Orthogonal polynomial-based nonlinearity modeling and mitigation for LED communications

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Abstract
The light-emitting diode (LED) is the major source of nonlinearity in LED communications, and the nonlinearity needs to be effectively modeled and thereby mitigated through predistortion or postdistortion to avoid degradation of communication performance. A memory polynomial is often used for LED nonlinearity modeling and mitigation in the literature. However, the estimation of memory polynomial coefficients suffers from numerical instability, resulting in inaccurate modeling and poor performance in nonlinearity mitigation. In this paper, we propose an orthogonal polynomial-based nonlinearity modeling and mitigation technique for LED communications with pulse amplitude modulation (PAM) signaling and show that the proposed technique significantly outperforms the conventional memory polynomial-based techniques.

Keywords
nonlinearity, polynomial, led, orthogonal, mitigation, communications, modeling

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Orthogonal Polynomial-Based Nonlinearity Modeling and Mitigation for LED Communications

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Abstract: The light-emitting diode (LED) is the major source of nonlinearity in LED communications, and the nonlinearity needs to be effectively modeled and thereby mitigated through predistortion or postdistortion to avoid degradation of communication performance. A memory polynomial is often used for LED nonlinearity modeling and mitigation in the literature. However, the estimation of memory polynomial coefficients suffers from numerical instability, resulting in inaccurate modeling and poor performance in nonlinearity mitigation. In this paper, we propose an orthogonal polynomial-based nonlinearity modeling and mitigation technique for LED communications with pulse amplitude modulation (PAM) signaling and show that the proposed technique significantly outperforms the conventional memory polynomial-based techniques.

Index Terms: Light-emitting diode (LED) communications, nonlinearity, memory polynomial, orthogonal polynomial.

1. Introduction

In recent years, light-emitting diode (LED) technology, known as Green Illumination, is growing rapidly. Compared to traditional incandescent and fluorescent lights, LEDs have long life expectancy, high tolerance to humidity, low power consumption, small size, and minimal heat generation [1]. Another important feature of LEDs is that they are semiconductor devices capable of fast switching with the addition of appropriate drivers [2]. Visible light emitted by LEDs can be modulated for simultaneous illumination and data transmission. As LED lights are ubiquitous, LED-based optical wireless communications can provide communication service at low cost [3]. Also, the spectrum region of visible light is unregulated and interferences with radio bands can be avoided [4]. As white light cannot penetrate through walls, it is easy to achieve secure transmission within a certain room and prevent interferences from other places. Therefore, LED communication technology is a promising field which offers a novel scheme of high-speed data transmission for indoor communications.
In LED communications, signals in electric domain are modulated to optical domain using the light-intensity of LEDs. At the receiver side, the intensity of the light is detected by photo detector (PD) and converted to electric signals [5]. However, the inherent nonlinearity of LEDs causes nonlinear distortions and LED is the major source of nonlinearity of LED communication systems [6], [7]. The nonlinear behavior of the LED transfer function distorts the amplitude of the signal, and lower peak signals are forced to be the minimum LED turn-on voltage, while higher peak signals are clipped to avoid reaching the maximum permissible voltage [8]–[10]. Moreover, due to transport delay, rapid thermal effects, as well as biasing circuits, such nonlinear behavior has memory effects and may vary with time [11].

Nonlinear system modeling and mitigation methods are essential for solving the distortions. In [12], LED nonlinearity is modeled using memoryless polynomials and then a predistorter which has the inverse characteristic of the polynomial model is designed to compensate for the nonlinearity. Because LEDs often exhibit nonlinearity with memory effects, especially for high speed transmission, memory-less polynomial are inadequate to model the LED nonlinearity. In [13], memory polynomial is used and a predistorter is constructed based on the least square method. However, when finding the model coefficients matrix inversion is needed, which is prone to numerical instability problem [14]–[16]. This results in inaccurate system modeling and thereby severely degrades the performance of nonlinearity mitigation. Compared with single carrier modulation, the nonlinearity is a more serious issue for optical-orthogonal frequency-division multiplexing (OFDM) [5] due to the inherent high peak-to-average-power (PAPR) ratio of OFDM signals. Moreover, the nonlinearity also causes subcarrier interference [17]. It is shown in [18] that single carrier system (e.g., with L-ary pulse amplitude modulation (L-PAM)) delivers better performance than OFDM. It is worth highlighting that, single carrier system with frequency domain equalization has the same complexity with OFDM system in detection. Hence, in this work, we focus on single carrier system with PAM rather than the OFDM system.

In this paper, an alternative LED nonlinearity modeling technique based on orthogonal polynomials is investigated. As pulse amplitude modulation is often used for single-carrier LED communications, we design orthogonal polynomials for L-PAM LED signals. Although orthogonal polynomial based techniques have been investigated for power amplifier modeling and predistorter design, it is the first time to apply orthogonal polynomial based technique to LED nonlinearity modeling where the input signals are real and non-negative valued. We developed a set of orthogonal polynomial basis for PAM LED signals for effective system modeling. Thanks to the use of orthogonal polynomials, the columns of the basis matrix for our new model are quasi-orthogonal, so the problem of numerical instability in finding the model coefficients is avoided, leading to accurate LED nonlinearity modeling. Furthermore, a predistorter is employed to mitigate the LED nonlinearity. Simulations show that, compared to the conventional memory polynomial based technique, our proposed orthogonal polynomial based techniques can significantly reduce the nonlinear distortion and remarkable system performance improvement can be achieved.

The organization of the paper is as follows. In Section 2, the conventional memory polynomial technique is introduced. In Section 3, the orthogonal memory polynomial for LED nonlinearity modeling is proposed, and then, a polynomial linearization technique is presented in Section 4. Simulation results are provided in Section 5 to verify the effectiveness of our proposed method. The paper is concluded in Section 6.

2. Conventional Memory Polynomial Based LED Nonlinearity Modeling

Memory polynomial proposed by Kim [19] and Ding [20] is one of the most popular methods for nonlinear system modeling and predistortion [21]. The memory polynomial model can be expressed as

\[ z(n) = \sum_{k=1}^{K} \sum_{m=0}^{M} a_{k,m} x(n - m) |x(n - m)|^{k-1} \]  

(1)
where \( x(n) \) is the input signal, \( K \) is the order of the polynomials, \( M \) is the memory length, and \( \{a_{km}\} \) are 2-D coefficients that represent both the nonlinear and memory effects. Noting that \( x(n) \geq 0 \) in LED communications, we have

\[
z(n) = \sum_{k=1}^{K} \sum_{m=0}^{M} a_{k,m} x(n-m)^k \tag{2}
\]

which can be rewritten as the following matrix form:

\[
z = \Phi a
\]

where

\[
z = [z(M), z(M+1), \ldots, z(N)]^T, a = [a_{1,0}, \ldots, a_{K,0}, \ldots, a_{1,M}, \ldots, a_{K,M}]^T, \quad \Phi = [\Phi_0, \ldots, \Phi_K, \ldots, \Phi_{M,0}, \ldots, \Phi_{M,K}]\]

with \( \Phi_k = [x(M-m)^k, x(M+1-m)^k, \ldots, x(N-m)^k]^T \).

Fig. 1 illustrates LED nonlinearity modeling. The electrical signal \( x(n) \) is input to LED for light intensity modulation. According to the memory polynomial model in (1), signal \( y(n) \) is linear with the polynomial coefficients to be determined. Hence, with a training sequence \( \{x(n)\} \), the coefficient \( \{a_{k,m}\} \) can be estimated by using the least-squares (LS) method with the following model:

\[
y = \Phi a + n \tag{4}
\]

where \( y = [y(M), y(M+1), \ldots, y(N)]^T \), and \( n \) represents nonlinearity modeling error and measurement noise. The LS estimate for the model coefficients is given by [14]

\[
a = (\Phi^H \Phi)^{-1} \Phi^H y \tag{5}
\]

The coefficients may also be calculated and updated by using the recursive least squares (RLS) as in [13]. It has been shown that \( \Phi^H \Phi \) is often ill-conditioned, which causes numerical instability and noise enhancement with LS estimation in (4) [14]–[16]. The problem deteriorates with the increase of \( K \) and \( M \), and even for small values of \( K \) and \( M \), the error induced by the matrix inversion can still be significant, leading to severe performance degradation in LED nonlinearity modeling [14], which is also demonstrated in Section 5 of this paper.

3. Orthogonal Polynomial Based LED Nonlinearity Modeling

To circumvent the problem of numerical instability in the above conventional memory polynomial techniques, we propose to use orthogonal polynomials for LED nonlinearity modeling. Instead of using (4), we use

\[
y = \Psi b + n \tag{6}
\]
In model (6), the new basis matrix $\Psi$ spans the same space, as $\Phi$ in (3), and it is given by $\Psi = [\Psi_0^T, \ldots, \Psi_0^{K}, \ldots, \Psi_1^T, \ldots, \Psi_1^{K}, \ldots, \Psi_M^T, \ldots, \Psi_M^{K}]^T$ where $\Psi_m^k = [\Psi_m^k(x(M-m)), \Psi_m^k(x(M+1-m)), \ldots, \Psi_m^k(x(N-m))]^T$ and the polynomial $\psi^k(x)$ is defined as $\psi^k(x) = d_{k,k}x^k + d_{k,k-1}x^{k-1} + \ldots + d_{k,1}x$ (here we abuse the use of superscript “$k$” of $\psi^k(x)$, where $k$ represents the order of the polynomial, rather than its power). The LS solution to (6) is given by

$$b = (\Psi^H\Psi)^{-1}\Psi^Hy.$$  \hspace{1cm} (7)

The above solution still involves a matrix inversion operation. To avoid the numerical instability problem, we require that any two columns of matrix $\Psi$ are quasi-orthogonal. This may be achieved by properly choosing the values of the polynomial coefficients $\{d_{k,l}\}$ so that

$$E[\psi^k(x)\psi^l(x)] = 0, \quad k \neq l$$  \hspace{1cm} (8)

i.e., $\{\psi^k(x)\}$ are orthogonal polynomials [14]. The values of the polynomial coefficients $\{d_{k,l}\}$ depend on the distribution of $x$. There are a few special orthogonal polynomials for some distributions, e.g., Hermite Polynomials for real valued Gaussian process with zero mean and unit variance [16]. However, there are no existing orthogonal polynomials for non-negative and real-valued signal $x$ in LED communications. In the next, we investigate the orthogonal polynomials for L-PAM LED signals.

We take 4-PAM modulation with memory polynomial of order 3 and memory length 1 as example. For a length-N training sequence, the matrix $\Psi$ will be

$$\Psi = \begin{bmatrix}
\psi^1(x(1)) & \psi^2(x(1)) & \psi^3(x(1)) & \psi^1(x(0)) & \psi^2(x(0)) & \psi^3(x(0)) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\psi^1(x(N)) & \psi^2(x(N)) & \psi^3(x(N)) & \psi^1(x(N-1)) & \psi^2(x(N-1)) & \psi^3(x(N-1)) \\
\psi_0 & \psi_0 & \psi_0 & \psi_0 & \psi_0 & \psi_0
\end{bmatrix}. \hspace{1cm} (9)$$

We need to find the coefficients of the following polynomials:

$$\psi^1(x) = d_{1,1}x$$  \hspace{1cm} (10)

$$\psi^2(x) = d_{2,2}x^2 + d_{2,1}x$$  \hspace{1cm} (11)

$$\psi^3(x) = d_{3,3}x^3 + d_{3,2}x^2 + d_{3,1}x$$  \hspace{1cm} (12)

where we have six unknown coefficients in total. Define $\alpha = \{\alpha_i, i = 1, 2, 3, 4\}$ as the alphabet of the 4-PAM modulation. It is reasonable to assume that the probabilities of $x(n) = \alpha_i$ for $i = 1, 2, 3,$ and 4 are the same. Hence, to ensure that the first three columns of $\Psi$ [i.e., $\Psi_0$ in (9)] are quasi-orthogonal (the memory effects are not considered), we have the following six constraints:

$$\sum_{i=1}^{4} \psi^k(\alpha_i)\psi^l(\alpha_i) = \begin{cases}
0, & \text{if } k \neq l \\
1, & \text{if } k = l
\end{cases} \hspace{1cm} (13)$$

where $k = 1, 2,$ and 3, and $l = 1, 2,$ and 3. The above six constraints lead to six second-order equations with six unknowns, which can be solved by using the `fsolve` function in MATLAB. It is noted that, as the PAM alphabet is known, the determination of the unknown orthogonal polynomial coefficients can be carried out offline and they only need to be calculated once.

Considering memory affects, we need to take into account the last three columns of $\Psi$, i.e., $\Psi_1$ in (9). The use of the orthogonal polynomials guarantees that any two different column vectors in $\Psi_0$ or any two different column vectors in $\Psi_1$ are quasi-orthogonal. In addition, we still needs the quasi-orthogonality between any two column vectors: one from $\Psi_0$ and the other one from $\Psi_1$. However, our finding is that the first column in $\Psi_0$ (denoted by $\Psi_0^0$) and
the first column in $\Psi_1$ (denoted by $\Psi_1^1$) can be highly correlated. This issue is unique to LED input signal which must be non-negative valued, so that all the elements in the first order vectors $\Psi_0^1$ and $\Psi_1^1$ have the same signs [see (10)]. The high correlation between $\Psi_0^1$ and $\Psi_1^1$ needs to be addressed; otherwise it will still cause ill condition of matrix $\Psi^H \Psi$, thereby resulting in numerical instability and noise enhancement in the LS estimation in (7). To avoid the issue, we remove the first order vector $\Psi_1^1$ from $\Psi$, leading to a new basis matrix $\Psi^0$. It is interesting that model (3) can still be represented by using the new basis matrix $\Psi^0$ as revealed by Proposition 1 and its extension. For simplicity, we only consider 4-PAM signaling and nonlinear model with polynomial order 4 and memory length 1 in the above. The discussions can be extended to any PAM signaling and nonlinear models with different polynomial orders and memory lengths.

$\textbf{Proposition 1}$

For non-negative L-PAM with $L = K$, model (3) can be represented by using the orthogonal polynomial based basis matrix $\Psi^0$ which is constructed from $\Psi$ by removing its first order columns for memory parts, i.e., there always exists $\mathbf{b}'$ so that $\Psi^0 \mathbf{b}' = \Phi \mathbf{a}$.

$\textbf{Proof}$

Without loss of generality, consider 4-PAM signaling, and assume that the nonlinear model has polynomial order 4 and memory length 2. According to (2) [which is equivalent to (3)]

$$y_c(n) = \sum_{k=0}^{4} a_{k,0} x(n)^k + \sum_{k=1}^{4} a_{k,1} x(n-1)^k + \sum_{k=1}^{4} a_{k,2} x(n-2)^k. \quad (14)$$

The orthogonal polynomial model with basis matrix $\Psi'$ is given by

$$y_0(n) = \sum_{k=1}^{4} b_{k,0} \Psi^k(x(n)) + \sum_{k=2}^{4} b_{k,1} \Psi^k(x(n-1)) + \sum_{k=2}^{4} b_{k,2} \Psi^k(x(n-2)) \quad (15)$$

where $\{\Psi^k(x(n))\}$ are orthogonal polynomials, and the polynomial coefficients are determined as shown in Section 3. Next, we prove that, there exist $\{b_{k,i}\}$ which make the two models (14) and (15) equivalent. The elements of the 4-PAM alphabet are denoted by $\alpha_1$, $\alpha_2$, $\alpha_3$, and $\alpha_4$. Define

$$P_{c_0}(x) = \sum_{k=1}^{4} a_{k,0} x^k, \quad A_{0,i} = P_{c_0}(\alpha_i)$$

$$P_{c_1}(x) = \sum_{k=1}^{4} a_{k,1} x^k, \quad A_{1,i} = P_{c_1}(\alpha_i)$$

$$P_{c_2}(x) = \sum_{k=1}^{4} a_{k,2} x^k, \quad A_{2,i} = P_{c_2}(\alpha_i)$$

$$P_{c_0}(x) = \sum_{k=1}^{4} b_{k,0} \Psi^k(x), \quad B_{0,i} = P_{c_0}(\alpha_i)$$

$$P_{c_1}(x) = \sum_{k=2}^{4} b_{k,1} \Psi^k(x), \quad B_{1,i} = P_{c_0}(\alpha_i)$$

$$P_{c_2}(x) = \sum_{k=2}^{4} b_{k,2} \Psi^k(x), \quad B_{2,i} = P_{c_0}(\alpha_i).$$
As we are considering 4-PAM, each term in (16) or (17) has four values, and therefore, there are 64 combinations. It appears that we need 64 equations to make (16) and (17) equivalent. In fact, these 64 equations can be reduced to the following 10 equations:

\[ A_{01} + A_{11} + A_{21} = B_{01} + B_{11} + B_{21} \]  
\[ A_{02} + A_{12} + A_{22} = B_{02} + B_{12} + B_{22} \]  
\[ A_{03} + A_{13} + A_{23} = B_{03} + B_{13} + B_{23} \]  
\[ A_{04} + A_{14} + A_{24} = B_{04} + B_{14} + B_{24} \]
\[ A_{11} - A_{12} = B_{11} - B_{12} \]
\[ A_{11} - A_{13} = B_{11} - B_{13} \]
\[ A_{11} - A_{14} = B_{11} - B_{14} \]
\[ A_{21} - A_{22} = B_{21} - B_{22} \]
\[ A_{21} - A_{23} = B_{21} - B_{23} \]
\[ A_{21} - A_{24} = B_{21} - B_{24}. \]

It can be verified that, (18) together with (22)–(27) guarantees that (16) and (17) are equal for 16 combinations i.e., \( r = 1, s \in \{1, 2, 3, 4\} \) and \( t \in \{1, 2, 3, 4\} \). Similarly, (19)–(21) with (22)–(27) guarantee the other 48 combinations. Hence, the above 10 equations guarantee (16) and (17) are equal for all 64 combinations. As the number of unknowns \( \{b_{k,l}\} \) is ten, their values can be determined. The above proof can also be extended to larger memory length and higher order polynomials.

**Extension to \( L > K \)**

In the above proof, we assume that the size of the PAM alphabet \( L \) is equal to the order of the polynomial \( K \). In the case of high order PAM, \( L \) may be larger than \( K \). In the following, we consider the case of \( L > K \).

From (22)–(24), we can see that when \( x = \alpha_1, \alpha_2, \alpha_3 \) and \( \alpha_4 \), \( P_{\alpha_1}(x) - P_{\alpha_4}(x) = C_1 \) where \( C_1 \) is a constant. Similarly, we can also show that when \( x = \alpha_1, \alpha_2, \alpha_3 \) and \( \alpha_4 \), \( P_{\alpha_2}(x) - P_{\alpha_4}(x) = C_0 \) and \( P_{\alpha_2}(x) - P_{\alpha_4}(x) = C_2 \). Because (16) is equivalent to (17), \( C_0 + C_1 + C_2 = 0 \). When \( L > K \), we can select \( K \) elements \( \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_K \) from the L-PAM alphabet. Similar to the proof in Proposition 1, we have \( P_{\alpha_1}(x) - P_{\alpha_4}(x) = C_0 \), \( P_{\alpha_2}(x) - P_{\alpha_4}(x) = C_1 \), \( P_{\alpha_2}(x) - P_{\alpha_4}(x) = C_2 \), and \( C_0 + C_1 + C_2 = 0 \) when \( x = \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_K \).

According to [22], we have

\[ P_{\alpha_2}(x) = \Delta_0 P_{\alpha_2}(0) + \Delta_1 P_{\alpha_2}(\alpha_1) + \cdots + \Delta_k P_{\alpha_2}(\alpha_K) = \sum_{i=0}^{K} \Delta_i P_{\alpha_2}(\alpha_i) \]  
\[ P_{\alpha_4}(x) = \Delta_0 P_{\alpha_4}(0) + \Delta_1 P_{\alpha_4}(\alpha_1) + \cdots + \Delta_k P_{\alpha_4}(\alpha_K) = \sum_{i=0}^{K} \Delta_i P_{\alpha_4}(\alpha_i) \]

where

\[ \Delta_j = \prod_{i=0, j \neq i}^{K} \frac{x - \alpha_i}{\alpha_i - \alpha_j} \]
with \( \alpha_0 = 0 \). Then, we define

\[
    h_0(x) = P_{\alpha_0}(x) - P_{\alpha_0}(x) = \sum_{i=0}^{K} \Delta_i(P_{\alpha_i}(a_i) - P_{\alpha_i}(a_i)) = \sum_{i=0}^{K} \Delta_i C_0.
\]

(30)

For LED communications, the input voltage \( x \) is approximately within the range of 2.4 v–3.8 v [23]. Next, we show that when \( x \in [2.4,3.8] \), \( h_0(x) \) is still approximately a constant \( C_0 \). We can select the following \( K \) elements from \([2.4,3.8]\) as \( \alpha_1 = 2.4, \alpha_2 = \alpha_1 + 1.4(1/(K-1)), \alpha_3 = \alpha_1 + 1.4(2/(K-1)), \ldots, \alpha_K = 3.8 \). When \( K = 4 \), we have

\[
    h_0(x) = (-0.0115x^4 + 0.1423x^3 - 0.6554x^2 + 1.3287x)C_0 = g_0(x)C_0.
\]

It is not hard to show that the maximum value and minimum value of \( g_0(x) \) with \( x \in [2.4,3.8] \) are 1.0005 and 0.9997, respectively. Hence, \( h_0(x) \) is between 0.9997\( C_0 \) and 1.0005\( C_0 \), i.e., \( h_0(x) = P_{\alpha_0}(x) - P_{\alpha_0}(x) \approx C_0 \) when \( x \in [2.4,3.8] \). When \( K = 5 \), we can find that \( h_0(x) \) is between 0.9999\( C_0 \) and 1.0001\( C_0 \). We can see that, with the increase of \( K \), \( h_0(x) \) approaches to \( C_0 \) more accurately. Similarly, we have \( P_{\alpha_0}(x) - P_{\alpha_0}(x) \approx C_1 \) and \( P_{\alpha_0}(x) - P_{\alpha_0}(x) \approx C_2 \) when \( x \in [2.4,3.8] \). Therefore, \( \Psi'b' \approx \Phi a \) for any \( x \) within the LED input voltage range. So, \( \Psi'b' \approx \Phi a \) for L-PAM with \( L > K \).

With the basis matrix \( \Psi' \), the numeric instability problem can be avoided, which is crucial to achieve accurate LED nonlinearity modeling, as demonstrated in Section 5. Based on the orthogonal polynomial basis \( \Psi' \), we can get

\[
    b' = (\Psi'^T\Psi')^{-1}\Psi'^Ty.
\]

(31)

Then, the nonlinear LED model (1) can be obtained because \( \Psi'b' \approx \Phi a \).

4. LED Nonlinearity Mitigation Through Predistortion

As shown in Fig. 2, once the nonlinear model for LED is found, a straightforward method for nonlinearity mitigation is to use a predistorter (at the transmitter) which is an inverse function of the obtained nonlinear model [24]. With the use of the predistorter, the output of the LED

\[
    y(n) = ax(n) + b
\]

(32)

where \( a \) and \( b \) are two constants which can be determined with the nonlinear model as shown in Fig. 3. The polynomial linearization technique is illustrated in Fig. 3 where the memory effects are not considered. For an input \( x \), the output of the LED should be \( y \). As the nonlinear model is known, we can find \( x' \) as shown in Fig. 3. Hence the task of the predistorter is to shift \( x \) to \( x' \). When the memory effects are considered, we rewrite the nonlinear equation as

\[
    y(n) = \sum_{k=1}^{K} a_{k,0}x'(n)^k + \sum_{k=1}^{M-1} \sum_{m=1}^{M-k} a_{k,m}x'(n-m)^k = ax(n) + b.
\]

(33)

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The first term represents the non-memory part while the second part represents the memory part. It is noted that, at time instant \( n \), \( x(n) \) are known, so the second term \( p(n) \) in the above equation can be calculated and cancelled. Hence, we have

\[
y'(n) = ax(n) + b - p(n) = \sum_{k=1}^{K} a_k x(n)^k.
\]

Then, the memory case is reduced to the memoryless case and \( x'(n) \) can be found.\(^1\)

### 5. Simulation Results

In this section, we compare the performance of the conventional polynomial technique and the proposed orthogonal polynomial technique in terms of LED nonlinearity modeling error and symbol error rate (SER) of the LED communication. We assume that the memory length is 1. The LED nonlinear transfer function is given by Hammerstein model [25]

\[
z(n) = \sum_{k=1}^{K} a_k x(n)^k + \lambda \times \left( \sum_{k=1}^{K} a_k x(n-1)^k \right)
\]

where \( \lambda = 0.1 \), and the polynomial coefficients \( \{a_k\} \) are obtained based on a LED data sheet [23]. We choose \( K = 4 \) and 5, and the coefficients are \( a_1 = 34.11, a_2 = -29.99, a_3 = 6.999 \), and \( a_4 = -0.1468 \) for \( K = 4 \), and they are \( a_1 = 413.2, a_2 = -541.5, a_3 = 263.4, a_4 = -56.73 \), and \( a_5 = 4.639 \) for \( K = 5 \).

\(^1\)In order to avoid the case that \( x'(n) \) is smaller than the turn-on voltage, we need \( ax(n) + b - p(n) > 0 \), which can be satisfied by properly choosing the values of \( a \) and \( b \).

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**TABLE 1**

LED nonlinearity modelling error comparison (in terms of normalized mean square error in dB) with 8-PAM and modelling SNR = 30 dB

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<td>-26.9156</td>
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<td>25.8336</td>
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<tr>
<td>500</td>
<td>120.3250</td>
<td>25.6039</td>
<td>-33.9727</td>
</tr>
</tbody>
</table>

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Fig. 3. Polynomial linearization technique (without considering memory effects).
The nonlinear model parameters are estimated using training sequence with LS. To evaluate the modeling performance, we define the normalized mean square error (NMSE) as

\[
\text{NMSE}(\text{dB}) = 10 \log_{10} \frac{\sum_{n=1}^{N} |y(n) - \hat{y}(n)|^2}{\sum_{n=1}^{N} |y(n)|^2}
\]

where \(y(n)\) is LED output, and \(\hat{y}(n)\) is the output of our nonlinear modeler. The input \(\{x(n)\}\) are randomly generated 8-PAM signals. The channel is being considered as a line of sight channel and additive white Gaussian noise (AWGN) is added at the receiver side. The variance of the measurement noise in (4) and (6) is denoted by \(\sigma_n^2\). We define the SNR in nonlinearity modeling as \(\text{SNR} = \frac{E(z(n)^2)}{\sigma_n^2}\), where \(z(n)\) is given in (2).

Table 1 shows the results of system modeling with 4-PAM. It can be seen that the conventional polynomial based technique does not work. It is also noted that the orthogonal polynomial based technique with basis \(\Psi\) performs poorly because it still suffers from the ill-condition
problem discussed in Section 3. In contrast, the proposed technique with basis $\Psi'$ works very well. In the subsequent simulations, orthogonal polynomial technique with basis $\Psi$ will not be considered. Tables 2 and 3 show the results of system modeling with 8-PAM. It can be seen that our proposed method achieves much better performance than the conventional one. With longer training sequence, better performance can be obtained. We note that the conventional technique does not work when the order of the polynomial $K = 5$.

Figs. 4 and 5 show the SER performance of the LED system with the predistorter in Section 4, where the polynomial coefficients are obtained using the conventional and proposed techniques with training length 500. The SNR for SER simulation is defined as $\text{SNR} = E(\frac{y(n)^2}{\sigma_w^2})$, where $y(n)$ is shown in Fig. 2 and $\sigma_w^2$ is the variance of the AWGN. The performance of the system without nonlinear distortion is also shown in Figs. 4 and 5 for reference, which is the lower bound of the SER performance. It can be seen from Fig. 4 (where $K = 4$) that both techniques work, but the proposed one outperforms the conventional one, especially when SNR is high. We can still see a small gap between the proposed technique and the lower bound, which is due to the residual nonlinear distortion. In Fig. 5 (where $K = 5$), the conventional technique does not work. However, the proposed one still works well and the performance is very close to the lower bound. The reason is that significant errors are induced due to the problem of numerical instability. In contrast, our approach works very well thanks to the use of orthogonal polynomials. Accurate nonlinearity modeling leads to high SER performance. Figs. 6 and 7 show the
performance of different techniques in terms of error vector magnitude (EVM) [13]. Figs. 6 and 7 have the same simulation settings as Figs. 4 and 5, respectively. The results again demonstrate the advantage of our proposed technique.

6. Conclusion
In this paper, we have investigated LED nonlinearity modeling and mitigation for LED communications with PAM signaling. An orthogonal polynomial based technique has been proposed for real-valued non-negative LED signals. It is shown that the proposed technique significantly outperforms the conventional technique in terms of LED nonlinearity modeling error and system SER performance.

References


