic of low-level controller 1.

424 The PVT system can provide the thermal energy independently or can operate in conjunction
 425 with the heat pump (PVT Direct mode and PVT pre-heating/pre-cooling the air for the AHU).

426 The heat pump can also operate without the PVT (Normal Conditioning mode).

427 The resistances R_{m11} , R_{m12} , R_{m1n} represent the heating/cooling delivery to the building using
 428 the various air mass flow rates.

429 The set of the constraints associated to the low-level controller 1 are as follows:

- 430 ■ Two fan speeds are not activated at the same time;

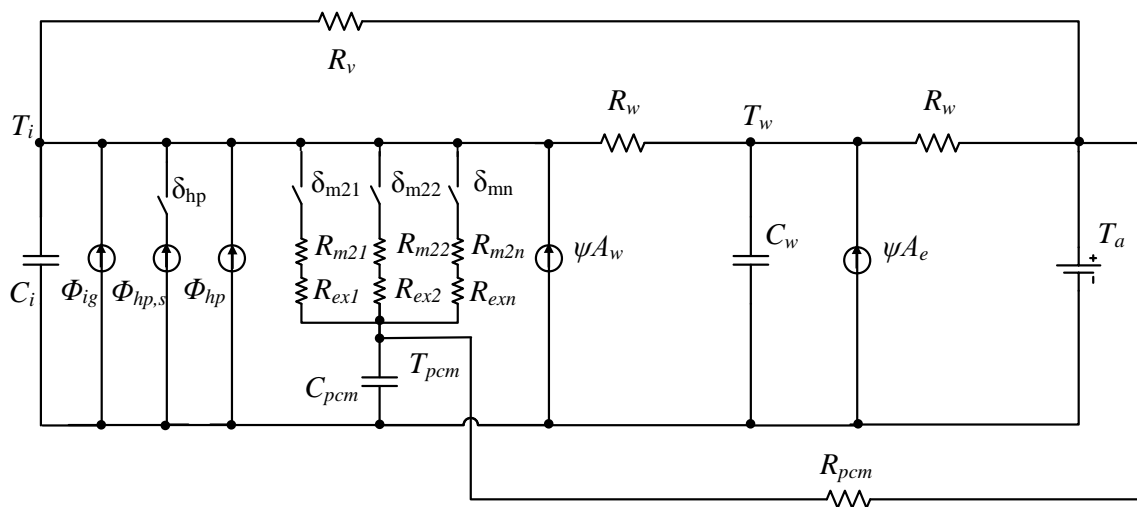
- 431 ▪ The heat pump thermal input respects the physical limits of the real unit; and
- 432 ▪ The comfort conditions are maintained, following the trajectory defined by the high-
- 433 level controller.

434 The limits of the air conditioning thermal capacity (Φ_{hp}) are the same as that of the high-level
 435 controller. Logical conditions are:

436 $\sim(\delta_{m11} \& \delta_{m12}) , \sim(\delta_{m11} \& \delta_{m1n}), \dots, \sim(\delta_{m12} \& \delta_{m1n}), \sim(\delta_{hp} \& \Phi_{hp} \neq 0)..$

437 5.3 Low-Level Controller 2 – PCM Discharging and Normal Conditioning

438 Low-level controller 2 (Figure 8) is selected when the Boolean variable δ_{m2} of the high-level
 439 controller is active. This controller sets the system in mechanical ventilation and operates the
 440 PCM Discharging Mode in conjunction with the operation of the heat pump. This controller
 441 can select various Boolean variables that correspond to the discrete fan speed levels.



442

443 Figure 8: System schematic of low-level controller 2.

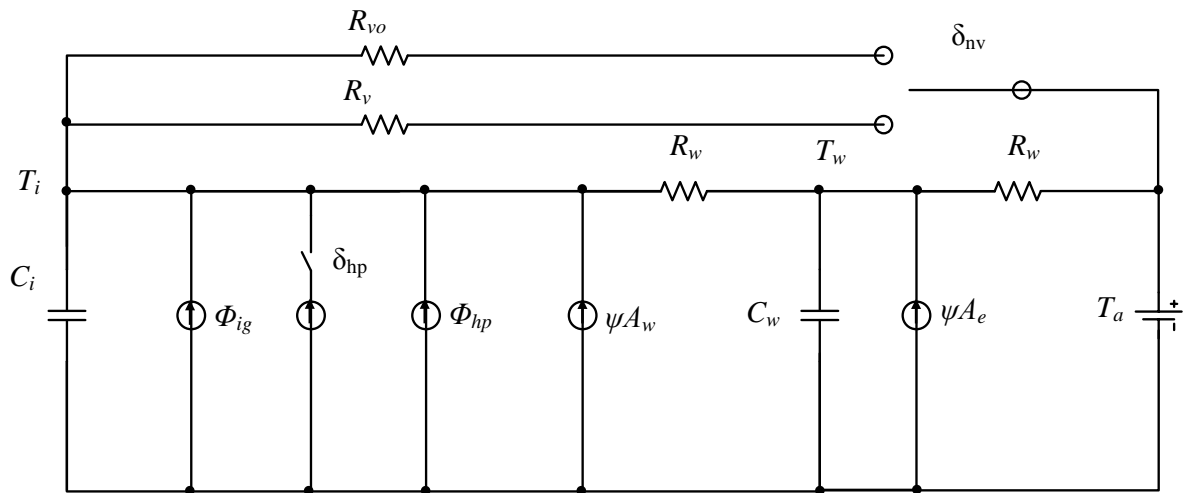
444 The resistances $R_{ex1}, R_{ex2}, R_{exn}$ represent the heat exchange rate into the PCM unit
 445 corresponding to the various air flow rates and $R_{m21}, R_{m22}, R_{m2n}$ represent the heating and
 446 cooling delivery to the building using the various air mass flow rates.

447 The set of the constraints associated to the low-level controller 2 are similar to the ones of the
 448 low-level controller 1.

449 $\sim(\delta_{m21} \& \delta_{m22}) , \sim(\delta_{m21} \& \delta_{m2n}), \dots, \sim(\delta_{m22} \& \delta_{m2n}), \sim(\delta_{hp} \& \Phi_{hp} \neq 0)..$

450 **5.4 Low-Level Controller 3 – Natural Ventilation and Normal Conditioning**

451 Low-level controller 3 (Figure 9) is selected when neither the Boolean variable δ_{m1} nor δ_{m2} of
 452 the high-level controller are active. This controller sets the mechanical ventilation system
 453 Normal Conditioning Mode and operates the heat pump depending on the value of the
 454 optimal thermal input Φ_{hp} calculated at each control step, and operates the high-level operable
 455 windows of the house. This controller utilises two Boolean variables that correspond to the
 456 operation of the building in mechanical or natural ventilation and the activation of the heat
 457 pump.



458
 459 Figure 9: System schematic of low-level controller 3.

460 The set of the constraints associated to the low-level controller 3 are as follows:

- 461 ▪ Natural ventilation and air conditioning are not to be activated at the same time;
- 462 ▪ The heat pump thermal input respects the physical limits of the real unit; and
- 463 ▪ The comfort conditions are maintained, following the trajectory defined by the high-
- 464 level controller.

465 The limits of the air conditioning thermal capacity (Φ_{hp}) are the same as that of the high-level
 466 controller. The logical conditions are that natural ventilation and the air conditioning do not
 467 operate at the same time and the heat pump cannot provide additional heating if it is not
 468 switched on.

469 $\sim(\delta_{hp} \& \delta_{nv}); \sim(\delta_{hp} \& \Phi_{hp} \neq 0)$.

470 Given the objective of the low-level controllers, it is possible that the controller will
471 intermittently heat and cool the building to follow a certain trajectory. To avoid this problem,
472 each low-level controller was developed in two versions, i.e. one only allows heating and the
473 other only allows cooling. The type of the controller used is dictated by the request for
474 heating or cooling by the high-level controller and for that hour the appropriate low-level
475 controller is selected.

476 **5.5 Weather Forecast**

477 In this paper, an adaptive weather-prediction model developed in [37] was used to forecast
478 external dry-bulb temperature and solar radiation over the 24 hours prediction horizon. These
479 predictions serve as the inputs to both the building model and the PVT analytical thermal
480 model.

481 Among the various approaches presented in [37], the Deterministic-Stochastic method was
482 adopted to predict both external dry-bulb temperature and solar radiation.

483 **6 Experimental and Simulation Results**

484 The experiments were conducted using the Illawarra Flame house at Innovation Campus,
485 Wollongong, in March and April 2015. The two tests were carried out using the HMPC
486 strategy presented in this paper, mainly in cooling dominated periods. In the first test, only
487 mechanical ventilation was used, while in the second one natural ventilation was also
488 included. Tests were also conducted during the heating dominated periods, but the results are
489 not presented in order to avoid a lengthy paper. Difficulties in implementing the target R=3.0
490 insulation to the PCM storage unit could not be fully achieved, and therefore affected the
491 PCM heat loss. The HMPC strategy successfully controlled the system, despite the issues
492 with the real system.

493 The tests were performed using a variable comfort band, which was considered as a variable
 494 soft constraint for the average indoor temperature, where the upper boundary \underline{T}_i and the
 495 lower boundary \overline{T}_i for the cooling and heating are defined as

$$496 \quad \overline{T}_i(k) = \begin{cases} 24^\circ C & \text{if } 9 \leq \text{hour}(k) \leq 18, \\ 25.5^\circ C & \text{otherwise.} \end{cases} \quad (8)$$

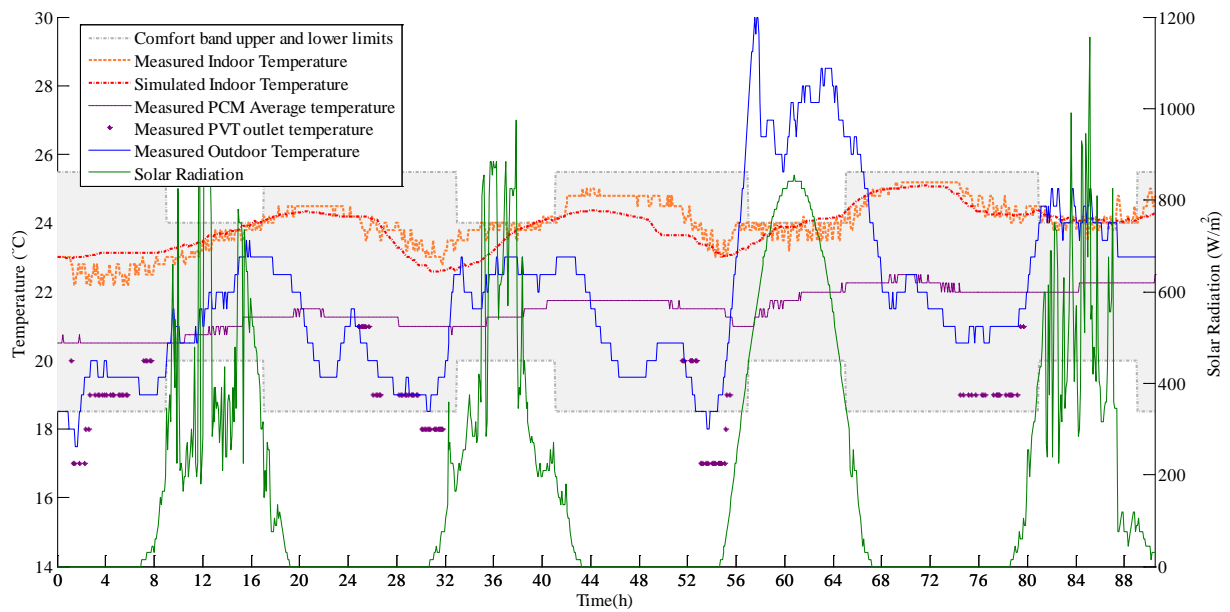
$$497 \quad \underline{T}_i(k) = \begin{cases} 20^\circ C & \text{if } 9 \leq \text{hour}(k) \leq 18, \\ 18.5^\circ C & \text{otherwise.} \end{cases} \quad (9)$$

498 Once the tests were completed, the same weather data was then used in a simulated
 499 environment in which the models of the building and equipment were the same as those
 500 previously presented.

501 The simulated controller has to deal with the same mismatch in the weather prediction, but
 502 there is no mismatch between the model used in the HMPC controller and the controlled
 503 system.

504 **6.1 Cooling Test - Mechanical Ventilation Only**

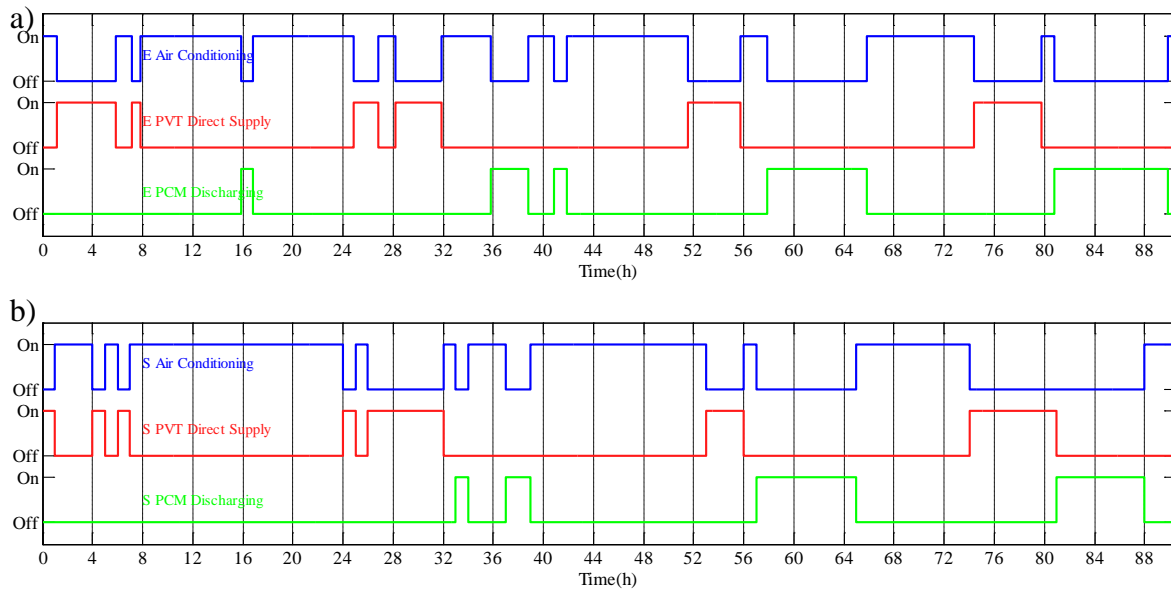
505 This test was conducted between 6th and 11th of March 2015. To stay within the defined
 506 comfort band, cooling was mostly required. The experimental and simulated indoor
 507 temperature profiles, outdoor temperature, global horizontal radiation and the objective
 508 temperature range are presented in Figure 10. In this test the daytime comfort band ended at
 509 5pm instead of 6pm. One can notice that both the simulated and experimental controllers led
 510 to the similar indoor temperature profiles, where it is noticeable the attempt to pre-cool the
 511 building at the end of the night, and drift as much as possible inside the comfort band. The
 512 reason behind this choice of the HMPC controller is that the most efficient cooling generation
 513 came from the Direct PVT supply mode, especially at the end of each night, when the
 514 temperature difference between the indoor and outdoor was greater.



515

516 Figure 10: HMPC experimental test compared to the simulated test, HVAC only, March
 517 2015: Temperature profiles and solar radiation.

518 When the pre-cooling provided by the PVT was not sufficient, the discharging of the PCM
 519 unit and the normal air conditioner were compensating to keep the temperature inside the
 520 comfort band. The profile of the activation of the various operating modes coinciding with
 521 the activation of the three low-level controllers is presented in Figure 11. In this case it is
 522 noticeable that both controllers tried to achieve their objectives with a similar sequence of the
 523 selected operating modes with night time PVT Direct pre-cooling followed by a combination
 524 of PCM Discharging and Normal Conditioning during the daytime.



525

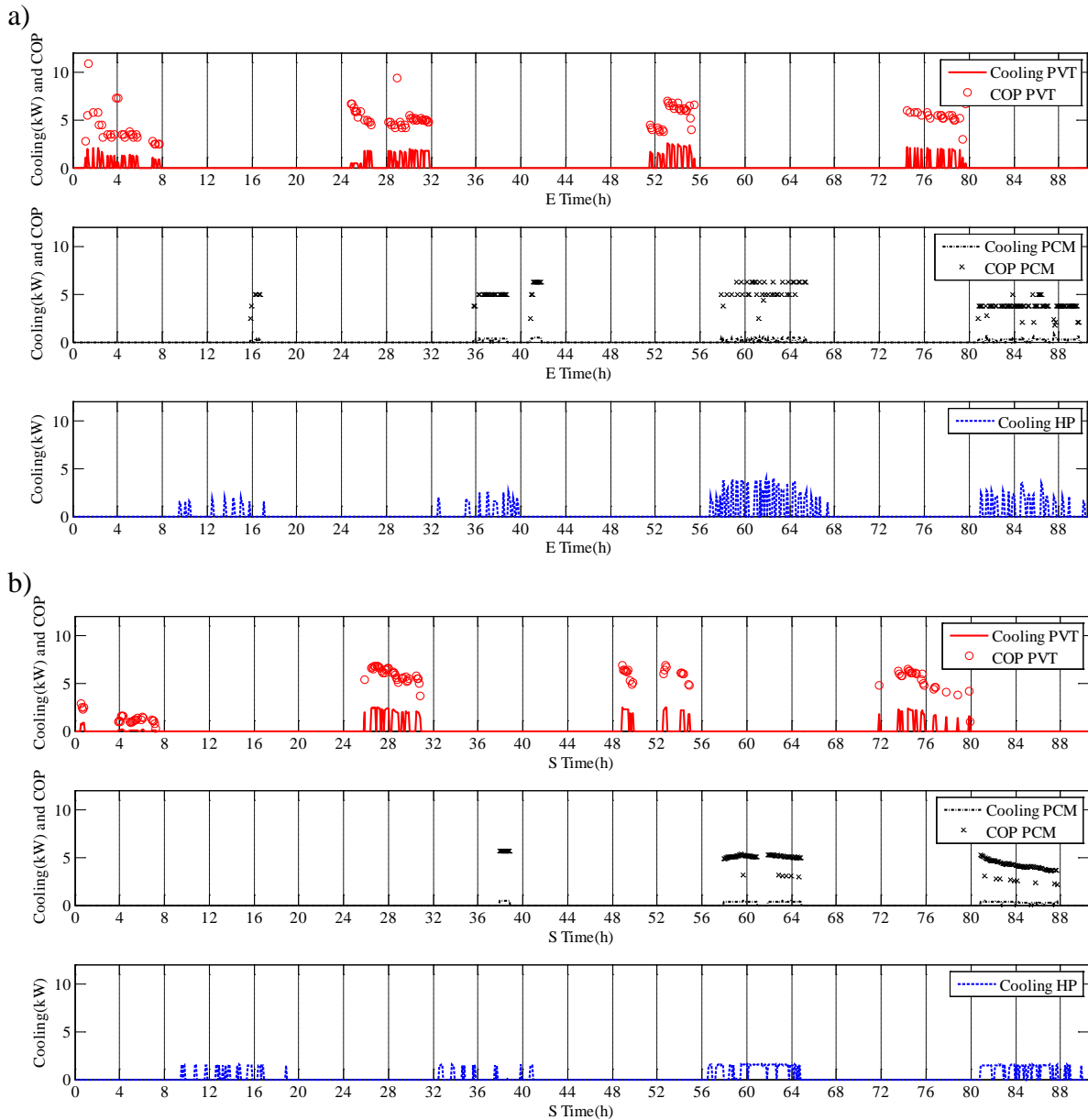
526 Figure 11: HMPC experimental test compared to the simulated test, HVAC only, March 2015: a)
 527 experimental and b) simulated test operating mode selection.

528 During the experimental test, the HVAC system provided a total of 45.2 kWh of cooling to
 529 the building, among which 14.9 kWh was from the PVT Direct Supply, 4.7 kWh was from
 530 the PCM Discharging and 25.6 kWh was from the use of the air conditioning unit. The
 531 average COP of the PVT system over the whole test, defined as the cooling provided divided
 532 by the electrical energy used by the fan, was 5.04, and the average COP of the PCM
 533 Discharging was 4.48. In both cases, the controller can maintain the performance above the
 534 reference COP of the air conditioning system, identified to be 2.1 on average.

535 The simulated test generated the similar results, with a total of 41.2 kWh of cooling provided
 536 to the building, among which the cooling provided the PVT, PCM unit and the air
 537 conditioning unit was 13.9, 4.7 and 22.6 kWh, respectively. The COP of the PVT system and
 538 the PCM unit discharging was 4.59 and 4.55 respectively, higher than the reference COP.

539 The instantaneous experimental and simulated cooling generation and COP of the various
 540 operating modes are presented in Figure 12.

541 It can be seen that the cooling of the house by both the PVT and PCM occurred in the similar
 542 ways in the experimental and the simulated tests, keeping the instantaneous COP of the
 543 system well above the given reference.



546 Figure 12: HMPC experimental test compared to the simulated test, HVAC only, March 2015: a)
 547 experimental and b) simulated instantaneous cooling generation and COP.

548 The overall performance of the experimental and simulated tests are also summarised in
 549 Table 3.

550

551
552

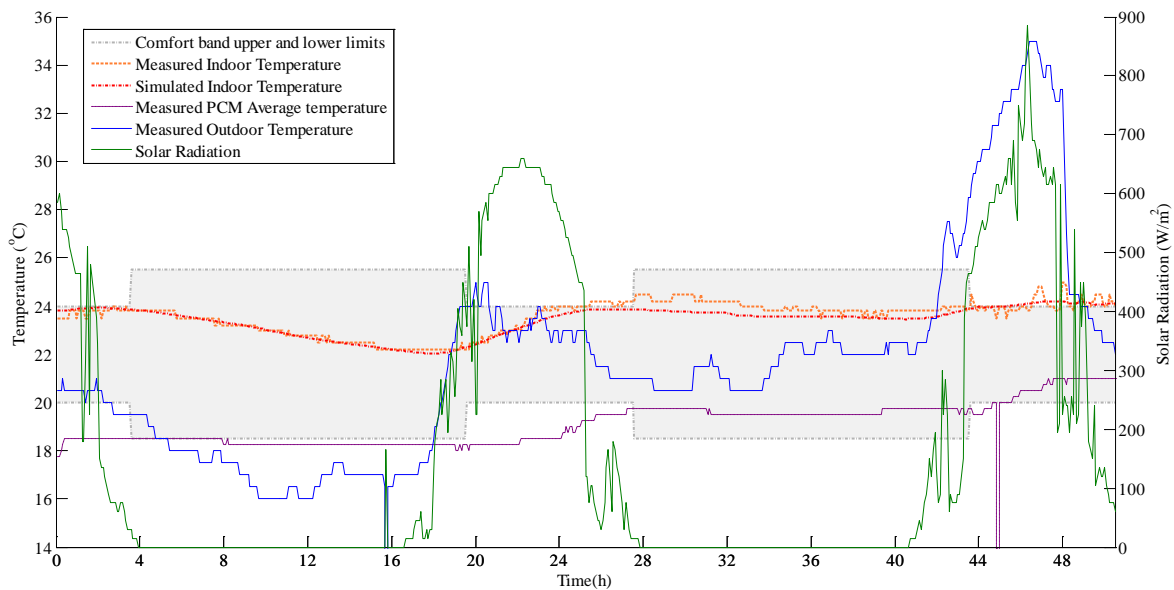
Table 3: Summary of the HVAC average performance, HMPC experimental and simulated test, HVAC only, March 2015.

	Experimental	Simulated	Relative difference
PVT Direct (kWh/day)	3.97	3.71	6.5%
Average COP PVT Direct	5.04	4.59	8.9%
PCM Discharge (kWh/day)	1.25	1.25	0.0%
Average COP PCM Discharge	4.48	4.55	1.6%
Normal Conditioning (kWh/day)	6.82	6.03	11.6%
Total Electrical Consumption (kWh/day)	4.74	4.23	10.8%

553

554 **6.2 Cooling Test - Mechanical and Natural Ventilation**

555 A similar experiment was undertaken, using both the HVAC system and natural ventilation
 556 between 14th and the 16th of April 2015. The cost associated with the windows opening was
 557 very small so that their operation was the most preferable conditioning solution for the
 558 controller to meet the thermal comfort targets, without using them if not necessary and the
 559 house can stay in the comfort band free-running with the windows closed.

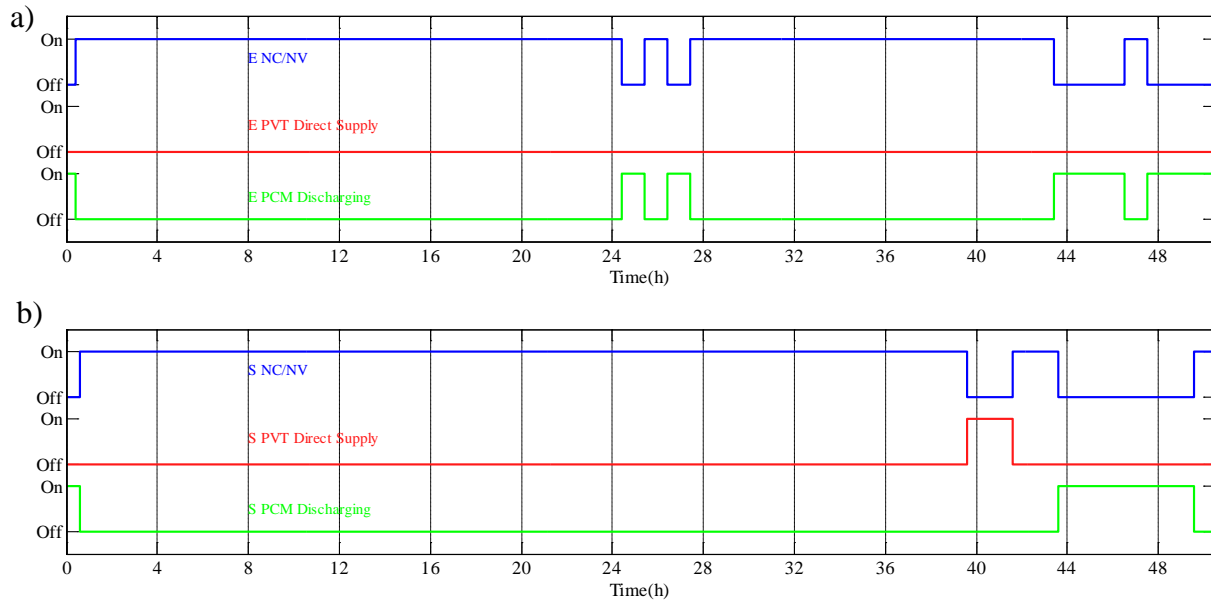


560

561 Figure 13: HMPC experimental test compared to the simulated test, HVAC and Natural Ventilation,
 562 April 2015: Temperature profiles and solar radiation.

563 The experimental and simulated indoor temperature profiles, outdoor temperature, global
 564 horizontal radiation and the objective temperature range are presented in Figure 13. It was be
 565 found that the simulated and the experimental trajectories were similar and the controller was

566 capable to effectively manage the indoor temperature even in very hot conditions. Compared
567 to the case where only the HVAC system was used, the Low-level Controller 3 was more
568 frequently used, since it is associated to both normal conditioning and natural ventilation
569 modes. The possibility to cool the building using natural ventilation was preferred by the
570 controller, since the cost associated to this mode is negligible compared to the other cooling
571 means. This behaviour of the controller can be observed in Figure 14 noting that mostly used
572 mode was Normal Conditioning and Natural Ventilation, by both the simulated and
573 experimental controllers. During this test PCM Discharging mode was only utilised at the end
574 of the second day (hours from 24 to 27) and during the third day (hours from 43 to 50). The
575 small difference between experimental and simulated operation is mostly due to a mismatch
576 between the model and the real system, and the need to correct the trajectory to keep the
577 temperature in the comfort band. The simulated controller activated the PVT direct supply for
578 a small portion of time at the end of the second night, whereas the experimental one used
579 natural ventilation, achieving a similar result in terms of remaining in the comfort band. It
580 can also be noted that this difference in operation is minimal if the results in Figure 15 are
581 observed, since a very small amount of cooling was provided via the Direct PVT mode.



582

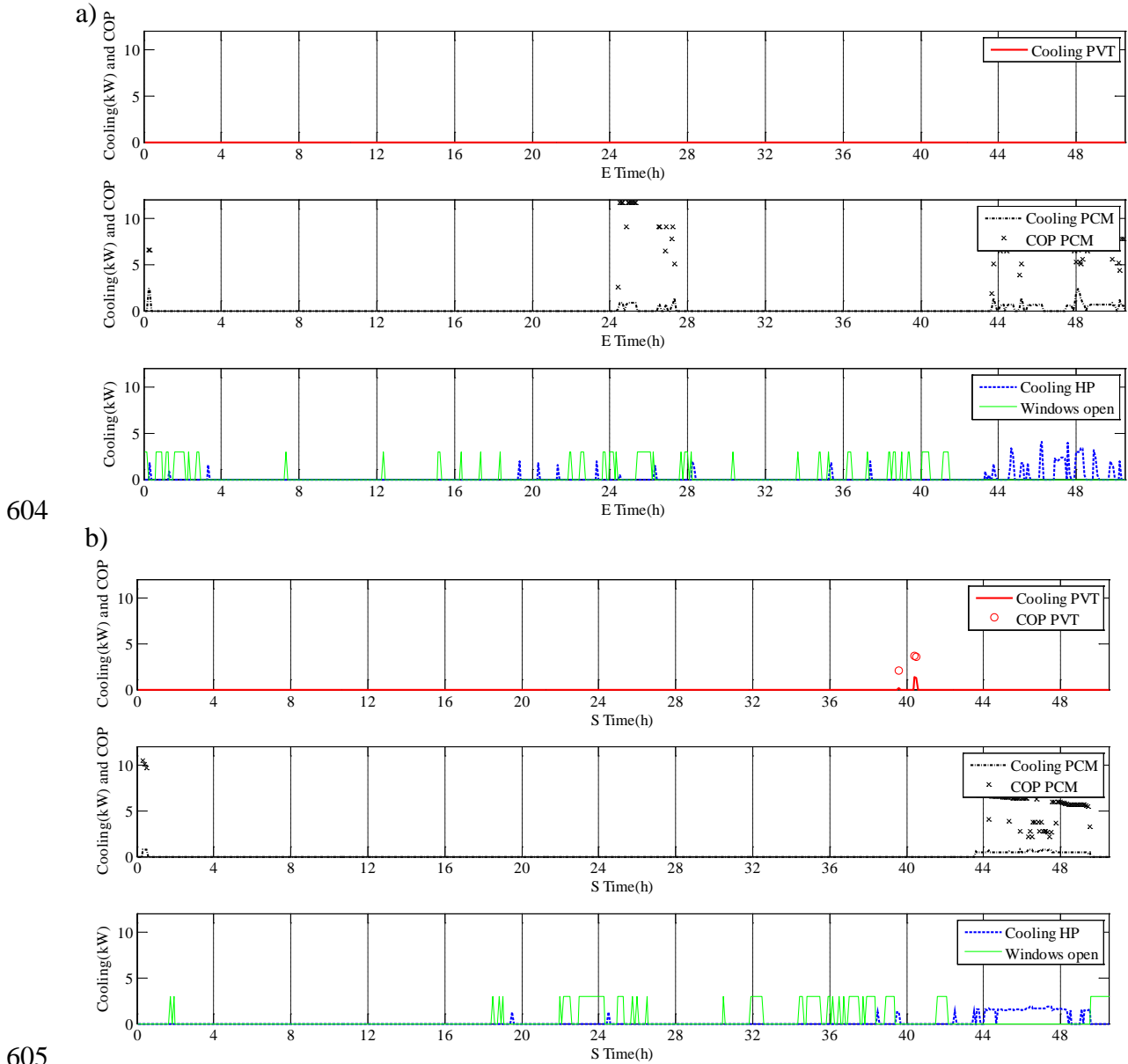
583 Figure 14: H MPC experimental test compared to simulated test, HVAC and Natural Ventilation, April
 584 2015: a) experimental and b) simulated test operating mode selection.

585 During the experimental test the HVAC system provided a total of 12.9 kWh of cooling to
 586 the building, among which 4.9 kWh was provided by discharging the PCM unit (at an
 587 average COP of 7.1) and 8.0 kWh was supplied by using the air conditioning unit. Direct
 588 PVT mode was not used in the test. The natural ventilation mode kept the operable windows
 589 open for a total of 76 five minute control steps (i.e. 6 hours and 20 minutes) over the whole
 590 test (i.e. 50 hours and 30 minutes).

591 In the simulated test, the HVAC system provided 11.9 kWh of cooling, among which 0.2
 592 kWh was from Direct PVT mode (at an average COP of 3.1), 3.2 kWh was supplied by
 593 discharging the PCM unit (at an average COP of 5.5), and 8.5 kWh was provided by using
 594 the air conditioning unit. During this simulated test, the Natural Ventilation mode was active
 595 for 99 five minute control steps (8 hours and 15 minutes). The instantaneous experimental and
 596 simulated cooling generation and COP under the various operating modes are presented in
 597 Figure 15.

598 In this case, the experimental and simulated test resulted in a similar pattern in the utilisation
 599 of the various resources. Natural Ventilation mode was often used during the night and

600 during the day when the temperature was below the current indoor average temperature (e.g.
 601 around hour 24). Both of the tests utilised PCM discharging during the last hot day, but it was
 602 also utilised in the experimental test between hour 24 and hour 28 to correct the indoor
 603 temperature trajectory.



606 Figure 15: H MPC experimental test compared to the simulated test, HVAC and Natural Ventilation,
 607 April 2015: a) experimental and b) simulated instantaneous cooling generation and COP.

608 The summary of the performance of the system in the experimental and simulated tests is
 609 presented in Table 4.

610 Table 4: Summary of the HVAC average performance, HMPC experimental and simulated test,
 611 HVAC and Natural Ventilation, April 2015.

	Experimental	Simulated	Relative difference
PVT Direct (kWh/day)	0	0.10	-
Average COP PVT Direct	0	3.10	-
PCM Discharge (kWh/day)	2.35	1.54	34.5%
Average COP PCM Discharge	7.10	5.50	22.5%
Normal Conditioning (kWh/day)	3.84	4.08	6.3%
Total Electrical Consumption (kWh/day)	2.15	2.17	0.9%

612

613 7 Conclusions

614 This paper proposed a Hybrid Model Predictive Control (HMPC) strategy to manage a solar-
 615 assisted HVAC system, retrofitted to a residential building. The system features a
 616 Photovoltaic-Thermal (PVT) system and a phase change material (PCM) active storage unit,
 617 integrated with a standard ducted air conditioning system. Comprehensive experimental
 618 control infrastructure was designed, installed and commissioned in the Illawarra Flame
 619 house, allowing the implementation of virtually any typology of control strategies to the
 620 house system. An HMPC strategy with two levels of control was designed, implemented and
 621 its operation demonstrated through both simulations and experiments. To understand the
 622 effect of the mismatch between the models and the real system, the controller was simulated
 623 using an identical model for both the controller and the system while introducing the same
 624 error in the weather forecast.

625 The HMPC strategy shows an anticipative behaviour that could be noticed in the cooling
 626 scenarios presented in this paper, where the HMPC pre-cooled the building utilising the fact
 627 that bringing fresh air further cooled by the PVT system can lower the temperature of the
 628 indoor air and building fabric more efficiently than the heat pump. The HMPC strategy also
 629 demonstrated the value of using natural ventilation in both the experimental tests and

630 simulated environment, by giving priority to window opening for cooling purposes whenever
631 available.

632 The controller was able to keep the indoor temperature within the prescribed range, utilising
633 the PVT and PCM storage to generate cooling at a higher COP than the reference standard air
634 conditioner. The winter experiments highlighted the fact that appropriate insulation of an
635 active PCM unit is crucial. Excessive heat losses prevented the system operated with efficient
636 charging and discharging cycles.

637 Overall, the HMPC strategy demonstrated to be capable of effectively utilising the
638 knowledge of the system future dynamics allowing it to manage well the available resources
639 to maximise the efficiency of the system. The results presented in this paper show that the
640 methodology proposed is implementable and promising. An important advantage of MPC
641 techniques is that they enable the use of real-time measurements and forecasts to optimise the
642 system performance when confronted with changing conditions and disturbances, which is
643 particularly important when solar-driven generation is implemented on site and is only
644 available at certain times of the day.

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647 advice and help in the formulation of the control problem and its implementation in the
648 Hybrid Toolbox.

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