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# Finite element prediction of plate-end cover separation in FRP-strengthened RC beams

S.S. Zhang

*Hong Kong Polytechnic University, shishun@uow.edu.au*

J.G. Teng

*Hong Kong Polytechnic University, cejgteng@polyu.edu.hk*

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## **Abstract**

The flexural strength of reinforced concrete (RC) beams can be effectively increased using either externally bonded (EB) or near-surface mounted (NSM) fibre-reinforced polymer (FRP) reinforcement. A likely failure mode of such FRP-strengthened RC beams is the plate-end cover separation failure mode which involves debonding of the bonded FRP reinforcement and the cover concrete along the level of the steel tension reinforcement. Despite many existing studies on this failure mode, the accurate prediction of this failure mode using the finite element (FE) method has remained a great challenge. This paper presents a novel FE smeared-crack approach for accurate prediction of plate-end cover separation in which the radial stresses generated by the steel tension reinforcement are taken into account for the first time. The capability and validity of the proposed FE model are verified using existing experimental results.

## **Disciplines**

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# FINITE ELEMENT PREDICTION OF PLATE-END COVER SEPARATION IN FRP-STRENGTHENED RC BEAMS

S.S. Zhang and J.G. Teng

*Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, Hong Kong, China.*

**Abstract:** The flexural strength of reinforced concrete (RC) beams can be effectively increased using either externally bonded (EB) or near-surface mounted (NSM) fibre-reinforced polymer (FRP) reinforcement. A likely failure mode of such FRP-strengthened RC beams is the plate-end cover separation failure mode which involves debonding of the bonded FRP reinforcement and the cover concrete along the level of the steel tension reinforcement. Despite many existing studies on this failure mode, the accurate prediction of this failure mode using the finite element (FE) method has remained a great challenge. This paper presents a novel FE smeared-crack approach for accurate prediction of plate-end cover separation in which the radial stresses generated by the steel tension reinforcement are taken into account for the first time. The capability and validity of the proposed FE model are verified using existing experimental results.

**Keywords:** FRP, strengthening, RC beams, bonded reinforcement, debonding, cover separation, finite element modelling

## 1 INTRODUCTION

Strengthening of reinforced concrete (RC) beams with an externally bonded (EB) fibre-reinforced polymer (FRP) plate is now a well-established technique (e.g. Hollaway and Teng 2008). Such beams often fail by debonding which separates the plate from the RC beam (Teng and Chen 2009). Two types of debonding failures are possible: (1) plate-end debonding in which debonding initiates near a plate end and propagates towards the strengthened region of the beam; and (2) intermediate crack (IC) debonding in which debonding initiates at a major flexural crack and propagates towards a plate end.

Plate-end debonding failures can be further classified into two main modes: (a) plate-end cover separation in which the plane of debonding is at the level of the steel tension reinforcement (Teng and Chen 2009); (b) plate-end interfacial debonding in which the plane of debonding is in the concrete adjacent to the adhesive layer (Teng and Chen 2009). Plate-end interfacial debonding is much less likely compared to plate-end cover separation and may only occur when the plate width is much smaller than the beam width so that the plate-to-beam plane is weaker than the plane between the steel tension bars and the concrete. Many experimental and theoretical studies have

been conducted on plate-end debonding failures of RC beams flexurally-strengthened with EB FRP reinforcement (e.g. Hollaway and Teng 2008), with the development of a reliable debonding strength model being a main aim of these studies. The most advanced strength model for plate-end debonding appears to be that proposed by Teng and Yao (2007).

As an alternative to the EB FRP reinforcement technique, the near-surface mounted (NSM) FRP reinforcement technique has attracted considerable research attention in recent years. In this technique, FRP strips or rods are placed into grooves which have been pre-cut in the concrete cover and partially filled with a bonding adhesive (usually an epoxy). In RC beams strengthened with NSM FRP reinforcement, cover separation at the curtailment of the FRP reinforcement has also been observed in laboratory tests (Al-Mahmoud et al. 2009; Teng et al. 2006). For simplicity, the term “plate-end cover separation” is also used to refer to such failure modes even though the NSM FRP reinforcement is generally not in the form of a plate. No strength model has been proposed for plate-end cover separation in RC beams bonded with NSM FRP reinforcement due to the limited amount of test data and limited research available.

Debonding failures in RC beams strengthened with EB FRP reinforcement have been the subject of many nonlinear finite element (FE)

studies (e.g. Lu et al. 2007). However, most of these existing FE studies have been focussed on the modelling of IC debonding (Zhang and Teng 2010). For IC debonding, accurate modelling of the shear behaviour of concrete is not critical. The situation is different for the modelling of plate-end cover separation for which both the tensile and shear behaviour of cracked concrete need to be properly modelled. Of the small number of FE studies focussed on plate-end cover separation (Arduini et al. 1997; Rahimi and Hutchinson 2001; Yang et al. 2003; Supaviriyakit et al. 2004; Pham and Al-Mahaidi 2005; Camata et al. 2007), none provides accurate modelling of cracked concrete (particularly with regard to shear behaviour) or the bond-slip behaviour between steel and concrete. The modelling of the FRP-to-concrete bond-slip behaviour has also been inadequate, but this aspect is believed to be less critical to the accurate prediction of plate-end cover separation. Furthermore, in some of these papers, clear evidence for the correct prediction of plate-end cover separation failure mode is not available (Rahimi and Hutchinson 2001; Arduini et al. 1997). Finally, Yang et al.'s (2003) model required time-consuming and complicated re-meshing while Camata et al.'s (2007) model is not truly predictive as the experimentally-observed crack pattern needs to be known before the FE model may give reasonable predictions. No FE study of plate-end cover separation in RC beams strengthened with NSM FRP has been found.

This paper presents a novel FE model for plate-end cover separation in FRP-strengthened RC beams based on the smeared crack approach in which the radial stresses generated by the steel tension reinforcement, which are identified for the first time as an important factor, are taken into account. In this FE model, the behaviour of cracked concrete is more accurately modelled and the bond-slip responses between FRP and concrete and between steel and concrete are both accurately captured. For simplicity, only simply-supported beams are explicitly referred to in the paper although the proposed FE approach is equally applicable to other beams.

## 2 PROPOSED FE MODEL

### 2.1 General

The general purpose FE program MSC.MARC (2005) was adopted as the computational platform for the present study.

The present FE model is a plane stress model with the concrete represented using four-node plane stress elements (Element 3 in MSC.MARC) and the steel rebars and FRP reinforcement represented using two-node beam elements (Element 14 for steel rebars and Element 5 for FRP reinforcement respectively). All these elements employ a full Gauss integration. In addition, four-node cohesive elements (Element 186) are used to model interfaces between concrete and steel/FRP reinforcement. Only half the span of a simply-supported beam subjected to symmetric loading needs to be included in the FE model by taking advantage of symmetry. The FE model was implemented with MSC.MARC (2005) via user-defined subroutines.

Apart from accurate modelling of cracked concrete subjected to tension and shear, the bond-slip behaviour between steel reinforcement and concrete also needs to be properly modelled as it has a significant bearing on the predicted crack pattern, which indirectly affects the cover separation failure process. In addition, special attention needs to be paid to the following two weakening effects in modelling the debonding plane between steel tension reinforcement and concrete: (a) the total width of concrete at this plane is smaller than the beam width as part of the width is taken up by the steel reinforcement; and (b) more importantly, the radial stresses generated by the deformed surfaces of the steel tension reinforcement (Figure 1) need to be included in the FE model. The second effect has not been considered in any previous study and is shown to be very important later in the paper through numerical results. The first effect can be easily included in a plane stress model, but the second effect needs an approximate representation in such an FE model.

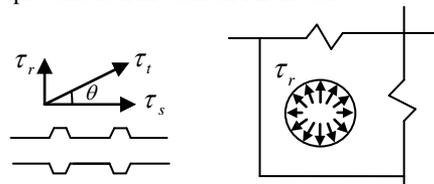


Figure 1. Bond stresses between steel and concrete

### 2.2 Modelling of Concrete, Steel and Interfaces

The yield surface of Buyukozturk (1977) with an associated flow rule is used to describe the compression-dominated behaviour of concrete, with the equivalent stress-plastic strain

behaviour being defined using data from the uniaxial compressive stress-strain curve for concrete proposed by Elwi and Murray (1979). The concrete is assumed to be linear-elastic before cracking. The behaviour of cracked concrete is modelled using the orthogonal fixed smeared crack model and the crack band concept with its post-cracking tension-softening behaviour represented by the exponential tension-softening curve of Hordijk (1991); the tensile strength and the tensile fracture energy of concrete are those given by CEB-FIP (1993). Therefore, the failure surface is a combination of the failure surface of Buyukozturk's (1977) plasticity model and the maximum tensile stress criterion. The shear retention factor model was deduced from the shear stress model proposed by Okamura and Maekawa (1991).

An elastic-perfectly plastic stress-strain relationship is assumed for the longitudinal steel reinforcement and the steel stirrups, but obviously other curves can be easily accommodated. The FRP strip is modelled as an isotropic linear-elastic material with brittle failure in tension. For more details, please refer to Zhang and Teng (2010).

The bond-slip model between steel and concrete proposed by CEB-FIP (1993) is adopted. For beams strengthened with EB FRP, the bond-slip model between FRP and concrete of Lu et al. (2005) (the simplified version) is used. For beams strengthened with NSM FRP, the bond-slip relationship needs to be obtained from the meso-scale FE analysis of a bonded joint test (Teng et al. 2009) as a theoretical model in closed-form expressions is not yet available; the NSM FRP is located at the mid-height of the groove.

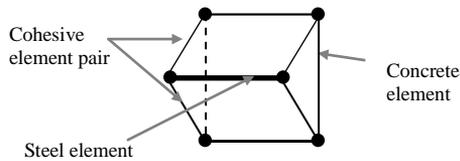


Figure 2. Cohesive-element-pair (CEP)

### 2.3 Modelling of the Radial Stress Effect

To represent the effect of radial stresses generated by the steel tension reinforcement, a special cohesive-element-pair (CEP) is proposed (Figure 2). The CEP consists of two 4-node cohesive elements each of which connects two nodes of the plane stress element representing

the adjacent concrete to the two nodes of the beam element representing the steel reinforcement located at the mid-height of the concrete element. The upper cohesive element is employed to simulate the shear bond behaviour between the steel tension reinforcement and the surrounding concrete, while the lower cohesive element simulates the interaction between the steel tension reinforcement and the concrete in the vertical direction.

Before any deformation occurs, the CEP overlaps with the adjacent concrete element whose height is equal to the diameter of the steel tension bars. When slips between the steel reinforcement and the surrounding concrete occur, the upper cohesive element will deform in shear to represent the shear bond-slip behaviour and does not experience any deformation in the normal (vertical) direction. In the meantime, from the shear bond force developed in the upper cohesive element, the associated normal (vertical) bond force in the lower cohesive element can be deduced; the lower cohesive element does not possess any shear stiffness or strength.

The determination of the normal (vertical) bond force is as follows. The radial stresses  $f_r$  from a steel rebar can be related to the shear bond stresses  $\tau_s$  if the angle  $\theta$  between them is known (Figure 1). Assuming that the radial stresses and the shear bond stresses are distributed uniformly around the circumference of a steel rebar, the internal force generated by the steel rebar in the vertical direction per unit length is given by

$$F_v = D \tau_r = D \tau_s \tan \theta \quad (1)$$

where  $D$  is the diameter of the steel rebars.

The angle  $\theta$  is a key parameter of the present FE model and has been studied by a number of researchers. Ferguson and Briceno (1969) recommended a value of  $45^\circ$  for simplicity; Tepfers (1973) found that the angle varied between  $30^\circ$  and  $45^\circ$ , using an FE model; Orangun et al. (1977) concluded that the angle ranged from  $38^\circ$  to  $53^\circ$ , by analysing the existing tests; and Tepfers and Olsson (1992) conducted a series of ring tests and reported that the angle was between  $30^\circ$  and  $65^\circ$ . In the present study, the lower-bound value of  $30^\circ$  is adopted, considering that flexural cracks may weaken the integrity of the concrete cover and hence reduce the normal stresses generated by slip actions of the steel reinforcement.

## 3 NUMERICAL RESULTS

Plate-end cover separation failures have been observed in a large number of tests on RC beams strengthened with EB FRP or NSM FRP reinforcement. In this study, a total of 43 beams that failed by plate-end cover separation were analysed to verify the proposed FE model. Among the selected beams, 40 specimens are RC beams strengthened with EB FRP, which can be found in Teng and Yao (2007) and Maalej and Bian (2001), and 3 specimens are RC beams strengthened with NSM FRP which were tested by Teng et al. (2006).

The FE investigation began with a convergence study for two of the selected specimens. One beam was strengthened with EB FRP [Beam 3 in Maalej and Bian (2001)] and the other with NSM FRP strips [Beam 1800 in Teng et al. (2006)]. Three representative widths of elements, which were used for most of the elements for the concrete in the beam, were examined: 30mm, 15mm and 5mm; the aspect ratios of these elements were about 1 except those elements used to model the part of concrete covering and below the steel tension reinforcement. From the convergence study, it was found that there was negligible difference among the selected element sizes in terms of load-deflection curves, indicating that the FE results were not sensitive to element sizes within such a reasonable range. Finally, the representative element size of 15 mm was adopted for all selected beam specimens, considering that it led to accurate load-deflection curves and clear crack patterns, without the computational effort becoming excessive.

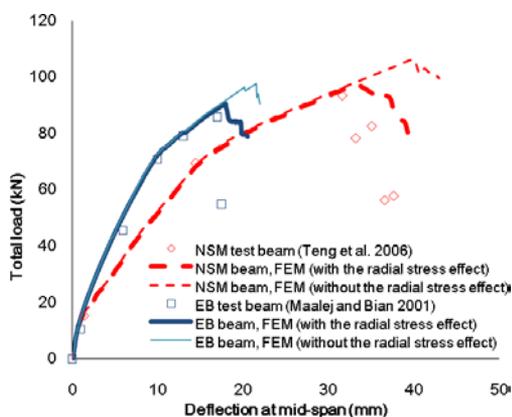
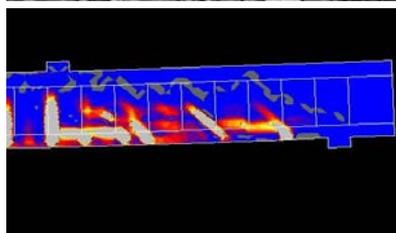
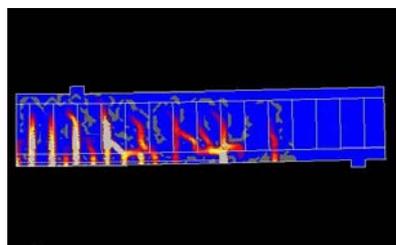
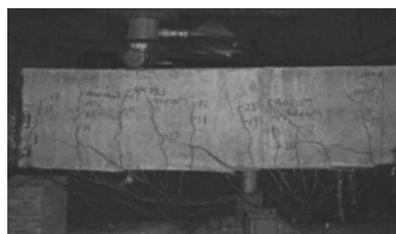


Figure 3. Load-deflection curves

The FE and the test load-deflection curves of the two selected specimens are compared in Figure 3, while the crack patterns are compared in Figure 4. A comparison of the ultimate loads of all selected specimens is given in Figure 5.



a) RC beam with EB FRP



b) RC beam with NSM FRP

Figure 4. Crack patterns at failure

In Figure 3, the unfilled squares and diamonds are the test results, the thicker curve is from the FE analysis with the effect of radial stresses taken into account, and the thinner curve is from the FE analysis with this effect ignored. If the radial stress effect is included in the FE analysis, the predicted load-deflection curve agrees closely with the test curve, with the FE ultimate load being a little higher than the test result. If this effect is ignored, the FE analysis significantly over-estimates the ultimate load and the corresponding deflection. This comparison indicates that the radial stress effect is very important.

In Figure 4, the crack patterns at the ultimate state from FE analysis with the radial stress effect included are contrasted with those from tests. It can be seen from Figure 4 that the crack patterns are closely predicted by the FE model. When the radial stress effect is not accounted for, the crack patterns from the FE model are far different from those from the tests (Zhang and Teng 2010).

As can be seen in Figure 5, the FE model including the radial stress effect gives close predictions of the test ultimate loads of all 40 beam specimens, with the average error being only 4.9%. This average error is small giving the complexity of the problem. On the contrary, the FE model with the radial stress effect ignored overestimates the test ultimate loads substantially, with an average error of 31.9%.

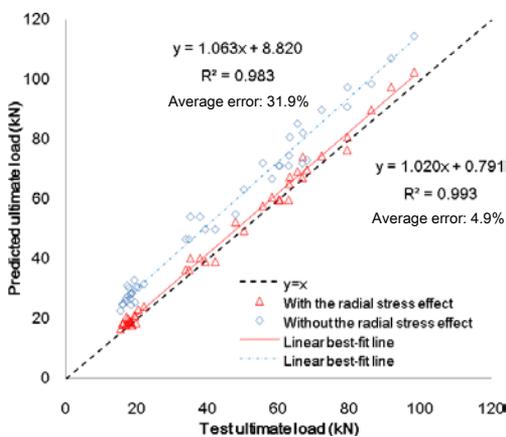


Fig.5 Comparison of ultimate loads

## 5 CONCLUSIONS

The paper has presented an FE model based on the smeared crack approach for plate-end cover separation failures in FRP-strengthened RC beams. In this FE model, the tensile behaviour of cracked concrete is modelled using the crack band and the fracture energy concepts while the shear behaviour of cracked concrete is modelled using Okamura and Maekawa's (1991) shear stress model. Bond-slip relationships between concrete and steel reinforcement (including both steel tension bars and stirrups) as well as between concrete and FRP reinforcement are accurately represented using cohesive elements. Most importantly, the radial stresses generated by the steel tension reinforcement are identified

to be important and rationally represented for the first time ever. This FE model has been shown to provide close predictions of load-deflection curves, ultimate loads, and crack patterns from tests, and can be employed in future studies to gain an improved understanding of the mechanism of plate-end cover separation failures and to generate numerical data for the improvement/development of debonding strength models.

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