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Vortex-Induced Motion of Nonlinear Compliant Low Aspect Ratio Cylindrical Systems

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Mooring systems employed for floating structures commonly result in nonlinear load-excursion characteristics. The nonlinear stiffness and the subsequent amplitude-dependent natural frequency influence the vortex-induced motion of the structure. The application of linear stiffness vortex-induced motion modelling and experimental data has been shown to produce significant uncertainties regarding the onset and vortex-induced response prediction of catenary moored cylindrical structures (Bjarke et al., 2003; Dijk et al., 2003). This study specifically investigated the 2-degrees-of-freedom vortex-induced motion of nonlinearly compliant elastically mounted rigid cylinders. Specifically considered was the 3rd-order polynomial hard-spring stiffness typical of catenary moorings. The linear and cubic compliance components were independently varied over the nonlinear compliance ratio range of 0 to 0.29. The study revealed that the analysis of nonlinear compliant structures, assuming the corresponding linear system response, is conservative with respect to mean inline structure displacement and the maximum inline and transverse oscillatory response. Relative to the nonlinear compliant system, the transverse and inline motions were overpredicted by adopting a linear restoring force model. The vortex-induced vibration initial lock-in conditions were also unaltered with increasing nonlinear stiffness. Conversely, the analysis of nonlinear compliant structures, assuming the corresponding linear system response, is not conservative with respect to the lock-out point and the maximum structure excursion (and consequently the maximum mooring line tension experienced) over the lock-in range. This has potential bearing on the way in which highly nonlinear compliant systems are modelled with regard to their vortex-induced motion response.

INTRODUCTION

Vortex-Induced Vibration (VIV) is a fluid-structure interaction phenomenon in which the structure is excited by forces induced by vortices shed alternately from the edges of a bluff object in the flow. The time-varying nonuniform pressure distribution around the object resulting from the vortex shedding (causing a time-varying lift force to be experienced by the object) creates the structural vibration. Near the natural frequency of the structure, the vortex shedding frequency synchronises with the natural frequency. The range of reduced velocity over which this occurs is known as the lock-in range. Mostly, these vibrations are undesirable, resulting in increased fatigue loading and component design complexity to accommodate these motions.

The shedding of vortices from cylindrical bluff objects is well documented for linear compliance cases (for example, Griffin, 1985). This idealised situation, however, often serves only as an approximation of the real system. For example, when dealing with physical restraint of floating structures, significant nonlinear compliance is commonly encountered. Bjarke et al. (2003) demonstrated the uncertainties that arise from the application of accepted linear compliance VIV design rules to the specific nonlinear load-excursion case of a catenary moored cylindrical FPSO riser/anchor buoy. The study concluded that further investigation was required to determine nonlinear compliance levels at which the linear system approximation remains valid. Spar platform model testing with soft linear springs has also been shown to produce significant deviation from catenary restraint results (van Dijk et al., 2003).

The purpose of this study then was to examine the influence of compliance nonlinearity on the vortex-induced vibration response of an elastically mounted rigid cylinder in steady, uniform current. Both cylinder translation motion transverse to the flow and inline with the flow were considered.

Equation of Motion

A simple single degree-of-freedom mass-spring-damper system with nonlinear stiffness characteristics $g(y)$ may be represented by Eq. 1. If the nonlinear stiffness may be described by a continuous function with continuous derivatives over the spring operational displacement, then the Taylor expansion of the system is shown in Eq. 2:

$$m\ddot{y} + c\dot{y} + g(y) = F(t) \tag{1}$$

$$m\ddot{y} + c\dot{y} + k_0 + k_1 y + k_2 y^2 + k_3 y^3 + k_4 y^4 \cdot = F(t) \tag{2}$$

With suitable choice of the y coordinate origin (i.e. such that the spring is unstretched in the zero static load condition), and the assumption that the spring properties in tension and compression are identical, this reduces to Eq. 3 when the series is truncated at the fourth term:

$$m\ddot{y} + c\dot{y} + k_1 y + k_3 y^3 = F(t) \tag{3}$$

Rearranging Eq. 3 then yields:

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y + \beta\omega_n^2 y^3 = \frac{F(t)}{m}. \tag{4}$$

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