1-1-2012

Analytical solution for interaction forces in RC beams strengthened with NSM rectangular bars

S S. Zhang
Hong Kong Polytechnic University, shishun@uow.edu.au

J G. Teng
Hong Kong Polytechnic University, cejgteng@polyu.edu.hk

Recommended Citation
Zhang, S S. and Teng, J G., "Analytical solution for interaction forces in RC beams strengthened with NSM rectangular bars" (2012). *Faculty of Engineering and Information Sciences - Papers: Part A*. 4952.
https://ro.uow.edu.au/eispapers/4952

Analytical solution for interaction forces in RC beams strengthened with NSM rectangular bars

Abstract

The flexural performance of an RC beam can be improved using the near-surface mounted (NSM) FRP strengthening technique. In such FRP-strengthened RC beams, debonding failure at the two bar ends may occur due to high interaction forces in the bar-end regions between the RC beam and the FRP bars. This paper presents a theoretical investigation into these interaction forces in RC beams strengthened with NSM FRP rectangular bars. By introducing and defining two interfacial stiffness parameters, an analytical solution for the tangential and the normal interaction forces is first obtained as an extension of the analytical solution developed by Smith and Teng [1] for beams strengthened with an externally bonded plate. The accuracy of the analytical solution is verified with predictions from a 3-D linear elastic finite element model.

Disciplines

Engineering | Science and Technology Studies

Publication Details

Zhang, S. S. & Teng, J. G. (2012). Analytical solution for interaction forces in RC beams strengthened with NSM rectangular bars. Proceedings, 6th International Conference on FRP Composites in Civil Engineering (pp. 1-6).

ANALYTICAL SOLUTION FOR INTERACTION FORCES IN RC BEAMS STRENGTHENED WITH NSM RECTANGULAR BARS

S.S. Zhang
Research Associate
The Hong Kong Polytechnic University
W712, The Hong Kong Polytechnic University
cezss@polyu.edu.hk

J.G. Teng
Chair Professor
The Hong Kong Polytechnic University
AG701, The Hong Kong Polytechnic University
cejgteng@polyu.edu.hk *

Abstract

The flexural performance of an RC beam can be improved using the near-surface mounted (NSM) FRP strengthening technique. In such FRP-strengthened RC beams, debonding failure at the two bar ends may occur due to high interaction forces in the bar-end regions between the RC beam and the FRP bars. This paper presents a theoretical investigation into these interaction forces in RC beams strengthened with NSM FRP rectangular bars. By introducing and defining two interfacial stiffness parameters, an analytical solution for the tangential and the normal interaction forces is first obtained as an extension of the analytical solution developed by Smith and Teng [1] for beams strengthened with an externally bonded plate. The accuracy of the analytical solution is verified with predictions from a 3-D linear elastic finite element model.

Keywords: finite element (FE) model, interaction forces, interfacial stiffness, near-surface mounted (NSM), RC beams

1. Introduction

The near-surface mounted (NSM) FRP strengthening of concrete structures involves cutting grooves in the concrete cover, filling the grooves with adhesive (usually epoxy adhesive), and embedding into each groove a circular or rectangular FRP bar. The NSM FRP method leads to better bond effectiveness with concrete and thus fuller utilization of the tensile capacity of FRP as shown by existing tests (e.g. Ref. [2]). Nevertheless, debonding of the FRP reinforcement from the concrete is still a likely failure mode. In RC beams flexurally-strengthened with NSM FRP bars on the beam soffit (Fig. 1), debonding at the ends of the NSM FRP bars (referred to as bar-end debonding in this paper), similar to plate end debonding in RC beams strengthened with a bonded soffit plate (i.e. plated beams) [3], has been observed in existing tests (e.g. Refs. [4] and [5]). This bar-end debonding failure mode is due to the high interaction forces (which are related to the interfacial stresses) between the concrete and the FRP near the bar ends. This paper presents briefly an analytical solution for these interaction forces in RC beams strengthened with NSM FRP bars; a full description of the solution is given in Ref. [6]. This analytical solution represents an extension of the analytical solution for interfacial stresses in FRP-plated beams developed by Smith and Teng [1].

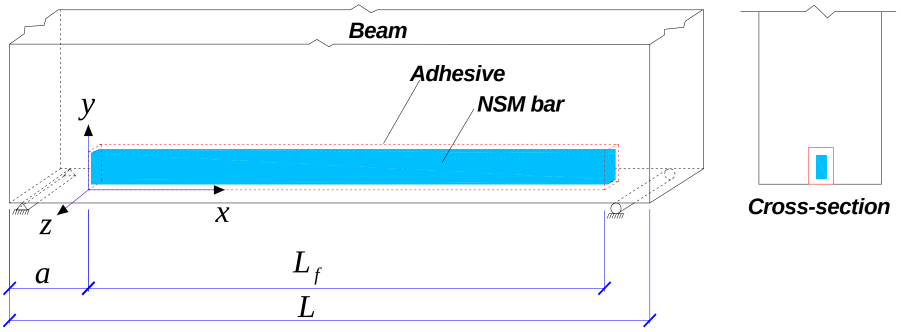
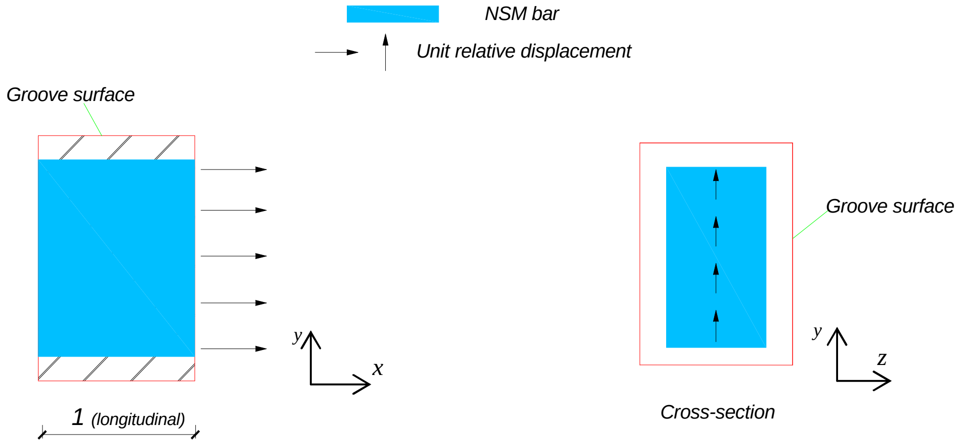


Figure 1. RC beam strengthened with an NSM bar



(a) Tangential interfacial stiffness k_t (b) Normal interfacial stiffness k_n

Figure 2. Interfacial stiffness parameters as interaction forces due to a unit relative displacement

For simplicity of analytical derivation without loss of generality, an RC beam strengthened with a single NSM bar of rectangular section (Fig. 1) is considered in this paper; the solution can be easily adapted for the case of multiple bars. To extend the solution of Smith and Teng [1] to the problem (Fig. 1), the RC beam strengthened with an NSM bar is also treated as a plane stress problem as was assumed for developing Smith and Teng's solution [1]. In RC beams strengthened with an NSM bar, the adhesive layer is not a flat layer, and the thickness of the adhesive layer is usually not constant around the NSM bar; thus a simple width of the adhesive layer, like in a plated beam, no longer exists. To overcome this difficulty, interfacial interaction forces (F_t and F_n) per unit length of the interface instead of interfacial stresses (² and ±) are introduced; F_t and F_n are the interaction forces in the tangential (i.e. longitudinal) direction and the normal (i.e. vertical) direction respectively, while ² and ± are the interfacial shear and normal stresses respectively; the subscript t and n are used to indicate the tangential and the normal directions respectively. Corresponding to the two interaction forces, two interfacial stiffness parameters are defined, including the interfacial tangential stiffness k_t and the interfacial normal stiffness k_n. These stiffness parameters are obviously equal to the interfacial interaction forces per unit length between the NSM bar and the concrete corresponding to a unit relative displacement between the NSM bar and the concrete in the designated direction, as illustrated in Fig. 2.

2. Governing equations and solution

The governing equations for interfacial stresses in plated beams under a uniformly distributed load q according to the solution of Smith and Teng [1] are given by

$$\frac{d^{2}2(x)}{dx^{2}}-\frac{G_a b_f}{t_a}\left(\frac{(y_b+y_f)(y_b+y_f+t_a)}{E_b I_b+E_f I_f}+\frac{1}{E_b A_b}+\frac{1}{E_f A_f}\right)2(x)-\frac{G_a}{t_a}\left(\frac{y_b+y_f}{E_b I_b+E_f I_f}\right)V_T(x)=0 \quad (1)$$

$$\frac{d^{4}\pm(x)}{dx^{4}}+\frac{E_a b_f}{t_a}\left(\frac{1}{E_b A_b}+\frac{1}{E_f A_f}\right)\pm(x)+\frac{E_a b_f}{t_a}\left(\frac{y_b}{E_b I_b}-\frac{y_f}{E_f I_f}\right)\frac{d2(x)}{dx}+\frac{E_a}{t_a}\frac{1}{E_b I_b}q=0 \quad (2)$$

where E_f, G_f, b_f, A_f and I_f are the elastic modulus, shear modulus, width, cross-sectional area and moment of inertia of the externally bonded plate respectively; E_b, G_b A_b and I_b are the elastic modulus, shear modulus, cross-sectional area and moment of inertia of the original beam respectively; E_a, G_a, and t_a are the elastic modulus, shear modulus and thickness of the adhesive layer respectively; y_b and y_f are the distances from the soffit of the original beam and the top surface of the bonded plate to their respective centroidal surfaces respectively; and $V_T(x)$ is the shear force. The subscripts b and f denote association of parameters with the original beam and the bonded plate respectively.

Eqs. (1) and (2) can be adapted for the prediction of interaction forces in RC beams strengthened with an NSM bar through the following modifications: 1) multiply the two sides of Eqs. (1) and (2) by the nominal width of interface b_n; 2) replace $b_n 2(x)$ and $b_n \pm(x)$ with $F_t(x)$ and $F_n(x)$ respectively; 3) replace $G_a b_n / t_a$ and $E_a b_n / t_a$ with k_t and k_n respectively; and 4) replace y_b+y_f with $y_b+y_f+t_a$ as t_a is usually two orders of magnitude smaller than y_b+y_f, and then let $d_{b-f}=y_b+y_f+t_a$ which is the distance between the centroid of the original beam and that of the NSM bar. As a result, the adapted governing equations for RC beams strengthened with an NSM bar are as follows:

$$\frac{d^{2}F_t(x)}{dx^{2}}-k_t\left(\frac{d_{b-f}^{2}}{E_b I_b+E_f I_f}+\frac{1}{E_b A_b}+\frac{1}{E_f A_f}\right)F_t(x)-k_t\left(\frac{d_{b-f}}{E_b I_b+E_f I_f}\right)V_T(x)=0 \quad (3)$$

$$\frac{d^{4}F_n(x)}{dx^{4}}+k_n\left(\frac{1}{E_b I_b}+\frac{1}{E_f I_f}\right)F_n(x)+k_n\left(\frac{y_b}{E_b I_b}-\frac{y_f}{E_f I_f}\right)\frac{dF_t(x)}{dx}+k_n\frac{1}{E_b I_b}q=0 \quad (4)$$

The solution to Eqs. (3) and (4) is in the following form:

$$F_t(x)=B_1\cosh(\mathrm{a}x)+B_2\sinh(\mathrm{a}x)+mV_T(x) \quad (5)$$

$$F_n(x)=e^{-\ x}\left[C_1\cos(\ x)+C_2\sin(\ x)\right]-n_1\frac{dF_t(x)}{dx}-n_2 q \quad (6)$$

The constants in Eqs. (5) and (6) are as follows:

$$\mathrm{a}^{2}=k_t\left(\frac{d_{b-f}^{2}}{E_b I_b+E_f I_f}+\frac{1}{E_b A_b}+\frac{1}{E_f A_f}\right) \ (7) \quad m=\frac{k_t}{\mathrm{a}^{2}}\left(\frac{d_{b-f}}{E_b I_b+E_f I_f}\right) \ (8) \quad =\sqrt[4]{\frac{k_n}{4}\left(\frac{1}{E_b I_b}+\frac{1}{E_f I_f}\right)} \ (9)$$

$$n_1 = \left(\frac{y_b E_f I_f - y_f E_b I_b}{E_b I_b + E_f I_f} \right) \quad (10) \quad n_2 = \left(\frac{E_f I_f}{E_b I_b + E_f I_f} \right) \quad (11) \quad B_1 = \frac{k_t y_b q a}{E_b I_b 2 \mathrm{a}} (L - a) - \frac{k_t}{\mathrm{a}^3} \left(\frac{d_{b-f}}{E_b I_b + E_f I_f} \right) q \quad (12)$$

$$B_2 = -B_1 = -\frac{k_t y_b q a}{E_b I_b 2 \mathrm{a}} (L - a) + \frac{k_t}{\mathrm{a}^3} \left(\frac{d_{b-f}}{E_b I_b + E_f I_f} \right) q \qquad (13)$$

$$\begin{aligned} C_1 = {} & \frac{k_n}{2 \quad {}^3 E_b I_b} \left[q(\frac{L}{2} - a) + \quad \frac{qa}{2} (L - a)) \right] - \frac{k_n}{2 \quad {}^3} \left(\frac{y_b}{E_b I_b} - \frac{y_f}{E_f I_f} \right) F_t(0) + \\ & \frac{k_t q \mathrm{a}^2}{2 \quad {}^3} \left(\frac{y_b E_f I_f - y_f E_b I_b}{E_b I_b + E_f I_f} \right) \left[\frac{y_b a}{2 E_b I_b} (L - a) - \frac{1}{\mathrm{a}^2} \left(\frac{d_{b-f}}{E_b I_b + E_f I_f} \right) \right] \left[\mathrm{a} \cosh(\mathrm{a} x) - \quad \cosh(\mathrm{a} x) \right] \end{aligned} \qquad (14)$$

$$\begin{aligned} C_2 = {} & -\frac{k_n}{2 \quad {}^2 E_b I_b} \frac{qa}{2} (L - a) + \\ & \frac{k_t q \mathrm{a}^2}{2 \quad {}^2} \left(\frac{y_b E_f I_f - y_f E_b I_b}{E_b I_b + E_f I_f} \right) \left[\frac{y_b a}{2 E_b I_b} (L - a) - \frac{1}{\mathrm{a}^2} \left(\frac{d_{b-f}}{E_b I_b + E_f I_f} \right) \right] \cosh(\mathrm{a} x) \end{aligned} \qquad (15)$$

---

where L and a are the span of the beam and the distance from the bar end to the nearest support respectively, as shown in Fig. 1.

3. Interfacial stiffness parameters

As is obvious from the preceding section, the interfacial stiffness parameters need to be determined first in order to evaluate the interfacial interaction forces from the analytical solution. The interfacial stiffness parameters can be obtained from linear elastic FE stress analysis as interaction forces per unit length due to a unit relative displacement in the designated direction (Fig. 2). However, direct FE analysis for each individual case to determine the two interfacial stiffness parameters is not suitable for easy exploitation of the analytical solution. Based on the results of an FE parametric analysis, simplified expressions for the tangential stiffness and the normal stiffness have been for rectangular NSM bars as follows:

$$k_t = (\frac{t_2}{2} + t_f) \frac{G_a}{t_1} + (t_1 + \frac{t_3}{2} + 2h_f) \frac{G_a}{t_2} \qquad (16)$$

$$k_n = (\frac{t_2}{2} + t_f) \frac{E_a}{t_1} \frac{1 - v}{(1 + v)(1 - 2v)} + (\frac{t_3}{2} + h_f) \frac{G_a}{t_2} \qquad (17)$$

where v is the Poisson's ratio of the adhesive, t_1 t_2 and t_3 are the thicknesses of the top, side and bottom adhesive layers respectively.

Table 1. Geometric and material properties of the reference beam

Component	Width (*mm*)	Height (*mm*)	Length (*mm*)	Elastic modulus (*MPa*)	Poisson's ratio
Beam	150	300	2400	20000	0.18
groove	12	12	1800	NA	NA
Square NSM bar	8	8	1800	200000	0.3
Adhesive	2 *mm* (thickness)		1800	3000	0.35

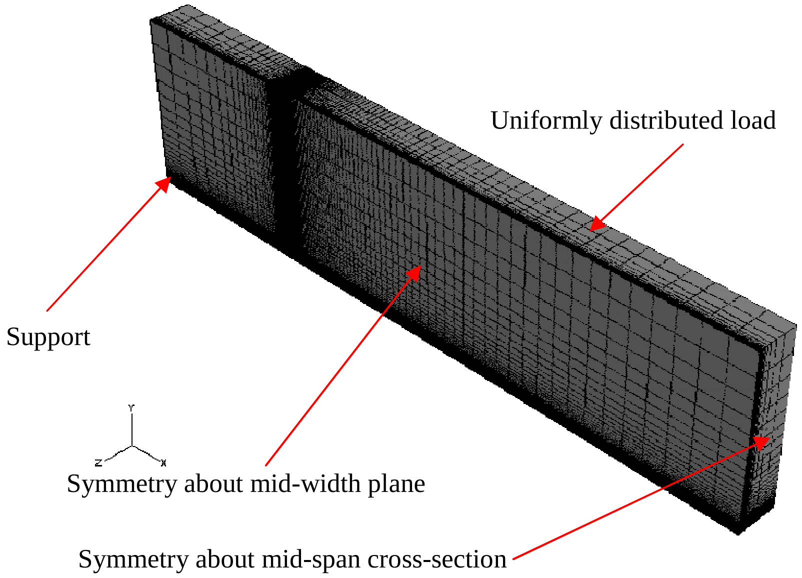
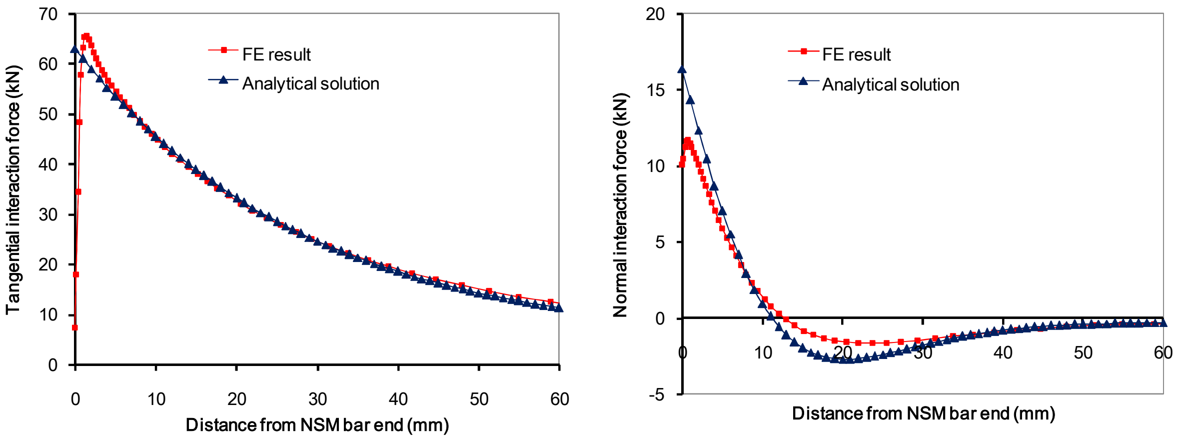


Figure 3. Finite element mesh of RC beam with an NSM bar



(a) Tangential interaction force (b) Normal interaction force

Figure 4. Comparison of interaction forces for a square NSM bar

4. Analytical predictions versus FE results

3-D FE analyses of RC beams strengthened with an NSM bar under a uniformly distributed load were conducted using the general-purpose FE software package ABAQUS; the eight-node brick element (Element C3D8) was used to model all components of the strengthened beam. The details of the strengthened beam are listed in Table 1. A graded FE mesh as shown in Fig. 3 was adopted, with the smallest elements being employed at the end of the NSM bar. From a convergence study, it was found that an element size of 1/6 mm is small enough for the square or nearly square elements near the bar end in present study. The interaction forces were obtained by integrating the relevant stresses on the middle-thickness surface of the adhesive layer. A comparison of interaction forces between the analytical method and FE analysis is given in Fig. 4. As can be seen from Fig. 4a, the analytical predictions of the tangential interaction force are in close agreement with the FE results; the peak value of the tangential interaction force from the analytical method occurs at the end of the NSM bar, but the peak value from the FE model is at about 1mm from the NSM bar end and is slightly higher than the analytical prediction. For the normal interaction force, Fig. 4b shows that the error of the analytical method is a little larger than that of the tangential interaction force. The normal interaction force from the analytical method is larger than the FE prediction at the NSM bar end, but the two values become almost identical at about 10mm from the end of the NSM bar, after which the FE values are larger than the analytical values. Beyond 40 mm from the end of the NSM bar, both values are small and are in close agreement.

5. Concluding remarks

This paper has presented a closed-form analytical solution for interaction forces in RC beams strengthened with a near-surface mounted (NSM) bar. The analytical solution represents an extension of the closed-form solution of Smith and Teng [1] for interfacial stresses in RC beams strengthened with an externally bonded FRP plate. The key element of the extension is the establishment of the tangential and normal interfacial stiffness equations. The predictions of the analytical solution have been found to be in close agreement with results from 3-D FE analysis. Both the analytical solution and the FE analysis predict large interaction forces near the ends of the NSM bar, which explains why bar-end debonding failure is commonly observed in tests on RC beams strengthened with NSM bars. The present analytical solution provides a simple tool for understanding these interaction forces and can potentially be used for checking or predicting the likeliness of bar-end debonding failure.

[1] SMITH, S. T., TENG, J. G., “Interfacial Stresses in Plated Beams”, *Engineering Structures*, Vol. 23, No. 7, pp. 857-871.

[2] El HACHA, R., RIZKALLA, S. H., “Near-Surface-Mounted Fiber-Reinforced Polymer Reinforcements for Flexural Strengthening of Concrete Structures”, *ACI Structural Journal*, Vol.101, No. 5, pp. 717-726.

[3] TENG, J. G., CHEN, J. F., SMITH, S. T., and LAM, L., “*FRP-strengthened RC Structures*”, West Sussex: Wiley, 2002.

[4] BARROS, J. A. O., FORTES, A. S., “Flexural Strengthening of Concrete Beams with CFRP Laminates Bonded into Slits”, *Cement & Concrete Composites*, Vol. 27, No.4, pp. 471-480.

[5] TENG, J. G., DE LORENZIS, L., WANG, B., RONG, L., WONG, T. N., LAM, L., “Debonding Failures of RC Beams Strengthened with Near-surface Mounted CFRP Strips”, *Journal of Composites for Construction*, ASCE, Vol. 10, No. 2, pp. 92-105.

[6] ZHANG, S.S., TENG, J.G., “Interaction Forces in RC Beams Strengthened with Near-Surface Mounted Rectangular Bars”, in preparation.