1-1-2006

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Recommended Citation

Chicharo, Jose F.; Xi, Jiangtao; Hu, Yingsong; Yang, Zongkai; and Li, Enbang: Discrete cosine transform based shift estimation for fringe pattern profilometry using generalized analysis model 2006, 6560-6567. <https://ro.uow.edu.au/engpapers/4209>

Discrete cosine transform based shift estimation for fringe pattern profilometry using generalized analysis model

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This paper presents a new analysis algorithm to carry out profile measurement with low computational complexity and less noise-sensitivity. Firstly, a discrete cosine transform (DCT) based representation method is introduced to express the height distribution of the three-dimensional surface. Then, a novel shift estimation algorithm, called DCT based shift estimation (DCT-SE) is presented to reconstruct three-dimensional object surfaces by using the proposed expression and the generalized analysis model. The advantage of DCT-SE is that without loss of measurement precise it provides lower computational complexity to implement three-dimensional reconstruction from distorted fringe patterns and at the same time survives from random noises. Simulations and experiments show that the proposed DCT-SE is a fast, accurate and efficient reconstruction algorithm for digital projection based fringe pattern profilometry technique. © 2006 Optical Society of America

OCIS codes: 120.6650, 120.2650, 150.6910.

1. Introduction

Fringe pattern profilometry (FPP) is one of the most popular non-contact approaches to measuring three-dimensional object surfaces. With FPP, a Ronchi grating or sinusoidal grating is projected onto a three-dimensional diffuse surface, the height distribution of which deforms the projected fringe patterns and modulates them in phase domain. Hence by retrieving the phase difference between the original and deformed fringe patterns, three dimensional profilometry can be achieved. In order to obtain phase maps from original and deformed fringes patterns, researchers contributed various analysis methods, including Fourier Transform Profilometry(FTP),¹⁻⁶ Phase Shifting Profilometry (PSP),⁷⁻⁹ Spatial Phase Detection (SPD),¹⁰ Phase Locked Loop (PLL)¹¹ and other analysis methods.^{12,13}

In recent years, because of the simplicity and controllability, digital projectors have been widely used to yield fringe patterns for implementing FPP.^{14–17} However, when generating fringe patterns by using digital projectors, nonlinear distortions are unavoidably introduced and result in visible measurement errors,^{16,17} which has been theoretically analyzed by Hu et al.¹⁷ In order to eliminate the reconstruction errors caused by nonlinear distortions, Guo et al.¹⁶ proposed a gamma correction based method to recover the distorted fringes. However, with this method, the precondition is that the projection system strictly matches the gamma distortion model. Moreover, as gamma coefficient varies with projection systems, the correction coefficient has to be estimated for different systems or whenever the system condition changes. Hu et al. introduced a generalized analysis model, which revealed the essential relationship between the projected and the deformed fringe patterns. This model does not depend on the nonlinear characteristics of projection systems.¹⁷ Meanwhile, based on the mathematical model, Gradient-based Shift Estimation (GSE) algorithm¹⁷ has been also presented to reconstruct accurate profiles from nonlinearly distorted fringe patterns.

However, all of the methods mentioned above, including GSE, are to calculate the height distribution of the surface point-by-point, but not to simultaneously reconstruct the whole object profile. Therefore, these methods will be sensitive to noises, moreover, low-pass or band-pass digital filtering is not a good choice to suppress random noises, as it effects on reconstruction results^{17,18} and introduce extra computational costs. In addition, although GSE provides the advantage to obtain precise surfaces without prior-knowledge of projection system, for some points of the surface, it needs very small learning rates and accordingly many times of iterations to achieve desired accuracy.

In order to reduce the influence from random noises and computational complexity, in this paper, we propose a parametrical estimation method based on DCT and generalized analysis model for fringe pattern profilometry, called DCT based shift estimation (DCT-SE) algorithm. With DCT-SE algorithm, we do not need to employ Fourier filters to suppress random noises before calculating the surface height distribution. Meanwhile DCT-SE provides optimal estimation of the height distribution in meaning of least mean square error with lower computational complexity and less noise-sensitivity. Additionally, because of the use of generalized analysis model, same with GSE, proposed DCT-SE algorithm can also accurately reconstruct object surfaces from distorted fringe patterns.

This paper is organized as follows. In Section 2, we review the principles of FPP technique and generalized mathematical model for fringe pattern analysis. In Section 3, we present a parametrical representation method for height distribution of object surfaces. Meanwhile, based on generalized analysis model, a parametrical estimation algorithm is derived. Simulation and Experimental results are demonstrated in Section 4. Section 5 concludes this paper.

2. Principles of FPP and Generalized Analysis Model

A schematic diagram of a typical FPP system is shown in Fig.1. For simplicity, we

Fig. 1. Schematic diagram of FPP system

consider a cross section of the object surface for a given y coordinate. Hence, the intensity of fringe patterns captured by CCD camera and the height distribution function can be expressed as a function with single variable x . Thus we use $s(x)$ and $d(x)$ to denote the intensity of the projected and deformed fringe pattern respectively and use $h(x)$ to represent the height distribution of the object surface.

Conventional fringe pattern analysis methods are based on Fourier expansion and fundamental component analysis, which is sensitive to nonlinear distortions and highly depends on the performance of filters and the characteristics of the profile.

In order to solve this problem and accurately reconstruct three-dimensional surfaces from nonlinearly distorted fringe patterns, Hu et al. presented a generalized analysis model to explain the principle of FPP technique:¹⁷

$$d(x) = s(x - u(x)) \quad (1)$$

$$h(x) = \frac{l_0 u(x)}{d_0 + u(x)} \quad (2)$$

where $u(x)$ represents the shift between the fringe patterns captured on the reference plane and the surface of the object, which varies with x coordinates. Eq.(1) reveals that the deformed signal $d(x)$ is a shifted version of $s(x)$, and the shift function $u(x)$ can be used to determine the object height distribution by Eq.(2). Because of the simplicity and generality, in following sections of this paper, we will use the generalized model for our algorithm derivation.

3. Parametrical shift estimation algorithms

As mentioned in Section 1, current fringe pattern analysis methods (such as FTP, PSP, SPD and even GSE), are measuring the height distribution point-by-point, which implies the reconstruction results will be very sensitive to noise. Although band-pass filters can eliminate the random noises, they will introduce undesirable distortions and extra computations. In this section, we propose a DCT based parametrical representation method for height distribution function and then based on generalized analysis model, DCT-SE algorithm is derived to implement fringe pattern profilometry.

3.A. DCT based parametrical representation method

According to generalized model, the key to reconstructing three dimensional surfaces is to retrieve shift function $u(x)$ from $s(x)$ and $d(x)$. However, if we still persist in directly measuring the function value of $u(x)$, the measurement has to be made independently for all points on the object surface, so that the computational complexity will not get improved and the reconstructed results will be still sensitive to the system noise. Hence, we have to make a change of representation method for shift function $u(x)$. In this paper, we propose a parametrical representation method based on discrete cosine transform (DCT).

If we regard discrete signal $u(x)$ as a column vector $\mathbf{u} = (u(1), u(2), \dots, u(N))^T$ (N is the length of signal $u(x)$ and $(\cdot)^T$ denotes transpose operation), its DCT can be

expressed as:¹⁹

$$\mathbf{w} = \mathbf{L} \times \mathbf{u} \quad (3)$$

where transform matrix $\mathbf{L}$ is a $N \times N$ matrix. i.e. $\mathbf{L} = (l_{ij})_{N \times N}$ where $i, j = 1, 2, \dots, N$ and

$$l_{ij} = \begin{cases} \frac{1}{\sqrt{N}}, & (i = 1) \\ \sqrt{\frac{2}{N}} \cos \frac{(i-1)(2j-1)\pi}{2N}, & (i \neq 1) \end{cases} \quad (4)$$

$\mathbf{w} = (w(1), w(2), \dots, w(N))^T$ is also a N -dimensional column vector.

As DCT is a unitary transform and $\mathbf{L}$ is an unitary matrix, IDCT (inverse discrete cosine transform) can be expressed as:

$$\mathbf{u} = \mathbf{L}^{-1} \times \mathbf{w} = \mathbf{L}^T \times \mathbf{w} \quad (5)$$

Meanwhile, it is well-known that DCT has a strong ability in compression. Therefore we do not need to use all the elements of vector $\mathbf{w}$ to recover $\mathbf{u}$ from $\mathbf{w}$. If some of the elements are small enough, we can discard them without loss of important informations. i.e. the N -dimensional vector $\mathbf{w}$ can be compressed to be a M -dimensional ($M < N$) vector $\mathbf{p}$:

$$\mathbf{p} = \mathbf{G} \times \mathbf{w}, \quad (6)$$

where $\mathbf{G}$ is a $M \times N$ 'selective' matrix and defined as:

$$g_{ij} = \begin{cases} 1, & \text{if } w(j) \text{ is the } i\text{-th selected element} \\ & \text{of } \mathbf{w} \\ 0, & \text{others} \end{cases} \quad (7)$$

Accordingly, the inverse transform matrix can be selected to be a $N \times M$ matrix $\mathbf{C}$ as well:

$$\mathbf{C} = (\mathbf{G} \times \mathbf{L})^T \quad (8)$$

Consequently, the original vector $\mathbf{u}$ can be expressed by a M -dimensional vector $\mathbf{p}$:

$$\mathbf{u} = \mathbf{C} \times \mathbf{p} = (\mathbf{G} \times \mathbf{L})^T \times (\mathbf{G} \times \mathbf{w}) \quad (9)$$

which implies that we can use less parameters to represent $\mathbf{u}$.

3.B. Derivation of DCT-SE algorithm

According to the representation method mentioned in Section 3.A, instead of directly calculating $\mathbf{u}$, we can estimate $\mathbf{p}$ before retrieve $\mathbf{u}$. Therefore, in order to estimate $\mathbf{p}$, we define the mean square error between $s(x)$ and $d(x)$ as an objective function.

$$J = E(e^2) = E([d(x) - s(x - u(x))]^2) \quad (10)$$

where $e = d(x) - s(x - u(x))$, $E(\cdot)$ denotes the expectation and recall $u(x)$ is the x -th element of $\mathbf{u}$. Similarly, in the following of this paper, for a vector $\mathbf{v}$, we use $\mathbf{v}(k)$ to

denote the k -th element of the vector $\mathbf{v}$. Meanwhile, for a matrix $\mathbf{A}$ we use $\mathbf{A}(k)$ to denote the k -th row vector of matrix $\mathbf{A}$. Hence, Eq.(10) can be expressed as:

$$J = E([d(x) - s(x - \mathbf{u}(x))]^2) \quad (11)$$

Substituting Eq.(9) into Eq.(11), we can have

$$\begin{aligned} J(\mathbf{p}) &= E([d(x) - s(x - (\mathbf{C} \times \mathbf{p})(x))]^2) \\ &= E([d(x) - s(x - \mathbf{C}(x) \times \mathbf{p})]^2) \end{aligned} \quad (12)$$

By minimizing this object function with respect to the parameter vector $\mathbf{p}$, $\hat{\mathbf{p}}$, the estimation of $\mathbf{p}$ can be estimated in sense of least mean square error. In order to minimize J , we use gradient-based method to obtain the estimation of $\hat{\mathbf{p}}$ in an iterative way:

$$\hat{\mathbf{p}}_{m+1} = \hat{\mathbf{p}}_m - \eta \cdot \nabla_{\mathbf{p}} J(\hat{\mathbf{p}}_m) \quad (13)$$

where $\nabla_{\mathbf{p}} J(\cdot)$ represents the gradient of function $J(\cdot)$ regarding to variable $\mathbf{p}$, η is the learning rate and the subscript m denotes the m -th time iteration. By using LMS algorithm, we can replace $\nabla_{\mathbf{p}} J(\hat{\mathbf{p}}_m)$ with $\hat{\nabla}_{\mathbf{p}} J(\hat{\mathbf{p}}_m)$, the estimation of the gradient $\nabla_{\mathbf{p}} J(\hat{\mathbf{p}}_m)$, for simplifying calculation. Namely,

$$\hat{\mathbf{p}}_{m+1} = \hat{\mathbf{p}}_m - \eta \cdot \hat{\nabla}_{\mathbf{p}} J(\hat{\mathbf{p}}_m) \quad (14)$$

where

$$\begin{aligned} \hat{\nabla}(J(\hat{\mathbf{p}}_m)) &= \nabla([d(x) - s(x - \mathbf{C}(x) \times \hat{\mathbf{p}}_m)]^2) \\ &= \nabla e_m^2 = 2e_m \nabla e_m = -2e_m \nabla_{\mathbf{p}} s(\hat{\mathbf{p}}_m) \\ &= -2e_m \nabla_u s(\hat{u}_m(x)) \cdot \nabla_{\mathbf{p}} u_m(x) \end{aligned} \quad (15)$$

where $u(x) = \mathbf{C}(x) \times \mathbf{p}$ and $\hat{u}_m(x) = \mathbf{C}(x) \times \hat{\mathbf{p}}_m$. So

$$\begin{aligned} \hat{\nabla}(J(\hat{\mathbf{p}}_m)) &= -2e_m \\ &\times \frac{s(x - (\hat{u}_m(x) + 1)) - s(x - (\hat{u}_m(x) - 1))}{(\hat{u}_m(x) + 1) - (\hat{u}_m(x) - 1)} \\ &\times \nabla_{\mathbf{p}} u_m(x) \\ &= -e_m (s(x - \hat{u}_m(x) - 1) - s(x - \hat{u}_m(x) + 1)) \\ &\times \mathbf{C}(x) \\ &= -[d(x) - s(x - \mathbf{C}(x) \times \hat{\mathbf{p}}_m)] \\ &\times [s(x - \hat{u}_m(x) - 1) - s(x - \hat{u}_m(x) + 1)] \\ &\times \mathbf{C}(x) \end{aligned} \quad (16)$$

Because every point contains efficient informations of parameter vector $\mathbf{p}$, we can sequentially and cyclically use all the point from 1 to N until the algorithm has

converged. Namely, for the m -th time iteration, we can use the point at $x = (m - 1)\%N + 1$, where $\%$ is the modulus operator, to calculate the gradient by using Eq.(16). Therefore, the derived DCT-SE algorithm can be expressed as:

$$\begin{aligned}\hat{\mathbf{p}}_{m+1} &= \hat{\mathbf{p}}_m + \eta \cdot [d(x) - s(x - \mathbf{C}(x) \times \hat{\mathbf{p}}_m)] \\ &\times [s(x - \hat{u}_m(x) - 1) - s(x - \hat{u}_m(x) + 1)] \\ &\times \mathbf{C}(x)\end{aligned}\tag{17}$$

3.C. Initialization of $\mathbf{C}$, $\mathbf{p}_1$ and η

In order to carry out the algorithm expressed by Eq.(17), we have to initialize $\mathbf{C}$, $\mathbf{p}_1$ and η . In fact, a good initialization can make convergence much faster. Meanwhile, we can note that though FTP can not give accurate reconstruction results, we can still use it to obtain a rough profile, which implies that the results obtained by FTP, $\mathbf{p}_{ftp}$, has been rather 'close' to the true value $\mathbf{p}$. Therefore, we can use FTP as a pre-estimate method for initialization and then adjust the values of $\mathbf{p}$ by using our proposed algorithm. Using FTP, we can reconstruct a rough profile and accordingly obtain $\mathbf{u}_{ftp}$ by inverting Eq.(2), a rough estimation of $\mathbf{u}$. Hence, making DCT on $\mathbf{u}_{ftp}$, we can have $\mathbf{p}_{ftp}$. Further, we can compose initial vector $\mathbf{p}_1$ by selecting M elements which have the biggest M absolute values in the elements of $\mathbf{p}_{ftp}$. Namely,

$$\mathbf{p}_1 = \mathbf{G} \times \mathbf{p}_{ftp}\tag{18}$$

$\mathbf{G}$ is the selective matrix defined as Eq.(7). How many elements are needed to be selected depends on the desired degree of approximation between $\mathbf{p}_1$ and $\mathbf{p}_{ftp}$. i.e. given a ratio $0 < \beta \leq 1$, we need the M elements of vector $\mathbf{p}_1$ to satisfy:

$$\frac{\sum_{k=1}^M |p_1(k)|}{\sum_{k=1}^N |p_{ftp}(k)|} \geq \beta\tag{19}$$

Obviously, M satisfying Eq.(19) always exists and the value of M depends on the profile of the object surface. Meanwhile, because of the powerful compression ability of DCT, practically M is much smaller than N with remaining the most energy of the original signals. This property is also confirmed by our simulation and experiment.

Accordingly, during initialization of $\mathbf{p}_1$, the selective matrix $\mathbf{G}$ has been determined at the same time. Thus, matrix $\mathbf{C}$ can be easily obtained by Eq.(8).i.e.

$$\mathbf{C} = (\mathbf{G} \times \mathbf{L})^T\tag{20}$$

Considering the differences of the elements of $\mathbf{p}$ might be large, it will be better to let the learning rates vary with the order of magnitude of elements for faster convergence. Consiquently, we can make the learning rate η be a vector $\eta = (\eta_1, \eta_2, \dots, \eta_N)^T$. As mentioned above, $\mathbf{p}_1$ is an initial estimation of $\mathbf{p}$, we can simply use $\mathbf{p}_1$ multiplied by a small learning step to construct η . i.e. let

$$\eta = \mu \cdot \mathbf{p}_1\tag{21}$$

where μ is a small scalar suited for learning.

4. Simulations and Experiments

4.A. Simulations

For simulation, we use a paraboloid object surface whose maximum height is 160mm, which is shown in Fig.2(d). Considering the second order harmonic usually has the strongest power in harmonics,¹⁷ the projected fringe pattern is generated from a cosinusoidal signal with the second order harmonic introduced, which is expressed as:

$$s(x) = 128 + 100 \cdot \cos(2\pi f_0 x) + 10 \cdot \cos(2\pi \cdot (2f_0)x) \quad (22)$$

where f_0 is the spatial frequency of the fringe pattern, which is assumed to be 10/meter in our simulation. i.e. the spatial period of the fringe pattern is assumed to be 100mm. Meanwhile, we assume l_0 and d_0 in Fig.1 equal to 5 meters and 2 meters respectively. The spatial resolution of the captured image is assumed to be 1 pixel/mm. Note for the projected fringe pattern given by Eq.(22), the second order harmonic only has -20db of power compared with the fundamental component. Further we assume when $s(x)$ is captured by CCD camera, it is distorted again by a nonlinear function:

$$\zeta(s) = 128 \frac{\tanh(\frac{3s}{128} - 3)}{\tanh(3)} + 128 \quad (23)$$

which is a nonlinear function mapping the values from range[0,256] to range [0,256]. The function is plotted in Fig.2(a). i.e. The captured fringe pattern of $s(x)$ is:

$$\hat{s}(x) = \zeta(s(x)) \quad (24)$$

Corresponding to Eq.(22), the deformed fringe pattern and its captured version can be respectively expressed as:

$$\begin{aligned} d(x) &= 128 + 100 \cos(2\pi f_0 x + \phi(x)) \\ &\quad + 10 \cos(2\pi \cdot (2f_0)x + 2\phi(x)) \end{aligned} \quad (25)$$

where $\phi(x)$ is the phase shift caused by the object surface. And

$$\hat{d}(x) = \zeta(d(x)) \quad (26)$$

The projected and deformed fringe patterns are shown in Fig.2(c) and (d) respectively. Additionally, for clarity of comparison, we will use the cross section of the surface to demonstrate reconstruction results.

4.A.1. Noise Free Situation

In the noise-free case, we can obtain $s(x)$ and $d(x)$ without any noise corruption. However, because of the nonlinear distortion of the fringe pattern, FTP algorithm is only able to work out a roughly reconstructed height distribution result represented by solid line in Fig.3(a) and dashed line is the true value of the height distribution. Corresponding to the roughly estimated height distribution, the pre-estimated shift

Fig. 2. Simulated fringe patterns and object

Fig. 3. Reconstruction results when noise free

$\mathbf{u}_{ftp}$ can be calculated by inverting Eq.(2). Then, making DCT on $\mathbf{u}_{ftp}$, $\mathbf{p}_{ftp}$ can be calculated. As defined by Eq.(19), in order to determine the dimension of the parameter vector $\mathbf{p}$, we can set a proper threshold for β . For instance, when $\beta = 0.95$ and $\beta = 0.99$, the determined dimensions of vector $\mathbf{p}$ are $M = 12$ and $M = 25$ respectively. For accuracy, in our simulation, we use $M = 30$ for performing our reconstruction. Hence, after determining M , $\mathbf{C}$ and $\mathbf{p}_1$ can be initialized. From the initial vector $\mathbf{p}_1$, we can adjust the parameter vector $\mathbf{p}$ by our proposed algorithm expressed by Eq.(17) until the algorithm converges to $\hat{\mathbf{p}}_{opt}$, the optimal estimation of $\mathbf{p}$, and at the same time an accurate reconstruction result can be obtained as shown in Fig.3(b) by solid line. In Fig.3(b), same with Fig.3(a), dashed line is the true value of the height distribution. We can see that the reconstruction result is almost identical with the true value. Meanwhile, it provides much more accurate and smoother surface compared with the result calculated by FTP algorithm. This result also confirms the proposed DCT-SE algorithm precisely converges to the optimal value. Additionally, as we only use a 30 dimensional vector to express the profile signal, compared with directly estimating the elements of vector $\mathbf{u}$ with $N = 1000$ dimensions, the parameter space has been significantly reduced. That is why the proposed DCT-SE algorithm can achieve much faster convergence.

4.A.2. Noisy Situation

In practice, random noise always exists when capturing the fringe patterns, i.e. the captured fringe pattern on the reference plane and object surface can be denoted as $\hat{s}_n(x)$ and $\hat{d}_n(x)$ and expressed by:

$$\begin{aligned}\hat{s}_n(x) &= \hat{s}(x) + n_1(x) \\ \hat{d}_n(x) &= \hat{d}(x) + n_2(x)\end{aligned}\tag{27}$$

where n_1 and n_2 are white noises. In our simulation, the noise power is set to be 20db and same with the noise free situation, the dimension of the parameter vector, M is still set to be 30. The corrupted fringe pattern $\hat{s}_n(x)$ and $\hat{d}_n(x)$ are shown in Fig.4 (a) and (b) respectively. With $\hat{s}_n(x)$ and $\hat{d}_n(x)$, reconstruction result using FTP is shown in Fig.5 by dotted line and the solid line represents the result by using proposed DCT-SE algorithm, dashed line the is true values of the height distribution. It can be seen that DCT-SE is not sensitive to noises and gives much smoother and more accurate reconstruction results even in case of that fringes are seriously corrupted by random noises.

4.A.3. Convergence Speed

We take $p(4)$ and $p(6)$ as two examples, which have significant relative errors between pre-estimated vector and ideal one, to show the fast convergence progress when using our proposed DCT-SE algorithm in Fig.6. We can see that DCT-SE algorithm converges around totally 30000 iterations. Meanwhile, because of using generalized analysis model, in terms of reconstruction accuracy, both DCT-SE and GSE have strong ability to accurately reconstruct three-dimensional surfaces from nonlinearly

Fig. 4. Captured fringes corrupted by noises

Fig. 5. Reconstruction results with noises

Fig. 6. Convergence of DCT-SE algorithm

distorted fringe pattern. However, DCT-SE wins in convergence speed. We define an average number of iterations, γ , as

$$\gamma = \frac{\text{the total number of the iterations}}{\text{the total number of sample points}} \quad (28)$$

By using FTP, GSE and DCT-SE, different reconstruction errors are compared in in Table 1, where ε_a and ε_s denote average absolute error and standard deviation respectively. Meanwhile, for similar reconstruction accuracy with GSE and DCT-SE, the average number of the iterations are also listed and compared. It can be seen that

Table 1. The average number of iterations

	γ	$\varepsilon_a(\text{mm})$	$\varepsilon_s(\text{mm})$
FTP	-	6.7768	8.2116
DCT-SE	37.5	0.7790	0.7397
GSE	2389.8	0.7903	1.6448

both GSE and DCT-SE can accurately reconstruct three-dimensional profiles from distorted fringe patterns and compared with GSE, DCT-SE algorithm can significantly reduce the iteration times so that much faster profilometry can be achieved.

4.B. Experiment

In our experiments, the fringes were projected using a InFocus LP530 DLP projector at a distance of 2m away from the reference plane. The images were captured by a DuncanTech MS3100-RGB 3CCD located 81cm away from the projector and connected to a computer, which is used to try to generate sinusoidal fringes. The object to be measured was a dome set on a flat board as shown in Fig.7(a). The maximum height of the dome is 22.8mm. The diameter of the bottom surface of the dome is 99mm. The thickness of the base board is 16mm. Fig.7(b) and (c) are the captured fringes on the reference plane and on the object surface respectively. As mentioned above, because of the nonlinear distortion, the captured fringe pattern on the reference plane becomes non-sinusoidal and non-periodic, which is shown in Fig.7(d), where the plotted signal is a cross section of Fig.7(b). In our experiment, the resolution of the CCD camera is 1392×1039 pixels. The filed of vision for CCD camera is $260\text{mm} \times 194\text{mm}$. Hence, the equivalent spatial resolution is 0.1868mm/pixel . The equivalent spatial period of projected fringe was 25.7mm .i.e. $f_0 = 38.9/m$.

The reconstruction result by using FTP is shown in Fig.8. We can see the surface measured by FTP appears jagged and rough. In contrast, by using DCT-SE, a very smooth reconstructed dome is shown in Fig.9.

5. Conclusion

This paper has presented a novel shift estimation method, called DCT based shift estimation (DCT-SE) for digital projection based fringe pattern profilometry. Because

Fig. 7. Object and fringe patterns in experiment

Fig. 8. Surface reconstructed by FTP

Fig. 9. Surface reconstructed by DCT-SE

of using generalized analysis model, both DCT-SE and GSE can accurately reconstruct three-dimensional surfaces from nonlinearly distorted fringe patterns. However, the proposed DCT-SE has more advantages over GSE. DCT-SE can significantly reduce computational complexity compared with GSE and has much faster convergence speed. Meanwhile, DCT-SE can survive from random noises without any low-pass or band-pass filters involved. Moreover, because of the fast convergence and the reduced dimension of the parameter vector, DCT-SE is more suitable for realtime profilometry. Simulation and experimental results shows that DCT-SE is a much faster, more efficient and reliable algorithm for fringe pattern profilometry.

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