1-1-2016

Optimal design of fixture layout in a multi-station assembly using highly optimized tolerance inspired heuristic

Satish Tyagi
Wayne State University, Satish.Tyagi@wayne.edu

Nagesh Shukla
University of Wollongong, nshukla@uow.edu.au

Sumant Kulkarni
University of Louisiana

Optimal design of fixture layout in a multi-station assembly using highly optimized tolerance inspired heuristic

Abstract

The multi-station assembly (MSA) process requires auxiliary devices such as fixtures and clamps to accurately locate and firmly hold the workpiece in a desired position. Improper positioning of these fixtures and clamps affects the dimensional integrity of final product. This study determines the optimal design of fixture layout that minimizes the product dimensional variations caused by the manhandling and aging of auxiliaries. In order to model variation propagation from one assembly station to another in the MSA, a state space model is employed. Further, an E-optimality based sensitivity criterion is proposed to mathematically formulate and measure the quality of the fixture layout design. In order to solve the mathematical formulation, a highly optimized tolerance inspired heuristic is proposed. The proposed approach takes its governing traits from local incremental algorithm (LIA) which was initially exploited to maximize the design parameter (yield) in the percolation model. LIA analogous to the evolution by natural selection schema, assists in suitably exploring the search space of the underlying problem. The assembly of Sports Utility Vehicle side frame has been used to illustrate the concepts and test the performance the proposed solution methodology. Further, robustness of the proposed heuristic is demonstrated by comparing its results with that of obtained from Basic Exchange Algorithm used in the literature.

Keywords

fixture, layout, multi, station, assembly, highly, optimized, tolerance, inspired, heuristic, optimal, design

Disciplines

Engineering | Science and Technology Studies

Publication Details

Tyagi, S., Shukla, N. & Kulkarni, S. (2016). Optimal design of fixture layout in a multi-station assembly using highly optimized tolerance inspired heuristic. Applied Mathematical Modelling: simulation and computation for engineering and environmental systems, 40 (11-12), 6134-6147.

Optimal design of fixture layout in a multi-station assembly using highly optimized tolerance inspired heuristic

Abstract: *The multi-station assembly (MSA) process requires auxiliary devices such as fixtures and clamps to accurately locate and firmly hold the workpiece in a desired position. Improper positioning of these fixtures and clamps affects the dimensional integrity of final product. This study determines the optimal design of fixture layout that minimizes the product dimensional variations caused by the manhandling and aging of auxiliaries. In order to model variation propagation from one assembly station to another in the MSA, a state space model is employed. Further, an E-optimality based sensitivity criterion is proposed to mathematically formulate and measure the quality of the fixture layout design. In order to solve the mathematical formulation, a highly optimized tolerance inspired heuristic is proposed. The proposed approach takes its governing traits from local incremental algorithm (LIA) which was initially exploited to maximize the design parameter (yield) in the percolation model. LIA analogous to the evolution by natural selection schema, assists in suitably exploring the search space of the underlying problem. The assembly of Sports Utility Vehicle side frame has been used to illustrate the concepts and test the performance the proposed solution methodology. Further, robustness of the proposed heuristic is demonstrated by comparing its results with that of obtained from Basic Exchange Algorithm used in the literature.*

Keywords: Fixture layout design, state space model, highly optimized tolerance, power law, local incremental algorithm.

1. Introduction

Among automotive industries, dimensional integrity a indicator of a high quality product, is a crucial factor in winning the market amid the acute competition. This fact has driven the organizations to design their assembly systems with higher precision to manufacture products with greater dimensional integrity. Fixture failures are recognized as the major contributor (approximately 72 percent) among all root causes of dimensional variation in an assembled product [1-3]. In the multi-station assembly (MSA) process, operations involve unification of two or more than two panels/sub-assemblies at more than one workstation. To provide physical support to a panel/subassembly, a 3-2-1 principle fixture layout design is generally employed. As illustrated in Fig. 1, 3-2-1 layout comprises of two locating pins and three net contact (NC) blocks. Locating pins are of two types: 4-ways pin (pin-hole locator, $P_{4\text{-ways}}$) to restrict motion of a panel in X-Z plane and 2-ways pin (pin-slot locator, $P_{2\text{-ways}}$) to prevent movement in Z-direction. Synchronization of these two pins restrains the rotation and translation motion of the panel in X-Z direction during assembly process. In addition, two principal locating points (PLPs) on each panel/sub-assembly restrict its movements in X and Z directions. Three NC blocks are used to constrain deformation in Y-direction. The current paper mainly deals with assembly process of rigid bodies in 2-D and deformation in Y-direction is a topic for future research.

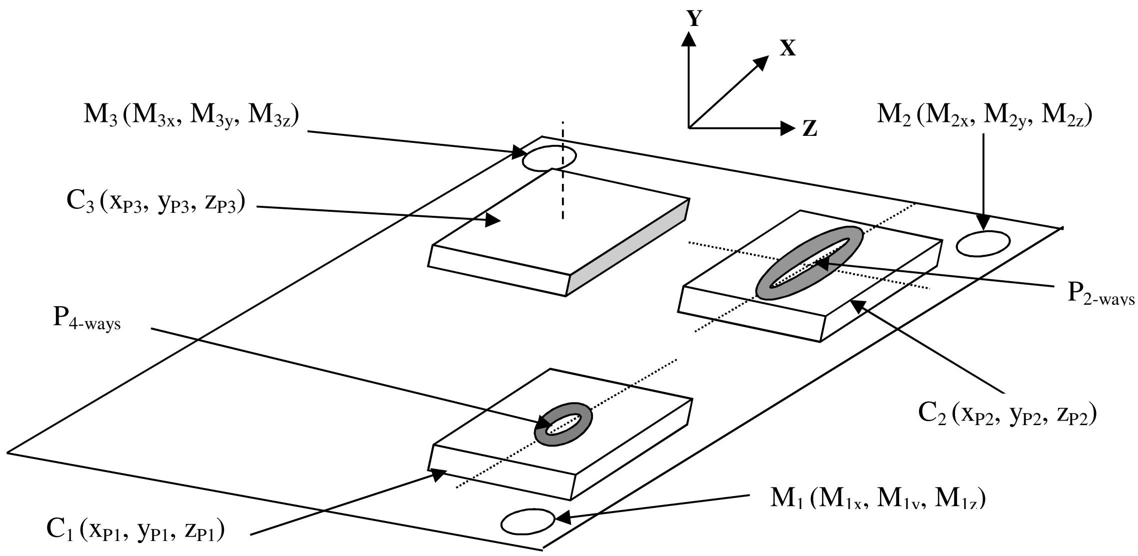


Fig. 1. Generic 3-2-1 fixture layout design for rigid parts

The complexity of fixture layout design problem is illustrated by considering Sports Utility Vehicle (SUV) side frame, which comprises of four panels viz. A-pillar, B-pillar, rail roof side panel and rear quarter panel (see Fig. 2). It is assumed that only two workpiece (panels or subassembly) can be assembled at each station. First two panels are assembled at station 1. Then, sub-assembly is passed on to the second station where it is assembled with the third panel. Fourth panel is assembled with the incoming sub-assembly (from station 2) on third station. Subsequently, assembled product is transferred to the fourth station where variations of product measurement points [M_1-M_{10}] are collected. Stepwise assembly of four panels at various stations can be represented in terms of PLPs as illustrated in Fig. 2. Fixture layout in Fig. 2 generically refers to an arrangement of 8 locators (2 locators on each panel). Assembly process can also be represented in terms of processing sequence as follows:

$$\left\{(P_1,P_2)^p,(P_3,P_4)^p\right\}^1 \Rightarrow \left\{(P_1,P_4)^s,(P_5,P_6)^p\right\}^2 \Rightarrow \left\{(P_1,P_6)^s,(P_7,P_8)^p\right\}^3 \Rightarrow \left\{(P_1,P_8)^s\right\}^4$$

Where, superscripts (1, 2, 3, and 4) indicate station number and P_1, P_2....P_8 stand for pair of locators employed. For example at station 3, the sub-assembly "A pillar + B pillar + rail-roof side panel" is restrained by locator pair P_1 and P_6 while new panel "rear quarter" is located by locator pair P_7 and P_8. Superscript '*p*' and '*s*' are used to indicate that locator pair is used to restrain the movement of a panel or a subassembly respectively.

Locators may be broken, worn, loose, or bent due to daily operations, which may result in depreciated product dimensional integrity during MSA. Moreover, variation generated at one station propagates to downstream stations in assembly line. In discrete part manufacturing, optimal design of fixture layout involves searching for position of PLPs such that the effect of these fixture variations on final product quality can be minimized. There can be infinite choices (candidate locations) to place locators in the continuous search space within each panel. In order to eliminate infinite possibilities, search space is reduced by discretizing each panel. In the current study, discretization distance is equal to the diameter of locator (10 mm). Based on the dimensions of each panel, the number of candidate locations to put one locator are N_1=697, N_2= 1038, N_3= 429, N_4= 6189 [4]. It is evident that even small number of panels can generate a large number of alternatives for fixture layout design. Therefore, efficient method is needed to identify the optimal fixture layout for MSA.

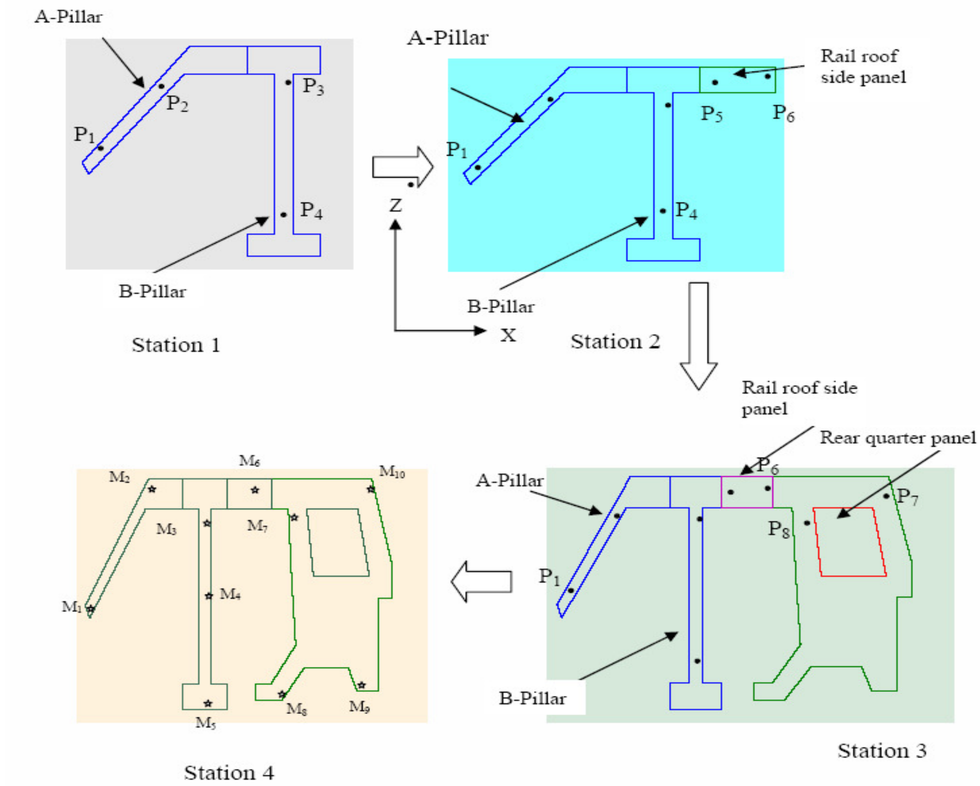


Fig. 2. Assembly of SUV at four stations

Introduced by Carlson and Doyle [5], Highly Optimized Tolerance (HOT) is inspired by the behavior of biological organism and advanced engineering technologies. Tradeoffs between yield and resource cost lead to the unpredictable event sizes in systems which are optimized by engineering design based frameworks. HOT is applied to study the behavior of complex systems in an uncertain environment. The characteristics associated with systems at HOT state are *power laws* and robustness against uncertainties, design flaws and rare perturbations. A HOT inspired heuristic is introduced in this paper to identify the optimal fixture layout design in MSA. In proposed heuristic, power law is applied in estimating the dynamic probability of placing locators on each panel. Furthermore, probability of placing a locator at the gravitational center (GC) is assumed zero and correspondingly candidate locations within each panel have assigned probabilities according to their Euclidean distance from GC. The assembly of a side frame has been used to illustrate the concepts. Further, robustness of heuristic is demonstrated by comparing its results with that of obtained from Basic Exchange Algorithm used in the literature.

The rest of the paper is organized as follows, relevant literature pertaining to fixture layout is detailed in the section 2. State space model for modeling the variation propagation is discussed in section 3. Background of HOT and proposed heuristic are described in Section 4. Section 5 details the computational experience and conclusive remarks are elaborated in Section 6.

2. Literature review

Previous efforts for fixture layout design were mainly concentrated on formulation of an objective function against definite constraints including ease with a workpiece can be loaded/unloaded, clamping and position stability, workpiece controlling capability in presence of external perturbations and uniqueness of its location. Asada and By [6] applied kinematical analysis to study the fixture layout problem. They build up a criterion to ensure the workpiece location and its loading as well as unloading capability on fixture layout. Ferreira *et al.* [7] proposed heuristic approaches for automatic construction of fixture configurations during assembly operations and aimed at minimizing the deflection and distortion of workpiece caused by locating pins. Additionally, finite element analysis and non-linear optimization algorithms have also been utilized to optimize the support position [8] and for sheet metal assembly [9]. Hockenberger and De Meter [10] introduced a heuristic to identify the optimal position of locators and clamps by considering min-max loading criterion. Abovementioned studies did not consider inevitable processing error(s) such as fault arbitrarily generated in fixture elements. Rong and Bai [11] studied the effect of locator error on accuracy of workpiece geometry by applying effective analysis approach against geometric plan constraints. Accuracy and repeatability is enhanced by using such improvements, however, major root cause of dimensional variation, fixture faults, has been neglected.

Ceglarek and Prakash [12] initiated works on diagnosis of fixture failures by adopting engineering models. However, their work was confined to single fixture, single fault assumption. Researchers extended the work to multiple faults, multiple fixture and optimal sensor distribution [13-15]. Unfortunately, researchers considered only single station instead of the multi-stations in modelling which is cumbersome owing to the station-to-station interactions. Mantripragada and Whitney [16] introduced the state transition model that considers variation accumulation in the assembly process. Actually, their focus was on modelling of variation accumulation caused by manufacturing deficiency. Lawless et al. [17] proposed a model to describe dimensional variation in both assembly and machining processes by employing the AR(1) model. Here AR (1) represents autoregressive model of of order 1. Nevertheless, these models were unable to define the relationship between fixture faults and part deviation particularly in MSA. Development of relationship among three main concepts: tooling locating error, part accumulative error, and part re-orientation error results in a state space model (SSM) that describes variation propagation in MSA [18-20]. Jin and Shi [18] work was confined to two fundamental assumptions: (1) only two panels are assembled at each workstation, and (2) in case of concurrent assembly their model fails as only single panel is assembled instead of a sub-assembly. These limitations has been overcome by Ding *et al.* [21] in station indexed SSM to model the variation propagation in MSA.

Lack of competent optimization algorithm further exacerbates the optimization of fixture layout design in MSA. Kim and Ding [22] used basic exchange algorithm (BEA) to identify the optimal design of fixture layout which was originally used in experimental design to resolve similar design problems. BEA becomes inhibitive approach in case of fixture layout design problem as satisfactory results are not obtained even after large computational time. Actually, no current method has potential to resolve the time complexity of abovementioned problem owing to computational complexity. This article focuses on developing an efficient algorithm that is capable of producing acceptable solution in a reasonable time. In their study, improved result was obtained in terms of computational time without significantly shifting the optimal value. Kim and Ding [4] presented a data mining method where small subset of design alternatives are selected and local optimization algorithm is adopted to identify the better design. Research has also been conducted for diagnosis of single fixture faults by using Principal Component Analysis (PCA) [23]. The focus of current work, is to identify optimal design of fixture layout in MSA process, which has received relatively little attention in the literature.

3. State space model (SSM)

Fixture malfunctioning is recognized as the major root cause of dimensional variation in MSA [17]. For example, locators P_1 and P_2 are employed to provide the support to a rectangular workpiece in X-Z plane (see Fig. 3(a)). Considering P_2 to be a faulty locator (fixture malfunction), the deviation of workpiece in Z-direction is shown in Fig. 3(b). The dimensional variation in the final part mainly occurs due to: (i) part locating error and/or (ii) part reorientation/relocation error. Part locating error arises due to fixture error at the current station whereas part reorientation error originates due to relocation of part around PLPs at downstream stations.

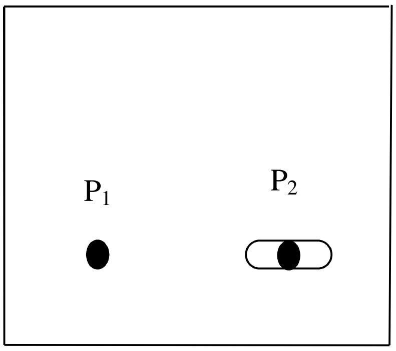
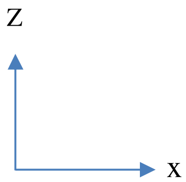


Fig. 3(a). Locators at nominal position



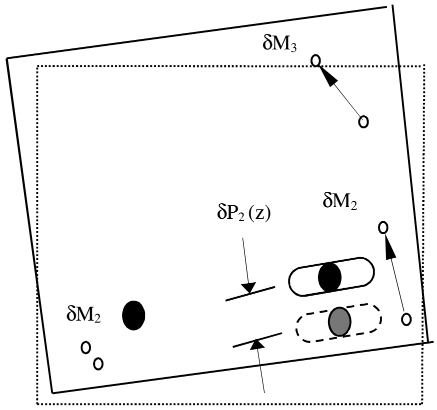


Fig. 3(b). Part variation due to fixture malfunction

3.1 Fixture error vector

Fixture locating error represented by $\Delta \boldsymbol{f}$ occurred at station *k* in X-Z plane is expressed as:

$$\Delta \boldsymbol{f}\,(\mathrm{k}) = \left(\Delta \mathrm{x}_{P_1},\ \Delta z_{P_1}, \Delta z_{P_2}\right)^{T} \tag{1}$$

Where, error of $P_{4\text{-ways}}$ (P_1) in X and Z directions are shown by Δx_{P_1} and Δz_{P_1} respectively. Δz_{P_2} is the fixture error for $P_{2\text{-ways}}$ (P_2) in Z-direction. Subsequently, error at each sub-assembly proceeds as an input for the downstream station. Thus, dimensional variation at any station is the accumulation of variations from the previous stations as shown by Fig. 4. Therefore, the dimensional variations may increase/decrease due to station-to-station interactions.

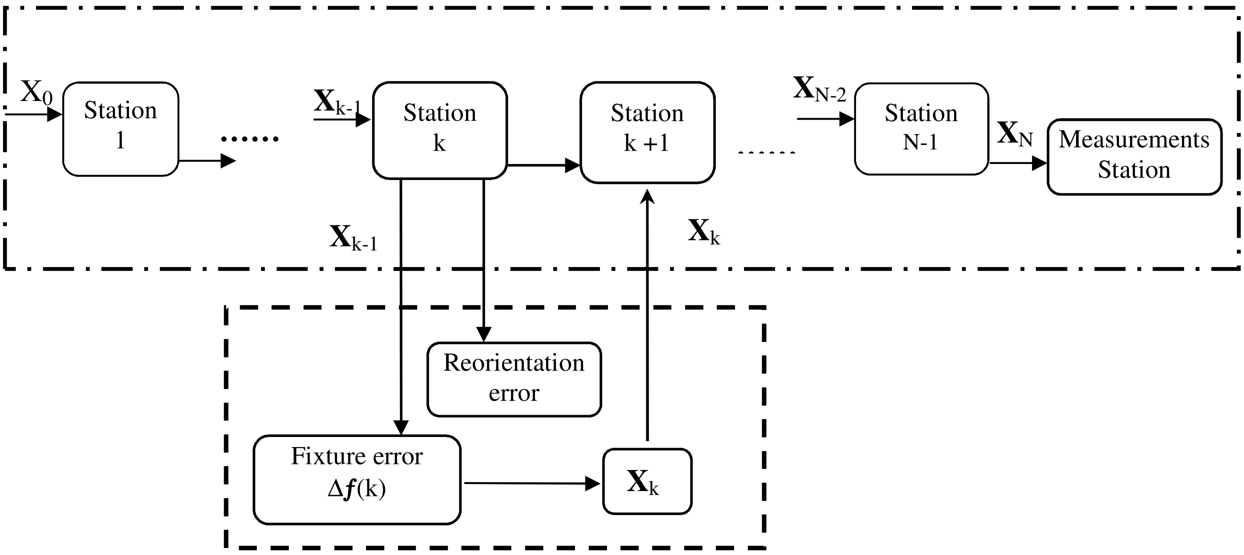


Fig. 4. Variation propagation in the multi-station assembly

3.2 Part variation vector

Part variation is defined as the deviation of a panel/sub-assembly from the nominal position in the assembly process. It can be in the form of rotation in X-Z plane and/or translation motion along X and Z axis. The part variation vector $\mathbf{X}_{\mathrm{P}}(\mathrm{k})$ is represented as:

$$\mathbf{X}_{\mathrm{P}}(\mathrm{k}) = \left[\Delta x_{\mathrm{P}}(\mathrm{k}), \Delta z_{\mathrm{P}}(\mathrm{k}), \Delta\phi_{\mathrm{P}}(\mathrm{k})\right]^{T} \tag{2}$$

Where, $\Delta x_{\mathrm{P}}(\mathrm{k})$ and $\Delta z_{\mathrm{P}}(\mathrm{k})$ are panel deviations in X and Z direction whereas $\Delta\phi_{\mathrm{P}}(\mathrm{k})$ is the rotation error of same panel at station *k*. Fixture error is the cause of part error. Part error represents the dimensional variation of final assembly. This part error is combination of part locating error and part reorientation error. The relationship between part error and fixture error can be expressed.

$$\begin{bmatrix} \Delta x_{\mathrm{P},k} \\ \Delta z_{\mathrm{P},k} \\ \Delta\phi_{\mathrm{P},k} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \dfrac{1}{\mathrm{P}_1(\mathrm{X}) - \mathrm{P}_2(\mathrm{X})} & \dfrac{1}{\mathrm{P}_2(\mathrm{X}) - \mathrm{P}_1(\mathrm{X})} \end{bmatrix} \begin{bmatrix} \Delta x_{\mathrm{P}_1} \\ \Delta z_{\mathrm{P}_1} \\ \Delta z_{\mathrm{P}_2} \end{bmatrix} + \omega_{\mathrm{P,k}} \tag{3}$$

Where, $\mathrm{P}_1(\mathrm{X})$ and $\mathrm{P}_2(\mathrm{X})$ are the nominal X coordinates of locators whereas un-modeled higher order terms are included in $\omega_{P,k}$. Eq. (3) represents a transfer function to calculate part error at one assembly station based on the error in the fixture.

3.3 State space modeling for MSA

Variation propagation in a MSA is modelled using state space approach with station number as its indices. In this paper, SSM proposed by Ding and Ceglarek [29] has been utilized for modelling represented as follows:

$$\mathbf{X}(k) = \mathbf{A}_{(k\text{-}1)} \times \mathbf{X}_{(k\text{-}1)} + \mathbf{B}_{(k)} \times \mathbf{U}_{(k)} + \mathbf{E}_{(k)},\ \ k = 1, 2 \ldots \mathrm{N} \tag{4}$$

$$\mathbf{Y}_{(k)} = \mathbf{C}_{(k)} \times \mathbf{X}_{(k)} + \mathbf{W}_{(k)},\ \{k\} \subset \{1, 2, 3 \ldots \mathrm{N}\} \tag{5}$$

Where, $\mathbf{X}_{(k)}$ represents the dimensional deviations occurring randomly as a result of assembly process on station *k*. Input vector $\mathbf{U}_{(\mathrm{k})}$ represents the random deviations associated with fixture locators. Process errors and unmolded higher order terms are represented by $\mathbf{E}_{(k)}$. $\mathbf{Y}_{(k)}$ and $\mathbf{C}_{(k)}$ denote product measurements and observation matrix. $\mathbf{W}_{(k)}$ is white noise representing measurement noise. Eq. (4) suggests that part deviation $\mathbf{X}_{(k)}$ at k^{th} station is influenced by the accumulated deviation up to station k-1($\mathbf{X}_{(k\text{-}1)}$) and deviation contribution at station k ($\mathbf{U}_{(k)}$). Observation vector $\mathbf{Y}_{(k)}$ in Eq. (5) is obtained at station k. $k \le$ N *i.e.* observation is carried out at some stations. In this study, end-of-line (EOL) observation strategy is applied *i.e.* inspection is carried out only at the last station. To be specific, SSM can be expressed as:

$$\mathbf{X}(1) = \mathbf{A}(0)\times\mathbf{X}(0) + \mathbf{B}(1)\times\mathbf{U}(1) + \mathbf{E}(1)$$

$$\mathbf{X}(k) = \mathbf{A}(k\text{-}1)\times\mathbf{X}(k\text{-}1) + \mathbf{B}(k)\times\mathbf{U}(k) + \mathbf{E}(k),\ \ k = 2,3$$

$$\mathbf{X}(4) = \mathbf{A}(3)\times\mathbf{X}(3) + \mathbf{E}(4)$$

$$\mathbf{Y}(4) = \mathbf{C}(4)\times\mathbf{X}(4) + \mathbf{W}(4) \tag{6}$$

The incoming part deviation $\mathbf{X}(0)$ from stamping process is considered negligible in this study. The matrices $\mathbf{A}(k)$, $\mathbf{B}(k)$, $\mathbf{C}(k)$ can be determined from the expression given in Ding *et al.* [31].

Table 1. **A**, **B** and **C** matrix

Symbols	**A**	γ(k, i)	**B**	**C**
Name	Dynamic matrix	State transition matrix	Input matrix	Observation matrix
Relationship	$\mathbf{X}_k = \mathbf{A}_{k-1}\cdot\mathbf{X}_{k-1}$	$\gamma(k, i)=\{\mathbf{A}_{k\text{-}1}\ldots\mathbf{A}_i$ if k> i	$\mathbf{X}_k = \mathbf{B}_k\cdot\mathbf{T}_k$	$\mathbf{Y}_k = \mathbf{C}_k\cdot\mathbf{X}_k$

4. Design criterion

The linear input-output relations between observation vector $\mathbf{Y}(k)$, and variation sources $\mathbf{U}(k)$, is illustrated based on the Stream-of-Variation Analysis model as shown in Eq. (4) and Eq. (5).

$$\mathbf{Y}=\mathbf{J}\cdot\mathbf{U} + \mathbf{J}(0)\cdot\mathbf{X}(0) + \mathbf{D} \tag{7}$$

Where, $\mathbf{Y}^T=[\mathbf{Y}^T(1)\ \ \mathbf{Y}^T(2)\ldots\ldots\mathbf{Y}^T(N)]$, $\mathbf{D}^T=[\mathbf{D}^T(1)\ \mathbf{D}^T(2)\ \ldots\ldots\mathbf{D}^T(N)]$ and $\mathbf{D}(k) \equiv \sum_{i=1}^{k}\mathbf{C}(k)\boldsymbol{\Phi}(k, j)\mathbf{E}(i) + \mathbf{W}(k)$. $\boldsymbol{\Phi}(i, j)$ is interpreted as change of fixture layout among multiple stations (from i^{th} to j^{th} station). The coefficient of first term of Eq. (7) **J** can be defined as:

$$\mathbf{J}=\begin{bmatrix} \mathbf{C}(1)\mathbf{B}(1) & 0 & \cdots & \cdots & 0 \\ \mathbf{C}(2)\mathbf{\Phi}(2,1)\mathbf{B}(1) & \mathbf{C}(2)\mathbf{B}(2) & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \mathbf{C}(\mathrm{N})\mathbf{\Phi}(\mathrm{N},1)\mathbf{B}(1) & \mathbf{C}(\mathrm{N})\mathbf{\Phi}(\mathrm{N},2)\mathbf{B}(2) & \cdots & \cdots & \mathbf{C}(\mathrm{N})\mathbf{B}(\mathrm{N}) \end{bmatrix} \tag{8}$$

and coefficient of $\mathbf{X}(0)$ as:

$$\mathbf{J}(0)=\begin{bmatrix} \mathbf{C}(1)\mathbf{\Phi}(1,0) \\ \mathbf{C}(2)\mathbf{\Phi}(2,0) \\ \vdots \\ \vdots \\ \mathbf{C}(\mathrm{N})\mathbf{\Phi}(\mathrm{N},0) \end{bmatrix} \tag{9}$$

The deviation due to stamping process $\mathbf{X}(0)$ is ignored as only part deviation during assembly processes is considered. Thus, the linear diagnostic model can be represented as:

$$\mathbf{Y}=\mathbf{J}\cdot\mathbf{U}+\mathbf{D} \tag{10}$$

As stated previously, in this study EOL observation strategy is applied, the observation equation can be expressed as:

$$\mathbf{Y}_{\mathrm{N}}=\sum_{k=1}^{\mathrm{N}}\mathbf{C}(\mathrm{N})\mathbf{\Phi}(\mathrm{N},k)\mathbf{B}(k)\mathbf{U}(k)+\mathbf{C}(\mathrm{N})\mathbf{\Phi}(\mathrm{N},0)\mathbf{U}(0)+\mathbf{D}(k) \tag{11}$$

$\sum_{k=1}^{\mathrm{N}}\mathbf{C}(\mathrm{N})\mathbf{\Phi}(\mathrm{N},k)\mathbf{B}(k)\mathbf{U}(k)$ of Eq. (11) represents fixture error inputs at all stations, hence is the focus of study. Eq. (11) is reformulated to determine the objective function.

$$\mathbf{Y}_{\mathrm{N}}=\sum_{k=1}^{\mathrm{N}}\mathbf{C}(\mathrm{N})\mathbf{\Phi}(\mathrm{N},k)\mathbf{B}(k)\mathbf{U}(k) \tag{12}$$

$$\hat{\mathbf{Y}}=\mathbf{MP}=\sum_{k=1}^{\mathrm{N}}\mathbf{C}(\mathrm{N})\mathbf{\Phi}(\mathrm{N},k)\mathbf{B}(k)\mathbf{U}(k)$$

$$\mathbf{M}=[\mathbf{C}(\mathrm{N})\mathbf{\Phi}(\mathrm{N},1)\mathbf{B}(1)\quad \mathbf{C}(\mathrm{N})\mathbf{\Phi}(\mathrm{N},2)\mathbf{B}(2)\quad \ldots\mathbf{C}(\mathrm{N})\mathbf{B}(\mathrm{N})]$$

$$\mathbf{U}^{\mathrm{T}}=[\mathbf{U}^{\mathrm{T}}(1)\quad \mathbf{U}^{\mathrm{T}}(2)\quad \ldots\mathbf{U}^{\mathrm{T}}(\mathrm{N})]$$

Here, $\hat{\mathbf{Y}}$ is the fixture-induced variation. Also, it is assumed that no error is induced at the fourth station, hence $\mathbf{P}(4)$ is zero. Therefore,

$$\mathbf{M}=[\mathbf{C}(4)\mathbf{\Phi}(4{,}1)\mathbf{B}(1)\quad \mathbf{C}(4)\mathbf{\Phi}(4{,}2)\mathbf{B}(2)\quad \mathbf{C}(4)\mathbf{\Phi}(4{,}3)\mathbf{B}(3)]$$

Kim and Ding [22] studied the similar kind of problem and adopted $\hat{\mathbf{Y}}^T\hat{\mathbf{Y}}$ (sum of squares of product deviations) to standardize the variations occurred from all stations. However, problem associated with $\hat{\mathbf{Y}}^T\hat{\mathbf{Y}}$ is its dependency on input variations and is not applicable to identify the optimal fixture layout design. Actually, a criterion that depends only on the fixture design (**M**) and is free from input variations (**U**) is desired. In the past, efforts have been made to establish a linear variation propagation model that links product dimensional variation to fixture deviation [23]. Based on variation model, a sensitivity index criterion is developed, which is influenced by fixture layout design [22]. The sensitivity index is defined as the ratio of output part variation to input fixture variation. Another way to define sensitivity index criterion is:

$$S = \frac{\hat{\mathbf{Y}}^T\hat{\mathbf{Y}}}{\mathbf{U}^{\mathrm{T}}\mathbf{U}} = \frac{\mathbf{U}^{\mathrm{T}}\mathbf{M}^{\mathrm{T}}\mathbf{M}\mathbf{U}}{\mathbf{U}^{\mathrm{T}}\mathbf{U}} \tag{13}$$

In Eq. (13), S is still not free from its dependency over input part variations. However being an important component of above equation, $\mathbf{M}^{\mathrm{T}}\mathbf{M}$ becomes a determinative factor that exploits information received from the fixture layout design and is independent from the input variations. Several researchers in the experimental design have specified a number of optimality criteria for such problem [22, 32]. Among them, D-optimality, A-optimality, and E-optimality criterion are generally used. Since **A**-matrix is singular, consequently $\mathbf{\Phi}(\mathrm{k}, i)$ is also singular [23]. Singularity issue of **A**-matrix imposes an additional limitation in employing D-optimality criterion to determine sensitivity index. Apparently, **M** is also singular matrix so D-optimality criterion has a same value (zero) for all possible combination of locator positions. In essence, D-optimality criterion is not capable of providing information about fixture design. Since E-optimality criterion tries to minimize the maximum Eigen value of matrix $\mathbf{M}^{\mathrm{T}}\mathbf{M}$ which is analogous to minimize the utmost value of sensitivity index whereas the A-optimality maximizes the summation of all eigenvalues of **M**. In comparison, both A-optimality and E-optimality can be considered for fixture layout design problem. However, E-optimality is a little conservative because it attempts to reduce the maximum sensitivity index. This makes E-optimality more acceptable to be used by practitioners the concept of E-optimality is very much relevant to the pareto principle in quality engineering. Hence, sensitivity index based on E-optimality criteria is used in this paper.

$$S = \frac{\hat{\mathbf{Y}}^T\hat{\mathbf{Y}}}{\mathbf{U}^{\mathrm{T}}\mathbf{U}} = \frac{\mathbf{U}^{\mathrm{T}}\mathbf{M}^{\mathrm{T}}\mathbf{M}\mathbf{U}}{\mathbf{U}^{\mathrm{T}}\mathbf{U}} \leq \lambda_{max}(\mathbf{M}^{\mathrm{T}}\mathbf{M}) \tag{14}$$

Where, $\lambda_{\max}\left(\mathbf{M}^{\mathrm{T}}\mathbf{M}\right)$ is the maximum Eigen value of $\mathbf{M}^{\mathrm{T}}\mathbf{M}$ which is equal to maximum value of sensitivity index (S_{max}).

$$S_{max} = \frac{\hat{\mathbf{Y}}^T\hat{\mathbf{Y}}}{\mathbf{U}^{\mathrm{T}}\mathbf{U}} = \frac{\mathbf{U}^{\mathrm{T}}\mathbf{M}^{\mathrm{T}}\mathbf{M}\mathbf{U}}{\mathbf{U}^{\mathrm{T}}\mathbf{U}} = ||\mathbf{M}||^2 = \lambda_{max}(\mathbf{M}^{\mathrm{T}}\mathbf{M}) \tag{15}$$

Thus, locators position becomes the design parameter and is shown as $\boldsymbol{\psi}(\ell)=\left[X_1 \;\; Z_1 \cdots\cdots X_\ell \;\; Z_\ell\right]^T$. Where, X_i and Z_i are the coordinates of i^{th} locator. Optimal design of fixture layout attempts to find $\boldsymbol{\psi}(\ell)$ that minimizes the sensitivity S_{max} while satisfying the geometric and other constraints. Hence, the optimization problem can be represented as follows.

$$\min_{\boldsymbol{\psi}(\ell)} S_{\max}\left(\boldsymbol{\psi}(\ell)\right) \equiv \lambda_{\max}(\mathbf{M}^{\mathrm{T}}\mathbf{M}) \tag{16}$$

$$\text{Subject to:} \qquad C\left(\boldsymbol{\psi}(\ell)\right) \geq 0 \tag{17}$$

Eq. (16) represents the objective function and C (.) is the geometrical constraint on the PLP locations.

Identification of an optimal design of fixture layout is a computationally complex problem due to (i) large number of alternatives, and (ii) non-linear objective function $\lambda_{\max}\left(\mathbf{M}^{\mathrm{T}}\mathbf{M}\right)$. Several methods have been proposed to solve such problems including Sequential Quadratic Programming, and Simplex Search. Problem associated with these methods is their inability to escape the local optima. To resolve this issue, random search methods have been proposed. They alleviated the problem of local entrapment; however, the convergence rate becomes slower [24, 25]. Proposed HOT inspired heuristic is discussed in following subsections.

5. Proposed optimization methodology: HOT inspired heuristic

5.1 Motivation

In complex systems, it is often observed that size of triggered events is independent to the size of initiating events [5]. It is an analogous phenomenon to MSA in which a small flaw can ultimately results in poor quality product. The goal is to make the assembly system more robust against the variation caused by faulty fixture at each step during MSA. The primary objective of HOT is to make the system robust at each step of optimization against the perturbations caused by random failure events. The similarity between the two motivated the authors to investigate and apply the salient features of HOT in MSA. This study uses power law distribution to form the basic mechanism for HOT heuristic.

5.2 Background information

Carlson and Doyle [6, 26,-27] proposed a mechanism to study the behavior of complex systems and consequences of system design on them. This mechanism referred as *Highly Optimized Tolerance* (HOT) is inspired by the biological organism and advanced engineering technology. It has been successfully applied to the different systems dealing with forest fire management, percolation model, sand pile model, biological cell survival systems and internet file transmission traffic [28]. These systems can be divided into two groups on the basis of number of particles occupied by the each site, *i.e.*, single particle (forest fire or percolation model) or multiple particles (sand pile model). Cascading failure event may occur in the system due to local external disturbances. The failure reduces the number of particles in a connected cluster area in the system. The affected region due to failure events and occurrence probability of external perturbations are governed by specific relations. These relations when represented in mathematical form are known as *power laws*. Power laws are common characteristics of various complex interconnected systems. Actually, it is assumed that the power laws are ubiquities in natural as well as in artificial systems. Power laws are assumed due to criticality in physics whereas in engineering, power law is generated due to parameters tuning and models optimization (forest fire or percolation model). HOT is fundamentally based on the control theory and power law.

HOT framework initially was applied on a forest fire management to define basic concepts [6]. Consider that a spark is dropped in a random system that has density equivalent to the designed system. Here, two cases arise: if spark hits vacant site nothing burns, however, in the other case, trees within connected cluster are burned. It is observed that the probability of occurrence of large events is less as compared to that of small events [6, 29]. HOT is described as the optimization of barriers patterns around the most sensitive areas so that region burned due to random event (fire) can be minimized and consequently, yield (objective value) of woods is maximized. In this scenario, yield is defined as the average density of remaining tress after failure event occurs. It is found in the various circumstances that yield obtained at HOT states are higher than that of obtained at other states. The complexity is also studied in terms of second example, percolation forest fire models which also act as preliminary foundation for HOT. Percolation models are defined as: In a two dimensional W×W model, sites are occupied with probability *P* and are empty with probability (*1-P*). It is an analogous to the forest fire model in which occupied sites corresponds to trees. Nearby occupied sites make a set to define the clusters in the percolation models. Influence of external perturbations like a spark as defined in forest fire model, the connected clusters are burned.

Structured internal configuration and robust external behavior are the major attributes of the designed systems at HOT state. These substantial alterations in the system attributes can be the outcomes of trivial design optimization in the sophisticated systems. The design optimization is carried out by leading initial random design towards the robust structures. To analyze the fundamentals between the random and designed systems, an alternative mechanism (HOT) has been proposed which results in following characteristics in the designed systems.

a High efficiency, performance, and robustness of a system that is designed for uncertainties.

b Robustness to the design flaws and unanticipated uncertainties;

c Power laws

In essence, HOT state reveals about the high efficiency, performance, and robustness of a system despite being in an uncertain environment. Optimization of an objective function against some specified constraints results in having abovementioned attributes in the designed solution of the system. To optimize the design criteria (yield) in percolation model, Carlson and Doyle [6] described a local incremental algorithm. This leads towards highly structured and efficient operating state of high yield value (objective value).

5.3 Local Incremental Algorithm (LIA)

Global optimization acquire HOT state by searching for improved local alteration in configuration after each step. In an engineering system, HOT state is clearly distinct as specific design that is free from any happenings. A system can be simplified to attain a specific state if design parameters such as density are optimized using an evolutionary algorithm. The evolution involves a large number of continuous configurations of the system corresponding to a particular yield at any stage. At this state, system follows *power law* distributions. Probability distribution and a constraint on the optimization are two basic ingredients of HOT state.

5.4 Minimum selection probability (T_p)

T_p is assigned to select any point from the candidate locations. It plays a key role in reducing the number of candidate locations that need to be analyzed, making proposed approach computationally economical. As T_p is increased, incurred computational time is reduced; however, resulting value of sensitivity index gets poorer. For example, if T_p is changed from 0.5 to 0.6 computations time is reduced by 10 percent but final solution quality deteriorates.

Moreover, to reduce the number of candidate locations after each iteration, $P_{R(x, z)}$ of locations is also updated. This alteration of selection probability is because the candidate locations have less tendency to get involved in next iteration to take system towards the HOT state.

5.5 Stopping criteria

The selection of stopping criterion is also an important factor for an efficient heuristic that should be applied judiciously. Following two stopping criteria are used in the proposed heuristic.

1. Once the number of iterations exceeds pre-assigned maximum value.
2. Another criterion is based on the value of sensitivity index. When value of sensitivity index does not improve after appropriate function evaluations, the heuristic is stopped.

5.6 Implementation procedure

As mentioned earlier, efficiency of BEA is poor due to large number of candidate locations. Hence, elimination of less probable candidate locations is essentially required. On each panel, there are certain regions such as geometrical central area where possibility to place a locator is very low [23, 30]. Therefore, the points in geometrical central area removed from the analysis. Additionally, if two locators are adequately apart from each other, part deviation is less affected as compared to when two locators are close. Therefore, points having a selection probability less than 0.5 are also eliminated directly. Further, candidate locations are also removed from a region up to 35 mm from all edges of each panel. Since placement of locators in this region does not provide sufficient strength to bear the vibrations during MSA. Elimination of these points reduces 41.73% candidate locations. This reduction increases effectiveness of proposed heuristic.

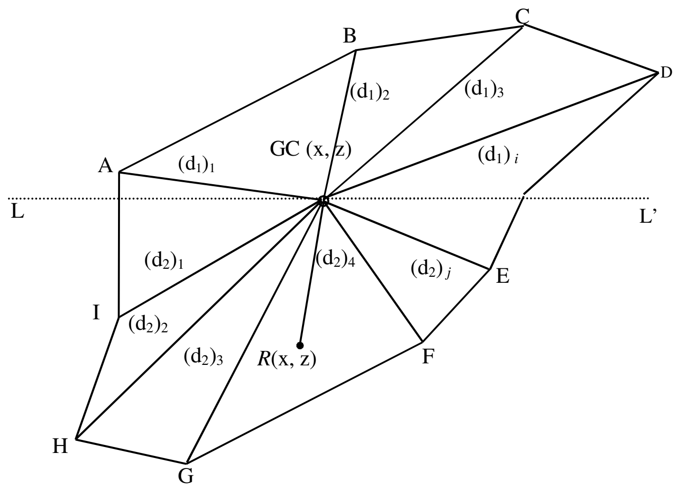


Fig. 5. Before placing the $P_{4\text{-ways}}$

In proposed heuristic, *power law* is generated based on the assumption of largest distance between two locators on each same panel. Let $P_R(x, z)$ is the probability of placing a locator at any point R(x, z) which is at a Euclidean distance d(x, z) from the Gravitational center (GC). Density of a panel is considered to be uniform, hence, GC and center of gravity coincide on each panel. In this context, power law conveys that selection probability of a coordinate is given by:

$$P_R(x,z) = \left(\frac{d(x,z)}{\ell}\right)^{\delta} \tag{19}$$

Illustrative example considered in this paper consists of four panels and each of which requires two locators on each panel/sub-assembly. Initially six locators in the order of $P_{4\text{-ways}}$, $P_{2\text{-ways}}$ on first three panels and then $P_{4\text{-ways}}$, $P_{2\text{-ways}}$ on fourth panel are placed. $P_{4\text{-ways}}$ on each panel is placed randomly at point R(x, z). $P_{R(x, z)}$ is assigned to the candidate locations according to their Euclidean distance (d{-,-}) from GC by using Eq. (20) and is demonstrated in Fig. 5.

$$P_R(x,z) = \begin{cases} \dfrac{d\{R(x,z)\}}{d_1} & \text{if } R(x,z) < LL' \\[2ex] \dfrac{d\{R(x,z)\}}{d_2} & \text{if } R(x,z) \geq LL' \end{cases} \tag{20}$$

Where, $p_1(x,z)$ is the location of $P_{4\text{-ways}}$ lying below GC. d{R(x, z)} is the distance of point R(x, z) from the GC. $(d_1)_i$ and $(d_2)_j$ are the distances from GC to upper and lower portion corners.

$$d_1 = max\{(d_1)_1, (d_1)_2, (d_1)_3 \cdots (d_1)_i\},\ d_2 = max\{(d_2)_1, (d_2)_2, (d_2)_3 \cdots (d_2)_j\}$$

Placement of $P_{4\text{-ways}}$ on each panel gives rise to a point having zero probability to place $P_{2\text{-ways}}$. Therefore, selection probability update to place $P_{2\text{-ways}}$ is essential. Placement of $P_{2\text{-ways}}$ on each panel is governed by Eq. (21). The whole scenario is shown in Fig. 6 where two lines LL' and L_1L_1' cross GC and $p_{1(x, z)}$.

$$P_R(x,z) = \begin{cases} \dfrac{d\{p_1(x,z), R(x,z)\}}{d_2} & \text{if } R(x,z) \geq L_1L_1' \\[2ex] \dfrac{\left| d\{p_1(x,z), R(x,z)\} - d\{GC(x,z), R(x,z)\} \right|}{d_2} & \text{if } LL' < R(x,z) < L_1L_1' \\[2ex] \dfrac{d\{GC(x,z), R(x,z)\}}{d_1} & \text{if } R(x,z) \geq LL' \end{cases} \tag{21}$$

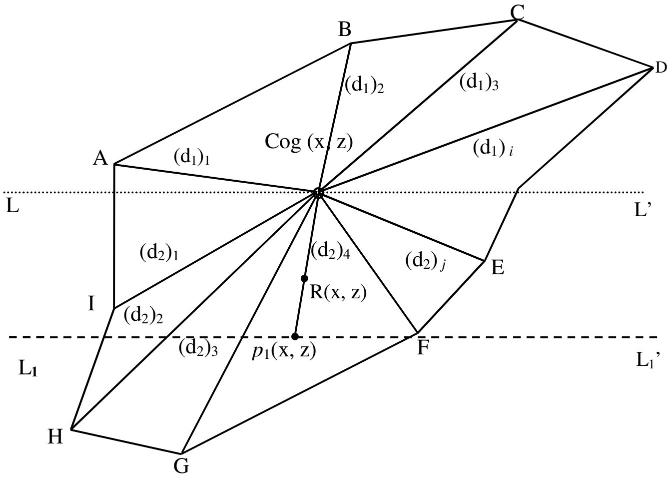


Fig. 6. After placing the $P_{4\text{-ways}}$

Now $P_{R(x,\ z)}$ is calculated which lies in between the region of these two lines. Graph of probability update before and after placing $P_{4\text{-ways}}$ on a single panel is shown in Fig. 7(a) and 7(b) respectively. In these figures, selection probability of a location and its distance from GC are shown on X and Y-axis, respectively. At the gravitational center the probability value to place the $P_{4\text{-ways}}$ locator is zero. The probability increases as the position moves away from the gravitational center (see Fig. 7(a)).

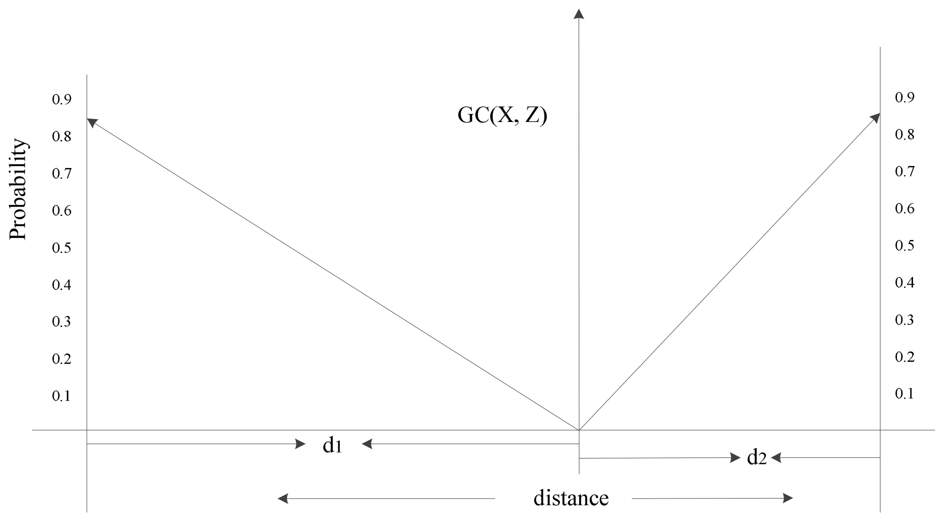


Figure 7(a). The representation of the probability distribution before placement

Once the $P_{4\text{-ways}}$ locator is placed, the probability needs be to updated to place a $P_{2\text{-ways}}$ locator. The distribution of updated probability is demonstrated in Fig. 7(b). The probability to place a $P_{2\text{-ways}}$ locator around GC increases but becomes zero after some distance. However, it starts to increase again.

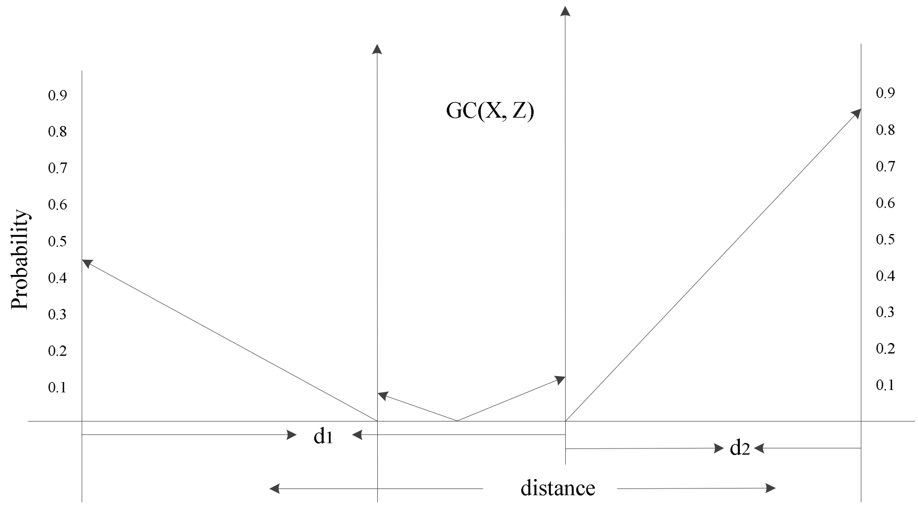


Figure 7(b). The representation of the probability distribution after placement

Random placement of all 7 locators is taken as the initial design and a position is selected for last locator ($P_{2\text{-ways}}$ on fourth panel) to calculate S_{max}. S_{max} for this random design is assumed as the smallest value. The position of last locator is altered and S_{max} is calculated again for the new design. If new S_{max} is less then initial value, then interchanged design of the fixture layout act as the initial design. In each step, the value achieved so far is stored which ensures that the best S_{max} found is returned at convergence. This is followed for all candidate locations on the fourth panel having $P_{R(x,\,z)}$ greater than T_p to explore the best location for $P_{2\text{-ways}}$. The process of placing the same locator on a panel is done to efficiently obtain the optimal S_{max}. The whole procedure is repeated individually for all other locators on different panels. The process continues until minimum S_{max} obtained from i^{th} iteration is not significantly better than that of obtained up to $(i\text{-}1)^{th}$ iteration. It leads the initial random design to a rare state that is known as the HOT state or optimal design of fixture layout. Pseudo code of HOT inspired heuristic is illustrated in Fig. 8.

```
Begin
 {Generate cand_loc
  Generate random design and calculate initial S_max
    Population Initialization
       {While (Termination Criteria! =True)
         --do--
              {Assign P_R(x, z) to each cand_loc
                 Select a location randomly for P_4-ways
```

Update $P_{R(x, z)}$

if $P_{R(x, z) > Tp}$

Select a location randomly for $P_{2\text{-ways}}$

End if

Calculate S_{max}

If (S_{max} < initial S_{max})

Interchange the initial design and S_{max} with current design and initial S_{max}

$P_{R(x, z)=} P_{R(x, z)} + \mu$

Set n=n+1

End do}

System has reached the HOT state.

End}

Fig. 8. Pseudo code of proposed HOT inspired heuristic to solve MSA problem.

This phenomenon is similar to LIA given in Carlson and Doyle [6]. LIA follows the law of natural selection in percolation system to attain HOT state. In percolation system, beginning takes place with an empty lattice [31]. All sites are occupied one by one by grains in such a way that after each step yield is maximum. Various configurations are obtained which are evolved in increasing order of the yield. The same analogous behavior is shown by heuristic that has been utilized here to determine optimal design of fixture layout. In our model, S_{max} is considered as a counterpart of yield. After application of proposed heuristic, the system attains HOT state that provides optimal value of S_{max} (yield).

6. Computational results and discussion

This section outlines the results obtained by applying the proposed HOT inspired heuristic on an example of SUV side frame from the literature. Comparative results with BEA are also summarized to prove the robustness of the heuristic. The heuristic has been coded in MATLAB 7.1 and experiments described throughout the paper have been performed on a Pentium IV-1.8 GHz processor. For computational experiments, the first task is to tune the heuristic parameters.

Parameters Tuning: T_p and μ are recognized as the key parameters that influence the final solution quality and efficiency of the proposed heuristic. To set optimal values for parameters, rigorous computational simulations are performed by varying parameter values. Obtained results are employed to evaluate the influence of parameters. Values assigned throughout the experiment in order to ensure the better performance of proposed heuristic are given in Table 2.

Table 2. The value of the pre-selected parameters

T_p	0.5
μ	0.05

Setting all parameters at their best values, all experiments are performed. The optimal position of locators obtained by HOT inspired heuristic is shown in Fig. 9 for underlined problem. In this figure, plus sign (+) shows position of the 4-ways locators; whereas to represent the 2-ways locators, a circle (●) is used. The position of locators for best solution is also provided in Table 3. The optimal value of S_{max} obtained is 11.30. The computation time taken to obtain optimal fixture design is 230.52 sec. On the other hand, BEA provides the best value of S_{max} as 11.28 in 1148 sec. Thus, proposed heuristic reduces computational time without shifting optimal value of S_{max} (see Table 4).

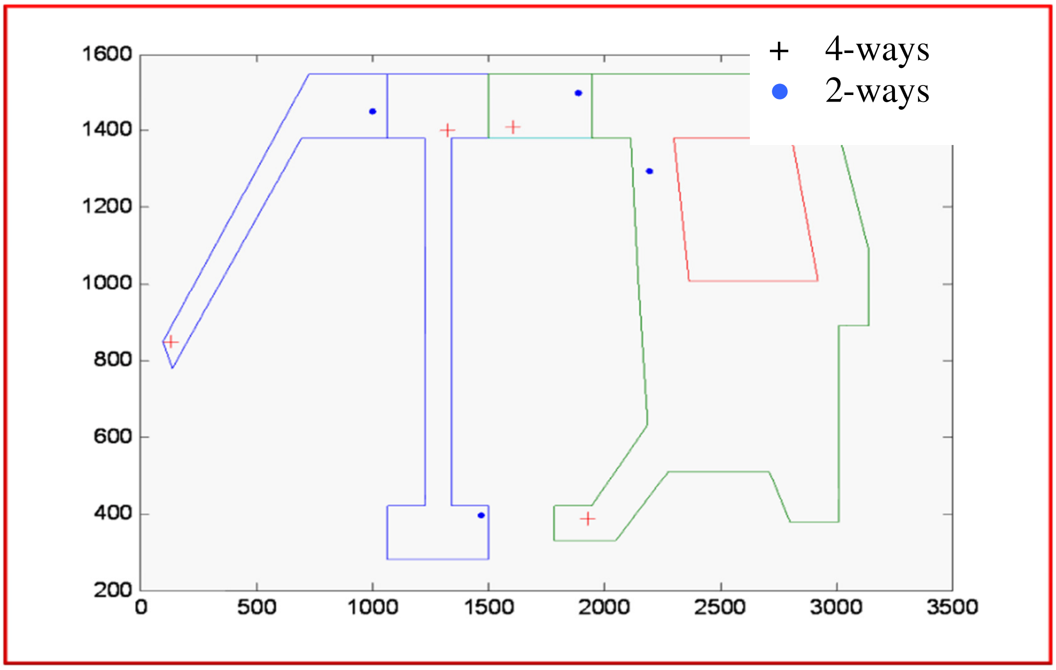


Fig. 9. Optimal position of the locators obtained by HOT inspired heuristic

The reduced burden of heuristic in terms of candidate locations per iteration helps the proposed heuristic to converge in a less CPU time. Elimination of frigid points from edges and geometrical central area of each panel plays a vital role in making proposed heuristic computationally economical. Another factor that facilitates proposed heuristic to reduce computational burden is regulated update of selection probability after placing $P_{4\text{-ways}}$ on each panel. This reduces the possibility of placing $P_{2\text{-ways}}$ in neighbourhood of first locator and helps second locator to be placed at an adequate distance from the first locator. Number of exchanges to be done in each iteration are decreased, which results in less computational time. The sensitivity value decreases as selection probability is increased, that in turn shows the optimum value of S_{max} obtained from the remaining candidate locations.

Table 3. Optimal design of fixture layout obtained from HOT inspired heuristic

PANEL #	PLP	X (IN MM)	Z(IN MM)
First	P1(4-ways)	135.2	850.7
	P2(2-ways)	1000.6	1450.8
Second	P3(4-ways)	1392.1	1486.6
	P4(2-ways)	1337.3	395.9
Third	P5(4-ways)	1614.4	1417.3
	P6(2-ways)	1899.2	1423.4
Fourth	P7(4-ways)	1936.2	405.7
	P8(2-ways)	2196.6	1299.8

6.1 Comparative analysis

In order to compare the results obtained from HOT heuristic and BEA a random initial design is used. The major advantage of using the random design is that it eliminates any predominance. Table 4 shows the comparative results of BEA and HOT heuristic. From this table , it is evident that the proposed heuristic performs better as compared to BEA in terms of computational time. In nutshell, aforementioned computational results not only prove the efficacy of the proposed heuristic but also provide a new dimension to the solution of complex problems.

Table 4. Comparison of basic exchange algorithm (BEA) and HOT inspired heuristic

Algorithm	S_{max}	Computational time (T)
Basic exchange algorithm	11.28	1148.0 sec
HOT inspired heuristic	11.30	230.52 sec

7. Conclusion

The current paper addressed the problem of determining the optimal design of fixture layout in a MSA. A state space model is utilized for modeling the variation propagation. Further, an E-optimality criterion is adopted to quantify the quality of fixture layout design. Optimal design of fixture layout is obtained after searching through a large number of candidate locations. For this purpose, an intelligent HOT inspired heuristic has been proposed. Power law is based on the fact that distance between two locators on each panel should be adequate to provide sufficient support to the workpiece. After every iteration, the selection probability of candidate points on each panel is reduced by a constant factor μ that enables proposed heuristic to get optimal S_{max} in a reasonable amount of CPU time. Solution quantity obtained from both HOT heuristic and BEA are approximately same, significant difference is found in the computational complexity. The enumerated results establish the superiority of proposed HOT inspired heuristic with over the BEA. Large computational time for BEA can be attributed to the numerous candidate locations among which optimal position of locators is to be determined. In essence, it can be concluded that proposed heuristic is promising for solving the complex optimization problems.

References

[1] Xie, W., Deng, Z., Ding, B., and Kuang, H.: Fixture layout optimization in multi-station assembly processes using augmented ant colony algorithm. Journal of Manufacturing Systems, (2014).

[2] Qin, G., Ye, H., & Rong, Y.: A unified point-by-point planning algorithm of machining fixture layout for complex workpiece. International Journal of Production Research, 52(5), 1351-1362 (2014).

[3] Jiang, K., Zhou, X., Li, M., and Kong, X.: A multi-objective optimization and decision algorithm for locator layout continuous searching in checking fixture design. The International Journal of Advanced Manufacturing Technology, **67(1-4)**, 357-366 (2013)

[4] Kim, P., Ding, Y.: Optimal engineering system design guided by data-mining methods. American Statistical Ass. and the American Society for Quality Techno metrics, **47**(3), 336-348 (2005)

[5] Carlson, J.M., Doyle, J.: Highly optimized tolerance: a mechanism for power laws in designed systems. Physical Review Letters **60 (2)**, 1412-1427 (1999)

[6] Asada, H., By, A.B.: Kinematics analysis of work part fixturing for flexible assembly with automatically reconfigurable fixture. IEEE J. Robot. Automat. **1**, 86–94 (1999)

[7] Ferreira, P.M., Kochar, B., Liu, C.R., Chandru, V.: AIFIX: An expert system approach to fixture design, in Proc. ASME Winter Ann. Meeting Computer-Aided/Intelligent Process Planning, Miami Beach FL, 1985, 73–82.

[8] Menassa, R.J., De Vries, W.R.: Optimization methods applied to selecting support positions in fixture design. ASME J. Eng. Ind. **113**, 412–418 (1991)

[9] Cai, W.J., Hu, S.J.: Optimal fixture configuration design for sheet metal assembly with spring back. Trans. NAMRI/SME. **24**, 229–234 (1996)

[10] Hockenberger, M.J., De Meter, E.C.: The application of meta-functions to the quasi-static analysis of workpiece displacement within a machining fixture. ASME J. Eng. Ind. **118**, 325–331 (1996)

[11] Rong, Y., Bai, Y.: Machining accuracy analysis for computer-aided fixture design verification. ASME J. Manufact. Sci. Eng. **118**, 289–300 (1996)

[12] Ceglarek, D., Prakash, P.K.S.: Enhanced piecewise least squares approach for diagnosis of ill-conditioned multistation assembly with compliant parts. Proceedings of the IMecE, Part B: Journal of Engineering Manufacture **226(3)**, 485-502 (2012)

[13] Apley, D.W., Shi, J.: Diagnosis of multiple fixture faults in panel assembly. ASME J. Manuf. Sci. Eng. **120**, 793 – 801 (1998)

[14] Rong, Q., Ceglarek, D., Shi, J.: Dimensional fault diagnosis for compliant beam structure assemblies. ASME J. Manuf. Sci. Eng. **122**, 773–780 (2000)

[15] Shukla, N., Ceglarek, D., Tiwari, M.K.: Key characteristics-based sensor distribution in multi-station assembly processes. Journal of Intelligent Manufacturing, 1-16 (2013)

[16] Mantripragada, R., Whitney, D.E.: Modeling and controlling variation propagation in mechanical assemblies using state transition models. IEEE Trans. on Robot. and Auto. **15**(1), 124-140 (1999)

[17] Lawless J.F., Machay R.J., Robinson J.A.: Analysis of variation transmission in manufacturing processes – part I. J. of Qua. Techno. **31**(2), 131-142 (1999)

[18] Jin, J. Shi, J.: State space modeling of sheet metal assembly for dimensional control. *ASME* J. Manu. Sci. Eng. **121**, 756-762 (1999)

[19] Abellan-Nebot, J.V., Liu, J., Romero S.F.: Quality prediction and compensation in multi-station machining processes using sensor-based fixtures. Robotics and Computer-Integrated Manufacturing **28(2)**, 208-219 (2012)

[20] Abellán, J.V., Liu, J.: Variation propagation modelling for multi-station machining processes with fixtures based on locating surfaces. Int. J. of Pro. Res. **51(15)**, 4667-4681 (2013)

[21] Ding, Y., Ceglarek, D., Shi, J.: Fault diagnosis of multi-station manufacturing processes by using state–space approach. ASME J. Manu. Sci. Eng., **124**, 313–322 (2002)

[22] Kim, P., Ding, Y.: Optimal design of fixture layout in multi-station assembly processes. IEEE Trans. on auto. Sci. and engine. **1**(1), 133-145 (2004)

[23] Ding, Y., Gupta, A. Apley, D.: Singularity issues of fixture fault diagnosis in multi-station assembly processes. ASME J. Manu. Sci. Eng. **126**, 200–210 (2004)

[24] Tyagi, S.K., Yang, K., Tyagi, A., Verma A.: A fuzzy goal programming approach for optimal product family design of mobile phones and multiple-platform architecture. IEEE Trans. on SMC, Part C: Applications and Reviews, **42(6),** 1519-1530 (2012).

[25] Tyagi, S.K., Yang, K., Verma, A.: Non-discrete ant colony optimisation (NdACO) to optimise the development cycle time and cost in overlapped product development, International Journal of Production Research **51(2)**, 346-361 (2013)

[26] Carlson, J.M., Doyle, J.: Highly optimized tolerance: robustness and design in complex system. *Physical Review Letters* **84**(11), 2529-2532 (2000).

[27] Carlson, J.S.: Quadratic sensitivity analysis of fixtures and locating schemes for rigid parts. ASME J. Manu. Sci. Eng. **123**, 462–472 (2001)

[28] Moritz, M.A., Morais, M.E., Summerell, L.A., Carlson, J. M., Doyle, J.: Wildfires, complexity, and highly optimized tolerance. PNAS **102(50),** 17912–17917 (2005)

[29] Zhou, T., Carlson, J.M., Doyle, J.: Mutation, specialization, and hypersensitivity in highly optimized tolerance. *PNAS* **99** (4), 2538-2545 (2002)

[30] Wang, M.Y.: An optimum design for 3-D fixture synthesis in a point set domain. IEEE Trans. Robot. Automat. **16**, 839–846 (2000)

[31] Ding, Y., Kim, P., Ceglarek, D. Jin, J.: Optimal sensor distribution for variation diagnosis in multi-station assembly processes. IEEE Transactions on Robotics and Automation, 19, 543–556 (2003).

[32] Ding, Y., Ceglarek, D., Shi, J.: Design evaluation of multi-station assembly processes by using state space approach. Journal of Mechanical Design, 124(3), 408-418 (2002).