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Turbulence structures in non-uniform flows

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ABSTRACT

This study investigates turbulence structures in steady and non-uniform flows. Equations of Reynolds shear stress and turbulent velocity fluctuations are derived and their physical interpretations are explained. The theoretical results show that, different from previous studies, the variation of water surface can generate the wall-normal velocity, resulting in deviations of Reynolds shear stress and turbulence intensities from those in uniform flows. A self-similarity relationship is found between the Reynolds shear stress and turbulence intensities in non-uniform flows. The existence of self-similarity indicates that the effect of non-uniformity does not influence the mixing length. An empirical equation has been proposed to express the relationship based on experimental data available in the literature. Good agreement is achieved between the measured and predicted turbulence intensities by applying the self-similarity relationship.

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1. Introduction and background

In natural streams, non-uniform flow is a ubiquitous phenomenon that is not fully understood even for the simplest case—gradually varied non-uniform flow in open channel. Turbulence structures are crucial for predicting sediment and contaminant motions in rivers, lakes, and coastal waters where the effect of non-uniformity is significant. Over the half-century, the turbulence structures of uniform flow have been studied extensively by many researchers, such as Laufer [6], Eckelmann [3], Grass [4], Nakagawa et al. [7], Steffler et al. [15], Dou [2], Nezu and Azuma [9], etc., but few researchers have studied the effect of non-uniformity on the turbulence characteristics. The turbulent structures in non-uniform flow are different from that in uniform flow. The accelerating flow generally hampers the turbulence whereas the decelerating flow strengthens the turbulence [13]. Unfortunately, the underlying mechanism for the phenomenon has not been well revealed, and the quantitative description for the turbulent characteristics is not available in the literature.

The uniform flow has been widely investigated theoretically and experimentally. Prandtl [11] proposed his well-known mixing-length theorem and obtained a rational formula for the velocity distribution. He postulated that, in an analogy with the molecular motion of a gas, when a small fluid element moves in a flow field, its momentum does not change until the fluid element has moved a distance termed as the mixing length. Physically, the

mixing length refers to the average path that fluid elements move freely without collision and momentum exchange.

The momentum equation in uniform flow yields

$$\left(\frac{-\overline{u'v'}}{u_*^2} \right)_{uf} = 1 - \frac{y}{h} \quad (1)$$

where $-\overline{u'v'}$ = Reynolds shear stress; u_* = shear velocity; y = distance normal to the wall; h = water depth. u' and v' are fluctuating velocities in streamwise and wall-normal directions, respectively, the subscript “uf” denotes the uniform flow.

Prandtl expressed u' and v' in the following way:

$$u' \propto l \frac{d\bar{u}}{dy} \quad (2)$$

$$v' \propto l \frac{d\bar{u}}{dy} \quad (3)$$

in which $\bar{u}$ = time-averaged velocity in the streamwise direction, l = mixing length = κy , and κ = Karman constant = 0.4.

The mixing-length theorem yields the log-law for mean velocity distribution and it has been proven to be valid in many cases. However, this theorem cannot be used to express the turbulence intensities (i.e., $\sqrt{u'^2}$ and $\sqrt{v'^2}$) directly because Eqs. (2) and (3) show the turbulence vanishes at points where $d\bar{u}/dy$ is equal to zero (at points of maximum velocity). This is certainly not the case in non-uniform flow because turbulent mixing does not vanish at points of maximum velocity. Therefore, researchers have to resort to the empirical way to express the turbulence intensities. By fitting experimental data, Nakagawa et al. [7] obtained the following equations:

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Nomenclature

h	water depth	$-\bar{u}'v'$	Reynolds shear stress
F	function	U	instantaneous velocity in the streamwise direction
G	function	V	instantaneous velocity in the wall-normal direction
l	mixing length	y	distance normal to the wall
S	energy slope	ε	eddy viscosity
$\bar{u}$	time-averaged velocity in the streamwise direction	τ	shear stress
$\bar{v}$	wall-normal velocity	κ	Karman constant
u_*	shear velocity	ν	kinematic viscosity
u' and v'	turbulent velocity fluctuations	ρ	fluid density

$$\left(\frac{\sqrt{\bar{u}'^2}}{u_*}\right)_{uf} = D_u \exp(-y/h) \quad (4a)$$

$$\left(\frac{\sqrt{\bar{v}'^2}}{u_*}\right)_{uf} = D_v \exp(-y/h) \quad (4b)$$

where $D_u = 2.3$ and $D_v = 1.27$.

However, the turbulent structures in non-uniform flow significantly differ from that in uniform flow, as observed by Cardoso et al. [1], Nezu et al. [10], Kironoto and Graf [5] and Song and Graf [14]. Song [12] measured the velocity and turbulent structures of non-uniform flows using an acoustic-Doppler-velocity profiler (ADVP) in a 16.8 m long, 0.60 m wide and 0.80 m deep flume. His channel tests were performed with aspect ratios of $3.0 < B/h < 4.0$ where B is the channel width and h is the flow depth at the measuring station.

ADVP is a non-intrusive measuring device to record the instantaneous distributions of streamwise ($\bar{u}$) and wall-normal ($\bar{v}$) velocities, as well as Reynolds shear stress ($-\rho\bar{u}'v'$). Error in Song's measurement by ADVP was about 15% [14]. The typical distributions of mean velocity, Reynolds shear stress and turbulence intensities measured by Song and his colleague are shown in Fig. 1. The empirical equations, as shown in Eqs. (5a), (5b) and (6), developed by Song and Chiew [13] are included for comparison:

$$D_u = 0.6(0.1\beta^2 + \beta) + 3 \quad (5a)$$

$$D_v = 0.3(0.1\beta^2 + \beta) + 1.5 \quad (5b)$$

$$\beta = \frac{gh}{u_*^2} \left(\frac{dh}{dx} - S \right) \quad (6)$$

where g = gravitational acceleration, S = bed slope, x = streamwise direction. For uniform flow (where $dh/dx = 0$, $\beta = -1$), the values of $D_u = 2.46$ and $D_v = 1.23$ are very close to that obtained by Nakagawa et al. [7]. However, the mechanism of deviation in D_u and D_v was not explained by Song and Chiew [13].

The most important characteristic in non-uniform flow is the existence of non-zero wall-normal velocity. The decelerating flow generates the upward wall-normal velocity whereas the accelerating flow yields the downward velocity [17,18]. The so-called "wall-normal velocity" refers to the time-averaged velocity in the wall-normal direction, which is zero at a solid wall but becomes non-zero in the main flow region under non-uniform flow. Even in uniform flow, this wall-normal velocity could be non-zero due to the existence of secondary currents.

The importance of non-zero wall-normal velocity in uniform flow has been recognized by Nezu and Nakagawa [8]. They summarized the influence of wall-normal velocity on the turbulence structures and other phenomena, as shown in Table 1.

It can be seen from Table 1 and Fig. 1 that the turbulent characteristics are totally different if the directions of wall-normal velocity are opposite. Therefore, the investigation of turbulence structures in non-uniform flows could be useful to understand the underlying mechanism of all odd phenomena listed in Table

1, as all of them are directly related to the direction of wall-normal velocity. In other words, regardless of non-uniformity or secondary currents, the wall-normal velocity is responsible for the re-distributions of Reynolds shear stress, turbulence intensities, etc. Hence, it is not surprising that the relationship of wall-normal velocity and the turbulent structures would be of interest to a wide circle of professionals, including hydraulic engineers, coastal engineers, geologists, hydrologists, geographers, and other scientists.

It is worthwhile to note that in Fig. 1, all data points of turbulence structures of decelerating flows locate on the right-hand side of the solid lines, whereas the data points in accelerating flow are on the left-hand side of the solid lines. The separation between data points implies that the mechanism for the deviation of turbulence intensities could be identical to the deviation of Reynolds shear stress. The mechanism for the deviation of Reynolds shear stress has been discussed by Yang and Lee [18]. As a continuous effort to investigate the effect of non-uniformity, this study concentrates on the turbulence intensities in non-uniform flow. The objectives of this paper include (1) to investigate the influence of the wall-normal velocity on turbulent structures; (2) to establish empirical relations between the wall-normal velocity, Reynolds shear stress, and turbulent structures; and (3) to compare the measured turbulence intensities with the developed model.

2. Reynolds shear stress distribution and wall-normal velocity

Consider a steady and non-uniform flow shown in Fig. 2, the bed slope is denoted by S and ΔS is the difference between the surface slope and bed slope. Obviously if $\Delta S = 0$, the flow is uniform; if $\Delta S > 0$ the flow is accelerated; if $\Delta S < 0$, the flow is decelerated. The fluid particle on the free surface cannot leave the free surface, thus it is understandable that the vector of velocity at $y = h$ in a non-uniform flow has an angle with x -axis. The component of surface velocity in y -direction is either positive or negative, as shown in Fig. 2. For the uniform flow where the surface line is parallel to the bed ($\Delta S = 0$), the wall-normal velocity must be zero.

As seen from the above analysis, the momentum flux caused by mean flow (i.e., $\bar{u} \cdot \bar{v}$) is non-zero in non-uniform flow. In the upper region, the value of $\bar{u} \cdot \bar{v}$ is very large relative to the Reynolds shear stress, where $-\bar{u}'v' \approx 0$ at $y \approx h$. However, at the boundary where $y = 0$, the momentum flux is zero as $\bar{v} = 0$. In the near wall region the Reynolds shear stress is maximum.

Yang and Lim [19] theoretically proved that in the field of turbulent flow the total shear stress should be expressed by $\tau_{(y)} = -\rho\bar{u}'v' - \rho\bar{u} \cdot \bar{v}$. By this definition, $\bar{u} \cdot \bar{v}$ plays crucial role for the distributions of velocity and turbulent structures in the surface region. In contrast, $\tau_{(y)} \approx -\rho\bar{u}'v'$, or the Reynolds shear stress dominate the turbulent structure in the bed region. This phenomenon has been well known for a long time, but the importance of $\bar{u} \cdot \bar{v}$ has been underestimated.

The above argument can be also derived from the continuity equation, that is

Fig. 1. Typical distributions of mean velocity (a), Reynolds shear stress (b) and streamwise and vertical turbulence intensities (c and d) in accelerating ($v < 0$) and decelerating ($v > 0$) flows where solid lines represent the values in uniform flows and the dashed lines are Eqs. (4a) and (4b), based on Song's [12] data.

Table 1

Summary of turbulent structures of cellular secondary currents in open channels, after Nezu and Nakagawa [8, p. 124]

Type	Upflow region ($\bar{v} > 0$)	Downflow region ($\bar{v} < 0$)
Primary mean velocity	Low	High
Bed shear stress	Low	High
Turbulence intensity farther from the bed	High	Low
Reynolds shear stress farther from the bed	High	Low
Suspended load	High	Low
Bed load	Low	High
On water surface	Boil lines, divergence	Foam lines, convergence
River bed form	Ridges	Troughs
Bed roughness	Fine sand, smooth bed strips	Coarse sand, rough bed strips

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \quad (7)$$

where $\bar{w}$ is the mean velocity in the spanwise direction. Along the channel central line, $\partial/\partial z = 0$ due to the symmetrical condition. Hence

$$\bar{v} = - \int_0^y \frac{\partial \bar{u}}{\partial x} dy \quad (8)$$

$\partial \bar{u}/\partial x > 0$ in accelerating flows, Eq. (8) shows the appearance of negative wall-normal velocity. In decelerating flow ($\partial \bar{u}/\partial x < 0$), a positive wall-normal velocity exists. The wall-normal velocity at the free surface can be expressed by Yang and Lee [18]

$$\bar{v}_h = -\bar{u}_h \Delta S \quad (9)$$

where $\bar{u}_h$ and $\bar{v}_h$ are velocities at the free surface in x - and y -directions, respectively.

Fig. 2. Sketch for non-uniform flow where y is normal to the solid wall.

Based on the total shear stress distribution in uniform flow, the corresponding deviation in non-uniform flow can be written as in Eq. (10) [18]

$$\left(-\frac{\overline{uv}}{u_*^2}\right) - \left(-\frac{\overline{uv}}{u_*^2}\right)_{uf} = f\left(\frac{y}{h}\right) \quad (10)$$

where $u = \bar{u} + u'$, $v = \bar{v} + v'$, and

$$f = \Delta S \left(\frac{\bar{u}_h}{u_*}\right)^2 \frac{y}{h} \quad (11)$$

As seen in Eq. (10), due to the negative $\bar{v}$ in accelerating flows, the total shear stress in non-uniform is smaller than that in uniform flow. In contrast, the total shear stress in decelerating flow would be higher than that in uniform flow.

Eq. (11) is a function of ΔS , suggesting that the perturbation on the free surface would lead to deviation of the total shear stress distribution from the classical Reynolds shear stress (i.e., Eq. (1)). Positive ΔS yields a positive deviation, or vice versa. If $\Delta S = 0$, the deviation is zero. In other words, Eq. (10) indicates that the total

shear stress in non-uniform flow could be treated as the sum of two linear distributions (the two dotted lines in Fig. 3). One is produced by the lower boundary and the other one is induced by the upper boundary, the summation of two shear stresses follows straight lines (see Fig. 4).

The deviation of Reynolds shear stress from the dotted line is identical to the deviation of momentum flux by mean flow. Fig. 3 illustrates that the summation of two independent linear distributions is same as that expressed by Eq. (10), or the variation of Reynolds shear stress can be expressed by

$$u_*^2 \left(1 - \frac{y}{h}\right) - \overline{u'v'} = -\left(\bar{u}_h \bar{v}_h \frac{y}{h} - \bar{u} \cdot \bar{v}\right) \quad (12)$$

The left-hand side of Eq. (12) is a function of velocity fluctuations, whereas the right-hand side depends on the mean velocities, indicating the mean flow structures and turbulent structures are interchangeable. In other words, the turbulence can be simply defined as a combination of decelerating/accelerating flows. Song and Chiew [13] obtained that

$$\bar{v} = \Delta S \bar{u} \frac{y}{h} \quad (13)$$

Also, the left-hand side of Eq. (12) is the deviation of Reynolds shear stress derived from that in uniform flow (see Fig. 3); therefore, Eq. (12) can be expressed by

$$\frac{d(-\overline{u'v'})}{dS} = (\bar{u}_h^2 - \bar{u}^2) \frac{y}{h} \quad (14a)$$

Eq. (14) is the alternative form of Eq. (12), and it expresses how the Reynolds shear stress varies with respect to the surface slope.

Likewise, the left-hand side of Eq. (10) is the variation of total shear stress and f is a function of ΔS . The derivative form of Eq. (10) can be deduced as

$$\frac{d(-\overline{uv})}{dS} = \bar{u}_h^2 \frac{y}{h} \quad (14b)$$

Fig. 3. Total shear stress $(-\overline{uv})$ distributions in accelerating and decelerating flows.

Fig. 4. Distribution of total shear stress difference based on Song's [12] experimental data, where U and V are instantaneous velocities in x and y directions.

From Eqs. (14a) and (14b), one has

$$\frac{d(-\bar{u} \cdot \bar{v})}{dS} = \bar{u}^2 \frac{y}{h} \quad (14c)$$

Eqs. (14a)–(14c) express the influence of surface slope on the Reynolds shear stress in non-uniform. Physically these equations are the same as that derived by Yang and Lee [18], but a derivative form is provided in this study.

Reynolds shear stress can be expressed by

$$-\bar{u}'v' = \varepsilon \frac{d\bar{u}}{dy} \quad (15)$$

where ε is eddy viscosity.

From Eqs. (2) and (3), one can be deduced that the velocity gradient and turbulence structures in decelerating flow will be higher than those in accelerating flow.

3. Distribution of turbulent fluctuating velocities

The typical profiles of turbulent intensities in decelerating/accelerating flows shown in Fig. 1. At the same level y , the measured turbulence in the decelerating flow is significantly higher than that in accelerating flow. Thus, one can conclude that the turbulence is promoted by upflow but depressed by downflow. These

phenomena are the same as that in uniform flow where the upward and downward motions are caused by secondary currents (see Table 1).

It would be interesting to know how the wall-normal velocity affects the turbulence intensity. To understand this, one may trace back to the expressions of turbulent velocity fluctuations or u' and v' . The mixing-length theorem shown in Eqs. (2) and (3) states that u' and v' are proportional to the product of mixing length and velocity gradient. The wall-normal velocity does not change the mixing length; therefore, one gets

$$u' \propto l \frac{d\bar{u}}{dy} = l_{uf} \frac{d\bar{u}}{dy} \bigg|_{uf} \frac{d\bar{u}/dy}{(d\bar{u}/dy)_{uf}} \propto u'_{uf} \frac{d\bar{u}/dy}{(d\bar{u}/dy)_{uf}} \quad (16)$$

Noticing that the wall-normal velocity does not change the eddy viscosity. One has

$$\frac{d\bar{u}/dy}{(d\bar{u}/dy)_{uf}} \propto \frac{-\bar{u}'v'}{(-\bar{u}'v')_{uf}} \quad (17)$$

Eqs. (16) and (17) show that the turbulent velocity fluctuation depends on the gradient of velocity. Eq. (15) and Fig. 3 demonstrate that the gradient is higher if $\bar{v} > 0$, and it becomes smaller if $\bar{v} < 0$. It is understandable that an up-flow enhances the turbulence, but a down-flow suppresses the turbulence. Thus the turbulence intensity can be evaluated by

Fig. 5. Self-similar relation between turbulence intensities and Reynolds shear stress in non-uniform flow.

Fig. 6. Comparison of measured and predicted turbulence intensities in accelerating flows based on Song's [12] experimental data, where the open circles represent the measured $\sqrt{u'^2}/u_*$, the solid circles are the measured $\sqrt{v'^2}/u_*$, the lines are the calculated values from Eq. (19).

$$\frac{\sqrt{u'^2}}{\sqrt{u'^2}_{uf}} = F \left(\frac{-\bar{u}'v'}{-\bar{u}'v'_{uf}} \right) \quad (18a)$$

Likewise, one can write

$$\frac{\sqrt{v'^2}}{\sqrt{v'^2}_{uf}} = G \left(\frac{-\bar{u}'v'}{-\bar{u}'v'_{uf}} \right) \quad (18b)$$

The experimental data from Song's [12] are plotted in Fig. 5, in the form of normalized turbulence intensity versus the normalized Reynolds shear stress. As expected by Eqs. (18a) and (18b), there is a one-to-one relation between the relative turbulence intensities and relative Reynolds shear stress, and this relation can be approximated by

$$F = G = 0.6 + 0.4 \frac{-\bar{u}'v'}{-\bar{u}'v'_{uf}} \quad (19)$$

The self-similarity shown in Eq. (19) states that the deviation of turbulence intensities in non-uniform flow from that in uniform flow depends on the deviation of Reynolds shear stress. The concave distribution of Reynolds shear stress (i.e., $-\bar{u}'v'/(-\bar{u}'v'_{uf}) < 1$) yields the concave distribution of turbulence intensities; conversely the convex distribution of Reynolds shear stress produces the convex profile of turbulence intensities. For a profile, if the measured Reynolds shear stress is linear, Eq. (19) predicts that the measured turbulence intensities can be expressed by Eqs. (4) and (5). Yang [16] found that Eq. (19) is also valid to express the turbulence intensities in uniform flow with presence of secondary currents:

In Eq. (19), if $-\bar{u}'v' = -\bar{u}'v'_{uf}$, $F = G = 1$, indicating that the turbulent intensities are identical to that in uniform flow ($\bar{v} = 0$). Obviously, if $(-\bar{u}'v')/(-\bar{u}'v'_{uf}) > 1$, then $F = G > 1$. This means that if the measured Reynolds shear stress is higher than that of uniform flow, then the corresponding turbulent intensities will be higher than the solid lines in Fig. 1. Likewise, if $(-\bar{u}'v')/(-\bar{u}'v'_{uf}) < 1$, Eq.

Fig. 7. Comparison of measured and predicted turbulence intensities in decelerating flows based on Song's [12] experimental data, where the open circles represent the measured $\sqrt{u'^2}/u_*$, the solid circles are the measured $\sqrt{v'^2}/u_*$, the lines are the calculated values from Eq. (19).

(19) gives $F = G < 1$, all data points in Fig. 1 are below the respective solid lines. Hence, the deviation of turbulence intensities from Eqs. (2) and (3) depends on the Reynolds shear stress derived from Eq. (1). As the Reynolds shear stress in non-uniform flow can be theoretically expressed by Eq. (10), one is able to assess the turbulence intensities in non-uniform flow. Figs. 6 and 7 show the comparison of measured and predicted turbulence intensities in accelerating and decelerating flows, respectively. The agreement is acceptable.

4. Conclusions

This study investigates the turbulence structures in non-uniform flow. It is found that the acceleration in longitudinal direction induces an up or down motion in the wall-normal direction. That motion in vertical direction is the most important characteristic of turbulence as observed by Reynolds. This study also explains the significant influence of the non-zero wall-normal velocity on the turbulence structures, such as the re-distributions of Reynolds shear stress, turbulence intensities, etc. The equations for Reynolds shear stress in non-uniform flow have been developed by taking the effect of non-uniformity into account, and the relationship between the Reynolds shear stress and turbulence intensity has been established. Good agreement between the measured and predicted turbulence intensities has been achieved. Based on this investigation, the following conclusions can be reached:

- (1) The upward velocity $\bar{v}$ enhances the Reynolds shear stress while the downward velocity $\bar{v}$ lessens the Reynolds shear stress. Subsequently the turbulence intensities are promoted by the upward velocity but depressed by the downward velocity. The distribution of total shear stress $(-\overline{u'v'} - uv)/u_*^2$ still follows a linear relation with relative water depth y/h .
- (2) The upward velocity results in convex distribution of Reynolds shear stress and turbulent intensities; the downward velocity leads to the concave distribution of Reynolds shear stress and turbulent intensities. The deviation of turbulent intensity in non-uniform flow from uniform flow depends on the ratio of Reynolds shear stress in the non-uniform flow to that in uniform flow, and this self-similar relation is obtained empirically in this investigation.
- (3) The existence of self-similarity indicates that the effect of non-uniformity does not influence the mixing length, and the fluctuating velocities are proportional to the mean velocity gradient. Good agreement can be achieved between the measured

and predicted turbulence intensities by using the self-similarity.

(4) The wall-normal velocity can be induced by non-uniformity. Accelerating flows produce downward velocity whereas decelerating flows generates upward velocity. The influence of wall-normal velocity on turbulent structures should not be underestimated. More theoretical and experimental studies are needed in the future.

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