Experimental evaluation of adaptive and variable structure control of piezoelectric actuation systems for micro/nano manipulation

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http://ro.uow.edu.au/engpapers/3232

Publication Details
Experimental Evaluation of Adaptive and Variable Structure Control of Piezoelectric Actuation Systems for Micro/Nano Manipulation

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Abstract—This paper proposes and evaluates an adaptive technique and a variable structure control approach for piezoelectric actuation systems to track specified motion trajectories. The proposed control methodologies are formulated to accommodate unknown or uncertain system parameters, non-linearities including the hysteresis effect, and external disturbances in the piezoelectric actuation systems without any form of feed-forward compensation. In this study, both control methodologies are demonstrated to possess a promising motion tracking ability experimentally. In comparison, the variable structure control approach is evaluated to be superior to the adaptive technique in the motion tracking control. With the ability to track motion trajectories under parametric uncertainties, non-linearities, and external disturbances, the proposed control methodologies are very attractive in realising the high-precision piezoelectric actuation systems for micro/nano manipulation.

I. INTRODUCTION

In micro/nano manipulation, piezoelectric actuators have been identified as one of the most effective means of accomplishing high-precision motion tasks. This is partly attributed to the properties of the piezoelectric actuator, which include high output force, fast response time, large bandwidth, compact size, zero backlash, zero stick/slip effect, and generally unlimited motion resolution. However, within the piezoelectric actuators, there exists a highly nonlinear relationship between the input (applied) voltage and the output displacement. This prevents the actuators from providing the desired high-precision motion resolution and accuracy. The non-linearities include the hysteresis and creep/drift effects, and the hysteresis effect is the major concern in piezoelectric actuation systems [1] in which piezoelectric actuators are employed for the motion actuation.

To resolve the drawback due to the nonlinear behaviour in the piezoelectric actuation systems, research studies have been conducted to model and compensate for the hysteresis effect. Compensation techniques in either open-loop or closed-loop feedback have been proposed to improve the positioning accuracy. Other research efforts have been focused on the enhancement of positioning performance by proposing closed-loop control to the piezoelectric actuation systems.

The examples of modelling techniques include a nonlinear dynamic model for piezoelectric actuators with hysteresis [2], a voltage-input electromechanical model [3], a charge steering model [4], and a model of physical hysteresis [5]. Generally, the hysteresis effect is very complex. It can never be modelled accurately and the model parameters are difficult to quantify in practice. Therefore, using merely the hysteresis model in an open-loop compensation, the resulting positioning accuracy is not guaranteed to be always repeatable. Furthermore, with only open-loop compensation, it is difficult to assess the positioning performance as the results cannot be measured without closed-loop feedback.

On the other hand, appropriate closed-loop control strategies can be formulated to treat the non-linearities (including the hysteresis effect) into account to achieve high-precision positioning of the piezoelectric actuation systems. Recent examples include a combination of a feed-forward model in a feedback control with an input shaper [6], an adaptive back-stepping approach [7], a tracking control of a piezo-ceramic actuator with hysteresis compensation [8], a new mathematical model for improving the positioning accuracy of piezoelectric actuators [9], and a nonlinear observer-based variable structure control [10]. The above control strategies have been developed for specific applications with unique arrangements of the physical hardware. They cannot be applied directly to any given system without an in-depth knowledge of both the system and the control formulation. Furthermore, in most closed-loop control studies, a complex hysteresis model has been commonly adopted to compensate for the hysteresis effect.

In this paper, two advanced control methodologies, namely adaptive and variable structure control, are proposed to track the desired motion trajectories and to accommodate the non-linearities in the piezoelectric actuation systems without any form of feed-forward compensation. For the proposed adaptive control methodology, it is an improvement on the original adaptive technique proposed by Slotine and Li [11]. In this approach, a saturation function derived from a special positive definite function [12] is employed to ensure both the position and velocity tracking errors converging to zero. For the proposed variable structure control, it is a refined approach
providing an improvement over a previous study [13]. This control concept relies on the specification of a target performance and the formulation of a target reaching scheme for the piezoelectric actuation system to reach the specified target performance by driving the tracking errors converging to zero.

For both proposed approaches, the controllers are expected to control a piezoelectric actuation system to closely track a desired motion trajectory in position, velocity, and acceleration. Implementation of the above-mentioned control methodologies are appropriate in practice. Control experiments conducted in a piezoelectric actuation system have demonstrated the effectiveness of both control methodologies with promising tracking performances.

This paper is organised as follows. The model of a piezoelectric actuator is presented in Section II. Section III and Section IV describe the formulation of the adaptive and the variable structure control methodologies, respectively. The experimental study is detailed in Section V followed by the results and discussion in Section VI. Finally, conclusions are drawn in Section VII.

II. MODEL OF PIEZOELECTRIC ACTUATOR

An electromechanical model of a piezoelectric actuator has been identified based on recent studies [3], [4]. For control purposes, this piezoelectric actuator model [14] can be expressed as

\[ m \ddot{x} + b \dot{x} + k x + v_h + f_e = v_{in}, \]  

where \( m \), \( b \), and \( k \) are the effective mass, damping, and stiffness, respectively, \( x \) is the actuator output displacement, \( v_h \) is the voltage due to the hysteresis, \( f_e \) is related to the force imposed by the external mechanical load, and \( v_{in} \) is the applied (input) voltage. The model (1) can also be extended to describe a piezoelectric actuation system. With this model of piezoelectric actuation system, advanced methodologies can be established to effectively control the system.

III. ADAPTIVE CONTROL METHODOLOGY

For the piezoelectric actuation system described by (1), an adaptive control methodology can be formulated for the purpose of tracking a desired motion trajectory \( x_d(t) \). Under the proposed control approach, the physical parameters of the system in (1) are assumed to be unknown or uncertain. The \( x_d(t) \) is assumed to be twice continuously differentiable and both \( \dot{x}_d(t) \) and \( \ddot{x}_d(t) \) are bounded and uniformly continuous in \( t \in [0, \infty) \). An adaptive technique is employed in the control law such that the closed-loop system will follow the required motion trajectory.

For the convenience of control formulation, the system model of (1) is rewritten in terms of a set of physical parameters,

\[ \varphi = [m, b, k, v_h]^T. \]  

As mentioned previously, the term \( v_h \) in (2) is the voltage due to the hysteresis, and its time derivative \( \dot{v}_h \) is assumed to be zero in this control formulation. The piezoelectric actuation system (1) becomes

\[ x^T \varphi + f_e = v_{in}, \]  

where \( x = [\ddot{x}, \dot{x}, x]^T \). A set of estimated parameters \( \hat{\varphi} \) of \( \varphi \) is defined as

\[ \hat{\varphi} = [\hat{m}, \hat{b}, \hat{k}, \hat{v}_h]^T. \]  

It must be noted that the external disturbances in the system can be included in the terms \( v_h \) and \( \dot{v}_h \) described in (2) and (4), respectively.

An adaptive technique [11] is employed to continuously update the control system through

\[ \dot{\hat{\varphi}} = -K^{-1} x_d y, \]  

where \( K \) is a \( 4 \times 4 \) constant positive definite diagonal matrix, \( x_d = [\ddot{x}_d, \dot{x}_d, x_d]^T \), and \( y \) is defined as

\[ y = \dot{e}_p + \alpha s(e_p), \]  

where \( e_p(t) = x(t) - x_d(t) \), \( \alpha \) is a positive scalar, and \( s(e_p) \) is a saturation function, which is the first derivative of a special positive definite function \( \rho(e_p) \) [12] defined as

\[ \rho(e_p) = \sqrt{\varepsilon^2 + e_p^2} - |\varepsilon|, \]  

where \( \varepsilon \) is an arbitrary constant with its absolute value chosen to be \( |\varepsilon| > 0 \), and the saturation function is given as

\[ s(e_p) = \frac{\rho(e_p)}{de_p} = \frac{e_p}{\sqrt{\varepsilon^2 + e_p^2}}. \]  

An adaptive control signal \( \dot{v}_{in} \) can therefore be established by

\[ \dot{v}_{in} = x_d^T \dot{\varphi}. \]  

The overall control input of the proposed adaptive control methodology is given as

\[ v_{in} = -k_p e_p - k_v \dot{e}_p + \dot{v}_{in} + f_e, \]  

where \( k_p \) and \( k_v \) are the proportional and derivative gains, respectively.

The stability of the proposed adaptive control methodology has been analysed [12], and the structure of the adaptive piezoelectric actuation control system is summarised in Fig. 1.
IV. VARIABLE STRUCTURE CONTROL METHODOLOGY

The motion tracking control in the piezoelectric actuation system can also be formulated as a target performance reaching scheme in establishing a variable structure control law such that the system described by (1) achieves the following target performance

\[
m_d \ddot{x}_p + b_d \dot{x}_p + k_d x_p = 0, \quad (11)
\]

where \(m_d, b_d,\) and \(k_d\) are the desired constant values of mass, damping, and stiffness, respectively. The control system will be designed to follow the desired motion trajectory \(x_d(t)\) in view of the uncertainties, which are modelled as

\[
\begin{align*}
\| \Delta m \| &= \| m - \hat{m} \| \leq \delta m, \\
\| \Delta b \| &= \| b - \hat{b} \| \leq \delta b, \\
\| \Delta k \| &= \| k - \hat{k} \| \leq \delta k, \\
\| v_h \| \leq \delta v_h,
\end{align*}
\]

(12)

where \(\Delta\) represents the modelling error of \(\dot{\bullet}\) and \(\dot{\bullet}\) represents the estimated values of \(\dot{\bullet}\). The symbol \(\| \|\) denotes an absolute value of \(\dot{\bullet}\) and the positive values \(\delta m, \delta b, \delta k,\) and \(\delta v_h\) represent the bounds of the variables. Generally, the bound \(\delta v_h\) is extended to include the external disturbances to the control system. In the proposed model of uncertainties, it is assumed that the exact values of \(m, b,\) and \(k\) in (12) are unknown. However, the estimated values and their corresponding bounds including the bound of the hysteresis effect and external disturbances are available. With this assumption, the variable structure control methodology can be realised.

In formulating the variable structure control methodology, a switching function \(\sigma\) is specified,

\[
\sigma = \dot{\xi} + \xi, \quad (13)
\]

where \(\xi\) is the state of a dynamic compensator used to shape the position tracking errors. The dynamic compensator can be chosen as

\[
\dot{\xi} = \alpha \xi + k_p e_p + k_v \dot{e}_p, \quad (14)
\]

where \(\alpha\) is a constant scalar, \(\alpha \leq 0, k_p,\) and \(k_v\) are the control gains which are related to the specified target performance (11). Differentiating (13) with respect to time, provides the following

\[
\dot{\sigma} = \dot{e}_p + \dot{\xi}. \quad (15)
\]

To examine the closed-loop dynamics of the system under the variable structure sliding mode control, the dynamic compensator (14) is substituted into (15) with the term \(\xi\) in (14) eliminated by using (13),

\[
\dot{e}_p + (k_v - \alpha) \dot{e}_p + k_v e_p = \dot{\sigma} - \alpha \sigma. \quad (16)
\]

By choosing

\[
\begin{align*}
k_p &= m_d^{-1} k_d, \\
k_v &= m_d^{-1} b_d + \alpha,
\end{align*}
\]

(17)

the closed-loop dynamics (16) becomes

\[
m_d \ddot{e}_p + b_d \dot{e}_p + k_d e_p = m_d (\dot{\sigma} - \alpha \sigma). \quad (18)
\]

During sliding motion where \(\dot{\sigma} = 0\) and \(\sigma = 0\), (18) achieves the target performance (11). A variable structure control law can therefore be formulated to drive the system to reach the sliding mode.

For the piezoelectric actuation system described by (1) under the modelled uncertainties (12), the system achieves the target performance (11) with the variable structure control law formulated as

\[
v_{in} = \hat{m} \ddot{x}_{eq} + \hat{b} \dot{x}_d + \hat{k} x_d + f_e - k_s \sigma - d \frac{\sigma}{\| \sigma \|}, \quad (19)
\]

where

\[
\ddot{x}_{eq} = \ddot{x}_d - \dot{\xi}, \quad (20)
\]

and the term \(d\) is governed by

\[
d \geq \delta m \| \ddot{x}_{eq} \| + \delta b \| \dot{x} \| + \delta k \| x \| + \delta v_h + \epsilon. \quad (21)
\]

The terms \(k_s\) and \(\epsilon\) in (19) and (21), respectively, are any positive scalars.

The structure of the proposed variable structure control methodology for the piezoelectric actuation system is summarised in Fig. 2. The feasibility study including stability analysis of this proposed control approach has been conducted [15].

In the implementation of the control law (19), the discontinuous function \(\sigma/\Delta\) will give rise to control chattering due to imperfect switching in the computer control. This is undesirable as un-modelled high frequency dynamics might be excited. To eliminate this effect, the concept of boundary layer technique [16] is applied to smooth the control signal. In a small neighbourhood of the sliding surface \((\sigma = 0)\), the discontinuous function is replaced by a saturation function which is defined as

\[
\text{sat}\left(\frac{\sigma}{\Delta}\right) = \begin{cases} 
-1 & : \sigma < -\Delta, \\
\sigma/\Delta & : -\Delta \leq \sigma \leq \Delta, \\
+1 & : \sigma > \Delta,
\end{cases} \quad (22)
\]

Fig. 2. Structure of the proposed variable structure control methodology.
where $\Delta$ is the boundary layer thickness, and the control law (19) becomes

$$v_{in} = \dot{m} \ddot{x}_{eq} + \dot{b} \dot{x} + \dot{k} x + f_e - k_s \sigma - d \text{sat}(\frac{\sigma}{\Delta}). \quad (23)$$

With the introduction of the saturation function (22) in the control law (23), the accuracy of $\sigma$ can only be guaranteed to stay within the boundary layer. From the closed-loop dynamics (18) of the control law, the steady-state value $\sigma_{ss}$ of the switching function within the boundary layer is given as

$$\sigma_{ss} = \frac{k_d e_{pss}}{m_d \alpha} \quad (24)$$

where $e_{pss}$ is the steady-state position error. As (24) describes the relationship between the steady-state position error and switching function, it can therefore be used to decide on the boundary layer thickness $\Delta$ in the control implementation.

The selection of a target performance for the control system is straightforward. By comparing (11) to a standard second-order characteristic equation

$$s^2 + 2 \zeta w_n s + w_n^2 = 0, \quad (25)$$

where $s$, $\zeta$, and $w_n$ are the Laplace operator, damping ratio, and undamped natural frequency, respectively, the desired parameters are obtained as

$$m_d = 1, \quad b_d = 2 \zeta w_n, \quad k_d = w_n^2. \quad (26)$$

As the desired response is selected through $\zeta$ and $w_n$, the control gains, $k_p$ and $k_v$, in (17) can therefore be calculated from (26).

V. EXPERIMENTAL STUDY

An experimental research facility, as shown in Fig. 3, has been established for the investigation of the proposed control strategies. The architecture of the experimental setup is detailed in the block diagram as shown in Fig. 4. It consists of a piezoelectric actuator with inbuilt position sensor, an amplifier module, a position signal processing unit, and a control PC installed with a digital-to-analogue (D/A) and an analog-to-digital (A/D) boards.

The piezoelectric actuator employed is a PI (Physik Instrumente) multi-layer PZT stacked ceramic translator, model P-843.30, capable of displacement of up to 45 $\mu$m corresponding to a range of operating voltage up to 100 V. The piezoelectric actuator is preloaded 300 N by an internal spring and is incorporated with a high-resolution strain gauge sensor for position feedback. The amplifier module is a PI model E-505.00 with a fixed output gain of 10 providing voltage ranges from -20 to +120 V. The position signal processing unit is housed in a PI servo controller, model E-509-S3. The PI servo controller is disabled and only the signal processing unit is used to interface with the position sensor. The control PC is equipped with a Pentium 4 2.8 GHz processor running on an operating system capable of hard real-time control. The D/A and A/D boards installed in the control PC are of 16-bit resolution, and they are used to generate the control signal and to read the actuator position, respectively. In the control experiments, the sampling frequency of the control loop is set at 2.5 kHz.

The control experiments serve not only to validate the theoretical formulation of the control algorithms but also to examine the effectiveness of the proposed schemes in a physical control system. In the experimental study, the closed-loop system is required to follow a desired motion trajectory, which is shown in Fig. 5 for position, velocity, and acceleration. The desired motion trajectory is formed by segments of quintic polynomials [17] for the implementation and analysis of the tracking and steady-state performances of the control system.

For the piezoelectric actuation system described by (1), the adaptive control law (10) and the variable structure control law (23) are implemented in the control PC for comparison. With the desired motion trajectory, tracking ability of the

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Fig. 3. Set-up of the experimental research facility

Fig. 4. Block diagram of the experimental architecture

Fig. 5. Desired motion trajectory
The steady-state position error in (24) is specified as $e_{psss} \leq 0.1 (\mu m)$, $\sigma_x \leq 0.16 (m/s)$, the boundary layer thickness $\Delta$ in (22) is chosen as the maximum value of $\sigma_{ss}$, i.e.

$$\Delta = 0.16 (m/s).$$

The positive scalar $\kappa$ in (21) is specified as $\kappa = 1 (V)$ and $k_v$ of the control law (23) is set to $k_v = 500 (V/s/m)$.

It is assumed that no external force is applied to the control system and the term $f_e$ in (10) and (23) is ignored in the control experiments.

## VI. Results and Discussion

In tracking of the desired motion trajectory as shown in Fig. 5, the piezoelectric actuator was commanded to travel in a range of $30 \mu m$ with a maximum velocity and an acceleration reaching $1.1 mm/s$ and $0.07 mm/s^2$, respectively. The resulting piezoelectric actuator positions and estimated velocities are shown in Fig. 6. Despite parametric uncertainties, hysteresis effect and external disturbances in the system, both control laws (10) and (23) showed a promising tracking performance.

For the variable structure control, the switching function, as shown in Fig. 7, indicates that the system operated within the boundary layer thickness given by (33), i.e. the system tracked the desired motion trajectory closely with the switching function $\sigma$ kept to a minimum.

For both control methodologies, the control input and position tracking errors are shown in Fig. 8. The resulting position tracking errors indicate that the control laws had successfully accommodated the non-linearities including the hysteresis effect in the control system.

In the experiments, the position errors during motion tracking were confined within $0.50 \mu m$ and $0.25 \mu m$ for the adaptive and variable structure control, respectively. At steady-state, the position errors of both control methodologies were similar and less than $0.03 \mu m$, which were almost at the noise level of the closed-loop systems. Fig. 9 shows the resulting actuator positions of both approaches when plotted against the desired position. The effectiveness of both proposed control methodologies were demonstrated as only a minimum hysteresis effect was observed in both control experiments.

In comparing the experimental results as shown in Fig. 8 and Fig. 9, the variable structure control approach appears to be superior to the adaptive technique for the tracking of motion trajectory in the experimental study.

In summary, the proposed adaptive and variable structure control methodologies for the piezoelectric actuation system are shown to be stable, robust, and capable of following the desired motion trajectory under the unknown or uncertain system parameters, nonlinear hysteresis effect, and external disturbances. Furthermore, implementation of both control methodologies is practical as only the estimated values of the system parameters are required.

## VII. Conclusions

An adaptive and variable structure control methodologies have been proposed for piezoelectric actuation systems to track specified motion trajectories. The control methodologies are formulated to accommodate unknown or uncertain system parameters, nonlinear hysteresis effect, and external disturbances without any form of feed-forward compensation.

In the experimental study, both control methodologies have been demonstrated to possess a promising motion tracking ability. In comparison, the variable structure control approach
has been observed to be superior to the adaptive technique in the motion tracking control.

ACKNOWLEDGMENT

This work is supported by an Australian Research Council (ARC) Linkage Infrastructure, Equipment and Facilities (LIEF) grant and an ARC Discovery grant.

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