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Phonon-limited mobility in two-dimensional semiconductors with spin-orbit coupling

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The authors demonstrate that the Rashba spin-orbit interaction in low-dimensional semiconductors can enhance or reduce the electron-phonon scattering rate by as much as 25%. The underlying mechanism is that the electron-phonon scattering phase space for the upper (lower) Rashba band is significantly enhanced (suppressed) by the spin-orbit interaction. While the scattering time decreases for the upper level, the mobility of the level increases due to an additional term in the electron velocity. © 2007 American Institute of Physics. [DOI: 10.1063/1.2783203]

The effects of spin-orbit interaction (SOI) on semiconductor nanostructures are currently attracting considerable interest due to the interesting physics and the potential application in spintronic devices. The Rashba and Dresselhaus SOIs (Refs. 2–4) play an important role in controlling and manipulating the spin and charge degrees of freedom in low-dimensional and nanoscale semiconductor systems. Experiments have confirmed that the strength of Rashba SOI can be tuned by the gate voltage in InGaAs-based two-dimensional electron gas and GaAs-based two-dimensional hole gas.

In addition, SOI can be used to modify and improve the electrical and optical properties of electronic systems. In narrow-gap semiconductor nanostructures, such as InAs and In$_{1-x}$Ga$_x$As quantum wells, the inversion asymmetry of the confining potential due to the presence of the heterojunction results in the spin splitting (or spontaneous spin splitting) of the carriers in the absence of any applied magnetic field. Most works in systems with Rashba SOI deal with the properties of spin transport, spin polarization, and optical absorption. For systems with two natural plasmon lines, SOI serves as a tuning mechanism of the optical spectral confinement among various collective excitations. We have shown recently that SOI can also induce a transverse electrical current in the presence of random impurity scattering as long as the time-reversal symmetry is broken.

The electron-phonon interaction plays a significant role in systems with SOI. It breaks the universality of the spin-Hall conductivity. The electron polaron effective mass is enhanced by SOI. In this work, we study the effect of SOI in the electron mobility due to the electron-phonon interaction. It is shown that SOI changes the electron-phonon scattering time and electron mobility in opposite ways; i.e., for the Rashba branch with a reduced scattering time, the mobility increases. Existing techniques for transport measurement can be employed to determine the scattering rate and electron mobility.

The results obtained from experimental studies indicated that for III-V compound based low-dimensional semiconductor systems at intermediate excitation levels (i.e., within the electron temperature range 10 K $<$ $T$ $<$ 40 K), the electron-acoustic-phonon interactions are mainly via coupling with deformation-potential acoustic phonons. In this work, we limit ourselves to the situation where electron-acoustic-phonon interaction is mainly via electron interactions with the deformation-potential acoustic phonons. For GaAs, only the longitudinal-acoustic-phonon mode is connected with the deformation potential.

We consider a two-dimensional electronic system in the $x$-$y$ plane in narrow-gap semiconductor nanostructures (e.g., InGaAs/InAlAs quantum wells). The Hamiltonian of a free electron is given as

$$H_0 = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{\alpha}{\hbar}(\sigma_1 p_y - \sigma_2 p_x),$$

where $m$ is the electron effective mass. The wave function can be written in the form of $\psi(x,y) = u_k(x,y)\xi_{\lambda,k}$, where $u_k(x,y) = \exp(ik_xx + ik_yy)$ and $\xi_{\lambda,k}$ is a spinor. The eigenvalue is

$$E_{\lambda}(k) = \frac{\hbar^2 k^2}{2m} + \lambda \alpha k,$$

where $k = (k_x,k_y)$, $k = |k|$, and $\lambda = \pm 1$. The wave function can be written in the form of

$$\xi_{\lambda,k} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \lambda e^{-i\varphi} \end{pmatrix},$$

where $\cos \varphi = k_y/k$.

We start with the Boltzmann equation for electrons under a weak probing field of $\mathbf{F}$. 

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\[-\frac{e}{\hbar} \cdot \frac{\partial f_x(k)}{\partial k} = \left( \frac{\partial f_x(k)}{\partial t} \right)_{\text{coll}} = -\frac{f_x(k) - f_{x0}(k)}{\tau(k)} \]  

and

\[-\frac{\partial f_x(k)}{\partial t} \bigg|_{\text{coll}} = \sum_{k', \lambda'} W_{\lambda \lambda'}(k', k) f_x(k)[1 - f_{\lambda'}(k')] - W_{\lambda \lambda'}(k, k') f_{\lambda'}(k')[1 - f_x(k)]. \tag{5} \]

where \( f_{x0}(k) = [1 + e^{(E_x(k) - \mu)/k_B T}]^{-1} \) is the Fermi-Dirac function, \( \mu = E_F - m^* \alpha^2 \hbar^2 \) is the chemical potential, and \( T_0 \) is the temperature. \( W_{\lambda \lambda'}(k', k) \) is the transition rate for scattering an electron from a state \( |k_x, k_y, \lambda \rangle \) to a state \( |k'_x, k'_y, \lambda' \rangle \), and it obeys the principle of detailed balance, \( W_{\lambda \lambda'}(k', k) = W_{\lambda' \lambda}(k, k') \). It can be written as

\[ W_{\lambda \lambda'}(k', k) = W_{\lambda \lambda'}^e(k', k) + W_{\lambda \lambda'}^{\text{ph}}(k', k). \tag{6} \]

The transition rate due to the electron-phonon interactions can be written in terms of Fermi’s golden rule,

\[ W_{\lambda \lambda'}^{\text{ph}}(k', k) = \frac{2 \pi}{\hbar} |H'_{\lambda \lambda'}(k', k)|^2 \delta(E_x(k') - E_x(k) - \hbar \omega_Q). \tag{7} \]

where \(+ (-)\) corresponds to the process of phonon absorption (emission). The matrix element is given as \( H'_{\lambda \lambda'}(k', k) = (k'_x, \lambda' | H_{\text{ph}} | k, \lambda) \). Here, \( H_{\text{ph}} \) is the electron-phonon interaction via deformation potential, given as \( H_{\text{ph}} = E_D \Delta(r) \) with \( E_D \) the deformation potential constant. By making use of Eqs. (1)–(3), we obtain

\[ |H'_{\lambda \lambda'}(k', k)|^2 = \sum_{q, q_0} \frac{E^2_F \hbar}{2M \omega_Q} [g^{\lambda \lambda'}(k', k) \delta_{\lambda', q_0} \delta_{q, q_0}] \times \left[ N_0 + \frac{1}{2} + \frac{1}{2} \right], \tag{8} \]

with \( Q = (q, q_0) \) and \( q = (q_x, q_y) \), where \( \omega_Q \) is the phonon frequency. \( M \) is the ion mass, \( N_Q = [\exp(\hbar \omega_Q/k_B T_0) - 1]^{-1} \) is the phonon distribution function, \( T_0 \) is the lattice temperature, and \( g^{\lambda \lambda'}(k \pm q, k) = [\delta_{\lambda \lambda'} \delta_{q, q_0}] = (1/2)[1 + \lambda \lambda' \cos(\varphi_{q_0} - \varphi_k)] \). We solve the scattering equation by employing the following approximations: (i) the distribution function is expanded in terms of the probing field \( F \), \( f_x(k) = f_{x0}(k) + f_{x1}(k) + f_{x2}(k) + \cdots \); (ii) the phonon energy is assumed to be much smaller in comparison to the electron energy. The inverse scattering time can be obtained as

\[ \frac{1}{\tau(k)} = \sum_{\lambda', \lambda''} W_{\lambda \lambda'}(k', k) \left[ 1 - \frac{U_{\lambda'}(k')}{U_{\lambda''}(k')} \right]. \tag{9} \]

In Fig. 1, the electron inverse scattering time is plotted. The following parameters of InAs-based structures have been used in our calculation: \( E_D = 5.08 \) eV, \( \rho = 5.66 \times 10^3 \) kg m\(^{-3}\), \( v_1 = 4.2 \times 10^5 \) m s\(^{-1}\), \( n_z = 4 \times 10^{12} \) m\(^{-2}\), \( m^* = 0.038 m_0 \), \((k_0^2) = 2 \pi n_0 \times 8 \pi \times 10^{14} \) m\(^{-2}\), and \( E_F = 25.267 \) meV. The SOI parameter is taken to be in the range of 0–5 \times 10^{-11} \) eV m. The correction of the scattering rate due to the SOI is of the gigahertz order. The rate increases (decreases) linearly with the Rashba parameter for the upper (lower) Rashba branch. The rate at zero SOI is around 6 GHz and the SOI induced correction can be as high as 25%. The increase and decrease of the rate for the two branches can be understood as follows. The electron-acoustic-phonon scattering is governed by the energy and momentum conservation laws, as well as the allowed phase space for the electronic transitions. For a given \( k = k_F \) and since \( k_F < k_F^0 < k_F \), all electrons in the upper branch in the vicinity of this \( k \) can be scattered by phonons via absorption and emission. However, the electrons in the lower branch can only make transitions via phonon absorption because states below that \( k \) value are nearly fully occupied. These corrections will decrease slowly with temperature.

The electrical current of the \( \lambda \) branch driven by a dc field along the \( x \) direction is given as

\[ j_{\lambda x} = -\frac{e}{h} \int v_{\lambda x}(k) f_{x0}(k) dk_xdk_y dy = \frac{e^2 F}{k_B T_0} \int \tau(k) f_{x0}(k)(1 - f_{x0}(k))[v_{\lambda x}(k)]^2 dk. \tag{10} \]

The electron velocity in the \( \lambda \) branch is given as

\[ v_{\lambda x} = \frac{1}{h} \frac{\partial E_x(k, k_x)}{\partial k_x} = \frac{h k_x}{m} + \lambda \alpha \frac{k_x}{h}. \tag{11} \]

For applications such as diffusive device modeling, what is needed is the total effect of scattering on the transport. This is expressed as a mobility. We identify the electron mobility in the \( \lambda \) branch as

\[ \mu_{\lambda x} = \frac{J_{\lambda x}}{e n_{\lambda x} F}. \tag{12} \]

with \( n_{\lambda x} = (k_0^2)/4 \pi \) and \( k_0^2/k_F^2 = \sqrt{1 - \alpha^2}/4 - \lambda \alpha/2 \).

The fractional changes of electron mobility in each branch as a function of SOI parameter at fixed temperature are shown in Fig. 2. It can be seen that the mobility depends on the Rashba parameter in an opposite way to that of the scattering time. While the \( \tau_+ \) decreases with \( \alpha \), \( \tau_- \) increases with \( \alpha \). This indicates that the SOI induced mobility changes are mainly due the additional velocities \( \delta v^\pm_{\lambda x} = \pm \alpha k_x/k \). \( \delta v^\pm_{\lambda x} = \pm \alpha k_x/k \) will be a dominant term at small \( k \). As a result, the mobility of the upper branch is greatly enhanced and that of the lower branch is reduced. Our results also indicate that the velocity effect overestimates the change of mobility. This overestimation is corrected by the SOI dependent electron-phonon scattering time. While the SOI can enhance or reduce the electron-phonon scattering rate and the electron mobility, it does not significantly alter the temperature dependence of phonon-limited mobility of the system at low temperatures. As a result, the fractional correction of the mobility has a
weak temperature dependence at low temperatures. As temperature increases (e.g., \(T > 80 \text{ K}\)), the mobility corrections show a stronger increase (decrease) with temperature for the upper (lower) branch. Such temperature dependence is shown in Fig. 3.

To make our discussion complete, we now introduce the Dresselhaus SOI term to the Hamiltonian, \(H = \beta / \hbar (\sigma_y p_x - \sigma_x p_y)\). The energy dispersion and the phase factor are now given as

\[
E_{k,\lambda} = \frac{h^2 k^2}{2m} + \lambda k \sqrt{\alpha^2 + \beta^2 - 2 \alpha \beta \sin 2 \theta}
\]

and

\[
\tan \phi = \frac{\alpha \cos \theta - \beta \sin \theta}{\alpha \sin \theta - \beta \cos \theta}
\]

where \(\cos \theta = k_x / k\) and \(\sin \theta = k_y / k\). In the case of \(\alpha = \beta\), the problem can be exactly diagonalized. The situation of \(\alpha = \beta\) can be achieved by applying an additional gate voltage. The scattering rate in this case exhibits strong angular dependence. Furthermore, the scattering rate of the two SOI branches are out of phase; i.e., along the direction where the electron-phonon scattering is the strongest for one branch, it is the weakest for the other branch, as shown in Fig. 4.

In conclusion, we have shown that SOI can be used to significantly change the electron-acoustic-phonon scattering time. However, the change of electron mobility is dominated by the electron velocity. The results presented here should be measurable in a dc transport experiment.

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