Modelling and control of electric vehicles with individually actuated in-wheel motors

Sean Christifor McTrustry
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Modelling and Control of Electric Vehicles with Individually Actuated In-Wheel Motors

Sean Christifor McTrustry

Supervisors:
Haiping Du, Phillip Commins

This thesis is presented as part of the requirements for the conferral of the degree:

Master of Philosophy (Mechatronic Engineering)

The University of Wollongong
School of Engineering and Information Sciences

March 2016
Declaration

I, Sean Christifor McTrustry, declare that this thesis submitted in partial fulfilment of the requirements for the conferral of the degree Master of Philosophy (Mechatronic Engineering), from the University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. This document has not been submitted for qualifications at any other academic institution.

Sean Christifor McTrustry

September 27, 2016
Abstract

Torque vectoring in electric ground vehicles (EGV) with individually actuated in-wheel motors (IAIWM) presents the opportunity to implement a wide range of control strategies for controlling vehicle yaw rate to improve vehicle stability and performance. The use of IAIWMs allows for alternative vehicle layout configurations which previously would have been unavailable to conventional internal combustion engine vehicles. The use of higher level control architectures to distribute torque amongst the two front wheel-drive, rear wheel-drive or four wheel-drive in-wheel motors of an electric ground vehicle has presented the opportunity to design characteristics of electric ground vehicles through active control of power trains. Previously in internal combustion engine vehicles, these characteristics have been indirectly tuned via common chassis parameters. The use of modern components such as in-wheel motors in electric ground vehicles also provides additional benefits such as precise torque generation, fast motor response and the capability to produce forward and reverse torque as well as regenerative braking to improve energy efficiency, and enabling the estimation or measurement of useful feedback information. This feedback information can be applied to direct yaw-moment control (DYC) strategies which can be used to improve vehicle performance. The application of these new vehicle configurations can allow for differential torque output to the left and right hand side of vehicles, generating a yaw moment, and hence directly affecting the yaw rate of the vehicle in a practice known as direct yaw-moment control. In addition to the potential electric ground vehicles possess for superior vehicle stability and performance, they are also a viable solution for the environmental concerns pertaining to transport needs and meeting lower emissions targets. In this thesis the process of converting an internal combustion engine vehicle to a fully electric vehicle with IAIWM will be presented. The first aim of this thesis is to conduct a literature review in which control strategies available for allocating torque to
individually actuated in-wheel motors on an electric ground vehicle are investigated, with the objectives of improving vehicle dynamics performance through control of yaw rate response. Secondly, this thesis will present the development of a simulation framework which models vehicle behaviour and addresses the major performance indicators relevant to evaluating vehicle dynamics performance with regards to torque vectoring (TV)/DYC strategies. Next, this thesis aims to show the effects of a traction control strategy, developed for active differentials, when adapted and extended for use as a direct yaw-moment control strategy on an electric ground vehicle with individually actuated in-wheel motors. This torque vectoring control strategy’s effect on a vehicle’s dynamic performance will be validated and analysed through use of simulations, using the platform developed as part of the work involved in this thesis. The simulation platform presented in this thesis is also intended for use as tool for investigation on future projects pertaining to the experimental electric vehicle. The next objective of this thesis is to establish the measurement and estimation techniques available and how they could be implemented through suitable hardware to measure and record the relevant performance indicators of vehicle dynamics in relation to a DYC strategy. Finally, this thesis aims to prove the accuracy of the simulation platform developed using experimental data acquired from sensors implemented on the experimental vehicle. The simulation platform is validated experimentally as an accurate representation of the experimental system and its performance in terms of realistic vehicle dynamics. Experimental data is used to recreate real-life driving manoeuvres in the simulation platform, and verify its performance by comparing results.
Acknowledgements

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- Haiping Du
- Uow & Innovation Campus Technical Staff
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# List of Abbreviations and Symbols

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<td>CAD</td>
<td>Computer Aided Design</td>
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<td>CAN</td>
<td>Control Area Network</td>
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<td>DYC</td>
<td>Direct Yaw-moment Control</td>
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<td>EGV</td>
<td>Electric Ground Vehicle</td>
</tr>
<tr>
<td>EV</td>
<td>Electric Vehicle</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<tr>
<td>GUI</td>
<td>Graphical User Interface</td>
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<td>IAIWM</td>
<td>Independently Actuated In-Wheel Motors</td>
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<td>OAEGV</td>
<td>Over-Actuated Electric Ground Vehicle</td>
</tr>
<tr>
<td>PI</td>
<td>Proportional-Integral</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-Integral-Derivative</td>
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<tr>
<td>PMSM</td>
<td>Permanent Magnet Synchronous Machine</td>
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<td>TV</td>
<td>Torque Vectoring</td>
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1 Introduction

The motivation for this project lies in the increasing demand for low environmental impact vehicles with adequate performance characteristics [4]. Internal combustion engine vehicles are a major contributor to greenhouse gas emissions, airborne pollution, carbon monoxide and air toxins [4]. With a significantly less profound impact on the environment than internal combustion engine vehicles [5], electric ground vehicles are identified as a potential technology for meeting the demand for low environmental impact vehicle [6]. Research and development on electric ground vehicles has been increasing and as a result, improvements in energy usage and control technology has been achieved [7].

The majority of passenger vehicles available on the market today make use of a single engine drivetrain, which distributes power to two or four wheels through a gearbox and differentials [8]. Electric ground vehicles may use different configurations, which can have a profound effect on the vehicle’s performance and efficiency characteristics. One configuration of electric ground vehicles which has shown improved results in vehicle performance characteristics is the use of four individually actuated in-wheel motors [9]. This setup features either a direct drive or reduction drivetrain on each wheel. These in-wheel motors allow for improved vehicle control, as they are part of the vehicles unsprung mass, and are used to actively design performance characteristics, as opposed to indirectly tuning them via the common chassis system [8]. Use of electric motors at each wheel also allows for improved accuracy in measuring vehicle characteristics, as each motor can be used as a measuring device for individual wheel speed. The major contributions of in-wheel motor technology to electric vehicle performance can be categorised into precise and fast torque generation, efficient feedback information on motor torque and speed output and ease of producing torque in both forward and reverse directions [10]. These characteristics allow for more accurate measurements and estimations to be made regarding vehicle dynamics.
behaviour and road surface conditions to allow the vehicle to adjust its torque allocation to improve performance.

The benefits of in-wheel motor technology as outlined above can be applied to the design of a control architecture with the purpose of distributing torque amongst the driving wheels of the electric vehicle. The measured and estimated data which can be acquired from this vehicle layout configuration allows for the design of a more effective torque vectoring control strategy. The practice of using differential torque outputs to generate a yaw moment for directly altering the vehicle’s motion about the vertical axis and improving vehicle stability is often referred to as direct yaw-moment control (DYC) [11]. An effective means of improving a vehicle’s dynamic performance is through the measurement and control of vehicle yaw rate. This strategy involves a controller which is designed with the objective of bringing the vehicle’s measured yaw rate into conformity with the optimal yaw rate by calculating and producing a corrective yaw moment via torque vectoring control [12]. Use of in-wheel motor technology in conjunction with direct yaw-moment control is a growing area of research in both academia and commercial research and development which has produced significant improvements in the handling and stability of passenger vehicles [13].

The first aim of this project is to convert an internal combustion engine vehicle, into an electric ground vehicle with individually actuated in-wheel motors. The objective being to function as a useful platform for future projects performing research into optimal energy efficiency and vehicle performance via higher level control. Secondly, this project aims to create a simulation model to function as an accurate representation of the experimental vehicle, serving as a robust and versatile platform for inspecting higher level vehicle control functions and their effect on vehicle performance. This project also aims to experimentally validate this model by measuring and comparing the key performance indicators of the simulation with experimentally obtained results on the experimental vehicle.
Thirdly, this project aims to establish a feasible strategy for the acquisition of useful data in real time which can be implemented into vehicle control, such as a torque vectoring control strategy. Finally, this project performs research into the current state of the art in torque vectoring control applications, with the aim to explore the practicality of developing a suitable torque vectoring control strategy, based on direct yaw-moment control (DYC) to improve vehicle yaw rate response and the handling and stability of the vehicle. The effectiveness of this strategy will be proved through simulations and its feasibility evaluated based on the resources required and process involved in implementing this strategy on a real system. The findings of this research, along with the results of simulations and experimental work performed will be detailed in this thesis, to establish a strategy and validate its feasibility in achieving this objective.
2 Literature Review

In recent years, in response to demand for commercial vehicles with reduced environmental impact [5], there has been an increased devotion of time and effort into the research and development of new configurations, technological developments and control strategies for electric ground vehicles [14]. The concept of using individually actuated in-wheel motors in electric ground vehicles has been explored by a number of institutions [13] the world over in pursuit of improving vehicle dynamics performance through torque vectoring and yaw rate control. In this chapter a review of the various methods of direct yaw-moment control via torque vectoring for electric vehicles is presented. Torque vectoring is based on the strategic distribution of torque to the driving wheels of a vehicle to improve the vehicle’s dynamic performance. Methods of torque vectoring and direct yaw-moment control in electric vehicles varies based on the configuration of vehicle hardware and the control variables in use. The vehicle hardware configuration may consist of active differentials, drivetrains with individual motors, or in-wheel motors to deliver the controlled torque to the vehicle’s driving wheels. Various torque vectoring strategies can be categorised depending on the control variable each utilises. This chapter will provide some background context on direct yaw-moment control via torque vectoring, review the important performance indicators relevant to applying an effective control strategy and review direct yaw-moment control strategies based on feedback of yaw rate, vehicle side-slip angle, and longitudinal slip ratio.

2.1 Background and Context

This section will provide background information and context relevant to the project. Concepts such as applications of torque vectoring and introducing feedback loops to yaw rate control helped to shape the current state of the art for DYC. Some examples which have been documented in literature will be discussed
in this section, along with important performance indicators relevant to the design of a DYC strategy.

2.1.1 Torque Vectoring and DYC Background

Torque vectoring refers to the distribution of torque from the engine to the wheels of a ground vehicle. Conventionally, torque vectoring in an internal combustion engine vehicle uses a differential to distribute torque from the engine to the axles of the vehicle. The research and application of torque vectoring control as a technique for the improvement of a vehicle’s dynamic performance is a growing area of interest in research and development in both industry and academia [15]. Active torque distribution is a relatively new concept, as the majority of literature on the subject has been published within the last ten years, but has progressed in both complexity and quality of results produced in recent years with the development of new technology such as hub motors (in-wheel motors).

Previously, applications of in-wheel motor technology were primarily for the use of vehicles such as bicycles and electric scooters [16] but this technology has seen significant development in recent years and as such their usefulness in passenger vehicle technology has been utilised [13]. The application of active torque vectoring control evolved from using a differential to distribute torque, to the use of active drive devices such as electronic limited slip differentials, on-demand centre couplings [17], front or rear electric axels with distributed torque [18] to the current state of the art which makes use of in-wheel motors in conjunction with a control architecture to distribute torque to each wheel. The application of in-wheel motors as a means of active torque distribution allows for the hub motors to be part of the vehicle’s unsprung mass, and allows for the active control of the distribution of torque, as opposed to indirectly tuning common chassis parameters to rely on the distribution of torque [8]. Utilising torque vectoring methods to deliver a torque differential across the left and right hand wheels of
a vehicle can be used to generate a yaw moment, which is used to directly alter the yaw rate of the vehicle. This practise is known as direct yaw-moment control (DYC) [11].

Prior to the use of in-wheel motors, active drive components have been utilised in electric vehicles to apply torque vectoring control techniques which produced improvements to vehicle handling and stability [18,19] due to the advantage provided by active differentials as they do not require brake or throttle intervention when compared to previously implemented yaw rate control strategies [20]. Work performed by Hancock et. al. [21] and Ikushima and Sawase [22] both developed direct yaw-moment control strategies based on the use of actively controlled mechanical differentials. Osborn et. al. performed studies into independent wheel actuation to explore its effectiveness in improving vehicle stability during critical steering manoeuvres whilst the vehicle is under acceleration. A strategy was adopted using a proportional-integral controller to apply yaw feedback to distribute torque to the front-rear wheels, and lateral acceleration feedback was used to adjust the torque distribution from the left and right hand driving wheels of the vehicle. A similar approach was adopted by De Novellis et. al. comparing feedback control techniques for a front-wheel-drive electric vehicle, with two individual power trains, one for each of the front wheels [9]. Yaw motion control through use of active differentials was also explored by Hancock et. al. [20] in which an active rear wheel drive system was shown to significantly modify a vehicle's dynamics performance through active control of lateral torque distribution on the vehicle’s rear axel. Osborn et. al. evaluated system performance by comparing a front-wheel-drive, and rear-wheel-drive both with a single open differential, and an all-wheel-drive model with three open differentials to the model with fully independent torque distribution, and front-rear torque distribution control implemented. These configurations were tested under a standard manoeuvre; response to steering input of 5 degrees. The results showed that of all the models evaluated, the model with independent torque control maintained the closest
conformity to the desired path of the vehicle, without affecting the acceleration of the vehicle. The limitations of this work performed include simplifications that were made to the vehicle model. The model is based on Newtonian equations and the Pacejka Magic Tyre Formula, however the model ignores heave, roll and pitch motion, has no suspension included, assumes the exact torque requested can be applied instantaneously to each wheel and assumes steering angles of each wheel are identical. Results were obtained using a seven degree of freedom vehicle model developed in Simulink using the Pacejka Magic Tyre Formula [23].

Another typical method of DYC is a control technique based on the Ackerman steering geometry. This technique involves calculating the desired angular velocity of each of the driving wheels in a front-wheel driven vehicle with an Ackerman steering mechanism. The relationship between wheel angular velocity and the Ackerman steering mechanism can be expressed as follows:

\[
\omega_L = \frac{v_L}{R} = \frac{v_r}{R} \left( 1 - \frac{d_r \tan \delta}{2l} \right) \tag{1}
\]

\[
\omega_R = \frac{v_R}{R} = \frac{v_r}{R} \left( 1 - \frac{d_r \tan \delta}{2l} \right) \tag{2}
\]

Where \( R \) is the radius of the tyre, \( \delta \) is the average of the front wheel’s steering angles, \( v_L \) and \( v_R \) are the velocity of the rear left and rear right wheels respectively, \( v_r \) represents the velocity of the rear axle at the centre and \( \omega_L \) and \( \omega_R \) represent the angular velocities of the front left and front right wheels respectively. Using this relationship, the angular velocity of the two front driving wheels are controlled based on their optimal values as set out by equations (1) and (2) when cornering [12]. The effectiveness of this method has been confirmed through simulated and experimental data for vehicles travelling at low speeds [24, 25]. The system is used to generate a yaw moment when cornering by reducing the angular velocity of the inner-wheel, and increasing the angular velocity of the outer-wheel [26]. However, it is important to note that this technique is only effective at low speeds, when vehicle side-slip does not occur. The effect of vehicle
side-slip which occurs during cornering at higher speeds creates lateral motion which affects the size of the turning radius of the vehicle, therefore making the equations which govern this control strategy inaccurate. This technique does not utilise feedback control, but it was a method which served to provide foundations for research into more advanced direct yaw-moment control techniques.

De Novellis et. al. explore the use of feedback control for vehicle yaw rate and perform an objective comparison of yaw rate control torque allocation techniques and compare four different approaches. A PID control with feedforward contribution, adaptive PID control with feedforward contribution, second order sliding mode control based on the sub-optimal algorithm and second order sliding mode control based on the twisting algorithm are compared with a baseline (uncontrolled) vehicle [9]. Generally vehicle side-slip angle can be maintained within the vehicle’s stability limits through implementation of a yaw rate controller, provided that friction coefficient of the the road surface and tyre are accurately measured/estimated, given a correct reference yaw rate is generated. Estimation is assumed to be implemented accurately, so as to focus on the comparison of the yaw rate controllers. Performance of the controller is assessed through a performance weighted function, which has been weighted to prioritise achievement of the reference yaw rate with respect to the minimisation of the control action required. Results are attained using a CarMaker vehicle model in which the front axle has two independently controlled drivetrains. The robustness of each controller is assessed by testing with two tyre topologies and by varying vehicle weight and friction coefficient whilst undertaking ramp steer, step steer, tip-in during cornering and frequency response (sinusoidal steering input) manoeuvres. The results of these analyses indicate that the PID algorithms produce good tracking performance and response to variations indicate a robust control system. Additionally, the use of the sub-optimal sliding mode has been shown to further enhance tracking performance. The relevance of this literature served to establish the potential of using feedback loops as a part of vehicle dynamic
control for active drive train technology.

The progression of hub motor technology along with the literature previously produced on active drive techniques focussing on controlling vehicle yaw rate has enabled the progression of research in this field and allowed for improvements in results. The use of electric motors and in-wheel motors’ enable sensing capabilities which provide information to the control system that is implemented into feedback loops, and offers a fast response to input of torque or speed demands [7]. Direct yaw-moment control is a prominent subject in literature concerned with improving the stability and performance of electric vehicles with individually actuated in-wheel motors. Research has revealed many approaches that have been applied to this technique with variations in design of hardware, logic and controllers [11,13,18,27–30].

Torque vectoring control of individually actuated in-wheel motors has also been shown to enhance the operational energy usage and efficiency of electric vehicles [31] and maximising their travel range [32]. Wang et. al. [32] obtained experimental results through use of an electric vehicle equipped with four in-wheel motors and tested on a dynamometer. The efficiency of the battery to motors, and efficiency of battery to ground is studied by Wang et. al. [32]. The strategy adopted essentially involves minimising the use of battery power during driving mode to reduce power consumption, and maximising battery power during braking to regenerate maximum power from the wheels at a given braking torque. Qian et. al. [31] explored an optimal driving torque distribution strategy to minimise the use of electrical energy, and validated results through simulation using Matlab, comparing the four independent-wheel drive vehicle with a single-engine driven electric vehicle, and successfully showed that the use of four independently actuated motors are more efficient than using one electric motor. Energy savings were measured from the simulation as high as 27.4% when comparing the optimal torque distribution strategy to a single-engine electric vehicle, however the
model explored accounted only for the vehicle under acceleration, and did not account for any improvements in optimising regenerative braking. The research performed by Wang et. al. also indicated there was potential to increase an electric vehicle’s energy usage through torque distribution strategies not only whilst under vehicle acceleration but also during regenerative braking.

An alternative to using feedback control of electric vehicles with individually actuated in-wheel motors to improve vehicle dynamics performance is model based control. Model based control utilises a mathematical model to predict the torque demand required to achieve the desired vehicle yaw rate [33]. This approach calculates a desired yaw rate as a result of steering angle, which is used as input to the system, and calculates the torque demand to each wheel to achieve the desired yaw rate to be used as system outputs. This is sometimes referred to as an inverse model approach. The benefits of applying model based approach as opposed to feedback approach include a more rapid response, reductions in time lag and undesired oscillations in response, provided that the model is an accurate approximation of the inverse of the system. Model based control has also been applied to improving the range of electric vehicles through optimisation of the distribution of driving and braking forces between the wheels [34]. The implications of this research are relevant as the range of electric vehicles when compared to internal combustion engine vehicles for the use of passenger vehicles is a limiting factor in their practicality. When applying a model based control strategy for optimisation of distribution of torque between front and rear wheels during acceleration and braking for straight line driving, an increase in range of 2.8 km was recorded. A limitation of this study however is that the model described in this source assumes that the left-right torque signals are even for straight line driving and accounts for only straight line driving. There is no solution proposed for utilising model based control for optimising torque distribution during driving which requires steering manoeuvres. An absence in literature is evident on model based systems for improving vehicle dynamics performance,
which provides a possible avenue of research to pursue.

2.1.2 Important Performance Indicators

In terms of evaluating an electric ground vehicle’s performance and stability, there are certain key performance indicators and performance objectives which are commonly measured and used in controller design [7,8,33]. Depending on the setup of the vehicle, and the control variables required for the controller, these values can either be measured, calculated using measured variables or acquired through use of estimation techniques. There are also other quantities relevant to the vehicle and its environment which can be useful in evaluating and improving the performance of an electric vehicle when designing and implementing a DYC strategy. This section of the paper will detail the relevance of longitudinal wheel-slip ratio, side-slip angle and yaw rate and how they are related to a vehicle’s dynamic performance.

**Longitudinal Wheel Slip** Longitudinal wheel slip represents the relationship between the tyre-road surface, forward velocity of the tyre, and its angular rotational speed. This relationship is not categorised by a single formula or definition, there are a number of accepted formulas used to define the longitudinal wheel slip of a vehicle [35]. One commonly accepted definition of this relationship is shown by:

$$\kappa = -\frac{V_x - r_e \Omega}{V_x}$$  (3)

Where $\kappa$ represents longitudinal wheel slip, $V_x$ is the longitudinal component of the total velocity vector of the wheel centre, $r_e$ is the effective rolling radius of the tyre, and $\Omega$ is the angular speed of revolution of the tyre [23]. Alternatively,
another representation of this relationship is expressed as [36]:

\[ \lambda = \left( \frac{\Omega R_c}{V} - 1 \right) \times 100\% \]  

(4)

Where in this definition, \( \lambda \) is the slip ratio (expressed as a percentage), \( \Omega \) is the wheel’s angular velocity, \( R_c \) is the effective radius of the tyre and \( V \) is the vehicle velocity. Both definitions are generally accepted and produce similar results for evaluating the slip ratio of a given tyre on a vehicle.

Longitudinal wheel slip is an important relationship, as the occurrence of significant wheel slip is representative of a loss of traction between the tyre and road surface, which can affect vehicle stability as well as the lateral and longitudinal dynamic performance of the vehicle. The tyres which are experiencing wheel slip may not be delivering sufficient torque to the road surface resulting in a loss of longitudinal acceleration. Similarly, tyres experiencing significant wheel slip and delivering reduced torque may alter the magnitude and direction of the total yaw moment generated by the vehicle affecting cornering behaviour, lateral performance and overall control of the vehicle [37].

**Vehicle Side-slip Angle**  Vehicle side-slip angle is often used as a performance indicator when measuring a torque vectoring strategy’s effect on the lateral performance of a vehicle, and is often used as the main control variable on which a controller is based on. If the magnitude of a vehicle’s side-slip angle increases to a large value, the vehicle loses capability to produce a yaw moment, which is resultant in the vehicle’s reduced stability. This occurs due to a decrease cornering stiffness in the tyres and yaw moment generated by lateral tyre forces [38]. Conversely, when the magnitude of vehicle side-slip angle is small, a consistency between vehicle heading and the vehicle’s forward velocity vector is inherent, which provides the driver with improved control of the vehicle during cornering manoeuvres [39]. Given these findings, common practice in DYC strategies which
employ side-slip angle as a control variable aim to maintain a side-slip angle of zero, or maintain vehicle side-slip angle within a stable region, to maintain vehicle stability and controllability. Lateral wheel slip is defined as the ratio of the lateral velocity and the forward velocity of the wheel. This value corresponds to the negative tangent of the slip angle \( \alpha \); hence, the vehicle side-slip angle \( \alpha \) is defined in (2) as:

\[
\tan \alpha = -\frac{V_y}{V_x}
\]  

(5)

where \( V_x \) is the longitudinal component of the total velocity vector of the wheel and \( V_y \) is the lateral component of the total velocity vector. Vehicle side-slip angle is commonly measured when evaluating the dynamic performance of a vehicle as the torque distribution strategy in place often has an objective to maintain the side slip as low as possible to ensure lateral stability of the vehicle [9]. The relationship between vehicle velocity and vehicle side-slip angle has been shown to have an effect on vehicle turning behaviour, depending on the steering behaviour of the vehicle i.e. if the vehicle is undergoing oversteer, understeer or neutral steer behaviour. For understeer, side-slip angle reaches a maximum value at larger velocities, for oversteer side-slip angle approaches negative infinity, and for neutral steer, vehicle velocity does not affect the magnitude of side-slip angle [40].

A general expression for obtaining the reference value for desired vehicle side-slip, in cases where controllers aim to track a desired vehicle side-slip response can be expressed as follows:

\[
\beta = \left(\frac{l_r - \frac{m_{l}v_x^2}{C_{\alpha r}}} {l (1 + Kv_x^2)}\right) \delta
\]

(6)

Where \( \beta \) is side-slip angle, \( l_r \) is distance from centre of gravity to rear axel, \( l_f \) is distance from centre of gravity to front axel, \( v_x \) is vehicle longitudinal velocity, \( C_{\alpha r} \) is total cornering stiffness of the rear tyres, \( l \) is the wheel base and \( K \) is the
stability factor (which will be explained in the following section).

**Yaw Rate** Yaw rate is another important performance indicator which reflects the effectiveness of the torque distribution strategy in place. Yaw rate control is one of the main aspects of vehicle stability control in modern passenger cars [9] and is commonly used as the main control variable in vehicles with direct yaw-moment control. The objective of the controller is to distribute torque to generate a corrective yaw moment to bring the vehicle’s measured yaw rate into conformity with the desired yaw rate [27]. Yaw rate control may be based on a number of properties, including the estimated friction coefficient of the road surface, the steering wheel angle and the vehicle velocity. A common strategy for yaw rate control is designing a controller which inspects the error between the measured yaw rate and reference (desired) yaw rate generated by the control system using the steady state yaw response derived from the two degree of freedom vehicle model [41]. A common form for this expression is shown below in equation (7):

$$ r_d = \frac{V_x}{l(1 + KV_x^2)} \delta $$

Equation (7) is a function of the driver’s steering wheel angle input and vehicle speed, where $V_x$ is the vehicle longitudinal velocity, $l$ is the length between front and rear axles, $\delta$ is the steering wheel angle and $K$ is the stability factor [27] derived from the vehicle’s properties of mass, wheel base and cornering stiffness of front and rear axles. The stability factor is used to adjust the optimal turning radius to neutral steering behavior, understeer or oversteer [12]. The stability factor $K$ is defined as:

$$ K = \frac{m}{l^2} \left( \frac{l_r}{C_{af}} - \frac{l_f}{C_{ar}} \right) $$

Where $m$ represents vehicle mass, $l_f$ and $l_r$ represent the distance from the vehicle centre of gravity to the front and rear axles respectively and $C_{af}$ and $C_{ar}$ represent the total cornering stiffness of the front and rear tyres respectively. Vehicle
turning radius is defined by the relationship:

\[ R = \frac{v}{r} \]  \hspace{1cm} (9)

Where \( R \) is the vehicle turn radius, \( v \) is the resultant velocity of the vehicle centre of mass (in cases where \( v_y \) is very small, this value can be approximated simply to \( v_x \)) and \( r \) is the vehicle yaw rate. Substituting this relationship into equation (7) results in the following relationship:

\[ R = \frac{l(1 + KV^2_x)}{\delta} \]  \hspace{1cm} (10)

Equation 10 demonstrates the relationship between vehicle velocity and turning radius, and how the sign of the stability factor is a representation of steering behaviour. Equation (10) shows that with a positive stability factor, turning radius increases with increasing vehicle speed, representing understeer behaviour. As the turning radius is increasing results in the vehicle turning less than expected. Similarly, a negative value for stability factor shows that turning radius decreases with vehicle velocity, representing oversteer behaviour, as the turning radius is decreasing, resulting in a tighter turn in which the vehicle is steering more than expected. Finally, when the stability factor is equal to zero, it can be seen in equation 10 that the value for turning radius is not affected by vehicle speed. The desired yaw rate for a vehicle exhibiting neutral steering behaviour i.e. when \( K = 0 \) is expressed as follows:

\[ r_d = \frac{V_x}{l} \delta \]  \hspace{1cm} (11)

This phenomenon is referred to as neutral steer behaviour, in which the vehicle steers with a constant turning radius, which neither increases or decreases regardless of vehicle speed. This formula is often utilised when calculating the desired or reference yaw rate for vehicle control, as neutral steering behaviour is
generally conceived by the driver as the most stable cornering behaviour.

2.2 Estimation Based Techniques for Vehicle Control

Acquiring vehicle dynamics information and data in real time is integral in the process of implementing a controller to improve vehicle yaw rate response. However there are certain characteristics of vehicle dynamics behaviour which although can be obtained easily through simulations, are difficult or impossible to directly measured through a hardware implementation on a vehicle in real time. Techniques can be implemented on the vehicle making use of known or measurable quantities to estimate values which are difficult to measure or immeasurable which can be used in the controller design to improve vehicle’s dynamic performance. This section will include a review of DYC strategies implemented based on estimation based techniques for acquiring useful vehicle dynamics data.

The principles behind torque vectoring control strategies for improving a vehicle’s dynamic performance via control of yaw rate work on the assumption that the torque output by the vehicle is working accurately. As such, traction control is an important aspect of vehicle stability as a means of ensuring the torque output of the vehicle is reliable and effective [7,42].

As identified by Yin et. al. maintaining traction control is dependent on controlling wheel slip i.e. anti slip control, as significant slip between the tyre and road surface is resultant in losses in longitudinal and lateral friction forces, which affect the acceleration/braking of the vehicle and response to steering input respectively [7]. As information about the tyre/road surface can be difficult or impractical to obtain in real driving situations, a solution is proposed in which the torque output of the motor is calculated from the motor current, which is used to estimate the force between the tyre and road surface in real time to use the maximum transmissible torque to control the vehicle yaw rate and improve stability. Hu et. al. also explore a similar technique, using maximum transmis-
sible torque estimation to provide information helpful to direct yaw control of electric vehicles [43]. An experimental vehicle is described by Yin et. al. in which the proposed control system is implemented and tested to validate results. The controller design is based on a torque limiter, in which the reference torque signal is allowed to pass unrestricted under normal conditions, but is constrained appropriately during low friction conditions. Results of this study verify that the controller is successful, demonstrating that when compared to an uncontrolled system, the controlled system maintains a reduced difference between the chassis velocity and wheel velocity. A limitation identified is that delays in the control system are causing differences in wheel and chassis velocity, however this can be rectified through use of a higher precision encoder. Hu et. al. studied the effects of a controller based on maximum transmissible torque through simulations tested on Carsim, and found similarly that this technique can be used to improve longitudinal and lateral friction force, which can be used to improve two-dimensional motion control. This technique is a useful contribution to be used in conjunction with additional control techniques for use of improving vehicle handling and performance.

Maeda et. al. propose a method of improving vehicle control and safety based on the estimation of values for wheel slip [44]. In this work an estimation method for wheel slip is presented to ensure traction for the vehicle. The slip estimation technique is also applied to driving force control, a concept also explored by the research group in previous works. A solution for improving vehicle safety during cornering is verified experimentally using slip ratio estimation. The controller estimates the wheel slip, and allows for permissible driving force via acceleration or braking during cornering manoeuvres to prevent understeer behaviour, improving the safety of the vehicle.

Similarly, estimation techniques for side-slip angle have been developed such as by Wang et. al. in which side-slip angle is estimated using an augmented sys-
tem of the traditional bicycle model and a simple visual model [45]. The issues encountered in this research was that the sampling rate for a normal camera is significantly slower than the sampling rate of the sensors typically used onboard electric vehicles, and that the image processing causes a delay in sampling time. This was overcome by designing a multi-rate Kalman Filter with intersample compensation. Utilising the side-slip estimation data as feedback a two degree of freedom controller was designed for the purpose of controlling vehicle yaw rate to improve performance. Simulations as well as results obtained from an experimental vehicle verified that the use of estimated values for side-slip angle could be used to improve vehicle performance, as the controller utilising estimated values via the augmented controller showed better results than using values obtained by the traditional bicycle model.

Yaw rate and lateral velocity are critical components of vehicle dynamics and as such are used as control variables in many contributions based on the availability of these values. In practise however, lateral and longitudinal values for vehicle velocity are generally estimated [46]. Cherouat et. al. propose a nonlinear observer of side-slip angle and yaw rate and an estimation technique for vehicle velocity. Geng et. al. propose a fuzzy logic controller based on side-slip angle as the main control variable for a direct yaw rate control system for an electric vehicle with individually actuated in-wheel motors [11]. An observer is constructed for the estimation of side-slip angle in this experiment.

2.3 Longitudinal Wheel Slip Based Techniques for Yaw Rate Control

As previously stated, the occurrence of longitudinal wheel slip reduces driving efficiency, cornering behaviour and stability of vehicles as a result of a loss of traction between the tyre/s and road surface [37]. Longitudinal wheel slip occurs as a result of driving torque exceeding a limit, and is more likely to occur when
the coefficient of friction between the tyres and road surface is reduced, such as on wet road surfaces. Contributions to developing control efforts to eliminate longitudinal wheel slip from occurring will be discussed in this section of the paper.

Lam et. al. propose a controller to suppress longitudinal wheel slip under various road surface conditions. The controller is applicable to vehicles with independent steering and individually actuated wheels, and uses longitudinal wheel slip as the main control variable [37]. The controller proposed in this work is a robust, low-cost and practical controller, requiring only sensing of wheel speed and an accelerometer and can be applied to vehicles with or without independent steering configurations and is arbitrary to number of driving wheels on the vehicle. The controller proposed is evaluated through simulations to determine its effectiveness in various scenarios, subjecting the vehicle to different target torque, steering command and road surface conditions. The simulation demonstrated an improvement in performance when compared to a conventional controller. The vehicle speed was maintained very close to the angular velocity of each wheel during the simulation, the slip ratio of each wheel was reduced and kept under 0.2, which is within the region of adhesion for the tyre. The the torque demands illustrated a “chatter” of torque when nearing their saturation limits, demonstrating the controller taking effect to restrict torque so as to avoid saturation of torque resulting in increased wheel slip. The proposed method allowed the vehicle to run faster than previously, without encountering significant slip.

In real life driving situations road surface conditions are not always ideal or consistent, for example if small patches of the road surface are wet or iced over. In such situations some of the tyres on the vehicle may be exposed to a road surface with a lower frictional coefficient, causing wheel slip which is distributed unevenly amongst the four tyres. Such road conditions can be detrimental to vehicle stability and performance. Road surface conditions where short patches
of low friction occur which are localised to the wheels on one side of the car are referred to as split slippery surface conditions [47, 48]. When referring to road surface patches shorter than the wheel base of the car this is referred to as instantaneous split slippery surface conditions [47, 48]. Driving force control methods have been proposed to improve the vehicle’s performance during split and instantaneous split slippery surface conditions utilising anti-slip control. The controller proposed maintains that on instantaneous split slippery surface conditions (when the slippery patch is shorter than the vehicle’s wheel base) total driving force is maintained by redistributing driving force from the wheel experiencing slip to the other three wheels of the vehicle. When the vehicle runs on extended split slippery surface conditions (when the length of the slippery patch exceeds of the vehicle’s wheel base) the vehicle yaw rate can be altered due to the inconsistent transmission of driving force to the road surface. In these instances the yaw rate is suppressed by setting the left and right driving forces to be equal. Essentially, the controller functions by maintaining driving force over instantaneous split slippery surface conditions, and suppressing yaw rate deviation in split slippery (non-instantaneous) surface conditions [48]. This controller was evaluated on an experimental vehicle and simulated model performing experiments on both split slippery and instantaneous split slippery surface conditions. The evaluations performed revealed that the controller was effective in both suppressing yaw rate on split slippery surface conditions and maintaining total driving force in instantaneous split slippery surface conditions. These results verified that with a controller such as this in place, electric vehicles can be driven with improved safety during adverse road conditions. This controller was developed and implemented using both an optical vehicle velocity observer [48] and using driving stiffness and slip ratio estimation [47]. Future work identified by the author group includes extending the strategy to be applicable to cornering and braking, as currently this method applies only to straight line driving.
2.4 Vehicle Side-slip Angle DYC Strategies

Controller design based on the measurement of side-slip angle as the main control variable will be reviewed in this section. As discussed previously, side-slip angle is an important performance indicator when inspecting the lateral stability of a vehicle, as DYC strategies which neglect side-slip are often functioning on controlling an inaccurate representation of the vehicle’s behaviour [12]. As described in the work set out by Shibahata et al, large values for side-slip angle are resultant in reduced cornering stiffness and lateral forces, decreasing magnitude of yaw moment generated [38]. The inability to generate yaw moments when large side-slip angle values are present indicates a loss of vehicle stability under these conditions. As such, many controllers have been designed based on using side-slip angle as the main control variable, in which a vehicle side-slip angle is maintained at zero, to a reference value, or within a stable region. Maintaining a low value for side-slip angle has the benefit of maintaining consistency of the vehicle’s heading with the longitudinal velocity vector of the vehicle, which is indicative of a vehicle stability, and a firm sense of control by the driver. This section will focus on the review of controller designs which focus on controlling vehicle side-slip as a means of improving vehicle stability.

Studies have shown that the relationship of turning radius and lateral acceleration of a vehicle remain linear up to a certain limit but once the lateral acceleration has increased beyond this limit the relationship becomes nonlinear [49]. In this nonlinear region, the general case is that accelerating during cornering results in increased understeer behaviour, and decelerating during cornering decreases understeer, and at high enough values can result in oversteer. Shibahata et al. propose a study to improve vehicle performance characteristics, which involves developing a new method of analysis for determining vehicle yaw rate, lateral force and vehicle side-slip within the nonlinear region and a basic yaw moment controller [38]. The major finding of the study performed into vehicle behaviour
analysis was that the side-slip angle taken at the vehicle centre of gravity can be used to indicate vehicle dynamic stability in all states of motion, i.e. in both the linear and nonlinear region. Therefore a single model can be used for vehicle analysis based on side-slip angle for all stages of vehicle motion. Another finding of this study was that the stabilizing yaw moment to bring about neutral steer cornering behaviour increases during longitudinal acceleration whilst cornering, and decreases during braking whilst cornering. By adding a corrective yaw moment controller, the linear region of cornering behaviour was increased from the region of approximately $0.1 - 0.4g$ depending on whether acceleration or braking is occurring during cornering to around $0.7g$ regardless of acceleration or braking during cornering.

Chunyun Fu explored the application of using vehicle side-slip as the sole control variable for improving vehicle turning stability for electric vehicles with independently actuated motors [12]. The controller design was based on maintaining a reference side-slip value of zero ($\beta^* = 0$) to maintain consistent vehicle heading with the vehicle’s direction of travel to ensure maximum perceived control by the driver and maintain neutral steer behaviour. The control law implemented is expressed as follows:

$$e_\beta(\tau) = \beta(\tau)^* - \beta(\tau)$$

Given that the desired side-slip is zero, this control law simplifies to:

$$e_\beta(\tau) = -\beta(\tau)$$

This control law was implemented and evaluated using MATLAB/Simulink, using a model based on a rear-wheel drive electric vehicle with independently actuated motors, performing step-steer manoeuvres and response to sinusoidal steering input. A PID controller was utilised with the side-slip error function as input, and corrective torque to each of the two rear driving wheels as output. The
proposed controller was evaluated in simulation, and compared against typical DYC techniques, such as the Ackerman method, and conventional equal torque distribution. Results indicated that the proposed method maintained a smaller error for side-slip angle and yaw rate for all steering manoeuvres. Additionally, the system indicated the benefit of in-wheel motors, as reverse torque (or braking torque) was evident on the inner-rear wheel during cornering manoeuvres. This proved to be beneficial for the purpose of both producing a greater yaw moment to correct steering behaviour, and allows that particular motor to act in regenerative braking mode, improving the efficiency of the vehicle.

2.5 Yaw Rate DYC Strategies

In this section, DYC strategies where yaw rate was used as the main control variable will be reviewed. The basic principal these controller designs are based on is maintaining a minimal error function between a vehicle’s desired and measured yaw rates, using a measured vehicle yaw rate feedback as the main control variable to generate a corrective yaw moment. There are many variations in controller design and control techniques which can be applied to yaw rate based DYC in which authors in the field have published results. This section will provide a review on such approaches.

Motoyama et. al. explored the effect of traction force distribution ratio via a yaw rate feedback controller using proportional and derivative gain terms [50]. The controller worked on the principal of controlling the distribution of traction force between front and rear as well as left and right wheels. The controller algorithm was based on achieving neutral steer cornering behaviour, as defined in equation (7) with a stability factor of zero. The control law employed by this controller is expressed as:

\[
\alpha = \dot{\alpha} + K_P (\Phi - \Phi^*) + K_D (\dot{\Phi} - \dot{\Phi}^*)
\]  

(14)

Where \(\alpha\) is the traction force distribution ratio, \(\dot{\alpha}\) is the last sampled traction force
distribution ratio, $K_P$ is the proportional gain term, $K_D$ is the differential gain term, $\Phi$, $\Phi^*$, $\dot{\Phi}$ and $\dot{\Phi}^*$ are the yaw rate and desired yaw rate and their derivatives respectively. This controller was validated on simulations and an experimental vehicle. Results showed that the method of controlling traction distribution across the left and right driving wheels in conjunction with yaw rate feedback bore a significant affect on the vehicle’s cornering behaviour, and improvements in vehicle performance. Vehicle cornering behaviour under left/right traction force distribution control behaved close to neutral steer behaviour, along with an improved steering response. This control technique was implemented through use of limited slip differentials, and it is expected that applying a similar control law with improved hardware such as individually actuated in-wheel motors could yield further improved results.

A similar method of yaw rate feedback DYC implemented through use of differentials was explored by Doniselli et. al. where a controller was proposed to limit longitudinal wheel slip and use yaw rate feedback as a control variable to calculate torque output to generate a corrective yaw moment [1]. The layout for the controller proposed by Doniselli et. al. is pictured in figure 1.

![Diagram of controller layout](image)

Figure 1: Controller layout for design proposed by Doniselli et. al. [1].

Similarly, this controller demonstrated an improved performance in vehicle cor-
nering behaviour through implementation of a DYC strategy using active differentials to direct torque output. Again it is expected that a further improvement in results could be obtained through use of individually actuated in-wheel motors and the benefits they provide while implementing a similar control law [10]. For further investigation of the effectiveness of this strategy, the control laws set out by Doniselli et. al. were implemented in Simulink as part of the work presented in this thesis and compared to an uncontrolled system. Significant improvements in vehicle cornering behaviour were observed employing the strategy, drastically reducing the understeer characteristic of the vehicle.

The utilisation of sliding-mode control in DYC strategies is a concept which has produced some promising results. Sliding-mode control has been identified as suitable for the application of yaw rate feedback controllers in electric vehicles, due to robust stability when subject to system disturbances and model uncertainty [9, 51].

The effectiveness of implementing sliding mode control as part of a torque vectoring control strategy is demonstrated by De Novelis et. al. in an investigation comparing the effects of a PID controller with feedforward contribution, adaptive PID control with feedforward contribution and two second order sliding mode controllers based on the sub optimal algorithm and the twisting algorithm in implementing a DYC strategy using yaw rate as the control variable. The designs are evaluated based on their performance relative to a baseline (uncontrolled) vehicle. The investigations were carried out using an experimentally validated simulation model, of a front-wheel-drive vehicle with two individually actuated electric motors. The main objective of the study, being to determine if there was any significant benefit to using adapative PID or sliding mode algorithms as opposed to traditional PID with constant gains. The PID controller with feedforward contributions utilises look-up tables for steering wheel angle, acceleration and coefficient of friction, obtained from [8]. The adaptive PID controller used for
vehicle yaw moment control’s design is set out as in [52]. The author also defines a sub-optimal second order sliding mode controller and twisiting second order sliding mode controller [9]. The comparison revealed that all of the controllers were capable of significantly changing the understeer characteristic when compared to the baseline vehicle. The comparison undertaken by the author indicated that taking into account ease of implementation and yaw rate response to steering manoeuvres, that conventional PID controllers are favourable for vehicle yaw rate control applications. It was shown that the sliding mode controllers could behave unpredictably, and while they were effective in minimizing variation in yaw rate acceleration during tip in manoeuvres, they can produce undesirable oscillations in yaw rate during step steer manoeuvres. A similar concept was also investigated by Chunyun Fu in which a controller was proposed to minimise the error function of the vehicle’s measured and desired yaw rate using a PID controller [12]. Desired yaw rate was calculated using (11) to maintain neutral steering behaviour. The controller was implemented in a MATLAB/Simulink simulation model of rear-wheel drive electric vehicle with independently actuated motors, in which the simulated vehicle was subject to step-steer manoeuvres and sinusoidal steering input, and compared to other typical DYC techniques such as the Ackerman control technique and a standard even torque distribution. Results of this simulation indicated that the proposed controller maintained a smaller error function between desired and measured yaw rate, and also displayed the capability of producing reverse (braking) torque during cornering, which as discussed previously is beneficial to both inducing a greater corrective yaw moment and improving vehicle efficiency through regenerative braking.

2.6 Integrated Control Systems

This section will review the effectiveness of controller designs which utilise a number of control variables such as both yaw rate and side-slip angle as a method
of yaw rate control for electric vehicles.

Fuzzy logic has been used to implement direct yaw-moment control with promising results. Author groups Boada et. al. and Tahami et. al. both explored integrated DYC strategies utilising fuzzy logic controllers to obtain results for improving vehicle performance and stability. An application of fuzzy logic in the use of yaw rate control is explored by Boada et. al. who propose a yaw moment controller based on fuzzy logic to improve vehicle handling and stability [27]. Tahami et. al. present a driver-assist stability system for electric vehicles with independently actuated in-wheel motors to aid with path correction, help cornering and straight line driving stability and to improve vehicle safety [28].

The system proposed by Boada et. al. generates a target yaw moment based on the difference of brake forces between the front wheels, and generates braking torque to assist the vehicle to maintain target yaw rate and side-slip values. The target yaw rate is calculated using equation (7) and the target side-slip is equal to zero at all times. Input to the controller was the error function for both the side-slip and yaw rate of the vehicle, expressed as:

$$e(\beta) = \beta - \beta_d$$

$$e(r) = r - r_d$$

Where $e(\beta)$ and $e(r)$ represent the error function for vehicle side-slip and yaw rate respectively, $\beta$ and $\beta_d$ represent the side-slip and target side-slip respectively and $r$ and $r_d$ represent the vehicle yaw rate and target yaw rate. The controller output is a corrective yaw moment, acting about the vehicle’s vertical axis, referred to as $M_x$. The corrective moment is generated through distribution of brake forces to induce a yaw moment. Five fuzzy sets are used for the controller inputs, and seven fuzzy sets are used for the controller outputs, ranging from “Negative Big” to “Positive Big”. The controller is tested using simulations, and is shown to perform
better than the uncontrolled system, showing a closer conformity to target yaw rate and side-slip angle values. The advantages of fuzzy logic control are their relative simplicity and good performance in controlling non-linear systems [27]. The controller design set out by Tahami et al. makes use of a yaw rate controller, slip controller and speed estimator. The yaw rate controller directly controls the yaw rate, by applying a differential torque input to the left and right side wheels based on input from a yaw reference generator. The slip controller works on the principle that the additional torque supplied by the yaw controller may saturate the tyre force, resulting in wheel slip. There is a fuzzy logic controller for wheel slip on each wheel, each of which use wheel slip and wheel angular acceleration of their respective wheel as inputs. The speed estimator is included as a means for determining wheel slip. This system makes uses of yaw rate control and estimation techniques to provide a wholistic torque vectoring strategy for yaw rate control. Tahami et al. evaluate their system using a simulated model based approach, which indicates an improved conformity to desired yaw rate and side-slip values when compared to an uncontrolled model.

As identified by De Novellis et al. two conditions are necessary allow the side-slip angle to be in a stable region for yaw rate control; an accurate estimation of the friction coefficient at the road-tyre surface and the generation of a correct reference yaw rate. However real life conditions may result in an inaccurate estimation for surface friction or the reference yaw rate may be unsuitable for the operating conditions, resulting in unstable vehicle behaviour. As such, controlling the side-slip angle in addition to the yaw rate can prove beneficial in controlling vehicle behaviour. The yaw moment controller design set out by the author group uses yaw rate regulation as the primary controller, and a side-slip angle controller that only operates when the values of side-slip are outside of a threshold defined by the controller [9]. Simulations have demonstrated that the integrated control structure of side-slip and yaw rate regulations worked more effectively than the
controllers evaluated regulating only yaw rate.

The E-VECTOORC Project (Electric-Vehicle Control of Individual Wheel Torque for On and Off Road Conditions) was a collaborative research effort with the objectives of developing side-slip angle and yaw rate control algorithms related to torque vectoring for the purpose of improving vehicle dynamics performance for electric vehicles with individually controlled drivetrains, and develop novel strategies of torque vectoring which enhance regenerative braking, anti-skid braking, and traction control functions [18].
3 Vehicle Dynamics and Modelling

This chapter will detail the mathematical vehicle model established to represent the vehicle dynamics of motion which are to be studied in this project. This project involves the development of an experimental vehicle, and as a part of this project, a computer simulation of the experimental vehicle has been developed. The mathematical model which this simulation is based on and validated by is presented in this chapter. As such, in this chapter the equations of vehicle motion, steering geometry and wheel and tyre dynamics are presented.

3.1 Experimental Vehicle Properties

As stated above, this project involves the development of an experimental vehicle. Relevant parameters of the vehicle’s geometric properties are expressed in table (1).

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle Mass</td>
<td>400kg</td>
</tr>
<tr>
<td>Wheel Base</td>
<td>2m</td>
</tr>
<tr>
<td>Wheel Track</td>
<td>1.165m</td>
</tr>
<tr>
<td>Wheel Radius</td>
<td>0.302m</td>
</tr>
</tbody>
</table>

The vehicle uses an Ackerman steering mechanism, an effect of this is that the steering behaviour of the vehicle is highly dependent on the vehicle’s geometry. As such, the vehicle model is designed to calculate steering wheel angles for each of the front wheels based on Ackerman geometry calculations. Accurate calculations of steering angles are required in order to implement a simulation which emulates the steering behaviour of the experimental vehicle accurately. A series of expressions for the relationships between vehicle turning behaviour, steering angles and vehicle geometry for vehicles with Ackerman steering mechanisms is presented by Genta et. al. in their model for a vehicle with Ackerman steering mechanism where only the front two wheels steer [53].
Figure 2: Diagram detailing the layout of a typical Ackerman steering mechanism.

The radius of the trajectory of the vehicle centre of mass can be expressed in the following formula [54]:

\[ R = \sqrt{b^2 + R_1^2} = \sqrt{b^2 + l^2 \cot^2(\delta)} \]  

(17)

Where \( R \) is the radius of the vehicle turning circle from the vehicle centre of gravity, \( R_1 \) is the radius of the vehicle turning circle taken from the centre of the rear axel, \( b \) is the distance from the rear axel to the vehicle centre of gravity, \( l \) is the wheel base and \( \delta \) is the steering angle. After obtaining a value for \( R \), the tangents of the steering angles for the left and right hand steering angles, \( \delta_1 \) and \( \delta_2 \) can be calculated using the following:

\[
\tan(\delta_1) = \frac{l}{R - \frac{t}{2}}
\]  

(18)

\[
\tan(\delta_2) = \frac{l}{R + \frac{t}{2}}
\]  

(19)

Where \( t \) represents the wheel track. Implementing these formulas in Simulink based on a given input for desired steering angle (\( \delta \)) to function as the “equivalent”
steering angle for the two wheel steering mechanism. As presented by Genta et. al. although it should be calculated by averaging the cotangents of the angles, it is very close to the direct average of the angles $\delta_1$ and $\delta_2$ [53].

3.2 Tyre and Wheel Model

This section presents the tyre and wheel model used in this project for studying vehicle dynamics response through simulations. The model of the tyre used for the evaluation of vehicle dynamics in the simulations discussed in this thesis use the TNO MF-Tyre model. MF-Tyre calculates the longitudinal and lateral tyre forces ($F_x$ and $F_y$) and moments ($M_x$, $M_y$ and $M_z$) acting on the tyre as well as slip conditions between the tyre and road surface. The Magic Formula tyre model developed by Pacejka et. al. provides a mathematical representation of the forces and moments acting between the tyre and road surface. The Magic Formula can be used to calculate an accurate representation of tyre behaviour [55], and is generally well accepted as a reasonable representation of tyre forces. The model uses lateral and longitudinal slip, camber and tyre vertical force ($F_z$) as inputs [56]. As such, the Magic Formula is utilised in the vehicle model to calculate the tyre forces for vehicle motion. The Magic Formula is expressed as follows:

$$y = D \sin\{C \arctan[Bx - E(Bx - \arctan Bx)]\}$$  \hspace{1cm} (20)

with

$$Y(X) = y(x) + S_V$$  \hspace{1cm} (21)

$$x = X + S_H$$  \hspace{1cm} (22)

Where $X$ is the input variable, being either slip ratio ($\kappa$) or wheel/tyre slip angle ($\alpha$). $Y(X)$ is the output variable for tyre longitudinal force ($F_x$) or tyre lateral force ($F_y$). $S_H$ and $S_V$ are horizontal shift and vertical shift, $B$, $C$, $D$ and $E$ are
the stiffness factor, shape factor, peak value and curvature factor respectively.

The rotational motion of each wheel is expressed in the formula below as:

\[ J_{ij} \dot{\omega}_{ij} = T_{ij} - F_{xij} r_{ij} \] (23)

Where \( J \) represents wheel inertia, \( \dot{\omega} \) represents wheel angular acceleration, \( T \) represents the torque delivered, \( F \) represents the driving force of the tyre and \( r \) is the effective radius of the wheel, \( i \) and \( j \) are to be substituted for \( f \) and \( r \) (front/rear) and \( l \) and \( r \) (left/right) respectively to represent which tyre is the subject of the formula e.g. \( T_{fl} \) to represent the torque delivered to the front left wheel. The wheel equation above can be used to calculate the driving force \( (F_{xij}) \) and angular velocity for each tyre, which are utilised in the Magic Tyre Formula, and also used to form the equations for vehicle motion.

The implementation of the Magic Tyre Formula in the TNO MF-Tyre model, the simulation model used for evaluating vehicle dynamics is utilised in this thesis to calculate the forces and slip effects acting on the tyres of the vehicle. From these values, vehicle motion can be modelled and calculated for further evaluation of vehicle dynamics effects.

### 3.3 Vehicle Model and Equations of Motion

The motion of the vehicle is modelled based on the Newtonian relationship of force being equal to the product of mass and acceleration i.e. Newton’s second law. This relationship can be used to represent the total longitudinal and lateral forces acting on the vehicle, expressed as:

\[ \sum F_x = m\ddot{U} - mVr \] (24)

\[ \sum F_y = m\ddot{V} + mUr \] (25)
Where $F_x$ and $F_y$ represent the total longitudinal and lateral forces respectively, $U$ and $V$ represent the longitudinal and lateral velocities respectively and $r$ represents the yaw rate. Applying the relationship above an expression for vehicle motion can be obtained. The model for vehicle motion includes the driving forces of each tyre, vehicle mass, lateral and longitudinal velocity/acceleration, yaw rate and steering angle of each wheel. As the vehicle model contains an Ackerman steering geometry the steering angle of each of the wheel will not be identical.

The values for the steering angle of each wheel can be acquired using the Ackerman formula outlined in the previous section. Although load distribution as a result of pitch and roll may affect vehicle traction these are neglected in the model as they are assumed to be quite small, and control efforts such as rollover prevention are not the subject of study in this thesis. As such, using the above relationships and taking into account steering wheel angles, the expression for total force acting on the vehicle in longitudinal and lateral directions can be expressed as:

\[
F_x = F_{xfl} \cos(\delta_{fl}) + F_{xfr} \cos(\delta_{fr}) + F_{xrl} \cos(\delta_{rl}) + F_{xrr} \cos(\delta_{rr}) \\
- F_{yfl} \sin(\delta_{fl}) - F_{yfr} \sin(\delta_{fr}) - F_{yrl} \sin(\delta_{rl}) - F_{yrr} \sin(\delta_{rr})
\] (26)

\[
F_y = F_{xfl} \sin(\delta_{fl}) + F_{xfr} \sin(\delta_{fr}) + F_{xrl} \sin(\delta_{rl}) + F_{xrr} \sin(\delta_{rr}) \\
+ F_{yfl} \cos(\delta_{fl}) + F_{yfr} \cos(\delta_{fr}) + F_{yrl} \cos(\delta_{rl}) + F_{yrr} \cos(\delta_{rr})
\] (27)

Where the steering angle of each wheel is denoted by $\delta_{ij}$. Note that for this project there is currently no implementation of active steering or four wheel steering. Only the front wheels of the experimental vehicle are controlled with steering inputs. As such, steering angle is always assumed to be zero for the rear wheels of the vehicle, therefore the equations of motion for the vehicle model can
be simplified and expressed as:

\[ F_x = F_{xfl} \cos(\delta_{fl}) + F_{xf r} \cos(\delta_{fr}) + F_{xrl} + F_{xrr} - F_{yfl} \sin(\delta_{fl}) - F_{yfr} \sin(\delta_{fr}) \]  

(28)

\[ F_y = F_{xfl} \sin(\delta_{fl}) + F_{xf r} \sin(\delta_{fr}) + F_{yfl} \cos(\delta_{fl}) + F_{yfr} \cos(\delta_{fr}) + F_{yrl} + F_{yrr} \]  

(29)

As such, by obtaining and applying the tyre forces and slip conditions from the TNO MF-Tyre model, and calculating steering angles from the Ackerman steering mechanism representation the simulation model has vehicle motion which takes into account slip conditions, steering angle and force generated between the tyre and road surface.
4 Research Design and Methodology

This chapter will detail contributions to the process involved in the delivery of the platform discussed in this project. Developing this platform required a representation of the system, the implementation of hardware and the acquisition and analysis of data relevant to the project. A representation of the system was required to aid in the design process of a control strategy and to observe vehicle behaviour in a simulated environment, as well as analysing spatial qualities pertinent to the design and implementation of hardware components in the development of the vehicle. Development of simulations, design and implementation of hardware and data acquisition and analysis will be discussed in this chapter.

4.1 Electric Vehicle Conversion

The platform being utilised in this project involves the conversion of a rear wheel drive internal combustion engine vehicle into an electric ground vehicle with four independently actuated in-wheel motors. The process of converting an existing internal combustion engine vehicle into an electric vehicle involves many factors which must be taken into consideration. Research into previous projects involving the conversion of an internal combustion engine vehicle into an electric ground vehicle sought to confirm that there were design considerations which must be payed attention to ensure the successful delivery of the project.

As previously discussed, the most economically and practically viable configuration for internal combustion engine vehicles is to have one centralised engine supply the output power to each wheel through use of drive trains and differentials. However due to the spatial qualities and output performance of electric motors, as well as developments in battery technology, there is room for re-evaluation in terms of vehicle layout design [57]. Current motor and battery technology can allow for anywhere from one to four electric motors to be included in the design of an electric ground vehicle.
Research brought to light certain qualitative issues, such that hub motors are quite close to the road surface and are exposed to dust and water [57]. Therefore the design must factor into consideration methods to minimise the risk of damage from these environmental factors. A conflict of interest in design raised here is that by sealing the hub motor off to protect it from these environmental effects, the possibility of convection cooling of the hub motor is no longer an option, but still retains the possibility for use of a liquid cooling system (water or coolant).

The first design objective encountered was to draft and implement a design concept to connect the motors in a suitable manner onto the vehicle. The objective was to accomplish a solution which allows for a firm and safe connection of the motors onto the vehicle that will not affect performance due to unnecessary vibrations, and to meet the spatial requirements available. That is, to attach the motors in the space available on the vehicle in an appropriate manner without compromising or damaging the structure of the vehicle. To aid in this task the CAD package CREO Parametric was used in conjunction with measurements taken of the vehicle to model and visualise a system for which the motors could be attached. CREO Parametric was very useful in this process, as once the key measurements of the vehicle had been taken, and a CAD model of the selected motor had been acquired, it was possible to visualise how the wheel hub and motor mated, and what was required to ensure a firm and safe connection. Due to a discrepancy in the outer diameter of the motor hub and the inner diameter of the vehicle hub, it was necessary to draft and manufacture a collar to house the motor hub and fit into the vehicle hub. Again, CREO Parametric was used for this purpose, and the components were manufactured in house by the university technical staff. The motor was secured to the vehicle via a bushing, put into place with a pressed fit and key.

After fitting the electric motors to the vehicle body, the conversion of the vehicle required the removal of obsolete mechanical components to make room for future
electronic components. Components such as the engine, radiator, fuel tank, gear box etc. all had to be removed as they were spatially demanding and provided additional unnecessary weight. With these components removed, the battery pack, battery management system and motor controllers were secured to the vehicle.

With the obsolete mechanical components removed and crucial electrical components now fitted to the vehicle, all that was required from this point was to include suitable sensors and control. These tasks will be discussed in Section 4.3.

4.2 Modelling of the System

This section will describe the process undertaken in which a simulation platform was developed for the representation of the experimental vehicle and torque vectoring control strategy. Developing an accurate representation of the system is a critical step in developing an accurate simulation framework. Developing a system representation and simulation aids in the design process and allows for the inspection and verification of the strategy proposed in this project. As such developing a system representation provides a platform which can be used for
the implementation of future work on the vehicle such as implementing an energy efficiency optimisation strategy, an alternative torque vectoring strategy or other avenues of research utilising the experimental vehicle. Various platforms were investigated as options for modelling a representation of the system which would provide a helpful simulation, which will be discussed in this section.

4.2.1 SimDriveLine

SimDriveLine is a block based modelling system which works in conjunction with Simulink. SimDriveLine uses physical inputs, such as torque or angular velocity, as well as mathematical inputs, where Simulink is used exclusively for mathematical inputs.

Using the ‘Vehicle Body’ block available in SimDriveLine, a hierarchical block based model has been developed to simulate results for a car with four in-wheel motors. The hierarchical model consists of four motor model subsystems, within four wheel subsystems connected to a system modelling the vehicle body and tyre dynamics. The model is configured to display the output of each motor in revolutions per minute (rpm), the current drawn by each motor in amps (A), the tyre slip ratio, normal forces applied to the front and rear tyres in Newtons (N) and the vehicle velocity in kilometres per hour (km/h). The model is designed to account for resistive and reactive power losses in each motor’s rotor, wind resistance and road incline. The model also accounts for a number of vehicle parameters, such as vehicle mass, number of wheels per axle, distance of centre of gravity from front and rear axles, height of centre of gravity from the ground, frontal area and the vehicle’s drag coefficient, using the ‘Vehicle Body’ block from the SimScape library to calculate the normal forces on the front and rear tyres, and vehicle’s longitudinal velocity and acceleration. The model also makes use of the ‘Tyre (Magic Formula)’ block from the SimScape SimDriveLine library. This block is used to model longitudinal dynamics of a vehicle’s wheel, tyre and
axel and road contact using the Magic Formula [23]. Use of this block within the model allows for measurement of tyre slip and longitudinal vehicle velocity.

Figure 4: Top level of SimDriveLine model of an electric ground vehicle with four in-wheel motors.

Pictured in figure 4 is the top level of the block based model used to evaluate the vehicle parameters listed above. Shown in figure 5 is the successive layer of the model, detailing the relationship between the vehicle body block, Tyre Magic Formula block, DC Motor models and power supply.

Using the model pictured in figure 5, values may be obtained such as discussed above. The main limitation of using SimDriveLine to evaluate vehicle dynamics however, is that it is for one dimensional evaluation i.e. the package can only
account for longitudinal motion. To investigate other vehicle dynamic characteristics relevant to implementing a DYC control strategy the lateral motion of the vehicle must be taken into account. Although this package is helpful for evaluating certain parameters such as acceleration, braking, maximum vehicle velocity and tyre slip, additional simulation packages are required to evaluate other important performance indicators such as yaw rate and side-slip angle, and inspect the lateral performance of the vehicle in a two or three dimensional model.
4.2.2 Simscape/SimMechanics

**Vehicle**  For the purpose of evaluating an electric ground vehicle with four independently actuated in-wheel motors, a vehicle dynamics testing simulation has been arranged. The blocks available in MATLAB SimMechanics in conjunction with MF-Tyre using the Delft Tyre standard implementation of the renowned Pacejka Magic Formula were used to simulate an electric ground vehicle with four independently actuated motors. The focus of this simulation is on combining the measurement of key performance indicators relevant to implementing a torque vectoring control strategy with the benefits of Simulink in testing control architectures in hardware and programming mode. The work presented in this section will focus on a simulation platform developed using MATLAB SimMechanics, with the intended use of investigating the vehicle dynamics of conceptual electric ground vehicles with individually actuated motors. Simscape/SimMechanics was utilised to develop a three dimensional simulation for inspecting the relevant vehicle dynamic properties and behaviour.

![Comparison of the modelled vehicle in simulation and the experimental vehicle.](image)

The simulation platform is developed to provide a baseline electric ground vehicle, in which higher level control-architectures can easily be implemented, interchanged and evaluated for the design of distributed drive systems and torque vectoring DYC control strategies. The platform allows for a visual representation of the electric ground vehicle, and inspection of relevant vehicle dynamics...
through measuring key performance indicators, such as tyre normal force, slip ratio, side-slip angle, yaw rate and rolling resistance over a series of commonly used driving manoeuvres such as J-turn and line change manoeuvres.

In the development of this platform, the focus is on measuring the performance of the vehicle’s response to steering input to evaluate the effectiveness of the DYC solutions being evaluated. The platform is not intended for the use of vehicle body design characteristics related to the aerodynamics of the vehicle. Therefore this platform is not intended for the use of measuring characteristics related to vehicle body design such as wind resistance and drag coefficient, and has been simplified to satisfy the requirements involved in developing a torque distribution or energy efficiency strategy. As such, the model has been simplified to include a visual representation of the vehicle containing a single point of inertia taken from the vehicles centre of gravity. However, to maintain robustness of the platform certain vehicle properties relevant to this area of design and research remain adjustable. The position of the centre of gravity and the magnitude of the inertia are both adjustable so that each system can be configured to accurately reflect the needs of the current project. Similarly, vehicle body parameters which affect vehicle performance are also adjustable. The vehicle length and wheel base are also adjustable, in conjunction with the vehicle mass, adjusting these values can be used to allow for the platform to make accurate calculations pertaining the performance indicators of a range of vehicles, adding to the robustness of the platform. All of these parameters are arranged into an accompanying script, which runs at the startup of the simulation to provide the model with the relevant vehicle properties and data required to perform.

The model discussed in this project was arranged from blocks available in the Simscape library and the Delft Tyre Model to resemble an OAEGV (Over-Actuated Electric Ground Vehicle) with in-wheel motors. The torque applied to each wheel was modelled by a Permanent Magnet Synchronous Machine (PMSM). These
PMSM models were integrated in the vehicle model to provide direct actuation to the wheels. The PMSM models incorporated the revolute joints connecting each wheel to the vehicle body via front and rear axles to represent the actuation of the in-wheel motors.

Figure 7: Simulink model of the electric vehicle with sensing and torque distribution control.

The motor models include a PMSM and power inverter from the SimPowerSystems library in Simulink as well as the appropriate control algorithms for torque control and angular velocity control. The standard PMSM model was modified to make the mechanical subsystem compatible with SimMechanics 2nd Generation. This allowed the PMSM block to integrate seamlessly with the rest of the vehicle model. The PMSM is driven by a voltage power inverter, which is controlled with a PWM duty cycle with a Space Vector Modulation technique. A complete integration of standard servo motor current control was implemented, using a PI controller for the direct (d) and quadrature (q) current components. Hall sensors at every 60 electrical degrees were used for the position feedback.
Figure 8: Flow diagram detailing the layout of the Simulink model of the electric vehicle.

The model also contains a low-fidelity variant of the motor model, which can be interchanged for examining vehicle properties where the type of motor is not essential to the outcome of the results. This is included for the purpose of reducing compilation and simulation time to streamline processes of testing where several iterations of running simulations are required.

As each wheel is powered with an in-wheel motor, the inertia parameter of the motor and the vehicle body are taken into account to reflect the distribution of sprung and un-sprung mass in the vehicle.

The end result of this work is a functional model and simulation of an OAEGV. The model has the capability of inspecting torque demand and response to torque input and vehicle dynamics response to steering inputs when undertaking performance evaluation manoeuvres. In addition to this, the developed model serves as a framework suitable for implementation of OAEGVs with higher-level DYC's containing control architectures for active drive solutions with hardware in the loop capabilities.
The model produces a visual output in which the user can view an animation of the vehicle undergoing a series of custom manoeuvres. For visualisation purposes only, the model can display the vehicle geometry from an -.stl file. It should be noted that the mass and inertia are not influenced by this geometry and is only for a visual aid.

The PMSM models can be switched to receive either a velocity or torque command. Therefore the model can be used to investigate the required torque demand to achieve certain speeds, or the system response given a torque input. As the PMSM models are high fidelity, the model framework also includes variant subsystems to support low fidelity blocks or other alternative motor models to maintain flexibility and to optimize the simulation time.

The Delft tyre model includes the feature of being able to adjust physical properties which affect the relationship between the tyres and driving surface. This feature allows the user to adjust the friction to simulate the vehicle undergoing manoeuvres under various road conditions [7], such as dry roads, icy roads and wet roads. It also allows for the vehicle’s performance to be evaluated under a wider range of conditions.

To develop a simulation model which represented the experimental vehicle it was essential to obtain measurements of the vehicle’s steering and suspension system so that they could be modelled accurately. The vehicle which is the subject in this project uses an Ackerman steering mechanism, and McPherson suspension. As discussed previously, the turning characteristics of a vehicle which uses an Ackerman steering mechanism are highly dependent on the geometry of the steering mechanism. To develop a model which accurately emulates the steering characteristics of the experimental vehicle, precise measurements were taken of the Ackerman steering mechanism, and applied to the properties of the Simulink/SimScape model. The vehicle steering and suspension systems were measured, and drawn in Tikz using vectors to have accurate diagrams to work
off of when modelling the system. The components comprising the front and rear suspension and steering systems when sketched in Tikz appear as in figure 10. The same concept of a low fidelity variant as applied to the motor model was applied to the suspension and steering. Pictured in figure 9 is an example of a low fidelity variant of the Ackerman steering mechanism for performing a J-turn manoeuvre as modelled in Simulink.

The overall composition of the front and rear suspension and steering systems can be simplified into the diagrams presented in figures 11 and 12.

These diagrams were used as the basis for modelling the vehicle’s suspension and steering to achieve an overall model for the vehicle in which vehicle behaviour and dynamic response could be modelled and simulated.

The vehicle’s overall displacement, slip ratio, side-slip angle, yaw rate, longitudinal, lateral and overall velocity and acceleration can be simulated. As discussed, these are important results required to inspect the vehicle stability and effectiveness of an implemented torque vectoring strategy.

**Environment** To evaluate the vehicle model developed in Simulink, certain environmental aspects were modelled to provide a more wholistic approach to investigating vehicle performance in real-life driving situations.

As discussed previously, split slippery surface and instantaneous split slippery surface conditions are highly applicable to real-life driving situations, in account-
ing for inconsistent road surface such as puddles, ice patches etc. on the road surface where the individual tyres may perform differently. To account for this, the tyre model was adapted to include split surface and instantaneous split surface conditions. Shown in figure 13 is an example of the Simulink model used to modify surface conditions to achieve split slippery surface or instantaneous split slippery surface conditions when required. The longitudinal friction can now be adjusted in the mask for each tyre to achieve split slippery surface conditions, and step times are included to extend the model to accommodate instantaneous split slippery surface conditions.
4.3 Data Acquisition and Estimation Techniques

The progression of available technology in recent years has allowed for the implementation of measurement devices and estimation techniques to be applied to receive fast and accurate data pertaining to the vehicle’s dynamic performance. This data can be applied in real time, based on the controller design for the vehicle’s torque vectoring control technique to provide feedback information to actively improve the vehicle’s dynamic performance when compared to control variables related to important performance indicators as mentioned previously. Based on the design of the controller, feedback information which can be used to improve vehicle performance can either be directly measured, or calculated using estimation techniques to provide an indication of how the vehicle is performing with respect to its target control variables. In this section the options explored for the acquisition of relevant vehicle dynamics data will be discussed.

4.3.1 VBOX

The VBOX20SL Dual Antenna is a data logging unit which utilises two GPS antennae to measure vehicle dynamics parameters. The VBOX20SL can be used to measure and log relevant parameters to implementing a DYC strategy such as
Figure 12: Tikz diagram of the layout of the vehicle’s rear suspension system.

slip angle, pitch angle and yaw rate as well as longitudinal and lateral acceleration and velocity. It is a compact unit, with a small size suited for implementation on small vehicles such as cars and off road vehicles. The VBOX20SL module makes use of two GPS signals to measure and calculate vehicle dynamics parameters. It is capable of logging at a rate of 20 Hz and has a positional accuracy of 40 cm (standard) or 20 cm (optimised with a base station). The VBOX20SL also includes two analogue outputs and two digital outputs, and a CAN Bus interface [2].

For data logging the VBOX20SL has the option to log data onto an SD card or connect to a computer via USB or RS232 serial port interface. This is a valuable and useful tool for acquiring experimental data for evaluating the lateral and longitudinal performance of the vehicle. The VBOX20SL can also log data in real time, via analogue and/or digital outputs, as well as a CAN Bus interface for logging data or use of Vbox input modules. For the purpose of implementing the VBOX20SL module as a data acquisition device for a torque vectoring DYC strategy these are useful features, as the output channels can be configured to
Figure 13: Modified mask for tyre models to extend vehicle testing to accommodate split and instantaneous split slippery surface conditions.

the required control variables for the DYC. For example, if yaw rate and side-slip angle are to be utilised by the controller as control variables, the output channels can be configured to yaw rate and side-slip angle, so that the controller can receive these values in real time at 20 Hz to be utilised in the DYC strategy.

The capabilities of measuring velocity, acceleration and position are useful in the process of validating the simulated model in the context of this project. This module will allow for the collection of experimental data of the vehicle’s dynamic performance, to be compared with that of the performance of the simulated model. Vehicle performance manoeuvres can be performed on the experimental vehicle and replicated in simulations to evaluate the consistency of the model. Parameters such as the vehicle’s acceleration curve, top speed, and uncontrolled yaw rate response to a given steering angle or steering manoeuvre can be assessed and compared. As stated previously, it is important to have a realistic and reliable model when utilising simulations as part of the design process, as such
the VBOX20SL is well suited for this purpose.

Vehicle side-slip angle is measured from a single point based on the positioning of the antennae. The two antennae are assigned as “Antenna A” and “Antenna B”, the VBOX20SL measures vehicle side-slip angle from the point where “Antenna A” is attached to the vehicle. This is a limitation in a sense due to the fact that the preferred method for some DYC controllers which utilise side-slip angle as a control variable measure the side-slip angle of each tyre individually so as to control vehicle side-slip more accurately. To implement a strategy where side-slip is measured from each tyre, the VBOX20SL would not be suitable, as it has insufficient antennae inputs to measure four differential GPS signals simultaneously. However there are also strategies which utilise only one value for side-slip angle measured from the vehicle’s centre of mass [58] which would only require two antennae to implement. Using a strategy such as this, the VBOX20SL module could be implemented into a feedback loop for a controller implementing a similar DYC strategy.

Due to the capability of the VBOX20SL module to output side-slip angle and yaw rate simultaneously, theoretically this module could be implemented in the use of a DYC strategy based on controlling side-slip angle, yaw rate or an integrated controller utilising both side-slip angle and yaw rate as control variables. The VBOX20SL is useful for both acquisition of experimental data and for use in measurement of vehicle dynamics parameters in implementing a DYC strategy.
4.3.2 Android/iOS Accelerometer and Gyroscopic Sensor Support from MATLAB

A mobile device with either an Android or iOS operating system can be used to connect to a computer running MATLAB for the acquisition of data from the mobile devices built-in sensors [3].

![Standard layout of measurable axes on a mobile device. Image courtesy of [3].](image)

Using the MATLAB Support Package for Android/iOs Sensors data can be logged or queried from the mobile device. Suitable mobile devices for this purpose are equipped with an accelerometer, gyroscopic sensor and GPS sensor. The MATLAB Support Package for Android/iOS Sensors enables the measurement of:

- Acceleration on three axes
- Magnetic field on three axes
- Angular velocity on three axes
- Azimuth, pitch and roll
- Latitude, longitude, altitude and velocity
This system operates by transmitting data between a computer running MATLAB and a mobile device running MATLAB Mobile. The values which are to be recorded can be controlled from either end, either from the MATLAB Mobile GUI or by sending commands from the MATLAB terminal once a connection has been established, to enable or disable sensors and logging.

Figure 16: Graphical user interface for Android/iOs Sensor Support on MATLAB Mobile. Image courtesy of [3].

A limitation is encountered, as this system only accommodates the logging of data to a computer running MATLAB. There is no simple solution for using any of the data acquired in real time which would not require the construction of a custom hardware system, therefore it would be impractical to implement this system as a means of acquiring data to be used as feedback information for a controller as part of a DYC strategy. The disadvantages of using the inbuilt sensors in a mobile device listed here can be overcome however, through use of a gyroscopic sensor. Gyroscopic sensors can be purchased and integrated into physical systems as required, and vary in accuracy and sensitivity based on
price. However, the usefulness of this system should not be overlooked. This system is useful for evaluating vehicle performance, and has use in confirming the accuracy of other measurement systems or validate the performance of the simulated model. The process of establishing the link and using the mobile device to log data is a quick and simple means for logging important vehicle dynamics data. As discussed above, integrating simulations into the design process is only beneficial if the simulation is an accurate reflection of the real life system, and as such the MATLAB Support Package for Android/iOs sensors is useful for this purpose. The accuracy of the sensors is dependent on the specifications of the mobile device being utilised.

4.3.3 Motor Controller

Certain information, such as the motor’s angular velocity, can be requested using through the CAN bus. The motor controller can transmit its angular velocity in revolutions per minute to within integer accuracy. As such, this is a simple and effective method for receiving data for the driven wheels of the vehicle’s angular velocity. As established previously, angular velocity of the wheels is required for calculating certain performance indicators such as longitudinal wheel slip.

4.3.4 Encoder

If the controller, when implemented is performing undesirably as a result of inaccurate values estimated for longitudinal slip, the possibility of installing an encoder onto the driving wheels can be explored to record higher accuracy values for the angular velocity of each wheel. A rotary encoder could be utilised to receive information on the angular position of the driving wheels to determine the angular velocity of each driving wheel. This data could be used in conjunction with information acquired through other means measuring the velocity of the vehicle to estimate values for longitudinal slip. With vehicle velocity and the
angular velocity of each driving wheel measured, it is possible to calculate the longitudinal slip ratio for each wheel of the vehicle using equation (3).

Figure 17: Rotary encoder employed for measuring steering wheel angle in the experimental vehicle.

Additionally, the rotary encoder is a useful tool for measuring steering wheel angle. As the steering wheel angle is an important factor in altering the course of the vehicle’s motion, it is necessary to measure the steering wheel angle when performing vehicle dynamics testing. Additionally, many DYC strategies require the measurement of the steering wheel angle as part of their algorithms for vehicle control. A rotary encoder was included in the vehicle modifications, by securing it parallel to the steering column with a shaft equal in diameter to the steering column, and linking with a pulley. To enable real time measurement so as to be useful for the purpose of both data logging and in the implementation of a DYC strategy, the encoder is connected to the CAN bus.

4.3.5 Controller Area Network (CAN) Bus

A Controller Area Network (CAN) bus is utilised within this experimental vehicle for consolidated, real-time transmission of critical data between vehicle systems. The CAN bus is characterised by a maximum signalling rate of 1 megabit per second, and unlike networks such as Ethernet or USB a CAN bus sends data in short messages over the entire network to provide consistent data to the whole
system [59]. All of the sensor data acquired from the experimental vehicle is to be broadcasted over the CAN bus. This is to ensure that all of the data critical to implementing a torque vectoring control strategy or for inspecting vehicle dynamics performance can be accessed from a single source and maintain a consistent time stamp for ease of analysing data.

4.4 Summary

Taking into account all of the data acquisition devices available to the project and the benefits they could provide, a final setup was decided upon and implemented on the experimental vehicle. A CAN bus interface was implemented to provide a single point of data acquisition, to allow for coallated data to be available from a single unit and consistently timestamped. A laptop running Matlab is utilised to receive the data on the CAN bus and send commands to request data when required from the sensors in use. A rotary encoder was used to measure and record steering wheel angle, as described previously by securing the unit parallel to the steering column. The VBOX20SL was also implemented due to its versatility and capability of measuring several important performance indicators, such as vehicle speed, side-slip angle and yaw rate. The motor controllers in use were utilised to obtain measurements for the angular velocity of each driven wheel, which can in turn be used to calculate longitudinal wheel slip. A current sensor was utilised to measure the current being delivered to the motor. By enabling current measurement it is possible to calculate the torque transmitted by utilising other measured variables such as the wheel radius and angular velocity of the wheel. All of the sensors included are interfaced via the CAN bus.
5 Controller Definitions and Simulated Results

In this section the process and methods involved in developing and implementing appropriate torque vectoring control strategies will be discussed, along with an evaluation of the effectiveness of the strategies investigated through use of simulations. Torque vectoring strategies will be assessed and compared based on their feasibility to implement in hardware, potential to improve vehicle performance and practicality of acquiring required feedback information as well as the potential to improve vehicle dynamics performance based on their simulated outcome. The torque vectoring control strategies presented in this chapter are designed for controlling either yaw rate or longitudinal performance and will be divided into relevant subsections.

5.1 Longitudinal Wheel Slip Regulation DYC for Front-Wheel Drive Electric Vehicle Controller Definition

The development of strategies pertaining to improving the lateral performance via yaw rate control of the vehicle will be discussed in this section of the paper. Longitudinal wheel slip regulation has been utilised in previous work as a means of traction control for front-wheel drive internal combustion engine vehicles to address the issue of traction force distribution between the wheels of a driving axle [1]. A control algorithm is presented, based on implementation for internal combustion engine vehicles with non-conventional differentials. The full set of control algorithms set out by Doniselli et. al. are expressed as follows:

\[ \Delta \lambda = \lambda_{FL} - \lambda_{FR} \]  
\[ \Delta \lambda^* = k_1(r - \frac{v_x \delta}{l}) \]  
\[ M_c = k_2(\Delta \lambda^* - \Delta \lambda) \]
\begin{align*}
M_L &= \frac{M}{2} + M_c k_L \\
M_R &= \frac{M}{2} - M_c k_R \\
k_L &= \frac{1}{|S_L| + 1} \\
k_R &= \frac{1}{|S_R| + 1}
\end{align*}

Where \( l \) is wheel base, \( r \) is yaw rate, \( \delta \) is steering angle, \( v_x \) is vehicle longitudinal velocity, \( \lambda \) is the slip ratio, \( \lambda^* \) is the desired slip ratio, \( M_c \) is the corrective torque, \( k_1 \) and \( k_2 \) are design parameters, \( M_L \) and \( M_R \) represent the torque delivered to the left and right hand side driving wheels respectively, \( k_L \) and \( k_R \) are functions of the slip ratio of the left and right wheels, \( S_L \) and \( S_R \) respectively \([1]\). The layout for the controller proposed by Doniselli et. al. is as discussed previously in the literature review.

The wheel slip regulation method proposed by Doniselli et. al. makes uses of differentials to distribute torque between the left and right hand side driving wheels of a two wheel-drive, front wheel-drive vehicle for improved traction control. The limitation incurred as a result of relying on a single internal combustion engine to deliver varying torque signals is identified by the author, however technological availability at the time did not allow for a solution such as independently actuated in-wheel motors to deliver differential torque signals to the vehicle’s driving wheels. Exploring the benefits of in-wheel motor technology when applied to this strategy is addressed in this section. In-wheel motors, as opposed to active differentials offer faster and more precise torque distribution as well as the capability to deliver torque in forward and reverse directions \([7]\). As part of this study, adapting this traction control method and modifying the control law for use as a DYC strategy for improving vehicle performance of EVs with IAIWM was undertaken. The controller presented in this section investigates adapting this traction control method for use as a DYC strategy by modifying
the control law to accommodate the use of in-wheel motors and explore the benefits they provide. The controller proposed in this section also investigates the effects of extending the solution so that the algorithm can be applied to four-wheel drive vehicles with independently actuated in-wheel motors. Additionally, further improvement to the controller is investigated by implementing differential and integral gain terms to the controller. Using the simulation framework developed as a platform for investigation, the modified control laws are modelled and implemented using Simulink to investigate the validity of these control laws as a DYC strategy.

The first step of extending on the control strategy set out by Doniselli et. al. [1] was to apply the control law to an electric vehicle with independently actuated in-wheel motors, as opposed to an internal combustion engine vehicle with active torque distribution. It is expected that the strategy should be more effective based on the advantages that the use of in-wheel motors offer [7]. For initial investigation, the control algorithms presented by Doniselli et. al. [1] were implemented in Simulink on the vehicle model for some indication of how effective this strategy would be on a front-wheel-drive electric vehicle with two independently actuated in-wheel motors.

The control architecture for the in-wheel motor design required some alterations to translate the concept set out by Doniselli et. al. from an internal combustion engine vehicle to an electric vehicle with in-wheel motors. The original controller makes use of an input torque from the motor, which is divided and delivered to the two driving wheels via a differential based on the feedback information obtained from the controller. The baseline vehicle implemented in Simulink consists of a top level controller to deliver the torque to each of the two motors driving the front wheels of the vehicle, with a PID feedback loop to simulate the driver’s throttle command to control velocity.

In this loop, the vehicle’s forward (longitudinal) velocity is used as the control
variable and is used to deliver an even torque signal to each of the vehicles driving wheels. This controller is not intended as a method of DYC implementation, but rather as a modelling solution to produce a driving torque signal for basic motion in place of a throttle command from the acceleration pedal by the driver. As the original algorithm accounted for the torque delivery from a single source (the internal combustion engine) and this implementation makes use of two sources for torque delivery (two in-wheel motors) equations (33) and (34) can be substituted for:

\[ M_L = T_L + M_c k_L \]  \hspace{1cm} (37) \\
\[ M_R = T_R - M_c k_L \]  \hspace{1cm} (38)

Where all terms are as previously expressed, with the exception of \( T_L \) and \( T_R \) which represent the torque delivered to the front left and front right wheels respectively by the velocity controller, replacing \( \frac{M}{2} \) as the torque split is no longer required as a result of the driving torque being delivered from independent sources.

The result of this work was a simulated implementation of the wheel slip regulation control law set out by Doniselli et. al. adapted for use as a DYC strategy on
Figure 19: Simulink model of Doniselli et. al. control law adapted for vehicles with in-wheel motors.

a front-wheel-drive electric vehicle with in-wheel motors. The expression of the adapted control laws in Simulink are set out as in figure 19.

5.1.1 Simulated Results

The results of the effectiveness of the strategy proposed in the previous section when compared to an uncontrolled front-wheel-drive EV performing the same manoeuvre are shown in figures (20) and (21).

Figure 20: Yaw rate response of uncontrolled front-wheel-drive EV performing J-turn manoeuvre.

The system was evaluated by performing a J-turn manoeuvre, with a steering
Figure 21: Yaw rate response of longitudinal wheel slip regulation algorithm adapted for front-wheel-drive EV with in-wheel motors performing a J-turn manoeuvre.

angle input of 6 degrees at time $t = 6\, \text{s}$. As demonstrated by the results in figure (20) and figure (21) there is a increased steady state yaw rate error when compared to the uncontrolled system. Even when tuning the design parameters (torque scaling and gain) for optimal results for a specific manoeuvre, the system displays a slow response to steering input and maintains a significant steady state error. In this case, using this particular method of control, the steering behaviour is affected negatively. It is noteworthy however that the controller did function to eliminate wheel slip from occurring on all tyres excluding the front-left tyre, which still experienced approximately equal wheel slip to the uncontrolled system, as shown in figures (22) and (23). The reduced wheel slip is indicative of reduced loss of traction, which in turn is an indication of improved vehicle stability and safety. So although in this case there was no improvement in yaw rate response, the controller did serve to reduce traction loss when performing the manoeuvre and contribute to the vehicle’s overall stability and safety.

5.2 Longitudinal Wheel Slip Regulation for Electric Vehicle with Four IAIWM Controller Definition

The next step in extending this DYC strategy was to investigate the effects of longitudinal wheel slip regulation when applied to vehicles with four independently
Figure 22: Uncontrolled longitudinal wheel-slip for front-wheel-drive EV (simulated).

actuated in-wheel motors. As the original control algorithms set out by Doniselli et. al. were intended for use on an internal combustion engine vehicle with a single source of torque delivery, yet again the algorithms had to be modified to accommodate the four in-wheel motor vehicle system. In the four-wheel-drive implementation wheel slip on all four wheels can be regulated, and used to generate corrective yaw moments as opposed to the two-wheel-drive implementations previously discussed. The control law was modified to accommodate the feedback information of wheel slip from all four wheels, and to deliver a corrective yaw moment via delivering torque signals to each of the four driving wheels instead of two. The comparison of the difference in slip ratio between the left and right driving wheels was extended to include both driving wheels on the left and right hand sides of the vehicle, as opposed to only the front two wheels. The adaptation of this control law for a four-wheel drive electric vehicle with independently actuated in-wheel motors is expressed in the following paragraphs. The measured longitudinal slip ratio difference is modified to represent the total difference in wheel slip between all driving wheels on the left and right hand sides of the vehicle. The modified expression for obtaining the measured longitudinal slip ratio
Figure 23: Controlled longitudinal wheel-slip for front-wheel-drive EV (simulated).

difference is expressed in equation 39:

$$\Delta \lambda = \lambda_{FL} + \lambda_{RL} - \lambda_{FR} - \lambda_{RR}$$

(39)

Where $\Delta \lambda$ represents the longitudinal slip ratio between left and right hand side driving wheels of the vehicle, $\lambda_{FL}$, $\lambda_{RL}$, $\lambda_{FR}$ and $\lambda_{RR}$ represent the longitudinal wheel slip of the front left, rear left, front right and rear left wheels respectively. Including this expression as part of the control law adds to the robustness of the model. As this value scales the magnitude of the error function, it serves as a rudamental form of adaptively tuning the proportional gain of the controller. Consequently, the functions $k_L$ and $k_R$ are extended to represent terms for all four driving wheels, and become $k_{FL}$, $k_{FR}$, $k_{RL}$ and $k_{RR}$ expressed as:

$$k_{FL} = \frac{1}{|S_{FL}| + 1}$$

(40)

$$k_{FR} = \frac{1}{|S_{FR}| + 1}$$

(41)

$$k_{RL} = \frac{1}{|S_{RL}| + 1}$$

(42)
\[ k_{RR} = \frac{1}{|S_{RR}| + 1} \quad (43) \]

Equations (40) through (43) represent a simple function to limit output torque delivered to the motors to prevent excessive wheel slip, by multiplying the corrective torque delivered to each wheel by their respective \( k_{ij} \) value. When wheel slip for a given wheel, \( \lambda_{ij} \) is a small value close to zero, \( k_{ij} \) will be close to one, resulting in minimal compensation to the torque signal. Conversely, if \( \lambda_{ij} \) happens to be a large negative or positive value this will result in a reduced value for \( k_{ij} \) resulting in a reduction in torque delivered to that particular wheel. These terms are extended to include all four wheels in the adapted design which reduce the torque signal sent to each wheel when longitudinal wheel slip is detected, in an effort to restrict excessive wheel slip from occurring.

This controller calculates the error between the desired yaw rate, \( r_d \), calculated using equation (11) and the measured yaw rate, \( r \), and scales this error signal based on the longitudinal slip ratio difference between the left and right hand side wheels, \( \Delta \lambda \), as expressed by equation (39). The controller design allows for design factor \( k_1 \), which serves as a proportional gain factor for scaling the error signal. As such, the error function is expressed as:

\[ M_c = k_1 (r_d - r) - (\Delta \lambda) \quad (44) \]

The following relationships represent how the system is adapted to an electric vehicle with four independently actuated in-wheel moments. As shown by the following expressions, the corrective yaw moment is delivered to the four driving wheels, via four separate torque signals. Each signal is a result of the scaled error function comprised of yaw rate error and slip ratio difference (\( M_c \))

\[ M_{FL} = T_{FL} - k_2 M_c k_{FL} \quad (45) \]

\[ M_{FR} = T_{FR} + k_2 M_c k_{FR} \quad (46) \]
The system when modelled in Simulink is pictured in figure 24.

\[ M_{RL} = T_{RL} - k_2 M_c k_{RL} \]  
\[ M_{RR} = T_{RR} + k_2 M_c k_{RR} \]

Figure 24: Simulink model of longitudinal slip ratio (Doniselli et. al.) control law extended to vehicles with four in-wheel motors.

5.2.1 Simulated Results

After extending the strategy to vehicles utilising four independently actuated in-wheel motors, the system was evaluated. As the main objective of this thesis is to improve vehicle stability and control, the effectiveness of the controller was evaluated in terms of minimising yaw rate error. The system was evaluated using a J-turn manoeuvre initially, and compared against a vehicle without yaw rate control performing the same manoeuvre. The results of a step steer manoeuvre, with a steering input of 6 degrees at time \( t = 7 \text{s} \) and vehicle speed of 20 km/h for the uncontrolled and controlled vehicle are shown in the figures 25 and 26.

As demonstrated by the results in figure 26, the rise time is reduced in the controlled system, indicating a system which is more responsive to steering input. There is no overshoot evident, however there is a steady state error occurring. The
Figure 25: Simulated yaw rate response of an uncontrolled vehicle performing a J-turn manoeuvre.

presence of a steady state error indicates that the controller could be improved by adding an integral gain term to eliminate this problem.

5.3 Longitudinal Wheel Slip Regulation with PI Controller for Electric Vehicle with Four IAIWM Controller Definition

As such, the next task for extending the wheel slip regulation DYC strategy was to introduce an integral gain term to the controller to investigate its effect on the system, particularly in eliminating steady state error. The adaptation of the system to include both proportional and integral gain terms when modelled in Simulink is shown in figure 27.

5.3.1 Simulated Results

This system was evaluated in simulation once again using the same J-turn manoeuvre for consistency. The results of which including the proportional and integral gain terms are shown in figure 28.

As demonstrated by the results in figure 28, by adding integral gain to the con-
controller, the steady state error is eliminated with zero overshoot, and maintains a reasonable response time. By examining the wheel-slip exhibited by each tyre in the simulation comparing the controlled and uncontrolled system response to a J-turn manoeuvre, as shown in figures (29) and (30), increased positive wheel-slip can be observed in the right hand side tyres, and increased negative wheel-slip can be observed in the left hand side tyres. This is resultant of controller intervention to regulate the wheel-slip difference between the left and right hand sides to induce a corrective yaw moment for the vehicle.

For further consistency of results, the uncontrolled system was compared against the PI controller in a lane change manoeuvre to evaluate the yaw rate response of the system. The results of the uncontrolled system are shown in figure 31.

As shown by figure 31 these results indicate a delayed yaw rate response to the steering input, lagging by almost 1 second. Additionally, the system is not achieving the peak values set out for target yaw rate. This indicates that the driver’s perception of control of the vehicle in performing such a manoeuvre would be reduced, resulting in a sluggish and unresponsive steering manoeuvre.

By including the controller proposed in this section with the same PI terms the results for a lane change manoeuvre are demonstrated in figure 32:
Figure 27: Simulink model of longitudinal slip ratio control law including proportional and integral gain terms.

As demonstrated by the results in figure 32 the system is a lot more responsive to steering input. There is a significant reduction in lag, indicating a much more responsive system with an increased sense of perceived control by the driver. The lag in peak values is reduced from almost 1 second to approximately 0.06 seconds. Additionally, the peak error values experienced in the line change manoeuvre are approximately 1.3 rad/s, indicating a significant understeer characteristic during cornering. This error is reduced in magnitude from approximately 1.3 rad/s to 0.015 rad/s with the controller in place, equating to a reduction of approximately 25% in magnitude of error, and a turning characteristic much closer to neutral steer behaviour, resulting in improved stability, and perception of control by the driver.

5.3.2 Summary

The strategy proposed in this section has demonstrated through simulations, using the framework established in Simscape, an improvement in yaw rate response and reduction in steady state error when performing standard testing manoeu-
The control strategy outlined in this section requires the measurement of longitudinal slip ratio and yaw rate to be implemented on a real life vehicle. To obtain values for longitudinal slip ratio, one method involves calculation using
Figure 30: Longitudinal wheel slip of each tyre for controlled EV with four IAIWM performing J-turn manoeuvre.

Figure 31: Simulated yaw rate response of uncontrolled system performing a line change manoeuvre.

the vehicle speed and wheel angular velocity of each driven wheel. Essentially, to implement this strategy on an experimental vehicle the measurement of vehicle speed, wheel angular velocity and yaw rate are required. As discussed previously, the sensor capabilities on the experimental vehicle developed as part of this project are capable of measuring all of these parameters, using the VBOX20SL module and motor controllers. Therefore implementing this control strategy on the experimental vehicle developed in this project is realistically achievable with the hardware and sensor setup currently in place.
5.4 Longitudinal Performance Control

This section of the paper will include contributions made towards a control strategy implemented for the purpose of improving the longitudinal performance of the vehicle. This section presents a torque vectoring control strategy for an electric ground vehicle with individually actuated in-wheel motors, and will outline the investigation of the effects of controlling longitudinal wheel slip as a means for improving vehicle longitudinal dynamics performance and stability under split slippery surface conditions. The controller presented here is evaluated using the simulation framework pertaining to relevant performance indicators used to benchmark the effectiveness of the control strategy proposed.

The effects of undesired wheel slip can result in a loss of power delivered from the driving wheels to the road surface, resulting in a loss of longitudinal acceleration and disruptions to vehicle velocity [7]. As such a torque vectoring strategy for maintaining longitudinal performance of the vehicle will be presented here. Controller design based on longitudinal performance control is highly applicable to real life driving situations. As discussed previously, under split slippery surface conditions, road surface conditions are not always consistent and the friction between each tyre and the road surface is not always the same. Split slippery surface conditions can prove to be detrimental to vehicle stability due to unevenly distributed loss of traction, which may result in producing an undesired
yaw moment or loss of longitudinal velocity/acceleration.

The purpose of this controller is to regulate wheel slip to maintain longitudinal acceleration and velocity by minimising disruptions to performance caused by undesired wheel slip under slippery and split slippery surface conditions. The controller proposed uses longitudinal wheel slip ($\lambda$) and vehicle longitudinal velocity as the control variables for achieving this purpose. The algorithm for this controller can be expressed as:

$$\Delta \lambda^* = k(v^*_x - v_x)$$  \hspace{1cm} (49)$$

Where $\Delta \lambda^*$ is the desired slip ratio, $v^*_x$ is the desired longitudinal vehicle velocity, $v_x$ is the measured or actual longitudinal vehicle velocity and $k$ is a proportional gain term.

![Simulink model of the control law](image)

Figure 33: Application of control law expressed as a Simulink model.

The relevance of this DYC solution pertains to real life driving situations where road surface conditions are not always optimal, and as such values for surface friction coefficient may vary between each tyre and their respective road surface contact area. For example, split-surface and instantaneous split-surface condi-
tions such as isolated puddles or ice patches on a road surface may result in inconsistent performance of tyres, and varying performance between the tyres of the vehicle at a given point in time, due to the composite road surface. As the purpose of this controller is to maintain longitudinal acceleration and velocity and minimise undesirable oscillations and disturbances to these values, longitudinal vehicle velocity is adopted as the control variable for this purpose.

5.4.1 Simulated Results

Fig. 34 demonstrated that when travelling on split-slippery surface conditions the uncontrolled system has a much slower acceleration curve, reaching the target velocity at time \( t = 13 \) s as opposed to the controlled system, which through redistribution of torque, managed to reach the target velocity at time \( t = 10 \) s. This translates to a difference in average rate of acceleration of approximately \( 16 \text{ m/s}^2 \) or an increase of 29%.

Fig. 35 demonstrates that even with no steering angle present the yaw rate response of the simulated vehicle under split-slippery surface conditions can affect vehicle heading and cause undesired fluctuations in vehicle yaw rate. Consequently, utilising the longitudinal slip regulation to maintain driving force, due to the redistribution of torque being biased towards one side of the vehicle, this can exasperate the effect on yaw rate that the vehicle experiences. Fig. 35 shows the yaw rate error increases significantly, from a peak value of approximately 0.018 rad/s to 0.04 rad/s. This increase in yaw rate error is a consequence of the controller correcting the loss of vehicle velocity due to low friction on some of the vehicle’s tyres. The trade off exists as the controller can only improve either yaw rate or velocity at any given point in time. Driver input however can be used to account for the yaw rate error with steering input.

Fig. 35 demonstrates that even with no steering angle present the yaw rate response of the simulated vehicle under split-slippery surface conditions can affect
Figure 34: Comparison of uncontrolled vehicle velocity under split-slippery surface conditions and controlled vehicle velocity under split-slippery surface conditions.

Vehicle heading and cause undesired fluctuations in vehicle yaw rate. Consequently, utilising the longitudinal slip regulation to maintain driving force, due to the redistribution of torque being biased towards one side of the vehicle, this can exasperate the effect on yaw rate that the vehicle experiences. Fig. 35 shows the yaw rate error increases significantly, from a peak value of approximately 0.018 rad/s to 0.04 rad/s. This increase in yaw rate error is a consequence of the controller correcting the loss of vehicle velocity due to low friction on some of the vehicle’s tyres. The trade off exists as the controller can only improve either yaw rate or velocity at any given point in time. Driver input however can be used to account for the yaw rate error with steering input.
5.5 Summary of Simulated Results

The DYC strategy based on longitudinal wheel slip ratio regulation was implemented through simulations on the model of the experimental vehicle. The control law as set out in equation (39) for four in-wheel motors and PI control was evaluated performing a J-turn and line change manoeuvre to inspect the system response. Firstly, the yaw rate response of the variations of control strategy of the simulated vehicle will be compared and inspected here. The yaw rate response of the uncontrolled system with front-wheel drive only can be seen in figure 36.

By adjusting the system to simulate an electric vehicle with four in-wheel motors, the results can be observed in figure 37.

As can be seen by figures 36 and 37, the introduction of four-wheel drive as compared to front-wheel drive does with no control law in place has a minimal
Figure 36: Simulated yaw rate response to J-turn manoeuvre for an uncontrolled, front-wheel drive vehicle.

Figure 37: Simulated yaw rate response to J-turn manoeuvre for an uncontrolled vehicle with four IAIWM.

effect on the system response when performing a J-turn manoeuvre. There is no significant change in rise time, settling time or overshoot/undershoot. There is a slight improvement in steady state error. The effects on system response of introducing a yaw rate feedback controller, which generates a corrective yaw moment based on longitudinal wheel slip regulation, as discussed previously, are shown in figure 38.

As demonstrated by figure 38, by including a simple DYC controller in the simulation, the system response has changed, and in this case, improved. This result confirms that the simulation is sensitive to DYC algorithms, and verifies that the simulation model can be used to examine the effect of a given DYC algorithm on
Figure 38: Simulated yaw rate response to J-turn manoeuvre for a vehicle with four IAIWM and longitudinal slip ratio controller implemented.

the system.
6 Experimentally Acquired Results and Validation

This chapter will include a summary of experimentally obtained results and calculations used for validating the simulated vehicle model. Methods of data acquisition and estimation of useful information will also be included in this chapter. A discussion of experimental results is included here. This section also includes a review of the validation process of the simulation platform used to obtain results and aid in the design process in this project.

6.1 Validation of Simulation Platform

This section will detail the process involved in verifying the validity of the simulation platform at use in the design and experimental process of this project. There are many benefits of integrating simulation into the process of design and experimentation. Simulating results can often be a lot more efficient in the design process as opposed to obtaining results experimentally. The design process may call for multiple iterations of the experiment to be run, with varying parameters and values. As such the use of simulation can streamline this process and allow for these design parameters to be easily manipulated. Additionally, there are often inherent risks in performing experiments, especially in a scenario such as this involving the operation of an electric vehicle performing cornering manoeuvres. The risk of injury or damage to property can be reduced by running tests in simulation, and minimising the number of experiments which need to be performed. However if the simulation platform is an inaccurate reflection of the real life system, the controller design may be based on false or inaccurate results and function ineffectively. Therefore it is important to validate the reliability and accuracy of the model before relying on it as a tool for design of real system components.
The validity of the simulation output was verified initially through observation, by inputting values and observing expected results. The simulation results indicate a functional platform and will be discussed in the following section. A simple torque vector control algorithm is to deliver different values of torque to either side of the vehicle, inducing a yaw moment about the vehicle’s vertical axis. This can assist the vehicle to turn, even when no steering angle is present. A generic torque input of 150 Nm was applied to the left side wheels and 50 Nm was applied to the right side wheels in the simulation, for a total of 400 Nm of torque distributed across the four driving wheels. This was compared to an even torque distribution of 400 Nm across the four driving wheels, with 100 Nm being delivered to each wheel. The simulation results of the biased and unbiased torque distribution are shown in figures 39 and 40.

Figure 39: Simulated system response of displacement over time to even torque distribution.

Figure 40: Simulated system response of displacement over time to a biased torque distribution.
The results in figure 39 and figure 40 display the longitudinal and lateral displacement of the vehicle given a steering angle of zero. The results are as expected, displaying straight line motion for the evenly distributed driving torque, and a deviation in path towards the right hand side of the vehicle when a torque bias was induced with increased torque to the left hand side driving wheels, evident as negative lateral displacement in figure 40.

The simulated results were also compared to theoretically calculated results to verify the model’s validity. Firstly, the forward velocity \( V_x \) of each tyre was calculated using the effective rolling radius \( r_e \) and angular speed \( \Omega_0 \) as described by equation (50) [23].

\[
V_x = \Omega_0 r_e
\]  

(50)

![Figure 41: Simulated and theoretically calculated forward velocity of the simulated vehicle model.](image)

Figure 41 shows the comparison between the calculated results and the simulated results of the forward velocity of the front left tyre given a constant torque signal. As demonstrated by the results, the expected forward velocity of the vehicle is a very close match to the simulation output, which is representative of a reliable system.

As discussed previously, the CAN bus can be used to request the motor angular velocity from the motor controller, however only to within 1rpm of accuracy.
Longitudinal wheel slip is used as feedback in some DYC strategies, and as such, it needs to be determined if 1 rpm accuracy is sufficient for use in a feedback loop to produce an effective result using a DYC controller. This was investigated by quantizing the data in the simulation model to within 1 rpm accuracy for wheel angular velocity, and calculating wheel slip. The results calculated were compared against the results simulated from the MF-Tyre model, and used as feedback in the proposed DYC strategy.

Figure 42: Comparison of simulated wheel slip acquired with unaltered and quantized data for wheel angular velocity.

As such, it was shown that the results calculated with the restrictions on accuracy for measuring wheel angular velocity, did not have a significant effect on the calculation of wheel slip. Both curves were representative of the same tyre behaviour, and all values had a very small margin of error, to within approximately 0.02%. As such, the DYC strategy was implemented using both sources for wheel slip feedback, and it was observed to have no effect on vehicle response as both yaw rates were identical. As such, it has been validated that the wheel slip which can be calculated from the measurements of wheel angular velocity obtained from the motor controllers are within an acceptable range of accuracy to implement a DYC strategy utilizing longitudinal wheel slip as feedback.
6.1.1 Experimentally Verified Results

Experimental data acquired from the various sensors previously discussed were used to verify the simulation platform’s validity in representing the vehicle as a real life system. Data acquired from the experimental vehicle’s sensors was used to create a comparison between the simulated vehicle and the experimental vehicle’s performance. Sensor data was used to create a torque signal and steering angle signal to be used as inputs to the simulation, and the outputs of the simulation were compared to the sensor data acquired from the electric vehicle to draw a comparison between the performance of the two systems.

As discussed previously, a rotary encoder attached to the steering wheel column was used to obtain measurements for the steering wheel angle of the experimental vehicle. The encoder was connected to the CAN bus so as to synchronise the information registered with the other signals so as to obtain more meaningful data for creating a wholistic representation of the vehicle manoeuvres completed and the resultant performance indicators to be measured. The encoder used to measure the steering wheel angle outputs data in “counts”. To convert this data into a meaningful measurement the steering wheel angle had to be calibrated. The steering angle was calibrated by turning the wheel for one or two full rotations, and adding the number of counts received. The calibration was repeated several times to ensure accuracy. The result of the calibration showed consistently that two full revolutions were comprised of 4456 counts, and a single revolution of the steering wheel was comprised of 2228 counts. Using this information, it could be determined that 1 degrees was equal to approximately 6.18 counts.

A major performance indicator in investigating and evaluating vehicle control and stability is the vehicle yaw rate. It is important to have a means of accurately measuring the yaw rate of the vehicle when implementing a DYC controller, as many controller designs require the measured yaw rate of the vehicle as a feedback signal. As such, it is necessary to ensure that the yaw rate response
of the simulated vehicle is reasonably consistent with that of the experimental vehicle, to ensure that the yaw rate response induced by the controller in simulations is an accurate reflection of how the experimental vehicle will behave. To validate the model’s yaw rate response was indeed an accurate reflection of the experimental vehicle, a test was performed, in which both the experimental and simulated vehicle drove at a constant speed, and were given the same steering input. Using the experimental vehicle and the method described above, a line-change manoeuvre was performed with important performance indicators being measured. Capturing the steering wheel position and torque output the manoeuvre was recreated on the simulation, and the performance was compared. The steering wheel angle and torque signals captured from the experiment and used as inputs to the simulation can be seen in figures 43 and 44.

![Simulated and Measured Steering Wheel Angle Input](image)

**Figure 43:** Steering wheel angle measured from encoder on experimental vehicle steering shaft and replicated signal built on Simulink for input to simulated vehicle.

The vehicle’s yaw rate was the first performance indicator to be compared, because as established, controlling this variable is the major focus of a DYC controller. As such, the result of the torque and steering signals recreated from the measured values from the experimental vehicle when used as input to the simulation can be inspected in figure 45.

![Simulated and Measured Vehicle Yaw Rate](image)

**Figure 45:** Vehicle yaw rate comparison between simulated and experimental vehicles.

After re-creating the torque and steering angles as input signals to the simulation, the vehicle was observed to perform the line change manoeuvre in the simulation, as performed earlier in the experimental vehicle. As recorded in figure 45 the
curves for the simulated and experimentally measured yaw rates indicate that the simulation is a reasonable representation of vehicle behaviour. Firstly, the curve in both the experimental and simulated vehicle is representative of a line change manoeuvre, with the yaw rate indicating steering action taken by the driver at the expected time. Secondly, the maximum yaw rate values obtained by both the simulated and experimental vehicle are reasonably close to each other, peaking at 0.48 rad/s for the simulated vehicle and 0.45 rad/s for the experimental vehicle. The margin of error in this instance is approximately 6%, which is within a reasonable degree of accuracy. This margin of error may be explained by the steering angle input for the simulation being slightly variant to the real life steering angle. As can be viewed in figure 43 the signal builder used to create the steering input for the simulation produced a much sharper curve than the
steering input measured from the encoder on the experimental vehicle. This was resultant in a slight variation in yaw rate, but overall still maintained a reasonably accurate representation of what the steering input was. The simulated response also produces a much “smoother” curve than the experimentally acquired yaw rate response. This disturbance in the signal can be explained by the sensor on the experimental vehicle is sensitive to several small oscillations in the yaw rate of the vehicle, which the simulation is not subject to. For example, the road surface in the simulation is evaluated as a smooth road surface, however in real life experiments the road surface is not perfectly smooth, so small oscillations in yaw rate occur as a result of “road noise”. Secondly, there are vibrations within the experimental vehicle, such as where the VBOX antennae are attached to the vehicle as a result of the vehicle driving on a road surface which is not completely smooth. As a result of these vehicle vibrations, small oscillations in the yaw rate are a consequence, which are recorded by the VBOX module. Thirdly, the steering wheel input signal in Simulink allows for a “perfect” signal, i.e. the output will be exactly what it is programmed to be, whereas in the experimental vehicle, the steering wheel angle is controlled by human hands, and is therefore subject to human error, resulting in slight variations to the steering wheel angle. However as the peak values are within a reasonably close range (error values < 10%) and the curve is representative of the manoeuvre performed, the oscillations can be disregarded to acquire a reasonably accurate representation of the yaw rate response from the simulated electric vehicle.

The next measurable variable to compare was the vehicle velocity. It is important that when given a similar torque input, the experimental vehicle and simulated vehicle achieve a similar rate of acceleration and top speed, to verify that the performance of the simulation is an accurate representation of the vehicle’s response to torque input. The torque signals displayed in figure 44 represent the torque delivered to the left front wheel of the experimental vehicle, and the signal used to deliver torque to the front left wheel of the simulated vehicle. The measured
torque signal was calculated from measuring the current delivered from one of the motor controllers to the front left motor. Due to the availability of sensors for the experiment, only one current signal could be measured, and the decision to measure current on the left hand side motor is arbitrary. As no torque distribution strategy for uneven torque signals is put in place here it is assumed that both torque signals are equal to a reasonable extent in this experiment, as the conditions for the simulation verification tests are treated as such, i.e. an identical torque signal is delivered to both the left and right hand side driving wheels. The results of this experiment when comparing vehicle speed can be observed in figure 46.

![Figure 46: Comparison of absolute vehicle speed for the experimental and simulated vehicles.](image)

As discussed above, a torque signal was created in Simulink signal builder and distributed to the two front driving wheels of the simulated vehicle, based on the torque calculated to be delivered to the experimental vehicle based on current measurements. Examining the velocity of the experimental and simulated vehicle, shown in figure 46 there is significant variation between the two curves during the deceleration of the vehicle. This can be explained because the experimental vehicle is at this stage, still equipped with mechanical brakes and there is no sensor in place for measuring the braking force from the mechanical brakes. As a result, at this stage the braking force acting on the vehicle cannot accurately be measured, only the torque being delivered to the electric motors can be measured.
Consequently, this allows for a torque signal mapped from the motor current to be used as torque input for the simulated vehicle, which does not account for mechanical braking, hence the variation in vehicle speed when the experimental vehicle begins to decelerate at approximately time \( t = 14 \) seconds. In the simulation, the torque delivered tapers off, but without the input of a braking force, the vehicle maintains forward velocity through its momentum. In future work when recreating this experiment, an idealized brake torque or a model for regenerative braking could be included. In this case however there was no means of accurately measuring the braking force and applying it to the model, and as the deceleration curve was not the focus of this investigation the braking torque was not considered crucial.

The next aspect of the curve which requires analysis is the vehicle top speed. For the simulated model to be an accurate representation of the experimental vehicle, a similar top speed should be attained given a similar torque signal as input to the motors. Comparing the experimentally obtained and simulated results, the experimental vehicle reached a top speed of approximately 21 km/h. When given the same torque input, the simulated vehicle reached a top speed of approximately 21.5 km/h. Comparing these two values, with an difference of 0.5 km/h the error for top speed is approximately 2.3%. This is within an reasonably small margin of error and indicates an accurate representation of the experimental system through a simulation in terms of top speed.

The final aspect of the curve to analyse is the rate of acceleration. It is important to determine whether the rate of acceleration of the experimental vehicle and simulated vehicle are reasonably similar (within ±10%) given the same input torque. Examining the curve at the time of acceleration, in this case from time \( t = 0 \) seconds to time \( t = 10 \) seconds, the average rate of acceleration for the experimental and simulated vehicles are calculated to be 1.90 m/s\(^2\) and 2.05 m/s\(^2\) respectively. This amounts to be a margin of error of approximately 7.9% for
average rate of acceleration. Again, this falls within a reasonable margin of accuracy (within ±10%) and is therefore determined to be a reasonably accurate representation of the system. Certain factors may affect the accuracy of the vehicle speed, such as wind resistance. Wind resistance can be adjusted in the simulated model, by setting a value for the damping coefficient for the vehicle centre of gravity in the six degree of freedom block which represents the vehicles geometric centre of mass. This factor however will not always be completely representative of the real life driving situation, as on different days, different conditions are apparent, for example if the vehicle is facing a strong headwind the acceleration and velocity will be slowed. Additionally, the variation in velocity and acceleration may also be a result of slightly different sized tyres used on the experimental vehicle and the simulated vehicle. The tyres available to the experiment were of a slightly larger radius than those utilised in the simulation, which is resultant in variant vehicle dynamic performance.

![Simulated Wheel Slip vs Measured Wheel Slip](image)

Figure 47: Comparison of simulated and experimentally acquired values for longitudinal wheel slip of the front-left tyre.

As discussed previously, longitudinal wheel slip ratio is an important performance indicator of vehicle dynamics, as it provides a strong indication of the traction of the vehicle’s tyres and the road surface. Consequently, it is important to have some form of sensor capabilities on the experimental vehicle which can allow for the calculation of the vehicle’s longitudinal wheel slip for both analysing vehicle performance and in the implementation of torque vectoring or traction control.
strategies. Shown in figure 47 are the longitudinal wheel slip ratios acquired from the simulated and experimental vehicles when performing the same manoeuvre. The wheel slip on the experimental vehicle is obtained by measuring the driven wheel’s angular velocity from the motor controller, using the radius of the tyre as a measured constant, and the linear velocity of the vehicle using the VBOX module. After measuring these values, the longitudinal wheel slip is calculated using Equation 3. There is some variance between the simulated and measured values for longitudinal wheel slip, this can be explained by the slight difference in wheel radius. The simulated vehicle and experimental vehicle are both using different sized wheels, hence resulting in a different result for wheel slip. Another source of variance in values is the road surface friction. In the simulation, a standard friction coefficient of 1 is used to represent a normal dry road surface. However, it cannot be determined if the road surface in the experimental trail is completely consistent and there may be some increased or decreased friction at certain parts of the road surface, which will affect the longitudinal slip of the tyres. Additionally, the motor controllers used on the experimental vehicle are only capable of outputting angular velocity to integer accuracy (RPM). Tests performed in the simulation mode in which the slip ratio was calculated using wheel angular velocity measurements quantised to within integer accuracy were compared to the default longitudinal wheel slip measurements. Results showed that the quantisation of angular velocity measurements did not bear a detrimental effect on the measurement of the wheel slip, or the performance of the control strategy put in place. Given that, and since the results for wheel slip are relatively close (maximum margin of error within 0.01%) it is assumed that this is an acceptable method for obtaining longitudinal wheel slip ratio measurements for use in a control strategy on the experimental vehicle.

As established, vehicle sideslip is another important performance indicator of vehicle stability and control. Sideslip angle is often controlled as part of a DYC strategy and as such, it is important that in this platform a means of measuring
and simulating vehicle sideslip is established. The VBOX20SL module was used to obtain experimentally measured values for vehicle sideslip, with the antennae positioned so that the point of measurement is to be taken from the vehicle centre of gravity (C.O.G.). Shown in figure 48 and figure 49 are the experimentally measured and simulated sideslip respectively. The simulation and experimental test were synchronised as outlined previously, with the torque and steering inputs to the simulated vehicle modelled based on the values measured from the experimental vehicle to recreate the same manoeuvre performed under similar conditions.

As shown by the results in figures 48 and 49 there appear to be more oscillations in sideslip in the experimentally acquired data in figure 48. This can be explained similarly to the oscillations in yaw rate, as vibrations which occur in the experimental system due to vehicle vibrations and road noise are not evident in the simulated model. The shape of the curve is a close match for both the experimental and simulated values, both reaching negative maximum and positive maximum at approximately \( t = 12.5 \text{s} \) and \( t = 14 \text{s} \) for the experimental data and at approximately \( t = 12 \text{s} \) and \( t = 13.5 \text{s} \) for the simulated model.

![Measured Vehicle Sideslip (C.O.G.)](image)

Figure 48: Experimentally measured sideslip measured from the vehicle centre of gravity.

The slight time shift and discrepancy in magnitude is a result of the VBOX20SL measuring the vehicle sideslip from the vehicle centre of gravity in the experimental results, and in the simulation the sideslip was measured from the front...
left and front right tyres. Figure 49 shows that the magnitude for sideslip is greater on the outer wheel (initially the right wheel when turning left, and then the left wheel when turning right) throughout the simulation. The positioning of the sensors at different locations on the experimental and simulated vehicle will result in a time shift due to being at a different displacement at any given time. The magnitude also varies due to the placement of the sensors, as the same magnitude of sideslip cannot be expected to be identical at different locations of the vehicle. Examining the desired and actual yaw rate of the vehicle in figure 50 it is evident that the vehicle is experiencing understeer turning characteristic. This result maintains consistency with the output of the vehicle sidelslip values being higher in magnitude on the outer wheels during cornering, as this behaviour is also indicative of understeer behaviour. The difference in magnitude in vehicle sidelslip could also be explained by differing road surface conditions between the simulated and experimental environment. Inconsistent road surface friction in the real life driving situation may result in increased or reduced wheel slip and this may be reflected in the results.

![Vehicle Sideslip Angle (simulated)](image)

Figure 49: Simulated vehicle sideslip front left and front right tyres.

However, the the shape of the curve and magnitude of sideslip angle remains consistent in both the simulation and experimental setup and given that in the majority of cases, the reference value for vehicle sideslip is zero, for evaluation in simulation and implementation using hardware both cases are useful. The
controller objective for sideslip control is consistent in both cases by reducing the error to zero in the same direction and at the same point in time during the manoeuvre.

Figure 50: Desired and actual yaw rate of line change manoeuvre.
7 Future Work and Conclusions

7.1 Future Work and Applications

Active steering, in which the angle of the vehicle’s wheels are controlled independently of driver input or steering wheel angle is an option for future implementation on this project. Active steering can be applied to either the front, rear or front and rear wheels of the vehicle [60, 61]. Active steering can be integrated into a DYC strategy and has been shown to be a robust solution in terms of conforming to desired yaw rate and side-slip angles, especially during changes in road surface conditions and vehicle parameters such as cornering stiffness [62]. As such, the integration of an active steering solution to the DYC strategy is a possible avenue of future research to pursue on the project which if implemented correctly, could result in improved vehicle control and stability.

Similarly to active steering, four-wheel steering has been shown to be an effective measure in eliminating vehicle side-slip, which as previously discussed is a major contributor to reductions in vehicle stability [63]. As such, four-wheel steering solutions are also a possible avenue of future research which can be undertaken on this experimental vehicle to further improve vehicle stability and handling. The implementation of which would involve extending the capabilities of both the experimental vehicle and the simulation framework.

Given the environmental benefits of EGVs, improving the energy efficiency of electric vehicles is a prominent area of research. A major limitation of EGVs when compared to internal combustion engine vehicles is their range i.e. comparing how far the vehicle can travel on a single charge as opposed to a tank of fuel. The experimental vehicle and simulation model developed as part of this project form a platform which can be used in future work applications for investigating, developing and evaluating range extension technology and improving overall operational efficiency. Improving the the efficiency and range of EGVs is a problem
in research which can be approached from a number of angles which provide opportunities for future work in a number of areas. Solutions which may be explored in future work using this platform to address this problem include battery management, improved regenerative braking and energy efficient distribution of torque.

Another avenue of future research to be explored in relation to this project is to implement and evaluate the torque vectoring control strategies presented in this paper on hardware. Utilising the capabilities of Simulink to deploy code to hardware, the control algorithms presented in this paper could be deployed to hardware and used in conjunction with the sensor network set up on the experimental vehicle to experimentally validate the effectiveness of the strategies proposed. Similarly, the simulation framework developed and presented in this paper can be used in future work on the project in the development and testing of new control laws and control algorithms. In addition to this, future work may also include utilising the experimental vehicle for the experimental evaluation of future control strategies.

As demonstrated by the results, when examining vehicle dynamics performance under split slippery surface conditions, the current vehicle setup as expressed in the simulations is capable of only improving either the yaw rate or the longitudinal performance of the vehicle. Hence a “trade off” exists in which one performance indicator must be favoured over the other. As discussed above, active steering and four-wheel steering are possible avenues of future research which may present a solution to this problem which is open to further investigation using the experimental vehicle and simulation framework presented here. Alternatively, a solution could be developed and implemented in which the controller forms a decision making process to switch automatically between control variables to optimise vehicle performance.
7.2 Conclusions

This thesis presents the development of a robust and parametric simulation framework for modelling the dynamic performance of electric vehicles. The simulation framework maintains robustness, in the sense that the block-based design enables modifications to be made to the configuration of the vehicle, in terms of its method of actuation and systems being inspected. In addition to this, the framework maintains the capability for adding additional systems for inspection to the simulated vehicle, for example battery management systems or an experimental regenerative braking setup could be modelled, and added into the framework in their appropriate locations.

Another major outcome of the work presented in this thesis was the development of a torque vectoring control algorithm, adapted from a traction control strategy for internal combustion engine vehicles. This traction control method was originally developed for regulating longitudinal wheel-slip on an internal combustion engine vehicle through use of an active differential to “split” the torque signal from the engine to the two driven wheels of the front axle. This control strategy was adapted for use as a direct yaw-moment control strategy for electric vehicles and extended to include electric vehicles with individually actuated in-wheel motors and utilise proportional and integral gain terms to further improve results. The effectiveness of this strategy was validated through simulations in improving yaw rate response and longitudinal performance under split slippery surface conditions. Simulation results showed an improvement in both the yaw rate response of the vehicle, and a reduction in steady state error to the vehicle’s yaw rate. Examining the simulated vehicle under split slippery surface conditions showed an improvement in longitudinal acceleration and maintaining a constant velocity, albeit at the cost of magnified errors to yaw rate.

An experimental electric vehicle with front-wheel drive individually actuated in-wheel motors converted from an internal combustion engine vehicle is another
contribution from work performed in this paper. This thesis has established that
the experimental vehicle presented is equipped with sufficient sensor capabilities
for evaluating the dynamic performance of the vehicle, as it is capable of obtaining sufficient data for either the calculation or measurement of the important performance indicators outlined in this thesis. Also presented in this thesis are
the experimental procedures undertaken in validating the simulation framework presented. The experimental data serves to validate that the performance of the simulation framework is in all cases a reasonably accurate representation of the performance of the experimental vehicle under similar conditions. Therefore the simulation framework presented serves as a valid and useful indication for evaluating the performance of the experimental vehicle presented.

This thesis has presented a torque vectoring direct yaw-moment control strategy which has been proven through simulations to improve the vehicle’s dynamic performance and stability. Additionally, an experimental test bed serving as a useful platform for future research in this area has been presented consisting of an experimental electric vehicle with independently actuated in-wheel motors and full sensor capabilities and an accurate simulation framework representing said experimental vehicle. The experimental and simulated setup presented in this paper serves to function as a wholistic and validated platform for continuing research on electric vehicle technology in the field of torque vectoring and direct yaw-moment control, and maintains the capability of extending research to additional avenues of electric vehicle technology related research and development.
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