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Principles of charge and heat transport in thermionic devices

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ABSTRACT

We enunciate the general principles that govern the transport of charge and heat in a thermionic device. We illustrate the application of these principles to the subject of domestic refrigeration. A complementary application is power generation. We distinguish Class 2 devices, in which the potential barrier on the hot side plays a role, from Class 1 devices, in which this barrier is irrelevant. We show that the effect of heat backflow is to drastically reduce the efficiency of thermionic devices in both GaAs and InSb representative semiconductor systems. We conclude that practical devices are not likely with bulk, single-barrier devices.

Keywords: thermionic, refrigeration, cooling, power generation

1. INTRODUCTION

The simplest thermionic device consist of two electrodes separated by a potential barrier. In order for the shape of the barrier to be inconsequential no electrons involved in transport should interact with the material in the barrier. To achieve this the electrons must traverse the barrier material ballistically. For this to happen the barrier width must be at most of the order of the mean free path of electrons in the material. The thermal resistance of the barrier is dependent on its width which sets a maximum for the thermal resistance between electrodes which, in turn, sets an upper limit for the thermal efficiency. The additional simplifying assumptions we make to begin with are: there is no space charge accumulation in the device; any potential applied across the device is dropped only in the barrier itself and not in the electrodes; the barrier width is greater than the tunnelling width of carriers in the barrier; the system is assumed to be in, or very close to, equilibrium so that Richardson's equation may be applied.

2. ELECTRICAL AND HEAT CURRENTS IN THE DEVICE

Fig. 1 shows a single-barrier thermionic device. A positive bias of V (V) has been applied to electrode 2 with respect to electrode 1. Electrode 1 is at temperature T_c (K) $< T_h$ (K), the temperature of electrode 2. We assume that a net electrical and heat current leaves electrode 1 and moves from left to right. A thermal resistance, R^{th} , exists between the two electrodes. A potential barrier, ϕ_c , exists between electrode 1 and the barrier. A potential barrier, ϕ_h , exists between the barrier and electrode 2. If material 'A' is identical to material 'C' then $\phi_c = \phi_h$.

2.1. Net electrical current leaving electrode 1

The net electrical current leaving electrode 1 and travelling across the barrier is

$$J^{net} = J_c - J_h. \quad (1)$$

Richardson's expression is used for J_c . Although derived for a metal-vacuum interface, the same form can be used at a semiconductor-semiconductor interface by changing the effective mass of electrons in Richardson's

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Figure 1. Schematic of a single-barrier thermionic device under applied bias V .

constant. When the effective mass is different between the two materials, the smaller effective mass is to be used, as shown by Shakouri and Bowers¹ and Grinberg.² The thermionic electric current that leaves electrode 1 (where a potential barrier ϕ_c is present) is given by

$$J_c = AT_c^2 \exp\left(-\frac{q\phi_c}{k_B T_c}\right). \quad (2)$$

An electron at the chemical potential of electrode 2 needs to overcome a potential energy of $q(\phi_c + V)$ in order to contribute to current flow. This then becomes the barrier height, or work function, for electrons leaving electrode 2:

$$J_h = AT_h^2 \exp\left(-\frac{q(\phi_c + V)}{k_B T_h}\right). \quad (3)$$

The size of the barrier ϕ_h does not influence the magnitude of electrical current that flows in the device. This statement is valid as long as $\phi_c + V \geq \phi_h$. If this condition is not satisfied, then the operating mode of the device changes and care must be taken when modelling such devices. Operating modes will be dealt with further in section 4. Combining Eq. 2 and Eq. 3 we obtain

$$\begin{aligned} J^{net} &= J_c - J_h \\ &= AT_c^2 \exp\left(-\frac{q\phi_c}{k_B T_c}\right) - AT_h^2 \exp\left(-\frac{q(\phi_c + V)}{k_B T_h}\right). \end{aligned} \quad (4)$$

The condition for a positive current to leave the cold electrode to the right is given by

$$V > V_0^J = \frac{2k_B T_h}{q} \log\left(\frac{T_h}{T_c}\right) + \phi_c \left(\frac{T_h}{T_c} - 1\right). \quad (5)$$

If the applied voltage is less than this then the larger thermal energy of electrons on the right-hand side of the barrier causes a net current to enter the cold electrode. A particular device, with $\phi_c = 0.7$ eV, $T_c = 500$ K and $T_h = 700$ K, will be used to illustrate our general results. The current density versus applied bias for this device is shown in Fig. 2. It can be seen that the net electrical current saturates at an applied bias of around 0.7 V. This occurs because the exponential term in J_h becomes so large that no current flows into the cold electrode from the hot electrode. The saturation current is

$$J^{sat} = J_c, \quad (6)$$

which has a value of $2.64 \times 10^4 \text{ Am}^{-2}$ for this device.

Figure 2. Current density of single-barrier device showing switch in sign at applied bias V_0^J given by Eq. 5 ($\phi_c = 0.7$ eV, $T_c = 500$ K, $T_h = 700$ K).

Figure 3. Energy current density of single-barrier device showing switch in sign at applied bias V_0^{Jq} given by Eq. 10 ($\phi_c = 0.7$ eV, $T_c = 500$ K, $T_h = 700$ K).

2.2. Net heat current leaving electrode 1 ignoring conduction heat current

The energy current, in contrast to the electrical current, is not constant throughout the device. The energy current leaving electrode 1 to the right is

$$J_{qc}^c = \left(\phi_c + \frac{2k_B T_c}{q} \right) J_c. \quad (7)$$

The energy current entering electrode 1 from the right is

$$J_{qc}^h = \left(\phi_c + \frac{2k_B T_h}{q} \right) J_h. \quad (8)$$

Each electron travelling from the right deposits, on average, a potential energy of $q\phi_c$ in addition to a kinetic component given by the average thermal energy, $2k_B T_h$. Combining Eq. 7 and Eq. 8, the net heat current leaving electrode 1 is

$$\begin{aligned} J_{qc}^{out} &= J_{qc}^c - J_{qc}^h \\ &= \left(\phi_c + \frac{2k_B T_c}{q} \right) J_c - \left(\phi_c + \frac{2k_B T_h}{q} \right) J_h. \end{aligned} \quad (9)$$

As for the electrical current, there is a critical voltage at which the net heat current leaving the cold electrode becomes positive, allowing operation as a refrigerator. Due to the larger prefactor of J_h in Eq. 9, namely $(\phi_c + 2k_B T_h/q)$, than that of J_c , $(\phi_c + 2k_B T_c/q)$, this voltage is always larger than the critical bias, V_0^J , given in Eq. 5. For refrigerator operation,

$$\begin{aligned} V > V_0^{Jq} &= \frac{2k_B T_h}{q} \log \left(\frac{T_h}{T_c} \right) + \phi_c \left(\frac{T_h}{T_c} - 1 \right) + \frac{k_B T_h}{q} \log \left(\frac{q\phi_c + 2k_B T_h}{q\phi_c + 2k_B T_c} \right) \\ &= V_0^J + \frac{k_B T_h}{q} \log \left(\frac{q\phi_c + 2k_B T_h}{q\phi_c + 2k_B T_c} \right). \end{aligned} \quad (10)$$

The energy current density for our device is shown in Fig. 3. Like the electrical current, the heat current saturates:

$$J_{qc}^{sat} = J_{qc}^c. \quad (11)$$

For this device the saturation energy current density is 2.07×10^4 Wm⁻² and the two critical voltages, V_0^{Jq} and V_0^J , are quite close. For a given hot and cold temperature, the difference will increase as the barrier height of the cold electrode decreases as a greater amount of work must be put into the system before heat starts to be removed from the cold side.

2.3. Net heat current leaving electrode 1 including conduction heat current

Incorporating heat current due to conduction, $\Delta T/R^{th}$, into the model,

$$J_{qc}^{out} = \left(\phi_c + \frac{2k_B T_c}{q} \right) J_c - \left(\phi_c + \frac{2k_B T_h}{q} \right) J_h - \frac{\Delta T}{R^{th}}. \quad (12)$$

The conduction heat current always works to reduce the net heat current leaving electrode 1 thus reducing the performance of the device. The main issue in finding a useful device is to reduce this heat current as much as possible while still having ballistic transport of carriers.

2.4. Net heat current entering electrode 2 including conduction heat current

With respect to electrode 2, the net heat current entering from the left is

$$J_{qh}^{in} = \left(\phi_c + V + \frac{2k_B T_c}{q} \right) J_c - \left(\phi_c + V + \frac{2k_B T_h}{q} \right) J_h - \frac{\Delta T}{R^{th}}. \quad (13)$$

This expression is the same as that given by Eq. 12 except that the potential energy contribution of each electron is increased by V because of the applied bias. Potential energy in the hot barrier is measured from its chemical potential, which has been lowered by an amount V with respect to the chemical potential of the cold side.

2.5. Work put into the system

Fig. 4 gives a thermodynamic representation of a thermionic device. The heat current leaving the cold reservoir is J_{qc}^{out} and the heat current entering the hot reservoir is $J_{qh}^{in} = J_{qc}^{out} + W^{input}$. The work done on the system is the difference between these two heat currents:

$$\begin{aligned} W^{input} &= J_{qh}^{in} - J_{qc}^{out} \\ &= \left[\left(\phi_c + V + \frac{2k_B T_c}{q} \right) J_c - \left(\phi_c + V + \frac{2k_B T_h}{q} \right) J_h - \frac{\Delta T}{R^{th}} \right] \\ &\quad - \left[\left(\phi_c + \frac{2k_B T_c}{q} \right) J_c - \left(\phi_c + \frac{2k_B T_h}{q} \right) J_h - \frac{\Delta T}{R^{th}} \right] \\ &= V(J_c - J_h) \\ &= VJ^{net}, \end{aligned} \quad (14)$$

which is the electrical power put into the system. Fig. 4 also shows the parasitic component of energy current due to conduction. This reduces overall device performance.

3. DEVICE PERFORMANCE

3.1. Thermal Efficiency

Equations 4, 12 and 14 allow us to calculate the thermal efficiency or *coefficient of performance* (COP) of the device

$$\eta = \frac{J_{qc}^{out}}{W^{input}} = \frac{J_{qc}^{out}}{VJ^{net}}, \quad (15)$$

with both electrical and energy currents considered to be positive when moving to the right. The COP is the amount of heat energy removed from the cold surface compared to the amount of input energy needed. These equations can be solved directly. Using the same specific device parameters as before, $\phi_c = 0.7$ eV, $T_c = 500$ K and $T_h = 700$ K, and neglecting conduction heat current the efficiency of a single-barrier device is shown in Fig. 5 (see Mahan³). The maximum theoretical efficiency for any reversible refrigerator is given by⁴:

$$\eta_{max} = \frac{T_c}{T_h - T_c} = \frac{T_c}{\Delta T}, \quad (16)$$

so for this device operating between 700 K and 500 K the maximum theoretical efficiency is 2.5. The device in Fig. 5 has a maximum efficiency of 2.08 near 0.35 V, or 83% of the theoretical limit. By modifying device parameters (such as barrier heights) the device efficiency can be increased. This will be dealt with in more detail in later sections. We note that there is little point in having a highly-efficient device if it does not remove enough heat from the cold side, J_{qc}^{out} , to achieve appreciable cooling.

Figure 4. Thermodynamic representation of thermionic device.

Figure 5. Theoretical efficiency of single-barrier device vs. applied voltage ($\phi_c = 0.7$ eV, $T_c = 500$ K, $T_h = 700$ K).

4. DEVICE CLASSES AND OPERATING MODES

There are two classes of single-barrier device. For both classes it is assumed that the applied bias is positive. A negative voltage would increase heat current from the hot to cold side meaning that the device is not operating as a refrigerator.

4.1. Devices having $\phi_c \geq \phi_h$ (Class 1 Devices)

The device shown in Fig. 1 is operating with $\phi_c + V \geq \phi_h$. This condition will be referred to as *Mode 1*. This mode of operation can always be ensured by having a class of device in which $\phi_c \geq \phi_h$, operating with $V \geq 0$. This type of device will be referred to as a *Class 1* device. All equations developed so far are *always* valid for this class of device. In fact, Class 1 devices can only operate in Mode 1.

4.2. Devices having $\phi_h > \phi_c$ (Class 2 Devices)

Fig. 6 shows a device in which $\phi_h > \phi_c$ and $V > 0$. Case (a) is for $\phi_h > \phi_c + V$. Under this regime equations 4 and 12 are no longer valid. This mode of operation will be referred to as *Mode 2* and is defined explicitly by $\phi_h > \phi_c + V$ when $V > 0$. In Mode 2 the net current ‘leaving’ the cold electrode to the right and moving across the barrier will have a different form to that of Eq. 4. The term ‘leaving’ is used for consistency – the net current can be negative and may enter the cold electrode from the right, depending on the magnitude of the applied bias. The expression for this current is

$$J^{net} = AT_c^2 \exp\left(-\frac{q(\phi_h - V)}{k_B T_c}\right) - AT_h^2 \exp\left(-\frac{q\phi_h}{k_B T_h}\right). \quad (17)$$

Electrons leaving the cold side must overcome a potential of $(\phi_h - V)$ to contribute to current flow, while electrons leaving the hot side must overcome a potential of ϕ_h . Analogously to the electrical current derived in section 2.1, ϕ_c does not enter into this equation. Similarly, the net heat current leaving the cold electrode is

$$\begin{aligned} J_{qc}^{out} &= \left(\phi_h - V + \frac{2k_B T_c}{q}\right) AT_c^2 \exp\left(-\frac{q(\phi_h - V)}{k_B T_c}\right) \\ &- \left(\phi_h - V + \frac{2k_B T_h}{q}\right) AT_h^2 \exp\left(-\frac{q\phi_h}{k_B T_h}\right). \end{aligned} \quad (18)$$

Each electron travelling to the right takes, on average, a potential energy of $q(\phi_h - V)$ in addition to a kinetic component given by the average thermal energy, $2k_B T_c$. Each electron travelling from the right deposits a

Figure 6. Schematic of a Class 2 single-barrier thermionic device ($\phi_h > \phi_c$) where (a) $\phi_h > \phi_c + V$ (Mode 2) and (b) $\phi_c + V > \phi_h$ (Mode 1).

potential energy of $q(\phi_h - V)$ in addition to a kinetic component given by the average thermal energy, $2k_B T_h$. The condition $J^{net} > 0$, i.e. a net electrical current flows to the right, is fulfilled when the applied voltage exceeds

$$V_0^{J_{(Class\ 2)}} = \frac{2k_B T_c}{q} \log\left(\frac{T_h}{T_c}\right) + \phi_h \left(1 - \frac{T_c}{T_h}\right). \quad (19)$$

Similarly, for positive heat current (heat being removed from the cold electrode), the applied bias must exceed

$$\begin{aligned} V_0^{J_q}_{(Class\ 2)} &= \frac{2k_B T_c}{q} \log\left(\frac{T_h}{T_c}\right) + \phi_h \left(1 - \frac{T_c}{T_h}\right) \\ &+ \frac{k_B T_c}{q} \log\left(\frac{q\phi_h - qV_0^{J_q}_{(Class\ 2)} + 2k_B T_h}{q\phi_h - qV_0^{J_q}_{(Class\ 2)} + 2k_B T_c}\right) \\ &= V_0^J_{(Class\ 2)} + \frac{k_B T_c}{q} \log\left(\frac{q\phi_h - qV_0^{J_q}_{(Class\ 2)} + 2k_B T_h}{q\phi_h - qV_0^{J_q}_{(Class\ 2)} + 2k_B T_c}\right). \end{aligned} \quad (20)$$

This expression is similar to the one given by Mahan³ but differs due to sign conventions. As Mahan notes, because $V_0^{J_q}_{(Class\ 2)}$ appears in the $\log()$ term, this equation must be solved numerically. For the device in Fig. 6 to have a positive net electrical current and still be operating in Mode 2 ($\phi_h > \phi_c + V$) the following relationship between ϕ_h and ϕ_c must be satisfied, found by substituting $\phi_h > \phi_c + V$ into Eq. 19:

$$\phi_h > \frac{2k_B T_c}{q} \log\left(\frac{T_h}{T_c}\right) + \phi_c \frac{T_h}{T_c}. \quad (21)$$

If this condition is not satisfied then no applied bias will induce a positive net electrical current while in Mode 2. Instead the device will operate in Mode 1. Thus, Class 2 devices can operate in both Mode 1 and Mode 2.

4.3. Class 2 Device Modes

This section will illustrate a Class 2 device as it flips from Mode 2 to Mode 1 at the critical bias $V = \phi_h - \phi_c$. In order to compare with the Class 1 device already described, the same device parameters will be used. According to Eq. 21 the barrier height of the hot side, ϕ_h , must be larger than 1.021 V. We set $\phi_h = 1.2$ eV. This value permits the switch in operating modes to be seen clearly in Figs. 7, 8, and 9. The Mode 1 curve on each of the figures would be obtained for a Class 1 device ($\phi_c \geq \phi_h$) with $\phi_c = 0.7$ eV. These lines can be used as a direct comparison between the performance of a Class 1 and Class 2 device. The net electrical current for a Class 2 device is shown in Fig. 7. The device switches modes at $V = \phi_h - \phi_c = 1.2\text{ V} - 0.7\text{ V} = 0.5\text{ V}$. The bias at which the Mode 2 electrical current becomes positive, according to Eq. 19, is 0.372 V. As Eq. 20 implies, the bias at

Figure 7. Electrical current density of a Class 2 device showing mode switch at $V = \phi_h - \phi_c$.

Figure 8. Heat current density of a Class 2 device showing mode switch at $V = \phi_h - \phi_c$.

Figure 9. Efficiency of a Class 2 device showing mode switch at $V = \phi_h - \phi_c$.

Figure 10. Efficiency of a Class 2 device with ϕ_c varying from 0.1 V to 0.7 V.

which the net heat current becomes positive is larger than 0.372 V. In order to avoid a numerical singularity when calculating the efficiency the minimum bias was set at a value higher than $V_{0 (Class\ 2)}^{Jq}$. Numerically it was found that $V_{0 (Class\ 2)}^{Jq} < 0.38$ V for this device. The bias at which the Mode 1 electrical current becomes positive is still 0.323 V. The net heat current for the Class 2 device is shown in Fig. 8. The bias at which the Mode 2 equation becomes positive is again less than that for the Mode 1 equation. The efficiency of the Class 2 device is shown in Fig. 9. Because of the mode switch for Class 2 devices, the correct lines for these devices are given by the solid line in each of the figures.

4.4. Effect of ϕ_c on Class 2 Device Operation

By rearranging Eq. 21 we can obtain the maximum ϕ_c for a given T_c , T_h , and ϕ_h that keeps the Class 2 device in Mode 2. For the device being considered this is $\phi_{c (max)} = 0.82$ V. Fig. 10 shows the same Class 2 device but now with ϕ_c varied from 0.1 V – 0.7 V in 0.1 V increments. While remaining in Mode 2, each device has the same maximum efficiency occurring at the same bias. This is to be expected since ϕ_c does not appear in the equations for this mode. Therefore, the only difference between devices is the critical bias ($V_{crit} = \phi_h - \phi_c$) at which they switch modes. After the switch they operate as a Class 1 device with the same cold-electrode barrier height.

Figure 11. Efficiency of Class 1 and Class 2 devices with $\phi_c = 0.7$ eV and, applicable to a Class 2 Device only, $\phi_h = 1.2$ eV. The Class 1 device is only slightly less efficient, but removes much more heat.

4.5. Comparison between Class 1 and Class 2 Devices

As already stated, the bias at which Class 1 electrical and heat currents become positive is less than that for Class 2 devices. For a bias less than the critical voltage both the electrical and energy currents are larger for Class 1 devices, so that more energy is removed in Class 1 devices. Fig. 11 shows the efficiency of both classes of device versus applied bias. Both devices have a cold electrode with a barrier height of 0.7 eV. The Class 2 device has a hot electrode with a barrier height of 1.2 eV. In this figure the Class 2 device has a slightly higher efficiency but by referring to Fig. 8 it can be seen that the Class 1 device removes substantially more heat from the cold electrode*. By sacrificing a small reduction in efficiency a large increase in cooling power is achieved. For Class 1 devices the barrier is varied on the cold side. For Class 2 devices the barrier being varied is on the hot side. For the Class 2 device the cold-barrier height is set small enough that the device is always operating in Mode 2. For a given barrier height the applied bias is varied until the maximum efficiency possible is found. This optimal bias is then used to calculate the heat current being removed from the cold electrode. The thermal efficiency approaches the maximum theoretical efficiency as the barrier height increases. If a finite thermal resistance is used then as the barrier height is increased, $J_{qc}^{sat}(= J_{qc}^c)$ reduces until it is less than the heat current due to conduction and the device can never operate as a refrigerator. The barrier height at which the two devices have the same efficiency is different for the two classes. The Class 1 device has an efficiency of 50% of the maximum when the barrier height is 0.103 eV. Class 2 devices have the same efficiency at a barrier height of 0.222 eV. By comparing the heat current being removed at these barrier heights with an applied bias that maximises the efficiency at these points it can be seen that Class 1 devices remove more heat from the cold side, giving better overall device performance.

4.6. Conclusion – Class 1 Devices perform better

For a given efficiency Class 1 devices remove more heat from the cold side than Class 2 devices. For this reason, only Class 1 devices will be considered further. In Class 1 devices, ϕ_h does not enter into the transport equations as long as the devices operate in Mode 1 ($V \geq 0$). This condition will hold whenever a Class 1 device is operating as a refrigerator.

5. CONSIDERATION OF FINITE THERMAL RESISTANCE

As discussed previously⁵ (see Eq. 12), the net heat current removed from a surface is reduced by heat conduction represented by the term $\Delta T/R^{th}$. The thermal resistance, R^{th} , is directly proportional to the width, L , of the

*remembering that the Mode 1 equations give the currents for the equivalent Class 1 device with the same ϕ_c .

barrier. To increase R^{th} we must increase L accordingly. But, in order for carriers to move ballistically the barrier width must be less than the carrier mean free path. Mahan *et al.*⁵ state there is little data available on the mean free path of carriers travelling perpendicular to the layers of a heterostructure; there is substantial data for motion along the layers. Those authors reason that if these data are similar, the mean free path for electrons in the barriers is 50 – 100 nm.

5.1. Mobility, Mean Free Path of Carriers, Thermal Resistivity and Thermal Resistance

The mean free path in a bulk material can be calculated by starting with the definition of carrier mobility^{6, 7}

$$\mu = \frac{q\langle\tau_m\rangle}{m}, \quad (22)$$

where q is the charge, m is the mass of the carrier and $\langle\tau_m\rangle$ is the momentum relaxation time, the average time between collisions. Thus the momentum relaxation time is

$$\langle\tau_m\rangle = \frac{\mu m}{q}. \quad (23)$$

For an ideal gas, extended to the free-electron model, the mean velocity of carriers is⁷:

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{k_B T}{m^*}. \quad (24)$$

The mean distance a between collisions (mean free path) is

$$\text{MFP} = \langle v_x \rangle \langle \tau_m \rangle = \frac{\mu}{q} \sqrt{k_B T m^*}. \quad (25)$$

For example, for electrons in bulk gallium arsenide⁸ at 300 K, $\mu = 8500 \text{ cm}^2/\text{V}\cdot\text{s} = 0.85 \text{ m}^2/\text{V}\cdot\text{s}$ and $m = 0.063m_e$, gives $\text{MFP}_{GaAs} \approx 82 \text{ nm}$. By comparison, electrons in InSb have a mobility of up to $7.7 \times 10^4 \text{ cm}^2/\text{V}\cdot\text{s}$ ⁹ and effective mass of $m = 0.014m_e$, corresponding to $\text{MFP}_{InSb} \approx 349 \text{ nm}$. A larger mean free path means that the barriers can be larger which generally means a larger thermal resistance, depending on the thermal resistivity of the material. One of the design goals is to find a material with a large carrier mobility and also a large thermal resistivity. Usually, semiconductors with small band gaps have small effective masses due to a large curvature of E vs. v_k caused by repulsion of the eigenstates.¹⁰ The small effective mass results in a high mobility due to the inverse relationship in Eq. 22. Another design criterion is to have a large effective mass of carriers as this term appears in the numerator of Richardson's constant. A larger value results in a larger efficiency and/or power output, depending on the applied bias. To give a practical example of maximising thermal resistance, the thermal resistivity of GaAs is $1.82 \text{ cm}\cdot\text{K}/\text{W}$.¹¹ The maximum thermal resistance per unit area for GaAs, under conditions for ballistic transport, is then

$$\begin{aligned} R_{GaAs}^{th} &= 1.82 \times 10^{-2} \text{ m}\cdot\text{K}/\text{W} \times 82 \times 10^{-9} \text{ m} \\ &= 1.49 \times 10^{-9} \text{ m}^2\cdot\text{K}/\text{W}. \end{aligned} \quad (26)$$

For InSb, the thermal resistivity is $5.56 \text{ cm}\cdot\text{K}/\text{W}$.⁹ The maximum thermal resistance per unit area for a barrier that permits ballistic transport in InSb is

$$\begin{aligned} R_{InSb}^{th} &= 5.56 \times 10^{-2} \text{ m}\cdot\text{K}/\text{W} \times 349 \times 10^{-9} \text{ m} \\ &= 19.42 \times 10^{-9} \text{ m}^2\cdot\text{K}/\text{W}. \end{aligned} \quad (27)$$

This is a very significant result, as the thermal resistance of the barrier material is one of the most important factors in device performance.

Figure 12. Performance of a Class 1 device with varying thermal resistance ($T_c = 280 \text{ K}$, $T_h = 300 \text{ K}$).

Figure 13. Performance of a Class 1 device with varying hot temperature ($R^{th} = 19.42 \times 10^{-9} \text{ m}^2\text{K/W}$).

5.2. Effect of Thermal Resistance on Device Performance

Fig. 12 shows the efficiency and net electrical and energy currents removed from the cold side of a Class 1 single-barrier thermionic device with varying finite thermal resistance. The thermal resistance ranges from the value for InSb given in Eq. 27 to 10 and 100 $\times$ this value. A temperature gradient of 20 K is used with T_h set near room temperature (300 K). The barrier height of the devices ranges up to 200 meV. A realistic value for the thermal resistance (Eq. 27) reduces the thermal efficiency to less than 10% of the theoretical maximum at these temperatures. For a temperature gradient of 20 K there is a large conduction heat current per unit area in GaAs of

$$J_q^{cond} = \frac{20 \text{ K}}{1.49 \times 10^{-9} \text{ m}^2\text{K/W}} = 1.34 \times 10^{10} \text{ Wm}^{-2}. \quad (28)$$

For a single-barrier GaAs device with $\phi_c = 70 \text{ meV}$, $T_c = 280 \text{ K}$ and $T_h = 300 \text{ K}$, the saturation heat current, J_{qc}^{sat} , leaving the cold side is $6.11 \times 10^8 \text{ Wm}^{-2}$. This is 20 $\times$ less than the conduction heat current calculated above. Even with an unrealistic device of $\phi_c = 0 \text{ eV}$, the saturation heat current removed from the cold side is only $4.54 \times 10^9 \text{ Wm}^{-2}$. This means that for a single-barrier device of a material like GaAs, even when operating under saturation conditions, it is impossible to obtain 20 K cooling at room temperature.

6. EFFECT OF TEMPERATURE ON DEVICE PERFORMANCE

Fig. 13 shows the efficiency and net electrical and energy currents removed from the cold side of a Class 1 single-barrier thermionic device with varying temperature, T_h , of the hot electrode. The thermal resistance is

Figure 14. Equivalence of Scenario A, with negative applied bias and Scenario B, with positive applied bias. In both cases $\phi_R + |V_{A,B}| > \phi_L$.

the value for InSb given in Eq. 27 ($19.42 \times 10^{-9} \text{ m}^2 \cdot \text{K/W}$). A temperature gradient of 20 K is used with T_h set at 300 K, 500 K, and 700 K. Increasing the operating temperature of the device, keeping the temperature gradient constant, the devices perform better — the efficiency and heat removed from the cold side both increase.

7. HANDLING NEGATIVE APPLIED BIAS FOR CLASS 1 DEVICES

Because Eq. 10 states that the applied bias must have a minimum positive value in order for the devices operate as a refrigerator, negative applied biases may appear to be of little interest. But a negative applied bias is of interest when generating I - V characteristic curves. Referring to Fig. 14, care must be taken when $\phi_R - V_A > \phi_L$ for a given barrier. Then Eq. 4, Eq. 12, and Eq. 13 are not valid. Under Scenario A, carriers must overcome $\phi_R - V_A$ in order contribute to current flow, as opposed to the usual case when carriers must overcome $\phi_L + V = \phi_c + V$ as in Fig. 1. For the equations derived so far to remain valid, the equivalence of Scenario A and Scenario B is used. If we assume a Class 1 device we set $\phi = \phi_L = \phi_R$. In Scenario A, the net electrical and heat currents leaving the left electrode are

$$J_A^{\text{out}} = AT_1^2 \exp\left(-\frac{q(\phi - V_A)}{k_B T_1}\right) - AT_2^2 \exp\left(-\frac{q\phi}{k_B T_2}\right) \quad (29)$$

and

$$\begin{aligned} J_{qA}^{\text{out}} &= \left(\phi - V_A + \frac{2k_B T_1}{q}\right) AT_1^2 \exp\left(-\frac{q(\phi - V_A)}{k_B T_1}\right) \\ &- \left(\phi - V_A + \frac{2k_B T_2}{q}\right) AT_2^2 \exp\left(-\frac{q\phi}{k_B T_2}\right) \\ &- \left(\frac{T_2 - T_1}{R^{\text{th}}}\right). \end{aligned} \quad (30)$$

By switching the electrode temperatures, taking the negative of the applied bias and keeping ϕ constant we achieve the equivalent device of Scenario B. The net electrical and heat currents entering the right electrode are

$$J_B^{\text{in}} = AT_2^2 \exp\left(-\frac{q\phi}{k_B T_2}\right) - AT_1^2 \exp\left(-\frac{q(\phi + V_B)}{k_B T_1}\right) \quad (31)$$

and

$$\begin{aligned} J_{qB}^{\text{in}} &= \left(\phi + V_B + \frac{2k_B T_2}{q}\right) AT_2^2 \exp\left(-\frac{q\phi}{k_B T_2}\right) \\ &- \left(\phi + V_B + \frac{2k_B T_1}{q}\right) AT_1^2 \exp\left(-\frac{q(\phi + V_B)}{k_B T_1}\right) \\ &- \frac{T_1 - T_2}{R^{\text{th}}} \end{aligned} \quad (32)$$

which are the forms given by Eq. 4 and Eq. 13. Since $V_B = -V_A$ we have the equivalence

$$J_A^{out} = -J_B^{in}, \quad (33)$$

$$J_{qA}^{out} = -J_{qB}^{in} \quad (34)$$

so that Eq. 4, Eq. 12 and Eq. 13 may be used by switching the temperatures of the sides and making the applied bias positive. When a negative bias is applied the heat current no longer saturates because V appears in the linear part of the heat current equations. As bias increases more heat VJ is deposited in the electrode.

8. CONCLUSION

Two classes of thermionic device have been discussed. Class 1 devices only operate in Mode 1. Class 2 devices can operate in either Mode 1 and Mode 2, depending on the applied bias. Class 1 devices always have better performance than Class 2 devices. Assuming $\phi_c \geq \phi_h$, Class 1 devices are ensured and ϕ_h no longer appears in device equations. The parasitic heat current due to heat conduction can not be ignored in a real device. When the finite thermal resistance is included the overall device performance drastically reduces, as illustrated by model devices from two semiconductor materials, GaAs and InSb. Indeed, assuming bulk material properties, no single-barrier thermionic device made of GaAs can operate as a refrigerator with a temperature gradient of 20 K at room temperature. Even for a material with 'good' thermal resistance, InSb, a maximum efficiency of less than 10% of the maximum theoretical efficiency is possible for a single-barrier, bulk device.

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