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ROBUST ADAPTIVE CONTROL FOR INDUSTRIAL ROBOTS
--- A DECENTRALIZED SYSTEM METHOD

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ABSTRACT

This paper proposes an adaptive control approach for the tracking control of industrial robots utilizing the fact that a robot model can be described by equations that are linear in the system's unknown parameters. Taking uncertainties into account, the resulting controller has the property of robustness. Moreover the paper gives proof of stability and analytical results of the boundness of position tracking errors. By means of introducing filter operations in state measurements the approach avoids the difficulty of measuring the accelerations of the robots' actuators. Simulation results are also presented.

1. INTRODUCTION

It is well known that the difficulties arising in accurate motion control for industrial robots are caused by the nature of their complicated dynamics, i.e., non-linearities and strong couplings between different joints. In order to improve motion performance, efforts have been made to develop control algorithms using adaptive control technology to allow the following of reference trajectories at high speed especially in the cases where robots are required to deal with variable payloads. A reasonable extension is using the configuration of the Model Following Adaptive Control (MFAC)[9] to describe robot dynamics by "linear" models and then to apply adaptive design procedures directly [2][3][6][10][11][12][14][17]. Due to the non-linearities of robots these sorts of methods encounter difficulties in estimating time-varying parameters.

It is known that the dynamic equations of industrial robots can been written to be linear in model parameters such as inertia and payload [4][16][19][20]. In these formulae the system states are generalized to be nonlinear functions of positions and velocities of robot joints. Based on these formulations it is possible to employ linear adaptive control techniques provided positions and velocities are measurable and nonlinear functions are known. There is also further research on the stability and convergence of these techniques. Two types of approach have been proposed. One is based on model following [4] [5] and the other is on the passivity of robot dynamics [19][20].

In the case of model following, Craig proposed an adaptive control method based on the computed torque control law. This approach leads to an asymptotically stable closed-loop system in the sense of Lyapunov stability. But, a drawback is that it requires the inverse of the inertia matrix to be calculated and the accelerations to be measured. The investigations of this paper are aimed at relaxing these two restrictions and obtaining a robust result in the cases there are uncertainties. Based on an *a priori* estimation of the system equations, the computed torque control is applied so that the controlled states are driven to a neighborhood of desired trajectories. Then the resultant system can be treated as a set of multi-input single-output error models, in which the unknown parameters appear in a form linear in the generalized states, and interaction within the subsystem is regarded as a disturbance. For this decentralized system configuration, a robust adaptive controller is designed using the Lyapunov direct method to ensure bounded position tracking errors.

The paper is organized as follows: Section 2 introduces the notation of robot dynamics and lists assumptions made; Section 3 describes nonlinear compensation using computed torque control[15][18]. The robust adaptive controller design for the resulant decentralized system is presented in Section 4. In Section 5, some simulation results will be shown and finally the conclusions are given in Section 6.

The notations used in this paper are defined as the following: $v \in R^n$ is a vector defined on the n-dimension real vector field and its i-th element is noted by v_i; $A(\,) \in R^{nx1}$ is a matrix and its i-th row is given by A_i and its i-j-th element by A_{ij}; A^T means the transpose of A; $(\dot{})$ represents the derivative with respect to time t; $\lambda(A)$ is an eigenvalue of matrix $A\in R^{nxn}$. In the cases where A is real symmetric, $\max\lambda(A)$ ($\min\lambda(A)$) represents its maximum (minimum) eigenvalue. $\|.\|$ is used to represent the norm of a vector or matrix.

2. SYSTEM DYNAMICS DESCRIPTION

The dynamics of robot systems can be described by the Lagrangian Equations [1][18][21]

$$\frac{d}{dt}(\partial L(q,\dot{q})/\partial\dot{q}) - \partial L(q,\dot{q})/\partial q = u, \qquad (2\text{-}1)$$

where

$$L(q,\dot{q})=K(q,\dot{q})-P(q)\in R^1, \qquad (2\text{-}2)$$

is the Lagrangian function of the system; q and $\dot{q}\in R^n$, in the robot's joint coordinate space, are the generalized position and velocity vectors respectively; $u\in R^n$ is the input torque vector causing the motion of the arms. In (2-2), $K(q,\dot{q})=\frac{1}{2}\dot{q}^TD(q)\dot{q}\in R^1$ and $P(q)\in R^1$ represent the kinetic and potential energy functions of a robot with n degrees of freedom (DOF) respectively. $D(q)\in R^{nxn}$ is the inertia matrix which is positive definite, i.e., $D(q)=D^T(q)>0$ for all q.

Substituting (2-2) into (2-1), noticing $\partial P(q)/\partial\dot{q}=0$, and denoting the gravitational torque $\partial P(q)/\partial q=g(q)\in R^n$, (2-1) then becomes

$$D(q)\ddot{q}+h(q,\dot{q})+g(q)=u, \qquad (2\text{-}3)$$

where

$$h(q,\dot{q})=\dot{D}(q)\dot{q}+k(q,\dot{q}), \qquad (2\text{-}4a)$$

$$k(q,\dot{q})=-\frac{\partial}{\partial q}\left(\frac{1}{2}\dot{q}^TD(q)\dot{q}\right). \qquad (2\text{-}4b)$$

For this motion equation an uncertainty term $d_0(t)\in R^n$ is introduced, which could include frictional torques and coupled torques ignored in the modelling, disturbance torques from the environment, measurement noise, payload variations, etc.. Thus, (2-3) becomes

$$D(q)\ddot{q}+h(q,\dot{q})+g(q)+d_0=u. \qquad (2\text{-}5)$$

If the i-j-th element of D(q) is $D_{ij}(q)$, (2-5) can be written as

$$D(q)=\{D_{ij}(q)\}=\{\sum_{k=1}^{m_i} d_{ijk}f_{dijk}(q)\} \qquad (2\text{-}6a)$$

Similarly, $h(q,\dot{q})$, $k(q,\dot{q})$ and g(q) can be given by

$$h(q,\dot{q}) = \{h_i(q,\dot{q})\}=\{\sum_{k=1}^{m_{ih}} h_{ik}f_{hik}(q,\dot{q})\} \qquad (2\text{-}6b)$$

$$k(q,\dot{q}) = \{k_i(q,\dot{q})\}=\{\sum_{k=1}^{m_{ik}} k_{ik}f_{kik}(q,\dot{q})\} \qquad (2\text{-}6c)$$

$$g(q)=\{g_i(q)\} =\{\sum_{k=1}^{m_{ig}} g_{ik}f_{gik}(q), \qquad (2\text{-}6d)$$

where m_i, m_{ih}, m_{ik} and $m_{ig}\geq 1$ are integers, d_{ijk}, k_{ik} and g_{ik} are some constant parameters related to the mass, inertia and payload of the robot arm, $f_{dijk}(q)$, $f_{gik}(q)$, $f_{hik}(q,\dot{q})$ and $f_{kik}(q,\dot{q})$ are nonlinear functions in q and $\dot{q}$. It is worth noting that these functions are only determined by the geometrical configurations of robots and therefore they could be obtained by kinematics investigations. Because of (2-4), it can be shown that d_{ijk}, h_{ik} and k_{ik} are dependent. The parameters related to joint i can be written as

$$\theta_i=[d_{i11},d_{i12},\dots d_{i1m_i},d_{i21},d_{i22},\dots d_{i2m_i},\dots d_{in1}, d_{in2},\dots d_{inm_i},\dots,h_{i1},h_{i2},\dots h_{im_{ih}},g_{i1},\dots g_{im_{ig}}] \qquad (2\text{-}7)$$

and it can be shown that θ_i and θ_j may also be correlated.

For the above equation of motion we make the following assumptions:

A-1) All nonlinear functions $f_{dijk}(q)$, $f_{hik}(q,\dot{q})$ and $f_{gik}(q)$ in (2-6) are bounded and continuous in q and $\dot{q}$. Moreover, they are all known.

A-2) For constant coefficients d_{ijk}, h_{ik} and g_{ik} in (2-6) there exist *a priori* estimates, noted by $\hat{d}_{ijk}$, $\hat{h}_{ik}$ and $\hat{g}_{ik}$, such that the estimates of them may be represented as

$$\hat{D}(q)=\{\hat{D}_{ij}(q)\}=\{\sum_{k=1}^{m_i} \hat{d}_{ijk}f_{dijk}(q)\} \qquad (2\text{-}8a)$$

$$\hat{h}(q,\dot{q}) = \{\hat{h}_i(q,\dot{q})\}=\{\sum_{k=1}^{m_{ih}} \hat{h}_{ik}f_{hik}(q,\dot{q})\} \qquad (2\text{-}8b)$$

and

$$\hat{g}(q)=\{\hat{g}_i(q)\} =\{\sum_{k=1}^{m_{ig}} \hat{g}_{ik}f_{gik}(q)\} \qquad (2\text{-}8c)$$

according to A-1). Moreover, it is also assumed the relations (2-4) still hold for the estimate (2-8a, b).

A-3) The estimate (2-8) results in a positive definite $\hat{D}(q)$.

A-4) Let $q_d\in R^n$ be the reference signal for q to track, then it is assumed that reference trajectory q_d, together with $\dot{q}_d$ and $\ddot{q}_d$ are all continuous and bounded.

A-5) The norm of the i-th component of the uncertainty term d_0 is bounded by a known constant c>0, i.e.,

$$\|d_0\|\leq c, \quad \text{for } i=1,2,\dots,n. \qquad (2\text{-}8)$$

3. COMPUTED TORQUE CONTROL

Using the computed torque control law, u is implemented utilizing *a priori* estimates of the system parameters:

$$u=\hat{D}(q)[\ddot{q}_d+K_v(\dot{q}_d-\dot{q})+K_p(q_d-q)+u_a]+\hat{h}(q,\dot{q})+\hat{g}(q). \quad (3\text{-}1)$$

where $K_v=\text{diag}\{k_{vi}\}\in R^{nxn}$, $K_p=\text{diag}\{k_{pi}\}\in R^{nxn}$ with k_{vi}, $k_{pi}>0$, for $i=1,2,...,n$ and u_a is an adaptation law which will be determined shortly. The control law is implemented in such a way that during the real-time control, the controller parameters in (3-1) are fixed by *a priori* estimation $\hat{\theta}_i$ $i=1,2,...,n$ and the adaptive control law only updates parameters in u_a.

Substitution of (3-1) into (2-2), and considering A-3, leads to an error dynamic vector equation:

$$\ddot{e}+K_v\dot{e}+K_pe=\hat{D}^{-1}(q)[\tilde{D}(q)\ddot{q}+\tilde{h}(q,\dot{q})+\tilde{g}(q)+d_0]-u_a, \quad (3\text{-}2)$$

where $e=q_d-q$ is position error; $\tilde{D}(q)=D(q)-\hat{D}(q)$, $\tilde{h}(q,\dot{q})=h(q,\dot{q})-\hat{h}(q,\dot{q})$ and $\tilde{g}(q)=g(q)-\hat{g}(q)$ are estimation error vectors. It has been shown, by [5], that the control law (3-1) (set $u_a=0$) leads to a L_∞ input-output stable system provided the following conditions are satisfied (for details ref. [5]):

(1) A-3) holds;

(2) $k_{vi}^2=k_{pi}>0$;

(3) A-4) holds;

(4) d_0 is bounded and uncorrelated;

(5) $\beta_1\alpha_2+\beta_2\alpha_3+2\beta_2\sqrt{\alpha_1\alpha_4}<1$, in which

$\beta_1=1/k_p$, $\beta_1=4\exp(-1)/k_v$,

$\alpha_1=\|D^{-1}(\tilde{D}(q)\ddot{q}_d+\tilde{h}(q,\dot{q}_d)+\tilde{g}(q)+d_0)\|_\infty$,

$\alpha_2=\|(I-D^{-1}\hat{D})K_p\|_{i\infty}$,

$\alpha_3=\|(I-D^{-1}\hat{D})K_v-2D^{-1}\tilde{H}_m(q,\dot{q}_d)\|_{i\infty}$,

$\alpha_4=\|D^{-1}\|_{i\infty}\max\|\tilde{H}_i(q)\|_{i\infty}$,

where $\tilde{h}(q,\dot{q})=\tilde{h}(q,\dot{q}_d)-2\tilde{H}_m(q,\dot{q}_d)\dot{e}+\tilde{H}(q,\dot{e})$; for a vector h, $\|h\|_\infty=\max_i\sup|h_i|$; for a matrix H, $\|H\|_{i\infty}=\max_i\sup\sum_j|h_{ij}|$

(6) The initial conditions $e(0)=\dot{e}(0)=0$ are satisfied.

In fact, the existence of $\hat{D}^{-1}(q)$ is not a very restricted condition because of the physical meaning of D(q). If the non-linear functions of positions in its elements are all known (which depend on whether the arm's joints are revolute or prismatic), the unknown parameters corresponding to mass, inertia sensor and geometrical size of a given robot should all be positive values. If the estimate of these true values is not negative, the resultant D(q) must be positive definite.

Denote

$$\hat{D}^{-1}(q)=\text{diag}\{[\hat{D}^{-1}(q)]_{ii}\}+[\hat{D}^{-1}(q)]_0=J(q)+J(q)_0,$$

where $\text{diag}\{[\hat{D}^{-1}(q)]_{ii}\}=J(q)$ is a diagonal matrix consisting of all diagonal elements of $\hat{D}^{-1}(q)$. Considering (2-4), Eqn.(3-2) then becomes

$$\ddot{e}+K_v\dot{e}+K_pe$$
$$=J(q)[\frac{d}{dt}(\tilde{D}(q)\dot{q})+\tilde{k}(q,\dot{q})+\tilde{g}(q)]-u_a+d,$$

where

$$d=J(q)_0[\frac{d}{dt}(\tilde{D}(q)\dot{q})+\tilde{k}(q,\dot{q})+\tilde{g}(q)]+\hat{D}^{-1}(q)d_0\in R^n. \quad (3\text{-}3)$$

As mentioned above if the conditions (1)-(6) are satisfied, control law (3-1) ensures input-output stability and the error state will stay within a bounded region including the origin. It has also been shown by [5], that the right hand side of (3-2) is bounded, which implies, in our case, $\|d\|=\|J(q)_0(\frac{d}{dt}(\tilde{D}(q)\dot{q})+\tilde{f}(q,\dot{q})+\tilde{g}(q))\|$ is bounded as well. Suppose this bound is given by constant vector v>0, then we have

$$\|d(t)\|\le\|v+\hat{D}^{-1}(q)d\|\le v+\|\hat{D}^{-1}(q)d_0\|\le\rho$$

where $\rho>0$ is constant vector.

Rather than formulating an error equation (3-3) and recognizing that (3-3) is linear in the parameters,the whole system is considered as n MISO subsystems separately. Then the i-th subsystem becomes

$$\ddot{e}_i+k_{vi}\dot{e}_i+k_{pi}e_i$$
$$=J_{ii}(q)[\frac{d}{dt}(\tilde{D}_i(q)\dot{q})+\tilde{k}_i(q,\dot{q})+\tilde{g}_i(q)]-u_{ai}+d_i, \quad (3\text{-}4)$$

where $\tilde{D}_i(q)$ is the i-th row of matrix $\tilde{D}(q)$ and d_i is the i-th component of d.

In accordance with A1), known nonlinear functions and unknown estimation errors of the coefficients in the equalities above can be decomposed as two vectors so that (3-4) becomes

$$\ddot{e}_i+k_{vi}\dot{e}_i+k_{pi}e_i$$
$$=J_{ii}[\frac{d}{dt}(\delta_{i1}(q,\dot{q}))\tilde{\theta}_{i1}+\delta_{i2}(q,\dot{q})\tilde{\theta}_{i2}+\delta_{i3}(q)\tilde{\theta}_{i3}]-u_{ai}+d_i$$
$$=J_{ii}\delta_i(q,\dot{q},\ddot{q})\tilde{\theta}_i-u_{ai}+d_i, \quad (3\text{-}5)$$

where

$$\delta_i = [\ \frac{d}{dt}(\delta_{i1}(q,\dot{q}))\ \ \delta_{i2}(q,\dot{q})\ \ \delta_{i3}(q)\ ]$$
$$\tilde{\theta}_i = [\ \tilde{\theta}_{i1}^T\ \ \tilde{\theta}_{i2}^T\ \ \tilde{\theta}_{i3}^T\ ]^T$$
$$\delta_{i1}^T = [f_{i11}(q)\dot{q}_1,..,f_{i1m_i}(q)\dot{q}_1,f_{i21}(q)\dot{q}_2,..,f_{i2m_i}(q)\dot{q}_2,..,f_{in1}(q)\dot{q}_n,..,f_{inm_i}(q)\dot{q}_n],$$
$$\delta_{i2}^T = [f_{ki1}(q,\dot{q}),\ f_{ki2}(q,\dot{q}),...,\ f_{kim_{ik}}(q,\dot{q})],$$

$$\delta_{i3}^T = [f_{gi1}(q), f_{gi2}(q),..., f_{gim_{ig}}(q)],$$
$$\tilde{\theta}_{i1}^T = [\tilde{d}_{i11},...,\tilde{d}_{i1m_i},\tilde{d}_{i21},...,\tilde{d}_{i2m_i},...,\tilde{d}_{in1},...,\tilde{d}_{inm_i}],$$
$$\tilde{\theta}_{i2}^T = [\tilde{k}_{i1},\tilde{k}_{i2},...,\tilde{k}_{imik}],$$
$$\tilde{\theta}_{i3}^T = [\tilde{g}_{i1}, \tilde{g}_{i2}, ..., \tilde{g}_{imig}].$$

Being a constant vector, $\tilde{\theta}_i$ is the parameter estimation error caused by the computed torque control law (3-1).

System (3-5) can be considered as a global feedback system with $J_{ii}\delta_i(q,\dot{q},\ddot{q})$ and d_i being interactions among different subsystems. Clearly if $\tilde{\theta}_i=0$ for all i and d=0, then the right hand side will disappear and u_{ai} no longer necessarily exists. In the cases where there are parameter errors in the computed torque control law, the design objective is to derive an adaptive control law so that the tracking error e_i becomes as small as possible.

To avoid measuring accelerations $\ddot{q}$, a filter operator $\alpha_i/(s+\alpha_i)$[16], where $\alpha_i>0$ is a constant and s the differential operator specified by $s(\cdot)=d(\cdot)/dt$, is introduced into both sides of (3-5). In doing so it is also assumed that the change of $J_{ii}(q)$ is much slower compared with changes in $\delta_i(q,\dot{q},\ddot{q})$, as the latter is a function of velocities and accelerations, so that the output of the filter, to which $J_{ii}(q)\frac{d}{dt}(\delta_i(q,\dot{q},\ddot{q}))$ is input, can be represented as

$$\omega_{i1}(q,\dot{q}) = \frac{\alpha_i}{s+\alpha_i}[J_{ii}(q)\frac{d}{dt}(\delta_i(q,\dot{q},\ddot{q}))]$$
$$= J_{ii}(q)\frac{\alpha_i}{s+\alpha_i}\frac{d}{dt}(\delta_{i1}(q,\dot{q},\ddot{q})). \quad (3\text{-}6a)$$

Denote

$$\varepsilon_i = \frac{\alpha_i}{s+\alpha_i}e_i, \quad (3\text{-}6b)$$
$$\eta_i = \frac{\alpha_i}{s+\alpha_i}d_i, \quad (3\text{-}6c)$$
$$\tau_{ai} = \frac{\alpha_i}{s+\alpha_i}u_{ai}, \quad (3\text{-}6d)$$

then (3-5) becomes

$$\ddot{\varepsilon}_i+k_{vi}\dot{\varepsilon}_i+k_{pi}\varepsilon_i=\omega_i(q,\dot{q})^T\tilde{\theta}_i+\eta_i-\tau_{ai}. \quad (3\text{-}7)$$

Here $\tilde{\theta}_i$=constant, is the estimated parameter vector with z_i dimension, $\omega_i(q,\dot{q})$ is the filtered observed vector formed by a set of known nonlinear functions of the states and $z_i=nm_i+m_{ik}+m_{ig}$. In view of A5) and (3-6d), $\eta_i(q)$ is still bounded by ρ_i, i.e.,

$$\|\eta_i(q,t)\|\le(1-e^{-\alpha_i t})\sup\|d_i\|\le\rho_i. \quad (3\text{-}8)$$

4. ROBUST ADAPTIVE CONTROLLER DESIGN

Suppose τ_{ai} has the form of

$$\tau_{ai}=\omega_i(q,\dot{q})^T\theta_i, \quad (4\text{-}1)$$

where $\theta_i\in R^{z_i}$ is the controller parameter vector which will be determined shortly. Substituting (4-1) into (3-7) gives

$$\ddot{\varepsilon}_i+k_{vi}\dot{\varepsilon}_i+k_{pi}\varepsilon_i=\omega_i(q,\dot{q})^T(\tilde{\theta}_i-\theta_i)+\eta_i$$
$$=\omega_i(q,\dot{q})^T\phi_i+\eta_i, \quad (4\text{-}2)$$

where $\phi_i=\tilde{\theta}_i-\theta_i$. Let $x_i=[\varepsilon_i, \dot{\varepsilon}_i]^T$, then for subsystem i, we obtain the state space description of error equation

$$\dot{x}_i=A_ix_i+b_i\omega_i^T\phi_i+b_i\eta_i, \quad (4\text{-}3)$$

where

$$A_i=\begin{bmatrix}0 & 1\\ -k_p & -k_v\end{bmatrix} \qquad b_i=\begin{bmatrix}0\\1\end{bmatrix}.$$

The whole system then becomes an error state equation with dimension of 2n:

$$\dot{x}=Ax+B\omega\Phi+B\eta, \quad (4\text{-}3a)$$

where

$$A=\text{diag}\{A_1\ A_2\ ...A_n\}\in R^{2nx2n}$$
$$B=\text{diag}\{b_1\ b_2\ ...b_n\}\in R^{2nxn}$$
$$\omega=\text{diag}\{\omega^T_1\ \omega^T_2\ ...\omega^T_n\}\in R^{nx\Delta}$$

and

$$\Phi^T=[\phi^T_1\ \phi^T_2\ ...\phi^T_n]\in R^{\Delta},$$

where $\Delta=\sum_{i=1}^{n}z_i$. For each error system (4-3), the following control strategy is introduced [8]

$$\dot{\phi}_i=-\frac{1}{2}\beta_i\phi_i-\gamma_i\,b_i^T P_ix_i\omega_i \quad \text{for } i=1,2,...,n. \quad (4\text{-}4)$$

where β_i, $\gamma_i>0$ are constants, $P_i^T=P_i>0$ is a solution of the Lyapunov equation $A_i^TP_i+P_iA_i^T=-Q_i$ for a given $Q_i^T=Q_i>0$ with a restraint condition

$$\min\lambda(Q_i)\ge1, \qquad \text{for } i=1,2,...,n.(4\text{-}5)$$

From this arises the following theorem:

Theorem: (i) The solution x_i and ϕ_i of the i-th error equation (4-3) and adaptive controller (4-4), under the restraint of (4-5), is uniformly bounded;

(ii) Additional to (i), if β_i in(4-4) satisfies

$$0<\beta_i\le(\min\lambda(Q_i)-1)/\max\lambda(P_i), \quad (4\text{-}6)$$

then state x_i, ϕ_i will converge to the residual set

$$D_i=\{(x_i,\phi_i)|v_i(x_i,\phi_i)<\frac{1}{\beta_i}\max\lambda^2(P_i)\rho_i^2\} \quad (4\text{-}7)$$

with a rate at least as fast as $\exp(-\beta_it)$, where ρ_i, given by (3-8), is the boundness of the uncertainties in subsystem i, and P_i, given by (4-4), satisfies restraint (4-5).

(iii) Furthermore, according to (i) and (ii), the solution x and Φ

of the overall system (4-3a) will converge to the residual set

$$D_0=\{(x,\Phi)|v(x,\Phi)<\frac{1}{\min\beta_i}\sum_{i=1}^{n}\max\lambda^2(P_i)\rho_i^2\} \quad (4\text{-}7a)$$

with a rate at least as fast as $\exp(-\min\beta_i t)$,

Proof : Consider a candidate of the Lyapunov function for the i-th subsystem (4-3):

$$v_i(x_i,\phi_i)=x_i^T P_i x_i+\frac{1}{\gamma_i}\phi_i^T\phi_i. \quad (4\text{-}8)$$

Its total derivative along the solution trajectory of (4-3) is

$$\dot v_i(x_i,\phi_i)=-x_i^TQ_ix_i+2\, x_i^T P_ib_i\phi_i^T\omega_i+\frac{2}{\gamma_i}\phi_i^T\dot\phi_i+2x_i^TP_ib_i\eta_i \quad (4\text{-}9)$$

Applying (4-4) leads to

$$\begin{aligned}\dot v_i(x_i,\phi_i)&=-x_i^TQ_ix_i+2x_i^TP_ib_i\omega_i^T\phi_i\\&\quad+\frac{2}{\gamma_i}\phi_i^T(-\frac{\beta_i\phi_i}{2}-\gamma_ib_i^T P_ix_i\omega_i)+2x_i^TP_ib_i\eta_i\\&\le-\min\lambda(Q_i)\|x_i\|^2-\frac{\beta_i}{\gamma_i}\|\phi_i\|^2+2\max\lambda(P_i)b_i\|x_i\|\,\|\eta_i\|\\&\le(1-\min\lambda(Q_i))\|x_i\|^2-\frac{\beta_i}{\gamma_i}\|\phi_i\|^2+\max\lambda^2(P_i)\rho_i^2.\end{aligned}$$

Furthermore, in view of (4-5) and (4-6),

$$\dot v_i(x_i,\phi_i)\le-\beta_iv_i(x_i,\phi_i)+\max\lambda^2(P_i)\rho_i^2. \quad (4\text{-}10)$$

Then conclusion (i) and (ii) of the theorem follow. Similarly, consider

$$v(x,\Phi)=\sum_{i=1}^{n}v_i(x_i,\phi_i)$$

as a Lyapunov function for the overall system, In view of (4-10),

$$\begin{aligned}\dot v(x,\Phi)&=\sum_{i=1}^{n}\dot v_i(x_i,\phi_i)\\&\le\sum_{i=1}^{n}\{-\beta_iv_i(x_i,\phi_i)+\max\lambda^2(P_i)\rho_i^2\}\\&\le-\min\beta_i v(x_i,\phi_i)+\sum_{i=1}^{n}\max\lambda^2(P_i)\rho_i^2,\end{aligned}$$

which results in (iii).

Corollary: Associated with the Theorem 1, the position tracking errors e_i is uniformly bounded by

$$|e_i|<\frac{1+\alpha_i}{\alpha_i}\zeta_i, \quad (4\text{-}11)$$

where $\zeta_i>0$, is given by

$$\zeta_i=\sqrt{\frac{\max\lambda^2(P_i)\rho_i^2}{\beta_i\min\lambda(P_i)}-\frac{\phi_i^T\phi_i}{\gamma_i\min\lambda(P_i)}}.$$

Proof: From (4-7), it has

$$\min\lambda(P_i)\|x_i\|^2\le x_i^TP_ix_i<\frac{1}{\beta_i}\max\lambda^2(P_i)\rho_i^2-\frac{1}{\gamma_i}\phi_i^T\phi_i,$$

which means

$$\|x_i\|<\sqrt{\frac{\max\lambda^2(P_i)\rho_i^2}{\beta_i\min\lambda(P_i)}-\frac{\phi_i^T\phi_i}{\gamma_i\min\lambda(P_i)}}=\zeta_i.$$

In view of (3-6a), $e_i=e_i+\dot e_i/\alpha_i=x_i(1)+x_i(2)/\alpha_i$, where $x_i(k)$ with k=1,2 is the k-th component of x_i. As $|x_i(k)|\le\|x_i\|$, then

$$|e_i|\le|x_i(1)|+|x_i(2)|/\alpha_i<\frac{1+\alpha_i}{\alpha_i}\zeta_i,$$

and the corollary is proved.

5. SIMULATION

The robot used to evaluate the proposed method is a SCARA manipulator with four DOF. For its first two joints, the Lagrangian description gives [13]:

$$\frac{d}{dt}\{\begin{bmatrix} d_{11}(q) & d_{12}(q)\\ d_{21}(q) & d_{22}(q)\end{bmatrix}\begin{bmatrix}\dot q_1\\ \dot q_2\end{bmatrix}\}+\begin{bmatrix}h_1(q,\dot q)\\ h_2(q,\dot q)\end{bmatrix}=u, \quad (5\text{-}1)$$

where

$$\begin{aligned}&d_{11}(q)=c_1+c_3+c_4m_L+(c_2+2l_1l_2m_L)\cos q_2,\\&d_{12}(q)=d_{21}(q)=c_3+l_2^2m_L+(c_2/2+l_1l_2m_L)\cos q_2,\\&d_{22}(q)=c_3+l_2^2m_L,\\&h_1(q,\dot q)=0,\\&h_2(q,\dot q)=((c_2/2+l_1l_2m_L)\sin q_2)(\dot q_1\dot q_1+\dot q_1\dot q_2).\end{aligned}$$

In the equations above

$$c_1=l_1^2(m_1/3+m_2) \qquad c_2=l_1l_2m_2$$
$$c_3=l_2^2m_2/3, \qquad c_4=l_1^2+l_2^2$$

where l_1=0.5m, l_2=0.3m are the lengths of link 1 and 2 respectively; m_l is the mass of payload fixed at the end of link 2; and m_1=6kg, m_2=4kg are the masses of the first and second link respectively.

The *a priori* estimate of the parameters used in the computed torque are $\hat l_1$=0.48m, $\hat l_2$=0.28m, $\hat m_1$=4.5kg, and $\hat m_2$=3kg. The initial values of on-line estimated parameter vectors are $\theta_i(0)=0$ (for i=1 and 2). Parameters k_{pi} and k_{pi} are set by k_{pi}=25, k_{vi}=5 (for i=1 and 2), and P_i and Q_i in the Lyapunov equations are given by

$$Q_i=\begin{bmatrix}30 & 0\\ 0 & 20\end{bmatrix} \qquad P_i=\begin{bmatrix}56 & .6\\ .6 & 2.5\end{bmatrix} \qquad \text{for } i=1,2,$$

which give $0<(\min\lambda(Q_i)-1)/\max\lambda(P_i)=0.34$.

It should be noted that the payload is fixed as m_L=2kg initially, but is changed to 0 kg at the instant of t=4s to investigate the algorithm's performance in handling variable payloads. But in the computed torque law $\hat m_L$=0.6kg. The reference trajectories

q_{d1} and q_{d2} are plotted in Fig.(5-1). Fig.(5-2) and (5-3) give e_1 and e_2 respectively which are the position tracking errors of two joints. It can been seen that, from these plots, controlled system tracks the desired trajectories quite well. The influence of step change of the load mass, which occurs at t=4s, are almost invisible on the plots of position errors.

6. CONCLUSION

This paper proposes a robust adaptive control approach for industrial robots based on the Computed-Torque Method and the Lyapunov direct method. The adaptive control is implemented at each subsystem and by special treatment of the model, a filter operator can be introduced to avoid the measurements of the accelerations. Moreover the stability investigation and super boundness of the position errors are given. An evaluation of theoretical analyses using computer simulation results is also presented.

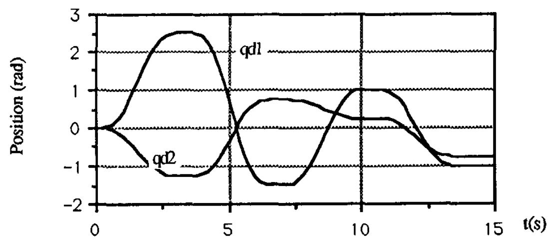

Fig.(5-1). Reference trajectories q_{d1} and q_{d2} .

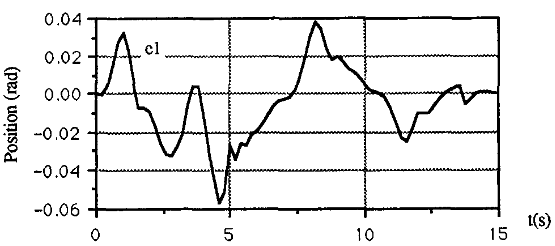

Fig.(5-2). Position tracking errors e_1.

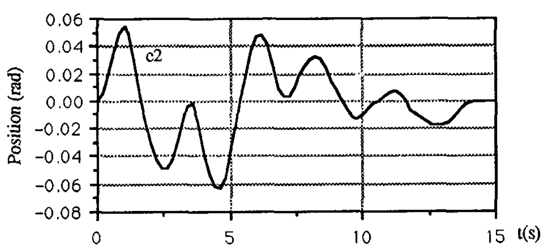

Fig.(5-3). Position tracking error e_2.

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