University of Wollongong

Research Online

Faculty of Engineering and Information
Sciences - Papers: Part A

Faculty of Engineering and Information
Sciences

1-1-2015

Briefing: effect of drain installation patterns on rate of consolidation

Cholachat Rujikiatkamjorn

University of Wollongong, cholacha@uow.edu.au

Buddhima Indraratna

University of Wollongong, indra@uow.edu.au

Follow this and additional works at: <https://ro.uow.edu.au/eispapers>

Part of the [Engineering Commons](#), and the [Science and Technology Studies Commons](#)

Research Online is the open access institutional repository for the University of Wollongong. For further information contact the UOW Library: research-pubs@uow.edu.au

Briefing: effect of drain installation patterns on rate of consolidation

Abstract

Prefabricated vertical drains (PVDs) are employed to accelerate consolidation by decreasing the drainage path length. In the present study, using analytical solutions, an attempt was made to evaluate and quantify the effectiveness of two non-conventional PVD installation patterns, involving a parallel drain wall compared with a circular drain ring pattern, in contrast to conventional PVD installation (square or triangular patterns). The governing equations are based on the equal strain theory including the smear effect, and they provide a relative comparison between the two newly proposed installation patterns and the conventional square PVD grid, in terms of both the consolidation time and the equivalent drain spacing. The comparisons between the new and conventional installation patterns are made based on a single drain analysis and the density of PVDs per unit area.

Disciplines

Engineering | Science and Technology Studies

Publication Details

Rujikiatkamjorn, C. & Indraratna, B. (2015). Briefing: effect of drain installation patterns on rate of consolidation. *Proceedings of the Institution of Civil Engineers - Ground Improvement*, 168 (4), 236-245. *Proceedings of the Institution of Civil Engineers - Ground Improvement*

Briefing: Effect of drain installation patterns on rate of consolidation

1 Chalachat Rujikiatkamjorn BEng, MEng, PhD
Associate Professor, Centre for Geomechanics and Railway Engineering, School of Civil Engineering, Faculty of Engineering, University of Wollongong, Wollongong City, NSW, Australia

2 Buddhima Indraratna BSc, MSc, DIC, PhD, FTSE, FIEAust, FASCE, FGS
Professor of Civil Engineering, Faculty of Engineering; Director, Centre for Geomechanics and Railway Engineering, University of Wollongong, Wollongong City, NSW, Australia

Prefabricated vertical drains (PVDs) are employed to accelerate consolidation by decreasing the drainage path length. In the present study, using analytical solutions, an attempt was made to evaluate and quantify the effectiveness of two non-conventional PVD installation patterns, involving a parallel drain wall compared with a circular drain ring pattern, in contrast to conventional PVD installation (square or triangular patterns). The governing equations are based on the equal strain theory including the smear effect, and they provide a relative comparison between the two newly proposed installation patterns and the conventional square PVD grid, in terms of both the consolidation time and the equivalent drain spacing. The comparisons between the new and conventional installation patterns are made based on a single drain analysis and the density of PVDs per unit area.

Notation

B	half-width of the influence zone (m)
b_s	half-width of the smear zone (m)
b_w	half-width of the drain wall (m)
c_h	horizontal coefficient of consolidation for undisturbed soil (m^2/day)
c_{hs}	horizontal coefficient of consolidation for remoulded soil in smear zone (m^2/day)
k_h	coefficient of permeability in the undisturbed zone (m/s)
k'_h	coefficient of permeability in the smear zone (m/s)
l	length of the drain (m)
m_v	coefficient of volume compressibility (m^2/kN)
r	radius (m)
r_e	radius of influence zone (m)
r_i	radius of circular drain (m)
r_s	radius of smear zone (m)
r_w	equivalent radius of drain (m)
S	radius of smear zone/equivalent radius of drain (r_s/r_w)
S_{PVD}	influence zone of PVDs
S_{RING}	influence zone of drain rings
S_{WALL}	influence zone of drain walls
$T_{h,\text{PVD}}$	time factor for PVDs

$T_{h,\text{RING}}$	time factor for drain rings
$T_{h,\text{WALL}}$	time factor for drain walls
t	time (days)
t_{PVD}	required time for 90% degree of consolidation for PVDs
t_{RING}	required time for 90% degree of consolidation for drain ring
t_{WALL}	required time for 90% degree of consolidation for drain wall
U_{PVD}	degree of consolidation by PVDs
U_{RING}	degree of consolidation by drain rings
U_{WALL}	degree of consolidation by drain walls
$\bar{u}$	average excess pore pressure (kPa)
u_i	excess pore pressure in undisturbed zone (kPa)
u'_i	excess pore pressure in smear zone (kPa)
V	volume of soil mass (m^3)
x	radius of influence zone/radius of circular drain (r_e/r_i)
x_{OPT}	optimum ratio between radius of the influence zone and radius of circular drain
γ_w	unit weight of water (kN/m^3)
ε	vertical strain
μ_A	dimensionless factor for zone A of drain rings due to drain spacing and smear zone
μ_B	dimensionless factor for zone B of drain rings due to drain spacing and smear zone

μ_{PVD}	dimensionless factor for PVDs due to drain spacing and smear zone
μ_{WALL}	dimensionless factor for drain walls due to drain spacing and smear zone

1. Introduction

Prefabricated vertical drains (PVDs) are employed to accelerate the rate of excess pore water pressure dissipation through radial drainage that curtails the drainage path length considerably. Following the early development of cardboard-wick drains for the stabilisation of Scandinavian soft clay (Balasubramaniam *et al.*, 2005; Indraratna *et al.*, 2010; Kjellman, 1948), modern-day PVDs are composed of a plastic core with longitudinal drain channels, and a geotextile sleeve as a filter for protecting the plastic core. The desired degree of consolidation is achieved by choosing a suitable drain spacing and an appropriate installation pattern (Choa *et al.*, 2001; Hawlader *et al.*, 2002; Indraratna *et al.*, 2014; Jamiolkowski *et al.*, 1983; Pothiraksanon *et al.*, 2010). Conventionally, PVDs can be installed in a triangular or a square pattern, with almost equal efficiency (Figures 1(a) and 1(b)), but the latter is often preferred by contractors for convenience.

At the embankment centreline where lateral displacement is

negligible and one-dimensional consolidation is valid, the unit-cell analysis is simple and accurate. The unit-cell theories for the axisymmetric condition were proposed by Barron (1948), Chai *et al.* (2001), Hansbo (1981) and Hird *et al.* (1992). Indraratna *et al.* (2005), Indraratna and Redana (2000) and Richart (1957) introduced equivalent two-dimensional (plane strain) unit-cell formulations, which could be more conveniently incorporated in finite-element modelling. In a unit-cell analysis for a single vertical drain, the equivalent circular area of the surrounding soil (converted from a hexagonal or a square influence zone) is considered in order to analyse the consolidation process. Apart from the conventional square or triangular installation patterns, a number of PVDs can be installed at a much closer spacing to mimic either a large circular drain ring or a drain wall (Figures 1(c) and 1(d)). The drain-ring and drain-wall patterns are selected, because: a potential reduction in consolidation time can be obtained via larger drain spacing, thereby avoiding the adverse possibility of overlapping smear zones; and the existing PVD installation rigs can be readily modified to execute these patterns in the field. The installation of a drain wall is expected to be straightforward (i.e. each PVD can be installed in a straight line), while when installing a drain ring the rig needs to be modified in such a way that the mandrel can be moved horizontally to form a

Figure 1. Plan view of drain installation patterns: (a) square pattern; (b) triangular pattern; (c) drain walls; (d) drain rings installed in a square pattern

circular shape. The installation cost will depend on the number of PVDs required and the thickness of the soft soil, but will be higher than for conventional PVD patterns. However, the much quicker consolidation time will amply compensate for the higher initial cost of installation.

2. Analytical solutions under a unit-cell condition

In the present study, the analytical solutions were based on the equal strain concept, and were divided into three categories: PVDs with a conventional installation pattern; the drain ring; and the drain wall. For each unit-cell analysis, the effect of constant permeability in the smear zone was considered, but the effect of well resistance was ignored, assuming sufficiently large discharge capacity of the PVD. Figure 2 illustrates the unit cells adopted for the analytical solutions described below.

3. PVDs with conventional installation pattern

Figure 2(a) shows the plan view of a soil cylinder with a central vertical drain, where r_w is the equivalent radius of the drain, r_s is the radius of the smear zone and r_e is the radius of the influence zone. The coefficient of permeability in the smear and undisturbed zones is k'_h and k_h , respectively. An analytical solution for the degree of consolidation by the PVDs U_{PVD} can be given by (Hansbo, 1981)

$$1. \quad U_{PVD} = 1 - \exp\left(\frac{-T_{h,PVD}}{\mu_{PVD}}\right)$$

where $T_{h,PVD}$ is a time factor for the PVDs, and μ_{PVD} is a dimensionless factor for PVDs due to drain spacing and the smear zone, given by

$$\begin{aligned} \mu_{PVD} = \frac{1}{8} \left\{ \frac{n^2}{n^2 - 1} \left[\ln\left(\frac{n}{s}\right) + \frac{k_h}{k'_h} \ln(s) - \frac{3}{4} \right] \right. \\ \left. + \frac{s^2}{n^2 - 1} \left(1 - \frac{s^2}{4n^2} \right) \frac{k_h}{k'_h} \frac{1}{n^2 - 1} \right. \\ \left. \times \left(\frac{s^4 - 1}{4n^2} - s^2 + 1 \right) \right\} \end{aligned}$$

where $n = r_e/r_w$, $s = r_s/r_w$ and $T_{h,PVD} = c_h t / 4r_e^2$ (where c_h is the horizontal coefficient of consolidation for undisturbed soil, and t is time).

3.1 Drain wall

The band drains can be installed to create a series of parallel drain walls, as shown in Figure 1(c). Figure 2(b) presents the associated unit cell, where b_w is the half-width of the drain wall, b_s is the half-width of the smear zone and B is the half-width of the influence zone. According to Indraratna and Redana (2000), an analytical solution for consolidation by a drain wall U_{WALL} is given by

Figure 2. Unit-cell analysis: (a) PVDs in a conventional installation pattern; (b) drain wall; (c) drain ring

$$2. \quad U_{WALL} = 1 - \exp\left(\frac{-T_{h,WALL}}{\mu_{WALL}}\right)$$

where $T_{h,WALL}$ is a time factor for drain walls and μ_{WALL} is a dimensionless factor for drain walls due to drain spacing and the smear zone, given by

$$\mu_{\text{WALL}} = \frac{1}{8} \left[\alpha + \frac{k_h}{k'_h} (\beta) \right], \quad \alpha = \frac{2}{3} \frac{(p-q)^3}{p^2(p-1)}$$

$$\beta = \frac{2(q-1)}{p^2(p-1)} \left[p(p-q-1) + \frac{1}{3}(q^2+q+1) \right],$$

$$p = \frac{B}{b_w}, \quad q = \frac{b_s}{b_w}, \quad T_{h,\text{WALL}} = \frac{c_h t}{4B^2}$$

3.2 Drain ring

PVDs can also be installed close to each other to form a relatively large-diameter ring to allow concurrent consolidation of both the inner and the outer portions of the soil (Figure 1(d)). Figure 2(c) shows a unit cell of a circular drain ring, where r_i is the radius of circular drain, x is the ratio between the radius of the influence zone and that of the circular drain, and r_s is the thickness of the smear zone, which is assumed to be equal to that of the interior (zone A) and exterior (zone B) regions of the circular drain. Detailed derivations are given in the Appendix.

Analytical solutions for consolidation by a circular drain in both zones A and B can be given as shown in the following sections.

3.3 Zone A

The degree of consolidation by drain rings in zone A $U_{A,\text{RING}}$ is given by

$$3. \quad U_{A,\text{RING}} = 1 - \exp \left(\frac{-T_{h,\text{RING}}}{\mu_A} \right)$$

where μ_A is a dimensionless factor for zone A of the drain rings due to drain spacing and the smear zone, and $T_{h,\text{RING}}$ is the time factor for the drain rings

$$\mu_A = \frac{1}{32} \left\{ \frac{k_h}{k'_h} [1 - (1-m)^4] + (1-m)^4 \right\},$$

$$m = \frac{r_s}{r_i}, \quad T_{h,\text{RING}} = \frac{c_h t}{4r_i^2}$$

3.4 Zone B

The degree of consolidation by drain rings in zone B $U_{B,\text{RING}}$ is given by

$$4. \quad U_{B,\text{RING}} = 1 - \exp \left(\frac{-T_{h,\text{RING}}}{\mu_B} \right)$$

where μ_B is a dimensionless factor for zone B of the drain rings due to drain spacing and the smear zone

$$\mu_B = \frac{x^2}{8} \left\{ \frac{x^2}{x^2-1} \left[\ln \left(\frac{x}{1+m} \right) + \frac{k_h}{k'_h} \ln(1+m) - \frac{3}{4} \right] \right.$$

$$+ \frac{(1+s)m^2}{m^2-1} \left[1 - \frac{(1+m)^2}{4x^2} \right]$$

$$\left. + \frac{k_h}{k'_h} \frac{1}{x^2-1} \left[\frac{(1+m)^4-1}{4x^2} - (1+m)^2 + 1 \right] \right\}$$

To optimise the consolidation time, it is assumed that the consolidation responses in both zones A and B are the same. By equating Equations 3 and 4, the optimum ratio between the radius of the influence zone and that of the drain ring (x_{OPT}) can be determined. If $1 < k_h/k'_h < 7$, $0.06 \text{ m} < r_s < 0.18 \text{ m}$ and $0.4 \text{ m} < r_i < 2.4 \text{ m}$, x_{OPT} varies between 1.50 and 1.55.

4. Parametric study and discussion

4.1 A Single drain analysis

The parameters assumed were as follows: $r_w = 0.03 \text{ m}$, $1 < k_h/k'_h < 7$, $0.06 \text{ m} < r_s < 0.12 \text{ m}$ and $b_w = 0.0015 \text{ m}$ (half-width of a PVD). Both the PVDs and the drain rings are installed in a square pattern. For the circular drain, the optimum ratio x_{OPT} was used. The corresponding PVD spacing in terms of the size of the influence zone of the PVDs (S_{PVD}), drain walls (S_{WALL}) and drain rings (S_{RING}) are $1.83r_e$, $2B$ and $1.83x_{\text{OPT}}r_i$, respectively. Figure 3 shows the variation in the equivalent spacing of the drain rings and drain walls, in comparison with PVDs installed in a square pattern. The equivalent spacing was determined when the consolidation curves for the three patterns coincided (i.e. equating Equation 2, 3 or 4 with Equation 1). As an example, for $S_{\text{PVD}} = 1.0 \text{ m}$, depending on the smear-zone characteristics, the equivalent drain spacings for the drain rings and drain walls vary in the range 4.5–6.6 and 1.97–2.75 m, respectively. It can be seen that both drain rings and drain walls that provide larger drainage boundaries can offer a much larger spacing, compared with the conventional PVD spacing installed in a square pattern, to achieve the same degree of consolidation. Figure 4 presents the plan view of the installation patterns and the time-dependent degree of consolidation curves obtained using Equations 1–4, based on a drain spacing of 1 m (i.e. $2x_{\text{OPT}}r_i = 2r_s = 2B = 1 \text{ m}$) and using the following parameters: $r_w = 0.03 \text{ m}$, $k_h/k'_h = 2$, $r_s = 0.12 \text{ m}$, $b_w = 0.0015 \text{ m}$ (half-width of PVD). As suggested above, PVDs with a typical width of 0.1 m can be installed closely to form a large circular drain ring or a drain wall. A circular drain ring with 1 m spacing installed in a square pattern would require a drain radius of 0.36 m (based on Equations 3 and 4). Therefore, with no spacing between adjoining PVDs, a total of 23 PVDs are required to form a drain ring, whereas ten PVDs can make a drain wall (Figure 4(a)). A sketch showing the plan

Figure 3. Equivalent drain spacing (a) and (b) Drain Ring, (c) and (d) drain wall

view of a vertical drain with PVDs installed at 1 m spacing in three patterns is shown in Figure 4(a). There are numerous factors affecting the installation cost of PVDs on a particular project, including the subsurface conditions, the size of the project, the prevailing wage rates and the type of drain used. In ideal conditions, the installation cost may be estimated as AU\$1.2 per metre length, plus mobilisation. For a single drain installed at 1 m PVD spacing, a total of 23 and 10 PVDs are required for a drain ring and a drain wall, respectively, and the installation cost can be 23- and 10-fold compared with a single PVD installed in a convention pattern.

Continuous installation of PVDs to form ring and wall patterns may create significant soil disturbance and practical difficulties, and would contribute to retarded consolidation. However, this can be overcome by the exacerbated consolidation attributed to the much larger drainage boundaries. The following two hypothetical cases were examined.

- Case A: all installation patterns have the same smear-zone characteristics and the same horizontal coefficient of consolidation in the undisturbed zone ($c_h = 0.005 \text{ m}^2/\text{day}$).
- Case B: the installation of drain rings and drain walls

Figure 4. Analysis of a single drain: (a) plan view of vertical drains installed at 1 m spacing in three installation patterns. Time-dependent degree of consolidation (not to scale): (b) $c_h = 0.005 \text{ m}^2/\text{day}$; (c) $c_h = 0.0025 \text{ m}^2/\text{day}$

decreases the overall c_h by 50% (i.e. $c_h = 0.0025 \text{ m}^2/\text{day}$), and the permeability in the smear zone reduces further to half of that in the undisturbed zone.

For a given degree of consolidation of 90%, in case A the time required for consolidation for the PVD, drain wall and drain ring are 14, 60 and 250 days, respectively (Figure 4(b)). Thus the consolidation time for the drain rings and drain walls can be significantly reduced by 18- and four-fold, respectively, compared with the conventional PVD square pattern. This is because drain rings provide the longest drainage boundary, with the drain walls having the next longest drainage boundary. Figure 4(c) shows the consolidation curves for case B. It can be seen that the required times for consolidation for the drain wall and drain ring increase to 27 and 120 days, respectively. The corresponding rates of consolidation are still faster than those for a traditionally installed PVD pattern, even though the smear zone is extended as far as the influence zone.

4.2 Comparison for the same number of PVDs

As shown by the single drain analysis, although the consolidation time can be decreased significantly using the new installation patterns, the material and installation costs can be higher than for conventional installation pattern. The drain installation patterns were then analysed with the same number of PVDs per given area, so that the material and installation costs were practically the same in each case. Two hypothetical cases were examined.

- Case C – drain wall: a single drain wall installed per unit width, with 10 PVDs at 1 m spacing, with the following parameters: $B = 1.0 \text{ m}$, $k_h/k'_h = 2$, $b_s = 0.18 \text{ m}$, $b_w = 0.0015 \text{ m}$ (half-width of a PVD). For the same number of PVDs (with a typical width of 0.1 m) per unit area as in case D used to create the desired drain-wall configuration, the corresponding influence zone radius r_e was calculated to be 0.17 m (Figure 5(a)).
- Case D – drain ring: a single drain ring with $r_i = 0.9 \text{ m}$, and 56 PVDs installed at a spacing of 2.51 m in a square pattern. For the same number of PVDs per unit area as in case C installed individually, the corresponding influence zone radius r_e was calculated to be 0.18 m (Figure 5(b)).

From the above, when PVDs are installed individually, the size of the drain influence zone becomes similar to that of the smear zone, implying that the entire soil region encompassing the PVDs is fully disturbed due to the close proximity of the drains (Walker and Indraratna, 2007). Shogaki and Kaneko (1994) showed that the soil compressibility due to disturbance can increase by about 10-fold of its original value. Therefore, the ratio of the coefficients of consolidation between the disturbed and undisturbed soil (c_{hs}/c_h) can vary between 0.02 and 0.5 of the original value. For the undisturbed soil, in both cases C and D, a value of $c_h = 0.005 \text{ m}^2/\text{day}$ with $k_h/k'_h = 2$ was adopted. For the cases of individual PVDs, $c_{hs} = 1 \times 10^{-4}$ to $2.5 \times 10^{-4} \text{ m}^2/\text{day}$ was adopted to capture the overlapping smear effect.

The ratios between the required time for 90% consolidation for a drain wall (t_{WALL}) or a drain ring (t_{RING}) and the PVDs (t_{PVD}) as a function of the variation in the consolidation ratio were used for the purposes of comparison (Figure 5(c)). When c_{hs}/c_h is less than 0.1, the time required to reach a certain degree of consolidation for PVDs installed individually becomes larger than the corresponding values for the drain ring and the drain wall. This confirms that an increasingly closer drain spacing can retard the consolidation process, although the effective drainage path is shorter. In this respect, the alternative drain installation patterns, such as drain ring and drain wall, would be preferred in order to mitigate this problem.

While the analytical model presented here implies the possible advantages of creating large drain rings and drain walls with substantially increased drainage boundaries and much wider spacing, the practical difficulties of installing individual PVDs very close to each other to form continuous drain rings and walls, with the inevitable soil disturbance, require further study in field trials. In particular, field investigations are imperative to characterise the increased soil disturbance in the case of drain rings and drain walls as proposed herein, in order to accurately predict the actual extent of consolidation attributed to these proposed non-conventional PVD patterns.

5. Conclusion

The effectiveness of non-traditional drain installation patterns (i.e. circular drain ring and drain wall) on the rate of soft-soil consolidation, compared with conventional PVD installation patterns, was examined using analytical formulations and unit cell conditions. It was found that when PVDs are installed at 1 m drain spacing in a traditional square pattern, the equivalent drain spacing for drain rings and drain walls varies between 4.5–6.6 and 1.97–2.75 m, respectively, to achieve the same consolidation response. This much larger increase in spacing for drain rings and drain walls can be attributed to the substantially longer drainage boundaries compared with conventional PVD patterns.

For consolidation of 90% with a 1 m drain spacing, the computed consolidation times for drain rings, drain walls and the conventional square PVD pattern were found to be 14, 60 and 250 days, respectively. However, a total of 23 individual 100 mm wide PVDs would be required to form an effective drain ring, whereas 10 PVDs could establish a drain wall. There are numerous factors affecting the installation cost of PVDs on a particular project, including the subsurface conditions, the size of project, prevailing wage rates and the type of drain used. In ideal conditions, the installation cost may be estimated as AU\$1.2 per metre length, plus mobilisation. For a single drain installed at 1 m spacing, the installation cost can be 23- and 10-fold that of a single PVD installed in a conventional pattern. For a predetermined drain density per given area, the consolidation time required when PVDs are installed individually increases when the ratio of the coefficients of consolidation is less than 0.1.

Figure 5. Comparison for the same number of PVDs: (a) case C – drain wall; (b) case D – drain ring; (c) variation in the ratios of the time required for consolidation with the ratio of the coefficients of consolidation (not to scale)

Although the proposed analytical solutions suggest a very effective way of achieving increased soil consolidation by two non-traditional drain installation patterns via drain rings and drain walls, field trials are imperative to examine the possible practical difficulties, as well as to characterise the increased soil disturbance that may undermine the validity of these analytical formulations.

Acknowledgement

Cholachat Rujikiatkamjorn gratefully acknowledges the financial support received from the Early Career Researcher Award through the ARC Centre of Excellence for Geotechnical Science

and Engineering. The authors would like to thank Mr Pankaj Baral for his assistance during the manuscript preparation.

Appendix

The derivation of the analytical solution for consolidation around a circular drain is given below.

A1.1 Zone A

Considering the inner cylindrical soil ($0 \leq r \leq r_i$), the excess pore pressure gradients in the smear and undisturbed zones can be derived as

$$5a. \quad \frac{\partial u_i}{\partial r} = -\frac{\gamma_w}{2k_h} \frac{\partial \varepsilon}{\partial t} r \quad 0 \leq r \leq r_i - r_s$$

$$5b. \quad \frac{\partial u'_i}{\partial r} = -\frac{\gamma_w}{2k'_h} \frac{\partial \varepsilon}{\partial t} r \quad r_i - r_s \leq r \leq r_i$$

where k_h and k'_h are the permeability in the undisturbed zone and smear zone, respectively; u_i and u'_i are the excess pore pressure in the undisturbed zone and smear zone, respectively; γ_w is the unit weight of water; V is the volume of the soil mass; ε is the vertical strain; and r is the radius.

Integrating Equations 5a and 5b in the r direction with the boundary conditions (a) $u'_i(r_i) = 0$ at $r = r_i$ and (b) $u'_i(r_s - r_s) = u_i(r_s - r_s)$, the distributions of excess pore pressures in zone A in can be expressed as

$$6a. \quad u_i = \frac{\gamma_w}{4k_h} \frac{\partial \varepsilon}{\partial t} [(r_i - r_s)^2 - r^2] + \frac{\gamma_w}{4k'_h} \frac{\partial \varepsilon}{\partial t} [r_i^2 - (r_i - r_s)^2] \quad 0 \leq r \leq r_i - r_s$$

$$6b. \quad u'_i = \frac{\gamma_w}{4k'_h} \frac{\partial \varepsilon}{\partial t} (r_i^2 - r^2) \quad r_i - r_s \leq r \leq r_i$$

The mean excess pore pressure $\bar{u}$ is determined using

$$7. \quad \bar{u} \pi r_i^2 l = \int_0^l \int_{(r_i - r_s)}^{r_i} 2\pi u'_i r dr dz + \int_0^l \int_0^{(r_i - r_s)} 2\pi u_i r dr dz$$

where l is the length of the drain.

Substituting Equations 6a and 6b into Equation 7, combining with the well-known compressibility relationship $\partial \varepsilon / \partial t = -m_v \partial \bar{u} / \partial t$, and integrating, the average excess pore pressure is given by

$$8. \quad \bar{u} = -\frac{m_v \gamma_w d_i^2}{k_h \partial t} \mu_A$$

where

$$\mu_A = \frac{1}{32} \left\{ \frac{k_h}{k'_h} [1 - (1 - s)^4] + (1 - s)^4 \right\}$$

Rearranging Equation A4 and then integrating by applying the boundary condition $\bar{u} = \sigma_1$ at $t = 0$ gives

$$9. \quad \frac{\bar{u}}{\sigma_1} = \exp \left(\frac{-T_{h,RING}}{\mu_A} \right)$$

where

$$T_{h,RING} = \frac{c_h t}{4r_i^2}$$

The average degree of consolidation $U_{A,CIR}$ can now be evaluated as

$$10. \quad U_{A,RING} = 1 - \exp \left(\frac{-T_{h,RING}}{\mu_A} \right)$$

A1.2 Zone B

Using the same procedures as for zone A, the average degree of consolidation in zone B, $U_{B,CIR}$, can be expressed as

$$11. \quad U_{B,RING} = 1 - \exp \left(\frac{-T_{h,RING}}{\mu_B} \right)$$

where

$$\mu_B = \frac{x^2}{8} \left\{ \frac{x^2}{x^2 - 1} \left[\ln \left(\frac{x}{1 + m} \right) + \frac{k_h}{k'_h} \ln(1 + m) - \frac{3}{4} \right] + \frac{(1 + s)m^2}{m^2 - 1} \left[1 - \frac{(1 + m)^2}{4x^2} \right] + \frac{k_h}{k'_h} \frac{1}{x^2 - 1} \left[\frac{(1 + m)^4 - 1}{4x^2} - (1 + m)^2 + 1 \right] \right\}$$

REFERENCES

- Balasubramaniam A, Phienweij N, Bergado D, Bolton M and Oh E (2005) Deep-well pumping in the Bangkok Plain and its influence on ground improvement development with surcharge and vertical drains. *Ground Improvement* **9**(4): 149–162.
- Barron RA (1948) Consolidation of fine-grained soils by drain wells. *Transactions of the American Society of Civil Engineers*, **113**: 718–754.
- Chai JC, Shen SL, Miura N and Bergado DT (2001) Simple method of modelling PVD improved subsoil. *Journal of Geotechnical Engineering* **127**(11): 965–972.

- Choa V, Bo MW and Chu J (2001) Soil improvement works for Changi east reclamation projects. *Proceedings of the ICE – Ground Improvement* **5(4)**: 141–153.
- Hansbo S (1981) Consolidation of fine-grained soils by prefabricated drains. In *Proceedings of 10th International Conference on Soil Mechanics and Foundation Engineering, Stockholm*. Balkema, Rotterdam, the Netherlands, vol. 3, pp. 677–682.
- Hawlder BC, Imai G and Muhunthan B (2002) Numerical study of the factors affecting the consolidation of clay with vertical drains. *Geotextiles and Geomembranes* **20(4)**: 213–239.
- Hird CC, Pryah IC and Russel D (1992) Finite element modelling of vertical drains beneath embankments on soft ground. *Géotechnique* **42(3)**: 499–511, <http://dx.doi.org/10.1680/geot.1992.42.3.499>.
- Indraratna B and Redana IW (2000) Numerical modelling of vertical drains with smear and well resistance installed in soft clay. *Canadian Geotechnical Journal* **37(1)**: 133–145.
- Indraratna B, Rujikiatkamjorn C and Sathananthan I (2005) Radial consolidation of clay using compressibility indices and varying horizontal permeability. *Canadian Geotechnical Journal* **42(5)**: 1330–1341.
- Indraratna B, Rujikiatkamjorn C, Kelly R and Buys H (2010) Environmentally sustainable soft soil improvement via vacuum and surcharge preloading. *Ground Improvement* **163(1)**: 31–42.
- Indraratna B, Perera D, Rujikiatkamjorn C and Kelly R (2014) Soil disturbance analysis due to vertical drain installation. *Proceedings of the ICE – Geotechnical Engineering*, <http://dx.doi.org/10.1680/geng.14.00052>.
- Jamiołkowski M, Lancellotta R and Wolski W (1983) Pre-compression and speeding up consolidation. *Proceedings of the 8th European Conference on Soil Mechanics and Foundations, Helsinki, Finland*, vol. 3, pp. 1201–1206.
- Kjellman W (1948) Accelerating consolidation of fine grain soils by means of cardboard wicks. *Proceedings of the 2nd ICSMFE*, vol. 2, pp. 302–305.
- Pothiraksanon C, Saowapakpiboon J, Bergado DT, Voottipruex P and Abuel-Naga HM (2010) Soft ground improvement with solar-powered drainage. *Proceedings of the ICE – Ground Improvement* **163(1)**: 23–30.
- Richart FE (1957) A review of the theories for sand drains. *Journal of the Soil Mechanics and Foundations Division, ASCE* **83(3)**: 1–38.
- Shogaki T and Kaneko M (1994) Effects of sample disturbance on strength and consolidation parameters of soft clay. *Soils and Foundations* **34(3)**: 1–10.
- Walker R and Indraratna B (2007) Vertical drain consolidation with overlapping smear zones. *Géotechnique* **57(5)**: 463–467.

WHAT DO YOU THINK?

To discuss this paper, please email up to 500 words to the editor at journals@ice.org.uk. Your contribution will be forwarded to the author(s) for a reply and, if considered appropriate by the editorial panel, will be published as a discussion in a future issue of the journal.

Proceedings journals rely entirely on contributions sent in by civil engineering professionals, academics and students. Papers should be 2000–5000 words long (briefing papers should be 1000–2000 words long), with adequate illustrations and references. You can submit your paper online via www.icevirtuallibrary.com/content/journals, where you will also find detailed author guidelines.