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FLOW RESISTANCE OVER MOBILE BED IN AN OPEN CHANNEL FLOW

Shu-Qing Yang¹ and Soon-Keat Tan²

Abstract: This paper deals with the underlying mechanism of flow resistance in an alluvial channel: the effects of sidewall and bed form on flow resistance. Einstein's divided hydraulic radius approach and Engelund's energy slope division approach are re-examined. These two approaches assume that the shear stress on a mobile bed is the summation of shear stresses caused by skin friction and bed-form. Using a different approach, this paper presents a theoretical relationship between the total bed shear stress with grain and bed-form shear stresses. The contribution of sidewall on the total bed shear stress is also discussed. The authors found that the size of bed-form plays a significant role for the flow resistance, and developed relevant expressions for the length of the separation zone behind the bed-forms. In addition, a systematical approach has been developed to compute the flow velocity in an alluvial channel. This approach is tested and verified against 5989 flume and field measurements. The computed and measured discharges are in good agreement and 83.0% of all datasets fall within the $\pm 20\%$ error band.

CE DATABASE SUBJECT HEADINGS: Boundary shear; Velocity distribution, Open channel flow; Turbulent diffusion.

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Introduction

Estimating flow resistance in an alluvial channel with reasonable accuracy is of great interest to hydraulic engineers. However, the problem remains unsolved despite numerous investigations over the past decades. The difficulty arises because the sidewalls affect the flow resistance. In addition, the bed in an alluvial channel is not fixed but continually undergoes changes in its characteristic geometry and dimensions as a result of the interaction between the flow and bed. Many expressions of side wall corrections have been proposed over the past 90 years but very few are popular among the researchers. The earliest works related to sidewall correction may be traced back to Horton (1933), Einstein (1933) and Keulegan (1938).

Einstein and Barbarossa's (1952) method for flow resistance in an alluvial channel is well-known, and has been termed as the "divided hydraulic radius" approach. This approach first assumes that the hydraulic radius in an alluvial channel can be divided into two major components, i.e., hydraulic radius related to the bank and floodplain, and the other related to the bed, i.e.,

$$R = R_b + R_w \quad (1)$$

where R = hydraulic radius = A/p ; $R_b = A_b/p_b$; $R_w = A_w/p_w$, in which A is the flow area and p is the wetted perimeter; the subscripts b and w denote the bed and wall, respectively. As for the flow in the bed region, the mobile bed resistance depends on many interrelated factors. One of these factors is the skin resistance - a resistance produced by the boundary surface - and is dependent on the depth of flow relative to the size of the roughness elements on the boundary surface. The other main component of

flow resistance is the form drag associated with the bed features that set up eddies and secondary circulations.

It is widely accepted that one needs to include both the skin (grain resistance) and form drags for estimating flow resistance in an alluvial channel with bed-form. According to Einstein's theorem, the bed hydraulic radius R_b can be further divided into two components, i.e.,

$$R_b = R' + R'' \quad (2)$$

where R' = hydraulic radius associated with grain friction and R'' = hydraulic radius associated with sand wave resistance. Therefore, one has

$$\tau_b = \rho g R_b S \quad (3)$$

where τ_b = bed shear stress, ρ = fluid density; g = gravitational acceleration; and S = flow energy. It can be seen from Eq. 3 that in a uniform alluvial channel flow, S , ρ and g are constant. Thus one is limited to only a single relationship to identify the many different roughness components and the corresponding shear forces (i.e., bank or bed force; grain and bed-form force), i.e. "dividing" the hydraulic radius. There is no rigorous basis to support that the hydraulic radius is divisible (Chien and Wan, 1999, p266).

Keulegan (1938) suggested that the bisectors of the internal angles of a polygonal channel could be used as the division lines for delineating the bed and the sidewall areas.

Thus the flow area must satisfy the following geometrical condition

$$A = A_b + A_w \quad (4)$$

Dividing both sides of Eq. 4 with p , one has

$$R = \frac{A}{p} = \frac{A_b}{p_b} \frac{p_b}{p} + \frac{A_w}{p_w} \frac{p_w}{p} = R_b \frac{p_b}{p} + R_w \frac{p_w}{p} \quad (5)$$

Comparing Eq. 5 with 1, one may conclude that Eq. 1 is unreasonable since clearly $p_b/p < 1$ and $p_w/p < 1$. In order to separate the channel and floodplain resistance in an alluvial channel, Einstein assumed that $V_{\text{whole section}} = V_{\text{flood plain}} = V_{\text{channel}}$ where V is the flow velocity. The roughness of a floodplain is normally much larger than that on the main channel, and the water depth on the floodplain is much shallower than that of the main channel. Consequently $V_{\text{flood plain}}$ should be slower than V_{channel} . Thus, Einstein's approach of divided hydraulic radius would not be reasonable (Chien and Wan 1999, p.271). Mindful of the short-comings of "hydraulic radius division" as shown in Eqs. 1 and 2, Meyer-Peter and Muller (1948), Engelund (1966) and Smith and McLean (1977) proposed an alternative approach by dividing the energy slope into two components

$$S = S' + S'' \quad (6)$$

where S' is the energy slope due to grain friction and S'' is the component due to the bed-form resistance. They argued that the additional energy loss S'' is caused by the "sudden expansion" of the flow at the downstream side of sand waves. These two noteworthy approaches (divided hydraulic radius and divided energy slopes) proposed by Einstein and Meyer-Peter and Muller, respectively can be written in a general form:

$$\tau = \tau_w + \tau_b = \tau_w + \tau' + \tau'' \quad (7)$$

where τ = mean boundary shear stress; τ_w = sidewall shear stress; τ_b = bed shear stress = $\tau' + \tau''$. According to Einstein's approach, $\tau' = \rho g R_b' S$ and $\tau'' = \rho g R_b'' S$. It is to be noted that Engelund's definition is $\tau' = \rho g R_b S'$ and $\tau'' = \rho g R_b S''$, respectively. The direct summation of shear stress shown in Eq. 7 is widely accepted by hydraulic engineers as a principle, and this approach of summation of resistance components has been extended to artificial resistance components on a rigid bed (Einstein and Banks, 1950) and natural

resistance components with flexible vegetation (Tsujiimoto 1996, for example). However, this approach has not been examined rigorously from the theoretical point of view.

The objectives of this study include 1) to re-examine the influence of sidewall on the flow resistance; 2) to establish the rational basis for summation of composite roughness, i.e., linear summation of different resistance components acting on the same boundary; 3) to re-examine the underlying mechanism of flow resistance over a mobile bed, i.e. the hypothesis of hydraulic radius or energy slope division; and 4) to establish the mechanics for different resistance components acting on the same boundary. All in all, validity of Eq. 7 will be investigated and an attempt for establishing flow resistance division corresponding to channel's shape, roughness composition and sand waves will be made.

The Underlying Mechanism for Sidewall Correction

Most natural rivers have floodplains that extend laterally away from the main river channel at a gentle gradient or in a series of terraces. There is always some flow in the main channel, but the floodplains may be dry most of the time except during times of flood. A number of experimental and numerical studies have been performed and a comprehensive review can be found in Shiono and Knight (1991).

Yang (1993), and Yang and Lim (1997, 1998) proposed that the turbulent energy is always transferred towards the nearest boundary for dissipation, i.e. over the minimum relative distance, which is defined as the ratio of geometrical distance between the source of energy and boundary to the boundary roughness. Thus the flow region can be divided into many elements by following this principle. Details of the procedure are presented as follows:

During the flood season, the water level rises above the bed of the floodplain and the flow region can be divided into several sub-regions as shown in Figs. 1, where the dotted lines are division lines and the dashed lines denote the boundary normal distance l_b and l_w , respectively. In other words, the sidewall region includes the sub-area A_{w1} , A_{w2} , A_{w3} and A_{w4} . The division lines at the edge of flood plain, i.e., the interface between A_b and A_{w2} (or A_{w3}) can be determined using the method developed by Yang et al. (2004). The division lines at the corner, i.e., the interfaces between A_{w1} and A_{w2} , or A_{w3} and A_{w4} , are determined by equating the relative distance to the sidewall and floodplain and the slope, i.e.,

$$\frac{l_b}{l_w} = \frac{\Delta_b}{\Delta_w} \quad (8)$$

where Δ_b and Δ_w are bed and sidewall roughness, respectively. One has $l_b = l_w$ when $\Delta_b = \Delta_w$, which indicates that the division line is the bisector of the base angle if the sidewall roughness is identical with the floodplain roughness. This is consistent with Keulegan's (1938) conclusion. In practice, the equivalent roughness Δ_w can be determined from the Manning coefficient of bank n_w ($\text{s/m}^{1/3}$) using the Strickler equation (Smart, 1999), i.e.

$$\Delta_w = \left(6.7\sqrt{g}n_w\right)^6 \quad (9)$$

However, in the presence of bed-form or vegetation on the floodplain, it is difficult to estimate the floodplain roughness Δ_b . Therefore, it is necessary to establish an equivalent roughness for the bed.

For the case of flow in the main channel only, as shown in Fig. 2, the sidewalls have significant influence on the river flow resistance due to the narrow and deep geometry. The physics of sidewall effect could be complicated when one considers the interaction

of sidewall and bed-form. The latter is not fixed but changes dynamically in its characteristic geometry and dimension of the bed forms as represented by Δ_b in Eq. 8.

Summation of Resistance for Different Roughness Elements

Consider the equivalent roughness Δ_b of a mobile bed caused by sand-waves or vegetation. Δ_b can be attributed to at least two factors: (1) the skin resistance - a resistance produced by the boundary surface – and is dependent on the depth of flow relative to size of roughness elements along the boundary surface; and (2) the form drag caused by sand waves or vegetations that set up eddies and secondary circulations. Figure 3 shows a two-dimensional bed-form, in which L = length of sand wave; δ = bed-form height; h = flow depth; L'' = length of separation zone behind the sand wave; and L' = length dominated by grain friction. Within the length of L'' , the turbulent energy is mainly dissipated by large eddies in the lee of the sand wave. The energy loss within the length L' is attributable to the small eddies behind the bed-load particles. Thus, the total energy loss over a sand wave can be expressed as follows:

$$h_f = h_f' + h_f'' \quad (10)$$

Equation 10 is free of any assumption and is different from that of Einstein (1933) and Meyer-Peter and Muller (1948). Dividing both sides of Equation 10 by the sand wave length L , one obtains the expression of energy slope:

$$S_i = \frac{h_f}{L} = \frac{h_f'}{L} \frac{L'}{L} + \frac{h_f''}{L} \frac{L''}{L} = S' \frac{L'}{L} + S'' \frac{L''}{L} \quad (11)$$

where S_i = calculated total energy slope, $S' = h_f' / L'$ and $S'' = h_f'' / L''$. It can be seen from Equation 11 that the energy slope division proposed by Engelund (1966), and Meyer-Peter and Muller (1948) is a more physically plausible expression than that of Einstein

(1933). This is probably why the approach of Engelund performs best among the existing models and is most commonly used (Bennett, 1995).

Multiplying $\rho g R_b$ on both sides of Eq. 11, one obtains

$$\tau_b = \tau' \frac{L'}{L} + \tau'' \frac{L''}{L} \quad (12)$$

where τ_b = bed shear stress = $\rho g R_b S$; τ' = grain shear stress = $\rho g R_b S'$; τ'' = bed form shear stress = $\rho g R_b S''$.

Equation 12 becomes Einstein and Engelund's result as shown in Eq. 7, i.e., $\tau_b = \tau' + \tau''$ if $L'/L = L''/L = 1$ is assumed; but this assumption is physically unreasonable because L' and L'' are always less than L . Therefore, one could deduce that the widely accepted assumptions or Eq. 7 need to be improved upon.

Expression of Grain and Bed-form Resistance

A comprehensive review of the literature pertaining to grain roughness can be found in Bennett (1995). Generally, the grain shear velocity can be expressed by the following relation, regardless of the size of the roughness elements:

$$u_*' = \sqrt{g R_b S'} = \frac{V_b}{2.5 \ln \frac{R_b}{k_s'} + B} \quad (13a)$$

and

$$B = 6 + (2.5 \ln k_s'^+ - 3) \exp(-0.11 \ln^{2.5} k_s'^+) \quad (13b)$$

in which V = mean velocity; k_s' = equivalent roughness related to grains, and $R_b = A_b/p_b$, $k_s'^+ = u_*' k_s' / \nu$. Equation 13a is valid for hydraulically smooth, transition and rough regimes.

Equation 13b was obtained by Cheng and Chiew (1998).

There is no apparent consensus on the definition of k_s' and not surprisingly, a large range of k_s' -values ($1.25d_{35} \leq k_s' \leq 5.1d_{84}$) have been suggested (Van Rijn, 1982). Nevertheless Millar (1999) found that there was no significant difference between using d_{35} , d_{50} , d_{84} or d_{90} . In this study, k_s' suggested by Yang and Lim (2003) will be used, i.e.,

$$k_s' = 2d_{50} \quad (14)$$

Millar (1999) also ascribed the variation of k_s' to the presence of form roughness because it was generally determined by the best fit of the measured data corresponding to experiments with both plane bed, and bed with sand-wave features. Similarly, Eq. 13a should be also suitable for flow over large roughness element—the protruding large rock, boulders, sand waves or vegetations on the bed. The only distinction between the grain resistance and the form drag is reflected in the size of eddies behind the roughness element in which the turbulent energy is dissipated. The form drag can be approximated by

$$u_*'' = \sqrt{gR_b S''} = \frac{V_b}{2.5 \ln \frac{11R_b}{k_s''}} \quad (15)$$

For a sand wave bed, k_s'' should be related to the bed-form geometry and Yalin (1972, p. 235) assumed the following functional relationship:

$$k_s'' = f_{(\delta, \delta/L)} \quad (16)$$

in which f is a function. Van Rijn (1982) considered the average bed-form geometry and obtained:

$$k_s'' = 1.1\delta \left[1 - \exp\left(-25 \frac{\delta}{L}\right) \right] \quad (17)$$

Eq. 17 can be simplified as follows:

$$\frac{k_s''}{\delta} = 1.5 \left(\frac{\delta}{L} \right)^{0.2} \quad (18)$$

Figure 4 shows the relationship between k_s''/δ and the steepness of sand wave δ/L . The data is reproduced from van Rijn (1984c). It can be seen that the agreement between Eq. 18 and the measured data is good and may be acceptable.

In a later development, van Rijn's (1984c) merged the grain roughness with the effective bed-form roughness empirically as follows:

$$\Delta_b = k_s' + 1.1\delta(1 - e^{-25\delta/L}) \quad (19)$$

As the length of separation zone behind the roughness element is proportional to the height of sand waves, thus the following assumption may be introduced, i.e.,

$$L'' = \alpha\delta \quad (20)$$

where α = factor. Then

$$\frac{L''}{L} = \alpha \frac{\delta}{L} \quad (21)$$

and

$$\frac{L'}{L} = \frac{L - L''}{L} = 1 - \alpha \frac{\delta}{L} \quad (22)$$

For ripples and dunes, Engel (1981) and Karahan and Peterson (1980) suggested that α has a value in the range of 4~10.

Determination of the length and height of sand wave

Equation 12 shows that the flow resistance of mobile bed is strongly related to the length (L) and height (δ) of sand waves, which unfortunately are usually not measured in field measurements. However, to analyze flow resistance of the mobile bed in rivers using the above mentioned method, we have to determine the size of the bed form. There are many

empirical equations of the sand wave geometry in the literature. In this study, the sand wave height is determined using Karim's (1999) empirical equations:

$$\frac{\delta}{h} = c \left[\frac{\left\{ S - 0.0168 \left(\frac{d_{50}}{R_b} \right)^{0.33} F_r^2 \right\} \left(\frac{L}{R_b} \right)^{1.2}}{0.47 F_r^2} \right]^{0.73} \quad \text{for ripples and dunes} \quad (23)$$

and

$$\frac{\delta}{h} = c \left[\frac{\left\{ S - 0.0168 \left(\frac{d_{50}}{R_b} \right)^{0.33} F_r^2 \right\} \left(\frac{L}{R_b} \right)^{1.2}}{0.085 F_r^2} \right]^{0.73} \quad \text{for transition, antidunes or standing waves} \quad (24)$$

where Fr = Froude number = $V_b / \sqrt{gh}$, c = correction factor. The wave length L or ripple length is estimated using the following relationship proposed by Yalin (1964):

$$L = 1000 d_{50} \quad (25)$$

The dune length developed by Julien and Klaasen (1995)

$$L = 6.25 R_b \quad (26)$$

is very close to van Rijn's (1984a) relationship of $L = 7.3 R_b$ and Yalin's (1964) theoretical derivation of $L = 2\pi R_b$.

The antidunes (or standing waves) wavelength given by Kennedy's (1963) relationship is:

$$L = 2\pi F_r^2 R_b \quad (27)$$

For transitional bed regime, Karim (1999) suggested the following equation to estimate the length of bed form:

$$L = 7.37R_b \left\{ 0.00139 \left[\frac{V_b}{\sqrt{g(\rho_s / \rho - 1)d_{50}^3}} \right]^{2.97} \left(\frac{u_*}{\omega} \right)^{1.47} \right\}^{0.295} \quad (28)$$

where ρ_s and ρ are density of sand and water, respectively; and ω = particle settlement velocity. The relationship for bed regime prediction developed by Karim (1995) is also used in this study. The two limiting Froude numbers are defined as:

$$F_t = 2.716 \left(\frac{h}{d_{50}} \right)^{-0.25} \quad (29)$$

$$F_u = 4.785 \left(\frac{h}{d_{50}} \right)^{-0.27} \quad (30)$$

The use of F_t and F_u for different bed regimes may be determined from Froude number as follows:

Lower regime (ripple, dunes):

$$F_r < F_t \quad (31)$$

Transition regime (washed out dunes):

$$F_t \leq F_r \leq F_u \quad (32)$$

Upper regime (plane bed, antidunes):

$$F_r > F_u \quad (33)$$

A relationship developed by Guy et al. (1966) is used to identify the ripple regime, and is based on the following parameter:

$$N_* = \frac{u_* d_{50}}{v} \frac{V_b}{\sqrt{g(s-1)d_{50}}} \quad (34)$$

and $N_* < 80$ defines the occurrence of ripples.

Length of separation zone behind sand waves

The coefficient α , i.e., ratio of the length of separation zone to the height of sand wave may be naturally perceived as a variable depending on the magnitude of the flow and the size of the bed forms. Parameters that possibly affect the value of α may include: water depth h , height of sand wave δ , median size of sediment d_{50} , gravitational acceleration g , kinematic viscosity coefficient ν and mean velocity V_b . The writers are not aware of any formula for the determination of α . We have to determine α indirectly based on readily available measurements/data by using the following equation:

$$\alpha = \frac{u_*'^2 - u_*''^2}{u_*''^2 - u_*'^2} \frac{L}{\delta} \quad (35)$$

It can be seen from Eq. 35 that if L is overestimated or δ is underestimated, the obtained α could be very large. In this model, the bedform height (δ) and length (L) are estimated using the empirical equations. Thus it is natural that some discrepancies would creep in estimating δ and L , but the discrepancies could be corrected by using a factor c introduced in Eqs. 23 and 24.

In the following illustration, the coefficient α is calculated using experimental data of Guy et al (1966) in wide channels ($b/h > 5$). This dataset is specially selected because hydraulic parameters such as velocity, energy slope had been measured and tabulated, facilitating ready determine of k_s' using Eq. 14 and k_s'' using Eq. 17. Then by using Eqs. 13 and 15, the friction velocities corresponding to the grain friction and bed-form u_*' and u_*'' are estimated using the mean velocity, hydraulic radius and roughness k_b' and k_b'' . The parameter u_*' can be determined using the measured energy slope S from $(gRS)^{0.5}$. Bedform length and height can be determined using Eqs. 23-28.

Figure 5 is prepared with $c = 2$. It can be seen that the parameter α in the ripple regime is closely related to the parameter H^* which is defined as $\delta[(\rho_s/\rho-1)g/v^2]^{1/3}$. This parameter is slightly different from the particle parameter D^* used by van Rijn (1984a, 1984b), and Yu and Lim (2003), who defined $D^* = d_{50}[g(\rho_s/\rho-1)/v^2]^{1/3}$. Both parameters indicate that, for ripples, viscosity plays an important role. Based on Fig. 5, the coefficient α is expressed as:

$$\alpha = 13.45 - 0.0059H^* \quad \text{for ripples} \quad (36)$$

The calculated α in the dune regime is plotted in Fig. 6a ($c=2$) and the data points can be approximately expressed by

$$\alpha = 7 - 4.5 \frac{\delta}{h} \left[\frac{u_*}{\sqrt{gd_{50}}} \right]^{0.5} \quad \text{for dunes} \quad (37)$$

The variation of α in the transition regimes ($c=1.5$) and antidune ($c=1$) are presented in Fig. 6b, the experimental data shows that α can be empirically expressed as:

$$\alpha = 2.5 - 0.001 \frac{h}{d_{50}} \quad \text{for transition and antidunes} \quad (38)$$

It can be seen from Figs. 5 and 6a and 6b that α decreases systematically with the bed form development; from 13 in the ripple regime to 1.3 in the transition/antidunes regimes.

Calculation procedure

Consider the case of a rectangular flow channel, and the channel width b , water depth h , median sediment size d_{50} and energy slope S have been measured. The unknown discharge or mean velocity V can be estimated as follows:

- 1) Estimate the sidewall Manning coefficient and determine the sidewall roughness

Δ_w using Eq. 9.

- 2) Assume an appropriate value for the bed roughness Δ_b (use d_{50} as a first estimate); then the slope of division lines is determined using Eq. 8. Each sub-region area, i.e., A_b and A_w , as well as the hydraulic radius R_b and R_w can be obtained.
- 3) Calculate the mean velocity in each sub-region area using

$$V_w = 2.5u_{*w} \ln \frac{11R_w}{\Delta_w} \quad (39a)$$

$$V_b = 2.5u_{*b} \ln \frac{11R_b}{\Delta_b} \quad (39b)$$

where $u_{*w} = (gR_wS)^{0.5}$, $u_{*b} = (gR_bS)^{0.5}$. Then the total discharge and over-all mean velocity can be calculated using

$$Q = V_b A_b + V_w A_w \quad (40)$$

$$V = Q/A \quad (41)$$

where $A = bh$.

- 4) Calculate F_t and F_u using Eqs. 29 and 30 based on V_b , h and d_{50} ; then identify the flow regimes using Eqs. 31-34.
- 5) Calculate the bed form wave length, L and height δ using Eqs. 23-28, and estimate the coefficient α use Eqs. 36, 37 and 38.
- 6) Estimate the energy slope related to grains S' from Eq. 13 and the energy slope related to bed forms S'' using Eqs. 15 and 18, in which $u_*' = (gR_bS')^{0.5}$ and $u_*'' = (gR_bS'')^{0.5}$.
- 7) Estimate the total energy slope S_i using Eq. 11 based on S' , S'' and the relationships $L'/L = 1 - \alpha\delta/L$ and $L''/L = \alpha\delta/L$;

- 8) Calculate bed roughness Δ_b using Eq. 39b based on V_b , R_b and $u_* [= (gRS_i)^{0.5}]$; solve for Δ_b iteratively, and repeat steps 2-8 if necessary. Normally less than 100 iterations are required to arrive at the solution.

Calibration

First the proposed model is calibrated using laboratory and field data. Yang (1996) found that for laboratory flume with smooth walls, the division line can be approximately expressed as follows:

$$\frac{l_b}{l_w} = \left(\frac{u_* \Delta_b}{\nu} \right)^{0.07} \quad (42)$$

where $u_* = (gRS)^{0.5}$, l_b and l_w are normal distance to the sidewall and bed from the division line, respectively. The sub-flow areas A_b and A_w may be determined if the bed roughness Δ_b is assumed. Obviously if the assumed Δ_b closely approximates the real value of Δ_b , then the calculated S_i will be close to the measured energy slope S .

The following datasets are specially selected for model calibration:

- 1) Williams's (1970) experimental data: this dataset of 177 flume tests was conducted in narrow and deep channels to determine how the sidewall influences flow resistance, and the experimental results have been widely used to compare various models, such as Karim (1995), Yu and Lim (2003) etc. In this experimental dataset, nearly uniform-size particles with 1.35mm median diameter were used. The water depth, discharge and energy slope had been measured. The flume widths were 7.6, 15.2, 30.4 and 60.5cm. For each of these widths a series of runs was made at depths of 3.4, 9.1, 15.2 and 21.3cm. Among 177 tests, 96 experiments were performed in channels with aspect ratio (width/depth) less than 2, and 45 runs were carried out in

supercritical flow condition ($V/\sqrt{gh} > 1$). Transitional regime, standing wave and antidune were observed.

- 2) Wang and White's (1993) experimental data: this dataset comprises 108 runs and is selected because the experiments had been deliberately performed in the transition regime that is characterized by the resistance coefficient decreasing rapidly with increasing flow strength.
- 3) Stein's (1965) 59 flume experiments. This dataset is selected because it includes observation of the upper flow regime (antidune or standing waves).
- 4) Guy et al.'s (1966) 337 experiments are also included.
- 5) The field data of Rio Grande River (293 runs) is also used for the calibration.

In total, 683 experiments are shown in Fig. 7 in which the abscissa is the estimated flow velocity in cm/s and the ordinate is the measured velocity in cm/s. The solid line represents the perfect agreement and the dash lines are the $\pm 20\%$ error band. Good agreement between the measured and calculated flow discharges can be seen. In the calculation process the bed form type was determined based on the Froude number, i.e., F_r , F_t and F_u as well as N^* .

For field data, only those of the lower regime (ripple and dune) are readily available. The writers found that the proposed model yields better results if $c = 3.5$ is applied. The difference is partially due to the fact that the flow channel in the field is not exactly straight and uniform, and consequently the bed-form will be underestimated when Eqs. 23 and 24 are used in the field.

After the sand-wave height is corrected, it can be seen from Fig. 7 that the proposed model generally yields good agreement. The detailed discrepancy is listed in Table 1. The

last three columns of Table 1 show a summary of the percentage predictability scores for the various groups of data. It indicates the overall score: 67.4% of the 683 predictions scatter within the $\pm 10\%$ error band, and 86.4% are within $\pm 20\%$ discrepancy error band, and 91.3% within $\pm 30\%$ error band. The large errors are mainly associated with data in the transition regime (144 out of 678 runs), out of these 144 runs, 77 predictions (or 53.5%) are beyond the $\pm 20\%$ error band. In the dataset used, 150 experiments were conducted in the antidune regime but only 13 of which are outside the $\pm 20\%$ error band. The remaining 384 tests were carried out in the lower regime (ripple or dune), and only 6 are out of the $\pm 20\%$ error band. For field data, 293 runs are used for the calibration, only 21 of which are out of the $\pm 20\%$ error band.

Verification

Following the calibration, the proposed model is verified using flume data compiled by Brownlie (1981). The data set in this database is comprehensive and has complete records of the flow discharge, channel width, flow depth, hydraulic slope, median sediment size, specific gravity of sediment and temperature. Because of the huge number of data points, the data are sub-divided into 4 groups. All data sources are not cited in the reference of this paper as they could be found in Brownlie's (1981) report.

The group 1 database consists of 1185 flume experiments from 14 sources that are listed in Table 2. The measured and calculated discharges are plotted in Fig. 8. On average, 77.2% of 1185 runs fall within the $\pm 10\%$ error band, 94.3% of predictions in the $\pm 20\%$ error band and 98% in the $\pm 30\%$ error band.

The group 2 dataset includes 870 sets from 17 sources. The basic hydraulic conditions are shown in Table 3 and the calculated and measured discharges are plotted in Fig. 9. It can

be seen that Kalinske and Hsia (1945) and Neill's (1967) experimental data are not well predicted. The median sediment size used by Kalinske and Hsia (1945) and Neill's (1967) were 0.011mm and 20mm, respectively. It appears that though it produces reasonable results the model is not expected to yield good accuracy for very fine and very coarse sediment. On the average, 80.9% of 870 runs falls within the $\pm 10\%$ error band, 78.4% of predictions within the $\pm 20\%$ error band and 93.7% within the $\pm 30\%$ error band.

The group 3 dataset includes 1569 sets from 12 sources. The basic hydraulic conditions are shown in Table 4 and the calculated and measured discharges are plotted in Fig. 10. It is worthwhile to note that Waterway Experiment Station (Waterway 1936c) used lightweight material as model sediment and the measured velocities were generally less than the measured values. On average, 73.8% of 1569 runs fall within the $\pm 10\%$ error band; 92.2% of predictions within the $\pm 20\%$ error band and 95.3% within the $\pm 30\%$ error band.

The group 4 dataset includes 1389 sets from 21 sources of field measurement. The basic hydraulic conditions are shown in Table 5 and the calculated and measured discharges are plotted in Fig. 11. In the calculation, the sidewall Manning coefficient n_w is assumed to be 0.02, or the equivalent Nikuradse roughness is 0.54cm when Eq.9 is applied. In natural rivers, the flow is generally tranquil for $Fr < 1$. The upper flow regime (transition, antidune or standing wave) was not reported in Brownlie's report. As natural rivers are generally very wide and shallow, the water depth h is used to replace the hydraulic radius R_b in the calculation. On the average, 19.7% of 1389 runs fall within the $\pm 10\%$ error band, 58.4% within the $\pm 20\%$ error band and 80.3% within the $\pm 30\%$ error band. It is obvious

that the discrepancy associated with the field measurement is larger than that in flume experiments. The larger discrepancy could be attributed to the following factors:

- 1) Measurement errors. It is certain that the measurement error in the field is large. For example, in the source of Mountain Creek, Einstein did not measure the energy slope everyday. It was estimated from observations made on every other day, or the average of two observations if slope was measured shortly before and shortly after other stream flow measurements.
- 2) Variable sidewall roughness. In this verification exercise, the sidewall roughness is assumed to be constant regardless of river geometry. It is apparent that the assumption may not be always true.
- 3) Non-rectangular channel shape. Since the channel shapes were not recorded in Brownlie's (1981) report, the writers assume that the channels were rectangular which may not correctly reflect the stream cross section.
- 4) Straight river is rare and a river typically exhibits certain degree of meanderings making the flow parameters considerably more complex and the correction factor would be different from that in laboratory flume experiments.
- 5) The model employed empirical equations for estimating the bedform length and height, the estimation of α -value is based on these parameters. Certainly the errors in the prediction of bedform size would incur the discrepancy of measured and predicted velocities. This is partially why the velocity in large rivers is over-estimated.

Considering the complexity of natural rivers, the agreement shown in Fig. 11 is indeed encouraging.

Applicability, accuracy and limitations of the proposed model

For easy reference, and to reflect the extensive coverage used in the verification of the proposed model, the range of the pertinent parameters is summarized as follows:

For flume data (4307 sets): $0.8\text{l/s} \leq Q \leq 4613\text{l/s}$, $0.07\text{m} \leq b \leq 2.4\text{m}$, $0.01\text{m} \leq h \leq 0.97\text{m}$, $0.019 \times 10^{-3} \leq S \leq 6 \times 10^{-2}$, and $0.011\text{mm} \leq d_{50} \leq 20\text{mm}$.

For field data (1682 sets): $0.06 \text{ m}^3/\text{s} \leq Q \leq 28825\text{m}^3/\text{s}$, $0.035 \leq b \leq 1109\text{m}$, $0.29\text{m} \leq h \leq 17.28\text{m}$, $0.0021 \times 10^{-3} \leq S \leq 1.26 \times 10^{-2}$, and $0.096\text{mm} \leq d_{50} \leq 54.9\text{mm}$.

Among the 5989 measurements, 60.2% scatter within $\pm 10\%$ error band; 84.0% within $\pm 20\%$ error band, and 91.5% within the $\pm 30\%$ error band. This indicates that the proposed model yields reasonable results considering the uncertainties in measurement, especially for the field data. Since the model has been tested extensively, and encompassed such a wide range of hydraulic conditions, one could conclude that the proposed model is reliable in predicting the flow resistance in alluvial channels for a wide range of flow regimes tested.

This model yields better results when compared with other alternative. White et al. (1981) compared the previous models based on Eq. 7 and concluded that amongst the calculated bed friction values which lie within a factor of 2 of the measured values, Einstein and Barbarossa (1952) scored 21%, Engelund (1966) 83%, and White et al. (1981) 89%.

The writers believe that the proposed model yields such good results because it is physically based, rigorous and includes the following improvements: (a) appropriate expression of skin roughness and bed-form roughness; (b) sound theoretical derivation of bed shear stress summation, i.e., Eq. 12; (c) appropriate sidewall correction.

However, it should be noted that there are limitations on the prediction of discharge or mean velocity: 1) the predictions in the transition regime are not as good as those in other flow regimes; 2) for lightweight material transport, the predicted results are not as good as the flow with natural sand. One of the possible reasons for these discrepancies could be attributed to the prediction of sand-wave geometry. Further study to improve the prediction of sand-wave geometry would be necessary.

Conclusions

This study investigates the flow resistance in alluvial channels. The authors developed a systematical approach to evaluate the flow velocity using the parameters of flow depth, width, energy slope and sediment size. The proposed model considers the effect of sidewall, sand-wave geometry, sediment size etc. From the results of the verification study, the writers conclude that the proposed method demonstrates a good predictive ability for the discharge or velocity over mobile beds.

The study leads to the following conclusions:

1. The overall hydraulic radius is not equal to the sum of sidewall and bed radiuses as suggested by Einstein. Instead, the flow area could be divided into sub flow area according to Equation 8.
2. The head losses caused by protruding roughness elements in the stream-wise direction may be summed. The total bed shear stress depends not only on skin and bed-form shears, but also on the bed-form geometry.
3. Since the distinction of grain and bed-form roughness depends on the size of roughness element or eddies behind the roughness, the universal log-law can be used

to describe both the grain and bed-form resistance. A value equal to $2d_{50}$ is suggested as the equivalent Nikurads roughness. In the case of form drag, the equivalent roughness depends on the height and steepness of sand wave.

4. The authors demonstrate that the proposed model and the method are particularly convenient for determining the river discharge (or velocity) when only the water level, shape of cross section, and energy slope are measured. The validity of the approach has been tested with 4302 flume measurements and 1682 field observations. The computed and measured energy slopes are in good agreement, in which 83% of all datasets fall within the $\pm 20\%$ error band.

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Appendix II. Notations:

b = width of channel;

c = correction factor;

h = flow depth;

f = function of bed-form steepness;

Fr = Froude number;

F_t = number;

F_u = number;

g = gravitational acceleration;

h_f = energy loss;

k_s' = equivalent roughness related to grains;

k_s'' = equivalent roughness related to bed-form;

L = length of sand wave;

L'' = length of separation zone behind the sand wave;

L' = length dominating by grain friction;

m = no. of tests;

N_* = number

n = empirical coefficient;

R = bed hydraulic radius;

R' = hydraulic radius related to grain;

R'' = hydraulic radius related to bed-form;

S = energy slope;

S' = energy slope due to grain friction;

S'' = energy slope due to the bed-form resistance;

V = cross-sectional mean velocity;

α = coefficient;

τ = bed shear stress;

$\tau' = \rho g R' S$ or $\rho g R S'$;

$\tau'' = \rho g R'' S$ or $\rho g R S''$;

ρ = fluid density;

ρ_s = density of sand;

δ = bed-form height;

ω = particle settlement velocity.

Table 1 Summary of hydraulic conditions of experimental data shown in Fig. 7 and 8

Researchers	runs	Q(l/s)	b(m)	h(m)	S*1000	D _{50(mm)}	Prediction in error ranges		
							±10%	±20%	±30%
Williams (1970)	177	1.048~162	0.15~1.19	0.03~0.22	0.6~35	1.35	68.3	88.1	94.5
Wang & White (1993)	108	22.5~348	0.92~1.2	0.085~0.253	0.0110.73	0.076~0.76	55	78.7	83.3
Stein (1956)	56	78~481	1.22	0.09~0.31	0.61~16.95	0.4	55.3	78.6	83.9
Guy et al (1966)	337	24~632	0.61~2.438	0.09~0.34	0.055~16.2	0.19~0.54	73.0	89.3	94.9
Rio Grande (1965)	293	498-286000	14-121.9	0.158-1.88	0.69-2.46	0.173-10.95	38	93	99

Table 2, Summary of hydraulic conditions of experimental data shown in Fig. 8

Researchers	runs	Q(l/s)	b(m)	h(m)	S*1000	D _{50(mm)}	Prediction in error ranges		
							±10%	±20%	±30%
Abdel-AAL (1969)	10	10-35	0.305	0.09-0.14	1.9-2.5	0.105	30	70	90
Bengal (1965)	18	0.89-21.52	0.457	0.0137-0.16	0.2-1.0	0.315	50	94	94
Brooks (1957)	21	5.66-14.72	0.267	0.047-0.091	1.3-3.3	0.088-0.145	86	90	95
Casey (1935)	92	0.8-57.79	0.4	0.009-0.20	1.19-5.19	1.0	88	100	100
Chyn (1935)	32	12.3-30.6	0.61	0.054-0.1	1.1-3.0	0.59-0.84	100	100	100
Costello (1974)	28	39-66.7	0.915	0.14-0.16	0.37-1.01	0.51-0.79	96	100	100
Davies (1971)	79	25.5-322.8	1.372	0.076-0.30	1.37-1.18	0.15	92	94	94
Einstein and Chien (1955)	16	73.9-82.9	0.307	0.108-0.139	12.4-25.8	0.27-1.3	19	100	100
Pakistan (1967)	68	14.16-254.8	0.38-1.22	0.1-0.305	0.29-3.8	0.16-0.44	79	99	100
Pakistan (1966, 1968, 1969)	56	17-254.8	0.38-1.22	0.1-0.387	0.047-2.6	0.15-0.33	77	88	96
Foley (1975)	12	3.7-7.5	0.267	0.03-0.046	3.81-10.63	0.29	83	92	100
Franco (1968)	19	35.9-52.9	0.914	0.125-0.16	0.23-1.69	0.23-2.2	100	100	100
Gibbs (1972)	9	158.6-198.2	1.22	0.17	2.9-5	4.374	100	100	100
Gilbert (1914)	725	2.6-31.68	0.13-0.597	0.011-0.225	0.44-31	0.305-7.01	77	95	98

Table 3 Summary of hydraulic conditions of experimental data shown in Fig. 9

Researchers	runs	Q(l/s)	b(m)	h(m)	S*1000	D _{50(mm)}	Prediction in error ranges		
							±10%	±20%	±30%
Hill (1969)	45	71.47-121.8	0.61	0.16-0.256	0.52-2.67	0.088-0.31	87	100	100
Ho (1939)	78	3.4-67.8	0.4	0.05-0.22	0.99-6.28	1.4-6.28	87	100	100
Jorissen (1938)	24	3.28-34.2	0.61	0.02-0.1	1.11-3.33	0.6	100	100	100
Kalinske & Hsia (1945)	9	19.8-90.6	0.686	0.11-0.2	0.25-1.3	0.011	22	56	67
Kalkanis (1957)	23	9-34.3	0.3	0.1-0.24	0.23-1.23	0.013-0.033	43	61	70
Kennedy (1961)	41	5.67-70.1	0.267-0.85	0.02-0.1	1.7-27.2	0.549-0.233	73	90	95
Kennedy & Brooks (1965)	9	39	0.851	0.0695-0.167	0.56-2.5	0.142	89	89	89
Laursen (1958)	23	24-181	0.914	0.095-0.28	0.43-1.52	0.04-0.11	60	65	74
Macdougall (1933)	67	3.79-63.9	0.61	0.0195-0.159	1.11-3.33	0.66-1.26	96	100	100
Mutter (1971)	25	9.34-26.3	1.22	0.013-0.085	0.75-7.5	0.26	44	96	100
Neill(1967)	47	19-174.2	0.89-0.914	0.03-0.61	1.2-27	6-20	28	77	79
Nomicos (1957)	27	4.8-15.9	0.267	0.07	1.9-2.75	0.09-0.152	89	100	100
Nordin (1976)	59	280-2213	2.38	0.3-0.85	0.29-5.77	0.25-1.14	66	83	85
Obrien (1936)	83	18.97-150	0.914	0.09-0.327	0.31-3.08	0.36	84	98	100
Onishi et al (1972)	12	24.1-51.54	0.914	0.075-0.135	1.09-2.67	0.25	100	100	100
Pratt (1970)	60	15-354	1.372	0.076-0.457	0.0287-2.31	0.487	83	98	100
Sato et al (1958)	238	43-502	0.78	0.11-0.518	0.22-2.52	1.03-4.58	95	100	100

Table 4 A summary of hydraulic conditions of experimental data shown in Fig. 10

Researchers	runs	Q(l/s)	b(m)	h(m)	S*1000	D _{50(mm)}	Prediction in error ranges		
							±10%	±20%	±30%
Singh (1960)	305	2.63-27.38	0.253-0.75	0.018-0.199	1.0-14.0	0.62	92	98	99
Soni (1980)	23	1.4-9	0.2	0.05-0.1	2.07-7	0.32	87	96	96
Straub (1954, 1958)	24	8-170	0.305	0.03-0.23	0.78-7.34	0.16-0.19	50	92	92
Taylor (1971)	39	3.7-84	0.267-0.851	0.06-0.18	0.3-2.09	0.19-1.07	69	100	100
Vanoni & Brooks (1957)	15	14.4-108.7	0.85	0.0725-0.169	0.39-2.8	0.137	93	100	100
Vanoni & Hwang (1965)	16	3.68-185.5	0.267-1.1	0.07-0.37	0.45-2.9	0.206-0.23	75	100	100
Willis (1979)	32	17.3-47.3	0.36	0.1-0.149	0.83-8.58	0.54	69	72	84
Willis et al (1972)	91	78.1-480	1.219	0.11-0.378	0.308-2.04	0.1	56	93	100
Waterway (1935)	310	2.78-63.4	0.705-0.736	0.016-0.2	1.0-4.5	0.18-4.1	85	98	98
Waterway (1936)	98	6.9-62.5	0.7	0.03-0.134	1.0-2.0	0.95	100	100	100
Waterway (1936B)	331	8.66-38.8	0.305	0.073-0.27	1.0	0.35-1.2	70	99	99
Waterway (1936c)	285	0.66-34.0	0.305	0.02-0.238	0.1-1.0	0.83-3.5	50	80	87

Table 5 Summary of hydraulic conditions of experimental data shown in Fig. 11

Researchers	runs	Q(l/s)	b(m)	h(m)	S*1000	D _{50(mm)}	Prediction in error ranges		
							±10%	±20%	±30%
Acop canal (1979)	151	52131-486823	35.4-140.2	0.76-4.3	0.06-0.166	0.085-0.715	10	32	76
Red river (1968)	30	4247399-28825680	542-1103	6.92-17.28	0.018-0.1336	0.187-0.684	0	2	5
American canal (1957)	11	1217-29420	3.2-15.1	0.8-2.6	0.058-0.302	0.096-7.0	18	90	100
Chitale canal (1966)	32	27523-427571	23.8-121	1.31-3.38	0.051-0.254	0.11-0.31	0	28	44
Mississippi River(1968)	165	1512074-28825680	455-1109	4.66-17.28	0.0183-0.1336	0.163-1.129	4	10	26
Atchafalaya River (1968)	65	637109-14186313	304-503	6.0-14.7	0.0021-0.05	0.091-0.03	6	26	45
Colorado river (1958)	105	77531-500925	92.6-254.6	0.95-3.89	0.06-0.389	0.155-0.695	32	62	81
HII River (1959)	38	0.94-4851	0.35-8.0	0.019-0.73	0.84-10.7	0.21-1.44	26	60	100
Leopold River (1969)	55	83333-454301	88.7-152.4	0.96-4.1	0.037-0.346	0.14-0.814	22	45	62
Loup River (1959)	38	9315-12855	37.49-46.63	0.292-0.376	0.928-1.496	0.267-0.429	0	76	100
Missouri River (1978)	23	892000-1838000	196.3-223.5	2.83-5.0	0.144-0.161	0.2-0.22	87	100	100
Mountain creek (1944)	100	64.4-1492	3.9-4.3	0.046-0.177	1.37-3.15	0.90	6	66	100
South American canal (1973)	113	24000-14260000	27-845	1.32-13.28	0.004-0.45	0.1-1.05	34	58	72
Niobrara River (1955)	40	5860-16000	21	0.4-0.59	1.13-1.70	0.212-0.359	28	100	100
Portugal River (1969)	219	29000-660000	69.7-188.9	0.457-2.44	0.62-0.94	2.2-2.6	1	63	100
Rio Grande (1976)	27	3596-39077	16.76-22.86	0.39-1.51	0.45-0.8	0.17-0.24	41	93	100
Rio Grande (1968)	38	35000-286000	81-195	0.33-1.46	0.74-0.89	0.284-0.315	13	89	100
Oak Creek (1973)	17	1330-3397	4.43-5.91	0.3-0.52	9.7-12.6	8.2-23	0	100	100
Saskatchewan (1971)	55	4700-39000	3.048-6.09	0.73-2.4	1.58-7.45	18.6-54.9	49	98	100
Chop canal (1970)	67	27523-427571	23.8-121	1.31-3.38	0.051-0.254	0.11-0.31	25	58	76

CAPTIONS OF FIGURES

Fig. 1, flow region division for a river with floodplain;

Fig. 2 Flows in a narrow-deep river;

Fig. 3. Definition sketch;

Fig. 4 Relative bed-form roughness k_s''/δ versus δ/L

Fig. 5 Calculated coefficient α in ripple regime

Fig. 6a Calculated coefficient α in dune regime

Fig. 6b Calculated coefficient α in transitional and antidune regimes

Fig. 7, Comparison between measured and calculated discharges

Fig. 8, Comparison of measured and calculated flow rate;

Fig. 9 Comparison of measured and calculated flow rate;

Fig. 10 Comparison of measured and calculated flow rate;

Fig. 11 Comparison of measured and calculated flow rate

Fig. 1

Fig. 2

Fig. 3.

Fig. 4

Fig. 5

Fig. 6a,

Fig. 6b

Fig. 7

Fig. 8,

Fig. 9,

Fig. 10

Fig. 11