1-1-2009

An age-based inspection and replacement policy for heterogeneous components

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An Age-Based Inspection and Replacement Policy for Heterogeneous Components

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***Abstract*—This paper considers a hybrid maintenance policy for a single component from a heterogeneous population. The component is placed in a socket, and the component and socket together comprise the system. The s-population of components consists of two sub-populations with different failure characteristics. By supposing that a component may be in a defective but operating state, so that there exists a delay time between defect arrival and component failure, we consider a novel maintenance policy that is a hybrid of inspection and replacement policies. There are similarities in this approach with the concept of "burn-in" maintenance. The policies are investigated in the context of traction motor bearing failures. Under certain circumstances, particularly when the mixture parameter is large, and the distribution of lifetimes for the two component types are well separated, the hybrid policy has significant cost savings over the standard age-based replacement policy, and over the pure inspection policy. In addition to the cost metric, the mean time between operational failures of the system under the hybrid policy can be used to guide decision-making. This maintenance policy metric is calculated using simulation, and using an approximation which assumes that operational failures occur according to a Poisson process with a rate that can be calculated in a straightforward way. The simulation results show good agreement with the approximation.**

***Index Terms*—Age based replacement, delay time, inspection maintenance.**

ACRONYM

MTBOF	mean time between operational failures

NOTATION

X	component age at defect arrival
f_X	probability density function (pdf) of X
F_X	cumulative distribution function (cdf) of X
H	delay time from defect arrival to subsequent failure
f_H	probability density function (pdf) of H
F_H	cumulative distribution function (cdf) of H
λ	mean delay time
Y	age at failure, so that $Y = X + H$
Δ	interval between inspections
K	number of inspections during the inspection phase
T	age at preventive replacement during the wear-out phase
C_{R}	cost of preventive replacement
C_{F}	cost of failure $(> \mathrm{C}_{\mathrm{R}})$
C_{I}	cost of inspection $(< C_{\mathrm{R}})$
U	cost of a renewal cycle (r.v.)
V	length of a renewal cycle (r.v.)
$\mathrm{E}(U)$	expected cost of a renewal cycle
$\mathrm{E}(V)$	expected length of a renewal cycle
$C_{\infty}(.)$	long run cost per unit time
$f_i(x)$	pdf of components from sub-population $i = 1, 2$
p	mixing parameter
η_i	characteristic life for components from sub-population $i = 1, 2$
β_i	shape parameter of pdf of components from sub-population $i = 1, 2$
μ	mean time between operational failures
ρ	probability that a replacement cycle ends with failure

Manuscript received March 30, 2008; revised October 16, 2008; accepted February 12, 2009. First published August 07, 2009; current version published December 01, 2009. Part of the work presented here was carried out during the visit of the second author to the University of Salford. This visit was supported by CAPES (the Brazilian National Mobility program), grant number: 1045/07-5. Associate Editor R. H. Yeh.

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Digital Object Identifier 10.1109/TR.2009.2026796

I. INTRODUCTION

CONSIDER a system comprising a single component, and a socket, which together provide some operational function (see [1]). Further, suppose that the distribution of time to failure for the component follows a mixture distribution so that the s-population of components comprises a sub-population of weak components, and a sub-population of strong components. The weak components have a short characteristic life, and give rise to early failure. The strong components have a long characteristic life, and are the subject of wear-out. What then is a suitable maintenance policy for such a system, when the new, replacement component is taken from the mixed population? Finkelstein & Esaulova [2] argue that periodic policies do not take account of systematic changes that occur in the pattern of components' aging from a mixed population. We propose a policy which is a hybrid of inspection maintenance and age based replacement: inspections are carried out over the early life of the system; preventive replacement is carried out during later life.

We will consider a system for which failure implies immediate cost-consequences. In this context, for inspection to be viable, we suppose that a defect may arise prior to failure, and that these defects are detectable at inspection. Thus, we suppose that the system can be in one of three states: good, defective, or failed. Following Christer [3], we call the time lapse between defect arrival and failure due to this defect the *delay time*. The cost-optimal hybrid policy can be determined given the costs of inspection, preventive replacement, and failure; knowledge of the s-distribution of the time to defect arrival; and knowledge of the s-distribution of the delay-time. Age based replacement [4] is a special case of the hybrid policy. A pure inspection model (periodically inspect over the system life, and replace if defective) is also a special case.

The model we propose is related to burn-in maintenance. In this policy, components are subjected to a burn-in test, a process used to improve the quality of products or systems after they have been produced [5]. The implication is that the s-population of components produced are heterogeneous, and poor quality components (with short operational lives) will be screened during the burn-in. This early life screening will be analogous to the inspection phase of the hybrid maintenance model that we propose. Burn-in maintenance modeling is concerned with determining, in combination, optima for the burn-in time, and the subsequent preventive maintenance policy for the operational phase [6], [7]. The burn-in process is not always suitable because a long burn-in period may be impractical. Our model is different in that both inspection and preventive replacement will be carried out during the operational phase.

Murthy & Maxell [8] proposed a policy for maintenance during the operational phase which is a mixture of age-replacement policies. This policy involved an additional test that could be performed to determine the nature of the component (whether it is type i, weak, or strong, say), and then preventively replaced components of type i at age T_i. Our model does not require the performance of such a test.

We suppose that the lifetime distribution for a component follows a mixture distribution. Such a mixture is illustrated in Fig. 1. Mixtures of this kind do not necessarily have an increasing failure (s-hazard) rate function [10]–[14], and a bathtub shaped hazard is one possible form [9]. Note that the fitting of mixture distributions to failure data will be problematic. This is because data will often possess an underlying structure that is not immediately apparent due to, for example, inspections, left and right censoring, or heterogeneity. It would be unfortunate to fit a two-parameter Weibull distribution to failures that arise from a mixture, and then adopt an age-based replacement policy based on the fitted two-parameter Weibull because the implied critical replacement age would be inappropriate for both sub-populations of components [8]. A full discussion of the fitting of Weibull distributions to data in this context is given in Jiang & Murthy [15].

Our failure model should be distinguished from a combined policy that addresses competing failure modes in a component. For example, given that a component possesses two failure modes, one might consider inspection as an appropriate intervention for one mode of failure, and age-based replacement as appropriate for a second mode of failure. The combination of these policies (inspection, and age based replacement) appears to be like the policy presented in this paper, but is in fact not designed to address the notion that a component arises from a mixed population of weak and strong components. The failure time distribution for a competing risks model is different from that for a mixture. On the other hand, if we assume that all components can fail according to failure mode 1, and some (a proportion) can also fail according to a second failure mode 2, then these latter components can be viewed as weak. The s-population as a whole is then a mixture of the weak and strong, and the failure time distribution is the same as ours in principal, but with a subtly different form.

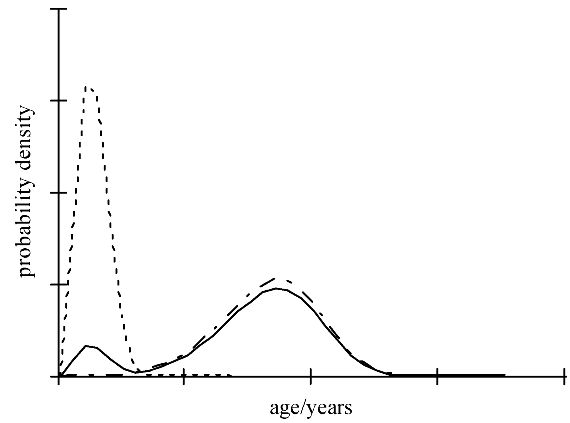

Fig. 1. Mixed distribution (——). Underlying Weibull distributions, $\mathrm{We}(\eta_1 = 3, \beta_1 = 2.5)$ (···), and $\mathrm{We}(\eta_2 = 18, \beta_2 = 5)$ (– – – – –), mixing parameter $p = 0.1$.

The layout of this paper is as follows. We first explicitly state our assumptions. Then we specify the policy, and calculate the long-run cost per unit time of the policy. We also discuss the reliability implications of the policy by considering the distribution of time between operational failures. The policy is illustrated using an example relating to the maintenance of traction motors used on the trains of a commuter railway. Finally, we present the conclusions of this work.

II. Assumptions

1) $f_X(x) = pf_1(x) + (1-p)f_2(x)$, where $f_1(x)$, and $f_2(x)$ respectively follow Weibull distributions with characteristic lives η_1, η_2, and shape parameters β_1, β_2.
2) Components may be in one of three states: good, defective, or failed.
3) The system is in the failed state iff the component is in the failed state.
4) The delay time, H, is s-independent of the time to defective arrival, X.
5) Inspections are perfect in that any defects present will be identified.
6) Defective components are replaced at inspection instantaneously, and the average cost of replacement of a defective component is C_{R}.
7) At the critical replacement age T, preventive replacement of a component is instantaneous, and again costs $C_{\mathrm{R}} < C_{\mathrm{F}}$.

8) Failed components are immediately apparent, cause operational failure of the system, and are replaced instantaneously with cost C_{F}.
9) The cost of inspection is $C_{\mathrm{I}} < C_{\mathrm{R}}$.
10) At replacement (whatever the state of the replaced component), the system is restored to the as-new state.

III. The Hybrid Maintenance Policy

To reduce early failures, the system is inspected with frequency $1/\Delta$ during the inspection phase up to age $K\Delta$. A component will be replaced at the time of the ith inspection if it is defective; it will be replaced on failure; and it will be replaced preventively at T if it survives to this age. The objective of inspection is to prevent early-life failures of weaker components. Inspections act as a natural, operational burn-in process, because the weak components will fail much earlier than strong components. The objective of preventive replacement, which takes place over a much longer time-scale, is to reduce wear-out failures in later life.

Failures are anticipated by a defective state. In the good and defective states, the system is operational. The notion of a defective state allows us to model inspections: if the delay-time is zero (two-state failure model), then, given our assumption that component failures lead to immediate operational failure, inspection is futile. Note that the model might be extended to consider a mixed population of delay-times, a proportion of which are zero [16]. This effectively relaxes the perfect inspection assumption because it implies that a proportion of failures cannot be prevented by inspection. We do not consider this model here however.

The decision variables in the model are K, T, and Δ. These are all age-related, so that on replacement, the inspection phase begins again. Thus, the maintenance model is analogous to age-based replacement. The as-new replacement assumption implies that we can use the renewal-reward theorem, and hence the long-run cost per unit time, as an objective function.

Within this framework, the length of a renewal cycle (time between renewals), V, can take different values, and

$$V = i\Delta \tag{1}$$

if $(i-1)\Delta < X < i\Delta \cap Y > i\Delta$, $(i = 1, \ldots, K)$. Thus, for example, $V = \Delta$, and the component is replaced at first inspection, if $X < \Delta \cap Y > \Delta$. Also,

$$V = Y,\ (i-1)\Delta < Y < i\Delta, \tag{2}$$

if $X > (i-1)\Delta \cap Y < i\Delta$, $(i = 1, \ldots, K)$;

$$V = Y,\ K\Delta < Y < T, \tag{3}$$

if $X > K\Delta \cap Y < T$; and

$$V = T \tag{4}$$

if $X > K\Delta \cap Y > T$.

The cost incurred in a cycle is given by

$$U = \begin{cases} iC_{\mathrm{I}} + C_{\mathrm{R}} & \text{if } V = i\Delta,\ i = 1, \ldots, K, \\ (i-1)C_{\mathrm{I}} + C_{\mathrm{F}} & \text{if } (i-1)\Delta < V < i\Delta,\ i = 1, \ldots, K, \\ KC_{\mathrm{I}} + C_{\mathrm{F}} & \text{if } K\Delta < V < T, \\ KC_{\mathrm{I}} + C_{\mathrm{R}} & \text{if } V = T. \end{cases} \tag{5}$$

Developing (1)–(4), we have that when $K > 0$ the expected renewal cycle length is

$$\begin{aligned} \mathrm{E}(V) = {} & \sum_{i=1}^{K} i\Delta \left[\int_{(i-1)\Delta}^{i\Delta} (1 - F_H(i\Delta - x))\, f_X(x) \mathrm{d}x \right] \\ & + \sum_{i=1}^{K} \int_{(i-1)\Delta}^{i\Delta} \int_{0}^{i\Delta - x} (x+h) f_H(h) f_X(x) \mathrm{d}h \mathrm{d}x \\ & + \int_{K\Delta}^{T} \int_{0}^{T-x} (x+h) f_H(h) f_X(x) \mathrm{d}h \mathrm{d}x \\ & + T \left[\int_{K\Delta}^{T} (1 - F_H(T-x))\, f_X(x) \mathrm{d}x + \int_{T}^{\infty} f_X(x) \mathrm{d}x \right]. \end{aligned}$$

In the same way, developing (5), we have that when $K > 0$, the expected cost per cycle is

$$\begin{aligned} \mathrm{E}(U) = {} & \sum_{i=1}^{K} (iC_{\mathrm{I}} + C_{\mathrm{R}}) \left[\int_{(i-1)\Delta}^{i\Delta} (1 - F_H(i\Delta - x))\, f_X(x) \mathrm{d}x \right] \\ & + \sum_{i=1}^{K} [(i-1)C_{\mathrm{I}} + C_{\mathrm{F}}] \int_{(i-1)\Delta}^{i\Delta} F_H(i\Delta - x) f_X(x) \mathrm{d}x \\ & + (KC_{\mathrm{I}} + C_{\mathrm{F}}) \int_{K\Delta}^{T} (F_H(T-x))\, f_X(x) \mathrm{d}x + (KC_{\mathrm{I}} + C_{\mathrm{R}}) \\ & \times \left[\int_{K\Delta}^{T} (1 - F_H(T-x))\, f_X(x) \mathrm{d}x + \int_{T}^{\infty} f_X(x) \mathrm{d}x \right]. \end{aligned}$$

Using the above expressions for $\mathrm{E}(U)$ and $\mathrm{E}(V)$, the optimal hybrid policy of inspection up to age $K\Delta$, and replacement at age T for components from a heterogeneous population, can be determined by minimizing the long-run cost per unit time:

$$C_{\infty}(T, \Delta, K) = \frac{\mathrm{E}(U)}{\mathrm{E}(V)}$$

with respect to K, Δ, and T.

The expressions simplify when $K = 0$ to

$$\begin{aligned} \mathrm{E}(V) = {} & \int_{0}^{T} \int_{0}^{T-x} (x + h) f_H(h) f_X(x) \mathrm{d}h \mathrm{d}x \\ & + T \left[\int_{0}^{T} (1 - F_H(T - x))\, f_X(x) \mathrm{d}x + \int_{T}^{\infty} f_X(x) \mathrm{d}x \right], \end{aligned}$$

and

$$\mathrm{E}(U)=C_{\mathrm{F}}\int_0^T (F_H(T-x))\,f_X(x)\mathrm{d}x$$
$$+C_{\mathrm{R}}\left[\int_0^T (1-F_H(T-x))\,f_X(x)\mathrm{d}x+\int_T^\infty f_X(x)\mathrm{d}x\right].$$

Expressions for $\mathrm{E}(U)$, and $\mathrm{E}(V)$ in the case $K=0$ could also be derived by noting that the hybrid policy reduces to age-based replacement with critical age T. Then, we have that $\mathrm{E}(V)=\int_0^T(1-F_Y(y))\mathrm{d}y$, and $\mathrm{E}(U)=C_{\mathrm{F}}F_Y(y)+C_{\mathrm{R}}(1-F_Y(y))$, where $Y=X+H$, and so $F_Y(y)=\int_0^y F_H(y-x)f_X(x)\mathrm{d}x$.

Also, when $K\to\infty$ (so that $T\to\infty$), we have a pure inspection model with replacement at failure, or at inspection when a defect is found; and $\mathrm{E}(U)$, and $\mathrm{E}(V)$ will have particular forms

$$\mathrm{E}(V)=\sum_{i=1}^{\infty}\left[i\Delta\int_{(i-1)\Delta}^{i\Delta}(1-F_H(i\Delta-x))\,f_X(x)\mathrm{d}x\right.$$
$$\left.+\int_{(i-1)\Delta}^{i\Delta}\int_0^{i\Delta-x}(x+h)f_H(h)f_X(x)\mathrm{d}h\mathrm{d}x\right]$$

and

$$\mathrm{E}(U)=\sum_{i=1}^{\infty}\left\{(iC_{\mathrm{I}}+C_{\mathrm{R}})\int_{(i-1)\Delta}^{i\Delta}(1-F_H(i\Delta-x))\,f_X(x)\mathrm{d}x\right.$$
$$\left.+\,[(i-1)C_{\mathrm{I}}+C_{\mathrm{F}}]\int_{(i-1)\Delta}^{i\Delta}F_H(i\Delta-x)f_X(x)\mathrm{d}x\right\}.$$

Note the hybrid inspection and replacement policies that have been developed to date [17] are based on the notion of increasing defect arrival rate, and minimal repair. For these policies, inspections will tend to be carried out with increasing frequency as the component reaches the critical age for replacement. This is in direct contrast to the policy developed in this paper.

IV. Distribution of the Time Between Operational Failures

For any system subject to some maintenance policy, it is interesting to determine the distribution of the time between operational failures. This distribution may be regarded as the system reliability [18]. For a simple system and policy, this distribution can be derived. For age replacement, for example, quantiles of the distribution can be explicitly obtained [19]. For the hybrid policy in this paper, the distribution of the time between operational failures may be obtained by Monte Carlo simulation (for a given set of parameter and policy values). We can also use the following approximation. The probability that a cycle (replacement interval) ends in failure, $\rho=\rho(K,\Delta,T)$, is given by

$$\rho=\sum_{i=1}^{K}\int_{(i-1)\Delta}^{i\Delta}F_H(i\Delta-x)f_X(x)\mathrm{d}x+\int_{K\Delta}^{T}F_H(T-x)f_X(x)\mathrm{d}x.$$

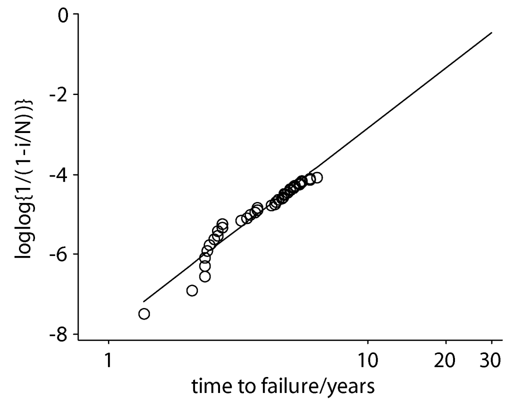

Fig. 2. Weibull plot for traction motor bearing failures. For a single s-population of components, this implies $\beta\approx 2.5$, and $\eta\approx 30$ years.

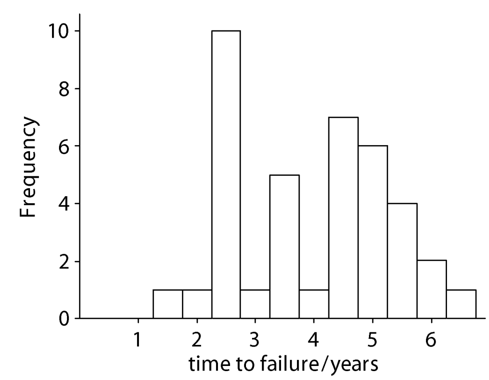

Fig. 3. Histogram of bearing failure times for 39 traction motors.

For small T and Δ, and large K, this probability will be small so that failures are rare events. Over some interval of time, $[0,t]$, the number of cycles is approximately $n=t/E(V)$, where $\mathrm{E}(V)$ is the expected cycle length. All intervals are s-independent, and so the number of failures in $[0,t]$ will have (approximately) a Poisson distribution with rate $r=n\rho/t=\rho/E(V)$. Thus, the times between operational failures are (approximately) exponentially distributed with mean

$$\mu=E(V)/rho. \tag{6}$$

The measure, μ, the mean time between operational failures (MTBOF), might be used in conjunction with some cost criterion (here the long-run cost per unit time), in a multiple criteria approach of the policy [20]. For example, given a reliability requirement expressed in terms of the MTBOF (say $\mu\geq\mu_R$), we can determine those policies for which this is true. That is, find that subset of the region $(K\geq 0,\Delta\geq 0,T\geq 0)$ such that $\mu\geq\mu_R$. Then, we can find the minimum cost policy in this subset. A reliability constraint could also be expressed in terms of some quantile of the distribution of the times between operational failures. Thus we may require $\Pr(Z>z_q)=q$ for specified z_q, and q, assuming $Z\sim\mathrm{Ex}\{\rho/E(V)\}$.

V. Case Study/Numerical Example

Scarf *et al.* [19] consider the lifetimes of bearings in 375V d.c. traction motors used by a commuter railway, and investigated preventive replacement policies for these bearings. The railway

Table I
OPTIMUM HYBRID POLICY FOR VARIOUS VALUES OF COST PARAMETERS, AND FAILURE MODEL PARAMETERS

β_1	η_1	β_2	η_2	p	λ	C_I	C_R	C_F	C^*	T^*	Δ^*	K^*	$C^*(K=0)$	T_0^*	$C^*(K,T=\infty)$	Δ_∞^*
2.5	3	5	18	0.1	0.5	0.05	1	10	0.224	10.25	0.85	5	0.240	10.49	0.373	0.38
2	3	5	18	0.1	0.5	0.05	1	10	0.225	10.25	0.87	5	0.240	10.49	0.373	0.38
3	3	5	18	0.1	0.5	0.05	1	10	0.222	10.25	0.83	5	0.240	10.49	0.373	0.37
2.5	2	5	18	0.1	0.5	0.05	1	10	0.216	10.25	0.6	5	0.242	10.51	0.374	0.37
2.5	4	5	18	0.1	0.5	0.05	1	10	0.228	10.81	1.44	7	0.238	10.46	0.371	0.38
2.5	3	4	18	0.1	0.5	0.05	1	10	0.253	10.45	0.99	10	0.273	9.75	0.376	0.38
2.5	3	3.5	18	0.1	0.5	0.05	1	10	0.269	10.23	0.88	11	0.298	9.35	0.378	0.38
2.5	3	3	18	0.1	0.5	0.05	1	10	0.288	10.21	0.75	13	0.331	8.94	0.381	0.40
2.5	3	5	12	0.1	0.5	0.05	1	10	0.313	7.51	0.78	9	0.348	7.05	0.479	0.28
2.5	3	5	24	0.1	0.5	0.05	1	10	0.171	13.67	0.84	5	0.183	13.90	0.311	0.46
2.5	3	5	18	0.1	0.25	0.05	1	10	0.243	10.29	2.74	1	0.246	10.32	0.479	0.32
2.5	3	5	18	0.1	1	0.05	1	10	0.198	11.44	1.33	8	0.231	10.80	0.290	0.49
2.5	3	5	18	0.05	0.5	0.05	1	10	0.185	9.86	2.96	1	0.188	9.89	0.362	0.38
2.5	3	5	18	0.15	0.5	0.05	1	10	0.254	10.64	0.64	7	0.294	11.01	0.384	0.36
2.5	3	5	18	0.1	0.5	0.025	1	10	0.202	11.25	0.68	16	0.24	10.49	0.297	0.25
2.5	3	5	18	0.1	0.5	0.075	1	10	0.233	10.4	1.75	2	0.24	10.49	0.430	0.50
2.5	3	5	18	0.1	0.5	0.05	1	2	0.103	14.56	0	0	0.103	14.56	0.126	
2.5	3	5	18	0.1	0.5	0.05	1	20	0.31	9.33	0.48	10	0.389	9.84	0.543	0.27

company uses 2296 traction motors (typically 32 per train), and over a period of study observed 39 bearing failures. These are shown in a Weibull plot, Fig. 2.

A histogram of bearing failure times is shown in Fig. 3. It is plausible that these failures are early life failures of bearings that arise from a mixed population; that is, we suppose that we are observing failures of the weak components only here. Furthermore, inspections and replacement may account for the reduced number of failures at ages 1, 2, 3, and 4, so that the mixture parameter is somewhat more than 39/2296. Plausible values may be in the range $p = 0.04$ to $p = 0.10$. We will assume a mixture with two sub-populations of bearings; the weak components with a Weibull distribution of time to defect (time in good state) with characteristic life $\eta_1 = 2.5$ (years), and shape parameter $\beta_1 = 3$; and the strong components with a Weibull distribution of time to defect (time in good state) with characteristic life $\eta_2 = 18$, and shape parameter $\beta_2 = 5$. Note that the parameters of the distribution of defect arrival time for the strong components are not based on the data here. The motors were preventively replaced at 7 years (this was the maintenance policy of the railway); therefore, we would expect to see only very few failures of long-lived (strong) components. An exponential distribution was arbitrarily chosen for the delay times, with mean $\lambda = 1/2$ (year). As the time to failure is the sum of the time to defective state plus the delay time, the actual failure time distribution (for weak components) will have a characteristic life of approximately 3 years. The variance of time to failure will of course be larger than that of the underlying time to defect arrival.

A hybrid maintenance policy for such a heterogeneous population of bearings is now investigated. For the purpose of this investigation, we take the preventive replacement cost as our unit of cost ($C_R = 1$), and consider a range of inspection and failure replacement costs. The results are shown in Table I. In this table, the long run cost per unit time of the optimum policy is C^*. The time unit is taken to be one year, although this is arbitrary. Minimum costs for the age based policy, $C^*(K = 0)$, and pure inspection policy, $C^*(K, T = \infty)$, are also shown along with the corresponding optimum values of the decision variables, T_0^*, and Δ_∞^*, respectively.

As can be observed in Table I, when the sub-populations are well separated (large β_2 or small η_1), a hybrid policy with $K^*\Delta^* < T^*$ is optimal. Two phases are then apparent: the early inspection phase, and the later non-inspection wear-out phase. As β_2 becomes smaller, and the sub-populations are less distinct, then $K^*\Delta^* \approx T^*$, and it is optimum to inspect over the entire life. When the cost of inspection is varied, the optimum policy behaves as expected: lower inspection costs lead to more inspections. Also, a longer mean delay time leads to more inspections, and vice versa, implying that inspections are only effective if there is sufficient delay between defect arrival, and consequent failure. The long run cost per unit time is presented in Fig. 4 (as a function of the age at preventive replacement), and Fig. 5 (as a function of the time between inspections during the inspection phase). Cost savings with respect to simpler policies can be observed in Table I. The saving with respect to pure age-based replacement $(C_\infty(K = 0, T^*) - C_\infty(T^*, \Delta^*, K^*))$ is between 5%, and 10%. The saving with respect to pure inspection $(C_\infty(K, T = \infty, \Delta^*) - C_\infty(T^*, \Delta^*, K^*))$ is larger.

Figs. 6–9 consider the MTBOF. Fig. 6 indicates that inspections should be spread over the life of the weak items; moderate values of the time between inspections are the most effective, and also, from Fig. 5, most cost effective. In Fig. 7, we observe that the MTBOF is higher for moderate values of the age at preventive replacement than for larger values of T. This result is true because, for $p > 0$, if we preventively replace at moderate T, then the new replacement component may be a weak item which can lead to early failure. Consequently, it would be better

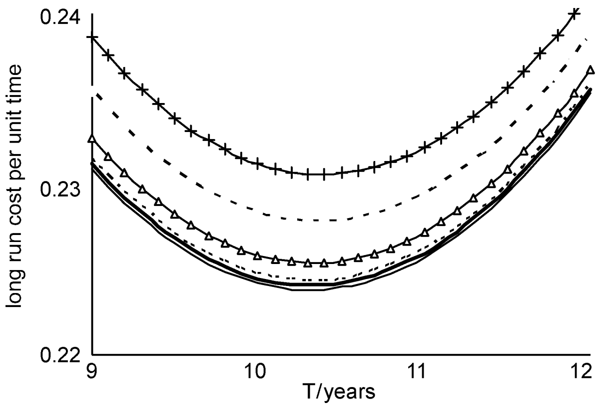

Fig. 4. Long run cost per unit time, $C_\infty(T, \Delta, K)$, as a function of age at preventive replacement, T, for $K = 1$ (— + —), $K = 2$ (– —— – ——), $K = 3$ (—Δ—), $K = 4$ (▬▬), $K = 5$ (——), $K = 6$ (– – – – –). $\Delta = \Delta^* = 0.86$. Parameter values: $p = 0.1$, $\eta_1 = 3$, $\eta_2 = 18$, $\beta_1 = 2.5$ $\beta_2 = 5$, $\lambda = 1/2$, $C_{\mathrm{R}} = 1$, $C_{\mathrm{F}} = 10$, $C_{\mathrm{I}} = 0.05$.

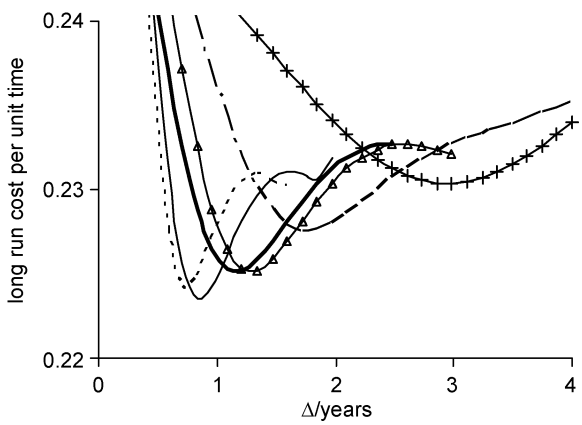

Fig. 5. Long run cost per unit time, $C_\infty(T, \Delta, K)$, as a function of inspection interval, Δ, for $K = 1$ (— + —), $K = 2$ (–— – —), $K = 3$(—Δ—), $K = 4$ (▬▬), $K = 5$ (——), $K = 6$ (– – – – –). T held at the optimum value specific for each value of K (for $K = 5, T = 10.248$). Parameter values: $p = 0.1$, $\eta_1 = 3$, $\eta_2 = 18$, $\beta_1 = 2.5$ $\beta_2 = 5$, $\lambda = 1/2$, $C_{\mathrm{R}} = 1$, $C_{\mathrm{F}} = 10$, $C_{\mathrm{I}} = 0.05$.

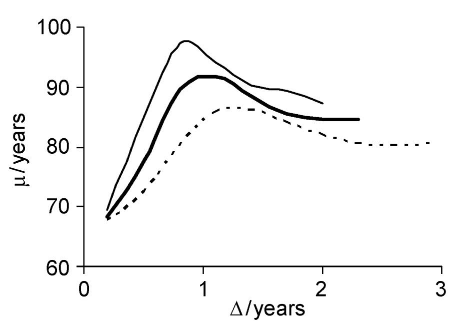

Fig. 6. For hybrid policy, MTBOF, μ, against inspection interval, Δ, for $K = 3$ (– – – – –), $K = 4$ (▬▬), $K = 5$ (——),. $T = 10.248$. Parameter values: $p = 0.1$, $\eta_1 = 3$, $\eta_2 = 18$, $\beta_1 = 2.5$ $\beta_2 = 5$, $p = 0.1$, $\lambda = 1/2$, $C_{\mathrm{R}} = 1$, $C_{\mathrm{F}} = 10$, $C_{\mathrm{I}} = 0.05$. MTBOF calculated using simulation of 10,240 cycles.

(lower cost, and higher reliability) to continue operating with the existing component, because, given that the existing component has survived to a moderate age, it is likely to be a strong item with large mean time to failure. In short, for the mixed defect arrival (and hence failure) distribution, the failure hazard at moderate t ($t \sim 5$ here) is larger than the failure hazard for larger t ($t \sim 10$ here). The size of this effect increases with the value of p, the mixing parameter (Fig. 8). A simple age based replacement policy will also display this behavior for a mixed failure distribution. Fig. 9 shows the distribution of times between operational failure for one simulation. Fig. 10 compares the MTBOF

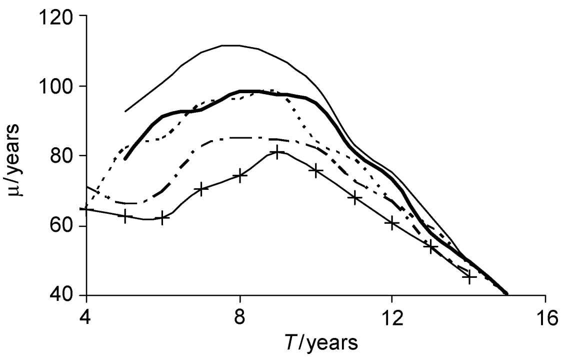

Fig. 7. For hybrid policy, MTBOF, μ, against age at preventive replacement, T, for $K = 1$ (— + —), $K = 2$ (– —— – ——), $K = 3$ (– – – – –), $K = 4$ (▬▬), $K = 5$ (——). $\Delta = 0.861$. Parameter values: $p = 0.1$, $\eta_1 = 3$, $\eta_2 = 18$, $\beta_1 = 2.5$ $\beta_2 = 5$, $\lambda = 1/2$, $C_{\mathrm{R}} = 1$, $C_{\mathrm{F}} = 10$, $C_{\mathrm{I}} = 0.05$. MTBOF calculated using simulation of 10,240 cycles.

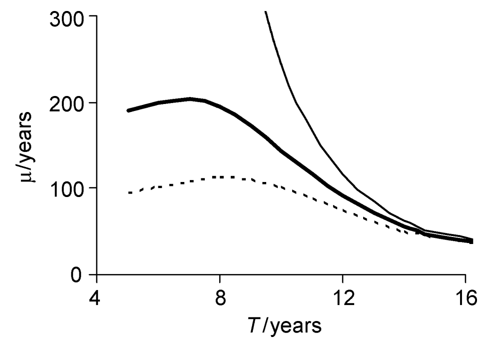

Fig. 8. For the hybrid policy, MTBOF, μ, against age at preventive replacement, T (with $\Delta = \Delta^* = 0.861$, $K = K^* = 5$) for various values of the mixing parameter, p, in the time to defect arrival distribution: $p = 0$ (——); $p = 0.05$ (▬▬); $p = 0.1$ (– – – – –). Parameter values: $\eta_1 = 3$, $\eta_2 = 18$, $\beta_1 = 2.5$ $\beta_2 = 5$, $\lambda = 1/2$, $C_{\mathrm{R}} = 1$, $C_{\mathrm{F}} = 10$, $C_{\mathrm{I}} = 0.05$. MTBOF calculated using the approximation (6), $\mu = \mathrm{E}(V)/\rho$.

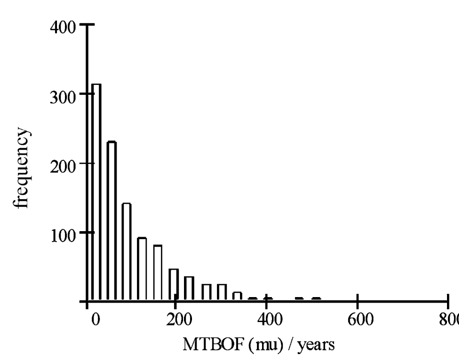

Fig. 9. Histogram of the distribution of time between operational failures for $T = 10.248$, $\Delta = 0.86$, $K = 5$. Parameter values: $p = 0.1$, $\eta_1 = 3$, $\eta_2 = 18$, $\beta_1 = 2.5$ $\beta_2 = 5$, $\lambda = 1/2$, $C_{\mathrm{R}} = 1$, $C_{\mathrm{F}} = 10$, $C_{\mathrm{I}} = 0.05$.

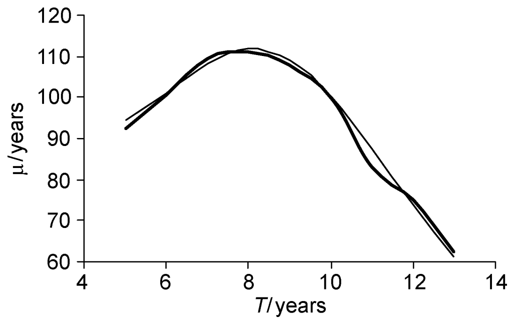

Fig. 10. For hybrid policy, MTBOF, μ, against age at preventive replacement, T (with $\Delta = 0.861$, $\mathrm{K} = 5$) for two cases: MTBOF calculated by simulation (▬▬); MTBOF calculated using the approximation (6), $\mu = \mathrm{E}(V)/\rho$ (——). Parameter values: $p = 0.1$, $\eta_1 = 3$, $\eta_2 = 18$, $\beta_1 = 2.5$ $\beta_2 = 5$, $\lambda = 1/2$, $C_\mathrm{R} = 1$, $C_\mathrm{F} = 10$, $C_\mathrm{I} = 0.05$.

calculated using simulation, and that calculated using the exponential approximation. This latter figure would appear to confirm that the approximation for the MTBOF (6) is a good one.

VI. Some Observations

A policy for which $K\Delta < T < \infty$ has a decreasing frequency of inspection; as the component ages, it is optimal to inspect less often. This decreasing inspection frequency may mirror maintenance practice. Other policies with decreasing frequency of inspection may be of interest (e.g. $\Delta \sim 1/\text{age}$), particularly if components arise from a mixture of more than two sub-populations.

Block type policies could be investigated, but the analytical calculation of the long-run cost per unit time will be difficult. Also, hybrid block-type policies for multi-component systems could be considered. If a series system comprises n components from a heterogeneous population, then, with shared set-up costs for inspection and preventive replacement, a hybrid block policy may be cheaper than individual (or collective) hybrid age-based policies for the components. It will be interesting to investigate circumstances when this occurs.

A hybrid block type policy for a single component would be to replace on failure, and preventively replace at times iT, $i = 1, 2 \ldots$; inspect at times $iT + j\Delta$, $j = 1, \ldots, K$, $(K\Delta < T)$; and replace defective components preventively at inspection. We might also consider a policy in which preventive replacements are time-based (block-type), but inspections are age-based, inspecting at ages $j\Delta$, $j = 1, \ldots, K$, provided $(i-1)T < t(j\Delta) < iT$, where $t(j\Delta)$ is the time of the jth inspection. In this policy, when a defective component is replaced at inspection, the inspections are rescheduled from the time of (defective) preventive replacement, but the subsequent preventive replacement (at time iT) is not rescheduled. In practice, when there are multiple components in a system, the rescheduling of the inspections following preventive replacement of a defective component would not cause inspections to become unsynchronized; instead it would imply that some components would continue to be inspected for longer. However, one could imagine a policy in which, if 1 component is preventively replaced at inspection, then the inspections of all components would be rescheduled. Such a policy could also be investigated.

VII. Conclusion

A hybrid maintenance policy for a simple system with a component from a heterogeneous population is proposed in this paper. A limited number of maintenance models for this kind of system have been developed to date. In particular, we consider components that arise from a mixture of 2 sub-populations. The first sub-population represents weak, low quality components (or possibly poor installation of components); while the second represents stronger, more long-lived components. The concept of delay-time in inspection maintenance is combined with that of age-based replacement in preventive maintenance into a hybrid policy that mitigates early failures of weak components, and extends the age at preventive replacement of strong components. The behavior of the policy is investigated for various values of the parameters of the underlying mixture, and costs of inspection, preventive replacement, and failure replacement. This behavior is as might be anticipated. Where the cost of failure is large, or the proportion of weak components in the mixed population is large, regular inspection in early life is recommended. Modest cost savings over a naïve age-based replacement model that assumes a single s-population of components can be achieved.

While cost savings may be modest, improvement in the reliability, as measured by the time between operational failures, can be significant. Furthermore, because of the nature of the failure hazard of the mixed population of components, broadly speaking there exist values of the decision variables at which, simultaneously, the cost criterion is minimized, and the reliability criterion is maximized.

If we relax the assumption regarding instantaneous replacement, then an availability criterion could be used with or in place of cost and reliability. It would be interesting to consider this in the context of a safety critical preparedness or protection system [21] in a future paper.

The policy mirrors maintenance practice in which preventive maintenance, using this term in the general sense of failure prevention, is more prudently applied to newer than to older systems. The similarity of the model to combined burn-in-replacement policies is discussed. The hybrid maintenance policy can be generalized and extended. Hybrid inspection and block replacement policies may be developed in a similar manner, although the calculation of the long-run cost per unit time will be more difficult. Extensions to repairable systems could also be considered.

Acknowledgment

The authors are grateful to the anonymous referees whose comments led to improvements in the paper.

References

[1] H. Ascher and H. Feingold, *Repairable Systems Reliability*. New York: Marcel Dekker, 1984.
[2] M. S. Finkelstein and V. Esaulova, "Why the mixture failure rate decreases," *Reliability Engineering and System Safety*, vol. 71, no. 2, pp. 173–177, 2001.
[3] A. H. Christer, "Developments in delay time analysis for modeling plant maintenance," *Journal of the Operational Research Society*, vol. 50, no. 1, pp. 1120–1137, 1999.
[4] R. E. Barlow and F. Proschan, *Mathematical Theory of Reliability*. New York: Wiley, 1965.
[5] J. H. Cha, S. Lee, and J. Mi, "Bounding the optimal burn-in time for a system with two types of failure," *Naval Research Logistics*, vol. 51, no. 8, pp. 1090–1101, 2004.

[6] A. Drapella and S. Kosznik, "A short communication combining preventive replacement and burn-in procedures," *Quality and Reliability Engineering International*, vol. 18, no. 5, pp. 423–427, 2002.
[7] R. Jiang and A. K. S. Jardine, "An optimal burn-in preventive-replacement model associated with a mixture distribution," *Quality and Reliability Engineering International*, vol. 23, no. 1, pp. 83–93, 2007.
[8] D. N. P. Murthy and M. R. Maxwell, "Optimal age replacement policies for items from a mixture," *IEEE Trans. Reliability*, vol. 30, no. 2, pp. 169–170, 1981.
[9] B. S. Dhillon, "A hazard rate model," *IEEE Trans. Reliability*, vol. 28, no. 1, pp. 150–154, 1979.
[10] R. E. Glaser, "Bathtub and related failure rate characterizations," *Journal of the American Statistical Association*, vol. 75, no. 7, pp. 667–672, 1980.
[11] P. L. Gupta and R. C. Gupta, "Ageing characteristics of the Weibull mixtures," *Probability in the Engineering and Informational Sciences*, vol. 10, no. 4, pp. 591–600, 1996.
[12] R. Jiang and D. N. P. Murthy, "Mixtures of Weibull distributions—parametric characterization of failure rate function," *Applied Stochastic Models and Data Analysis*, vol. 14, no. 1, pp. 47–65, 1998.
[13] M. S. Finkelstein and V. Esaulova, "On an inverse problem in mixture failure rates modeling," *Applied Stochastic Models in Business and Industry*, vol. 17, no. 2, pp. 221–229, 2001.
[14] H. W. Block, T. H. Savits, and E. T. Wondmagegnehu, "Mixtures of distributions with increasing linear failure rate," *Journal of Applied Probability*, vol. 40, no. 2, pp. 485–504, 2003.
[15] R. Jiang and D. N. P. Murthy, "Modeling failure-data by mixture of 2 Weibull distributions: A graphical approach," *IEEE Trans. Reliability*, vol. 44, no. 3, pp. 477–488, 1995.
[16] A. H. Christer and W. Wang, "A delay-time based maintenance model for a multi-component system," *IMA Journal of Management Mathematics*, vol. 6, no. 2, pp. 205–222, 1995.
[17] W. Wang and A. H. Christer, "Solution algorithms for a nonhomogeneous multi-component inspection model," *Computers & Operations Research*, vol. 30, no. 1, pp. 19–34, 2003.
[18] E. E. Lewis*, Introduction to Reliability Engineering*. New York: Wiley, 1987.
[19] P. A. Scarf, R. Dwight, and A. Al-Musrati, "On reliability criteria and the implied cost of failure for a maintained component," *Reliability Engineering and System Safety*, vol. 89, no. 2, pp. 199–207, 2005.
[20] A. T. Almeida, "Multicriteria modelling of repair contract based on utility and ELECTRE I method with dependability and service quality criteria," *Annals of Operations Research*, vol. 138, no. 1, pp. 113–126, 2005.
[21] J. K. Vaurio, "Unavailability analysis of periodically tested standby components," *IEEE Trans. Reliability*, vol. 44, no. 3, pp. 512–517, 1995.

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