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Abstract

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Keywords

unwrapping, method, multiple, phase, fringe, wavelengths, patterns, without, equivalent

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Phase unwrapping method based on multiple fringe patterns without use of equivalent wavelengths

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Abstract

In order to decrease the noise enhancement in the process of phase unwrapping, we present a new phase unwrapping approach for multiple wavelength phase shift profilometry. In this approach, a series of fringe images with only three wavelengths are captured by the 3D measurement system, and it does not need to calculate the equivalent wavelengths and the equivalent phases. As a result, the noise enhancement is avoided, leading to improved precision in the absolute phase map recovery. Experiment results demonstrate that the proposed method is more robust and effective compared to the existing TWSP method.

Keywords

Phase unwrapping; Equivalent wavelengths; Equivalent phases; Noise enhancement

1. Introduction

Fringe pattern profilometry (FPP) [1] has become an effective technology for non-contact measurement of 3D shapes, which has found various applications such as

manufacturing inspection, medical sciences, reverse engineering, quality control, etc. With FPP, Ronchi or sinusoidal grating patterns are projected on the object surface, which are deformed by the height distribution of the surface and acquired by a camera. These deformed patterns carry the 3D information of the object surface and hence can be analyzed to extract the information. In order to reconstruct the 3D shape, various fringe pattern analysis methods have been developed, e.g., Fourier transform profilometry (FTP) [2], phase shift profilometry (PSP) [3], modulation measurement profilometry (MMP) [4], phase-measuring profilometry (PMP) [5], etc.

Among these approaches, PSP is particularly commonly used due to the advantages such as high speed, high resolution, high accuracy and its non-contact nature. Moreover, the PSP is less sensitive to the surface reflectivity variations and the object can be measured on pixel-by-pixel basis. With PSP, multiple fringe stripes [6] with the same wavelength are usually utilized. However, the phase maps retrieved are wrapped ones, which must be unwrapped for 3D shape recovery. In order to unwrap the phase maps, many different algorithms are developed, e.g., spatial phase unwrapping [7] and temporal phase unwrapping methods [8]. Spatial phase unwrapping is based on the analysis of adjacent or neighboring pixels on an individual wrapped phase map, while the temporal phase unwrapping is a type of method based on analysis of multiple pattern images projected sequentially [9]. In contrast to the spatial approaches, the temporal ones are believed to be more robust, but they are less efficient in terms of speed due to the use of multiple pattern images.

The temporal phase unwrapping techniques can be classified as the Coding PSP method and the Multi Wavelength PSP (MWPSP) method. The coding PSP method tries to achieve phase unwrapping by associating the fringe strips with a digital codeword, which is usually implemented by projection of additional pattern images

such as Gray-code [10-11]. While the coding PSP is able to achieve robust phase unwrapping, they require projection of additional encoded pattern images.

The MWSP method is based on projection of multiple fringe patterns with different wavelengths. In MWSP method, a series of phase shift fringe images with different wavelengths are captured. Phase unwrapping is carried out, on one-by-one basis, from the phase map with the longest wavelength to the one with the shortest, where the unwrapped phase map is employed as the reference to recover the next phase map of the next shorter wavelength. Chen, et al. [12] described a novel technique to handle the 2π phase ambiguity problem through utilizing two properly chosen frequency gratings to synthesize an equivalent wavelength grating. Towers, et al. [13-14] proposed an optimal frequency selection for the multiple wavelength method. Later they verified, by simulation, an optimal three wavelength for 3D shape measurement. Li, et al. [15] presented a phase unwrapping approach based on multiple wavelength scanning for digital holographic microscopy, which unwrapped the ambiguous phase image layer by layer by synthesizing the extracted continuous components from a set of multiple phase images obtained by varying the optical wavelength. Ding, et al. [16-17] proposed an approach to recover absolute phase maps of two fringe patterns with selected frequencies, and later they provided a guideline for frequency selection. Long, et al. [18] proposed a method which can be applied for any two fringe patterns with different fringe wavelengths through choosing the fringe waveform lengths instead of using the two frequencies to describe the two fringe patterns.

Due to the existence of noise in the environment when we project the fringe patterns, multiple intermediate fringe patterns must be used, and so the result of the unwrapping will have error. In order to remedy the problem in PSP phase unwrapping

described above, we already developed a phase unwrapping algorithm based on three wavelength phase shift profilometry (TWSPSP) method [19]. However, the proposed technique in [19] still needs to calculate the equivalent phases through the equivalent wavelengths. The noise associated with the wrapped phase map will propagate to the unwrapped ones, leading to errors in the recovered phase maps.

In this paper, we present a new phase unwrapping approach based on the use of three fringe patterns with different wavelengths. Compared with the existing method in [19-20], the proposed method does not need to calculate the equivalent phases. As the noise propagation and enhancement are avoided, the resulting unwrapped phase is less influenced by the noise, therefore leading to improved phase unwrapping performance.

This paper is organized as follows. Section 2 presents the principle of the TWSPSP method and the proposed TWSPSP approach. Section 3 shows simulation and experimental results and the comparisons with the existing 3D reconstruction methods. Section 4 discusses the advantages of the proposed method compared with the existing TWSPSP method, the GCPSP method, and the MWSPSP method.

2. The principle of the Three Wavelength PSP (TWSPSP) method

2.1 The principle of the existing TWSPSP method

Taking the three classical wavelengths $\lambda_1=21, \lambda_2=18$ and $\lambda_3=16$ as an example to explain the TWSPSP method which uses three short wavelengths to generate an equivalent long wavelength. Assuming that the three short wavelengths are λ_1 , λ_2 and λ_3 , and the corresponding wrapped phase maps are $\phi_1(x, y)$, $\phi_2(x, y)$ and $\phi_3(x, y)$, phase maps $\phi_{12}(x, y)$, $\phi_{23}(x, y)$ and $\phi_{123}(x, y)$ can be obtained with their equivalent wavelengths $\lambda_{12}, \lambda_{23}$ and λ_{123} being calculated by:

$$\lambda_{12} = \left| \frac{\lambda_1 \times \lambda_2}{\lambda_1 - \lambda_2} \right|, \lambda_{23} = \left| \frac{\lambda_2 \times \lambda_3}{\lambda_2 - \lambda_3} \right|, \lambda_{123} = \left| \frac{\lambda_{12} \times \lambda_{23}}{\lambda_{12} - \lambda_{23}} \right|. \quad (1)$$

For the three classical wavelength values $\lambda_1=21$, $\lambda_2=18$ and $\lambda_3=16$, we can get $\lambda_{12}=126$, $\lambda_{23}=144$ and $\lambda_{123}=1008$. For the four steps PSP algorithm, four pattern images are projected, and the captured pattern can be represented in the following:

$$I_k(x, y) = a(x, y) + b(x, y) \sin(\phi(x, y) + \frac{\pi \times k}{2}), k = 0, 1, 2, 3 \quad (2)$$

where $a(x, y)$ is the background intensity, $b(x, y)$ is the intensity modulation, $\phi(x, y)$ is the wrapped phase.

We have showed in [19] that the wrapped phase value $\phi_i(x, y)$ can be calculated as:

$$\phi_i(x, y) = \arctan\left(\frac{I_0(x, y) - I_2(x, y)}{I_1(x, y) - I_3(x, y)}\right), i = 1, 2, 3. \quad (3)$$

From Eq. (1) and Eq. (3), the phase maps $\phi_{12}(x, y)$, $\phi_{23}(x, y)$ and $\phi_{123}(x, y)$ can be obtained by:

$$\begin{aligned} \phi_{12}(x, y) &= \arctan\left(\frac{\sin(\phi_2(x, y)) \times \cos(\phi_1(x, y)) - \cos(\phi_2(x, y)) \times \sin(\phi_1(x, y))}{\cos(\phi_2(x, y)) \times \cos(\phi_1(x, y)) + \sin(\phi_2(x, y)) \times \sin(\phi_1(x, y))}\right) \\ \phi_{23}(x, y) &= \arctan\left(\frac{\sin(\phi_3(x, y)) \times \cos(\phi_2(x, y)) - \cos(\phi_3(x, y)) \times \sin(\phi_2(x, y))}{\cos(\phi_3(x, y)) \times \cos(\phi_2(x, y)) + \sin(\phi_3(x, y)) \times \sin(\phi_2(x, y))}\right) \\ \phi_{123}(x, y) &= \arctan\left(\frac{\sin(\phi_{12}(x, y)) \times \cos(\phi_{23}(x, y)) - \cos(\phi_{12}(x, y)) \times \sin(\phi_{23}(x, y))}{\cos(\phi_{12}(x, y)) \times \cos(\phi_{23}(x, y)) + \sin(\phi_{12}(x, y)) \times \sin(\phi_{23}(x, y))}\right) \end{aligned} \quad (4)$$

We assume that there are W pixels along the direction vertical to the fringe strips. If the equivalent wavelength λ_{123} meets the condition $\lambda_{123} \geq W$, $\phi_{123}(x, y)$ varies from 0 to 2π without 2π discontinuities, and then the figures of the wrapped phase can be clearly expression. In this case, phase unwrapping is not required, because the absolute phase map is the same as the wrapped one, i.e., $\Phi_{123}(x, y) \equiv \phi_{123}(x, y)$.

Although $\phi_{123}(x, y)$ does not exhibit 2π discontinuities, reconstruction of the 3D surface directly from $\phi_{123}(x, y)$ will not achieve good results, because long wavelength usually results in low measurement accuracy. If the $\phi_1(x, y)$ can be unwrapped according to the equivalent phases, the result will be better. However, on $\phi_1(x, y)$, there are λ_{12} / λ_1 discontinuities with equal spacing λ_{12} , and within each λ_{12} there are $\lambda_{123} / \lambda_{12}$ discontinuities on $\phi_{12}(x, y)$. Hence, there are altogether $\frac{\lambda_{123}}{\lambda_{12}} \times \frac{\lambda_{12}}{\lambda_1}$ discontinuities on $\phi_1(x, y)$.

The absolute phase $\Phi_1(x, y)$ with wavelength λ_1 can be calculated as:

$$\begin{aligned} \Phi_1(x, y) &= \phi_1(x, y) + 2\pi \left(\text{int} \left(\frac{\phi_{123}(x, y)}{\frac{2\pi}{\frac{\lambda_{123}}{\lambda_{12}}}} \right) \times \frac{\lambda_{12}}{\lambda_1} + \text{int} \left(\frac{\phi_{12}(x, y)}{\frac{2\pi}{\frac{\lambda_{12}}{\lambda_1}}} \right) \right) \\ &= \phi_1(x, y) + 2\pi \left(\text{int} \left(\frac{\phi_{123}(x, y)}{2\pi} \times \frac{\lambda_{123}}{\lambda_{12}} \right) \times \frac{\lambda_{12}}{\lambda_1} + \text{int} \left(\frac{\phi_{12}(x, y)}{2\pi} \times \frac{\lambda_{12}}{\lambda_1} \right) \right) \end{aligned} \quad (5)$$

where $\text{int}(a)$ returns an integer close to a .

In the above, we did not consider the impact of measurement noise. Due to the existence of the measurement noise, the four pattern images can be represented as:

$$I_k(x, y) = a(x, y) + b(x, y) \sin\left(\phi(x, y) + \frac{\pi \times k}{2}\right) + \text{Gau}(x, y), k = 0, 1, 2, 3 \quad (6)$$

where $\text{Gau}(x, y)$ denotes white Gaussian noise. The measurement noise will be propagated and even enhanced in the calculations of the equivalent phases with Eq. (4), thereby leading to performance degradation in the calculations of the absolute phased with (5).

2.2 The principle of the proposed TWSPSP method

A new method is proposed in this section to avoid the above mentioned problem. We

still assume that there are W pixels in the direction vertical to the fringe strips, and three fringe patterns are projected with wavelengths $\lambda_1'=1008$ 、 $\lambda_2'=144$ and $\lambda_3'=16$ respectively, and we assume $\lambda_1' \geq W$. We still simulate four pattern images with Eq. (6), and add the same noise to the signal. For every wavelength, the corresponding wrapped phase value $\phi_1'(x, y)$, $\phi_2'(x, y)$ and $\phi_3'(x, y)$ can be calculated directly by Eq. (3), and Eq. (4) is not needed. In order to achieve this, the three wavelengths are chosen to meet the following relationships:

$$N_1 = \lambda_1' / \lambda_2', N_2 = \lambda_2' / \lambda_3' \quad (7)$$

where N_1 and N_2 must be integers. As ϕ_1' does not have any discontinuity, there are maximally N_1 discontinuities on ϕ_2' . Also, for each λ_2' pixels on ϕ_3' , there are N_2 discontinuities. Therefore, in total there are maximally $N_1 \times N_2$ discontinuities on ϕ_3' , which can be eliminated by phase unwrapping operation below, yielding the absolute phase $\Phi_3(x, y)$ as follows:

$$\begin{aligned} \Phi_3(x, y) &= \phi_3'(x, y) + 2\pi \left(\text{int}\left(\frac{\phi_1'(x, y)}{2\pi} \times N_1\right) \times N_2 + \text{int}\left(\frac{\phi_2'(x, y)}{2\pi} \times N_2\right) \right) \\ &= \phi_3'(x, y) + 2\pi \left(\text{int}\left(\frac{\phi_1'(x, y)}{2\pi} \times N_1\right) \times N_2 + \text{int}\left(\frac{\phi_2'(x, y)}{2\pi} \times N_2\right) \right) \end{aligned} \quad (8)$$

Note that when $\phi_3'(x, y) = 2\pi$, a discontinuity will occur. In order to remove such a discontinuity, the phase unwrapping equation of $\Phi_3(x, y)$ should be revised to:

$$\Phi_3(x, y) = \begin{cases} \phi_3'(x, y) + 2\pi \left(\text{int}\left(\frac{\phi_1'(x, y)}{2\pi} \times N_1\right) \times N_2 + \text{int}\left(\frac{\phi_2'(x, y)}{2\pi} \times N_2\right) \right), & \phi_3'(x, y) \neq 2\pi \text{ and } \phi_2'(x, y) \neq 2\pi \text{ and } \phi_1'(x, y) \neq 2\pi \\ \phi_3'(x, y) + 2\pi \left(\text{int}\left(\frac{\phi_1'(x, y)}{2\pi} \times N_1\right) \times N_2 + \text{int}\left(\frac{\phi_2'(x, y)}{2\pi} \times N_2\right) - 1 \right), & (\phi_3'(x, y) = 2\pi \text{ or } \phi_2'(x, y) = 2\pi) \text{ and } \phi_1'(x, y) \neq 2\pi \\ \phi_3'(x, y) + 2\pi \left(\text{int}\left(\frac{\phi_1'(x, y)}{2\pi} \times N_1 - 1\right) \times N_2 + \text{int}\left(\frac{\phi_2'(x, y)}{2\pi} \times N_2\right) \right), & \phi_3'(x, y) \neq 2\pi \text{ and } \phi_2'(x, y) \neq 2\pi \text{ and } \phi_1'(x, y) = 2\pi \end{cases} \quad (9)$$

Hence the formula for unwrapping the phase map ϕ_3 is obtained. Although Eq. (8) is similar to Eq. (5), the wrapped phase $\phi_1'(x, y), \phi_2'(x, y)$ can be directly calculated by Eq. (3), that is, the calculations of the equivalent wavelengths and their phases are not required, thereby improving the performance of absolute phase recovery, and the phase unwrapping is more robust.

3. Simulation and Experiment

In order to demonstrate the advantages of the proposed method, simulations and experiments have been carried out in our laboratory, where the fringe patterns covers 800 pixels along the direction vertical to the fringe strips, that is $W=800$. For the existing TWSP method, the wavelengths are set as $\lambda_1=21$, $\lambda_2=18$ and $\lambda_3=16$. For the proposed method, the three wavelengths are set to $\lambda_1'=1008$, $\lambda_2'=144$ and $\lambda_3'=16$.

In the simulations, we add the white Gaussian noises with three different SNRs (signal to noise ratio) to the four pattern images. The recovered wrapped phases of three wavelengths $\lambda=1008$, $\lambda=144$ and $\lambda=16$ are shown in Figs. 1-3 (with different SNRs: SNR=10dB in Fig. 1, SNR=15dB in Fig. 2 and SNR=20dB in Fig. 3.), where the wrapped phases by the existing TWSP method are also shown for comparisons .

Fig. 1. The wrapped phase with the three wavelengths when SNR=10dB. (a)The phases with $\lambda=16$ for the existing TWSPSP method, the proposed TWSPSP method and the ideal TWSPSP method (assuming no noise). (b)The zoomed view of Fig. 1(a). (c)The phases with $\lambda=144$ for the existing TWSPSP method, the proposed TWSPSP method and the ideal TWSPSP method. (d) The zoomed view of Fig. 1(c). (e)The phases with $\lambda=1008$ for the existing TWSPSP method, the proposed TWSPSP method and the ideal TWSPSP method. (f)The zoomed view of Fig. 1(e).

Fig. 2. The wrapped phase with the three wavelengths when SNR=15dB. (a)The phases with $\lambda=16$ for the existing TWSP method, the proposed TWSP method and the ideal TWSP method (assuming no noise). (b)The zoomed view of Fig. 2(a). (c)The phases with $\lambda=144$ for the existing TWSP method, the proposed TWSP method and the ideal TWSP method. (d) The zoomed view of Fig.

2(c). (e)The phases with $\lambda=1008$ for the existing TWPSP method, the proposed TWPSP method and the ideal TWPSP method. (f)The zoomed view of Fig. 2(e).

Fig. 3. The wrapped phase with the three wavelengths when SNR=20dB. (a)The phases with $\lambda=16$ for the existing TWPSP method, the proposed TWPSP method and the ideal TWPSP method

(assuming no noise). (b)The zoomed view of Fig. 3(a). (c)The phases with $\lambda=144$ for the existing TWSP method, the proposed TWSP method and the ideal TWSP method. (d) The zoomed view of Fig. 3(c). (e)The phases with $\lambda=1008$ for the existing TWSP method, the proposed TWSP method and the ideal TWSP method. (f)The zoomed view of Fig. 3(e).

The red lines of Fig. i(a) ($i=1,2,3$) denote the wrapped phase, for the existing TWSP method with λ_3 , and the green lines of Fig. i(a) ($i=1,2,3$) represent the wrapped phase for the proposed TWSP method with λ_3' . Because $\lambda_3 = \lambda_3'$ in the two methods, and the equations for calculating their wrapped phases are the same, their wrapped phases are almost the same. The wrapped phase ϕ_{12} is computed from patterns 1 and 2, and it is shown by the red lines of Fig. i(c) ($i=1,2,3$); while the wrapped phase of λ_2' for the proposed TWSP method is shown by the green lines of Fig. i(c) ($i=1,2,3$). The wrapped phase ϕ_{123} for the existing TWSP method is shown by the red lines of Fig. i(e) ($i=1,2,3$); while the wrapped phase ϕ_1 for the proposed method is shown by the green lines of Fig. i(e) ($i=1,2,3$), and the blue lines of Fig. i represent the ideal wrapped phases. To comparisons, parts of Fig. i(a) , Fig. i(c), Fig. i(e) ($i=1,2,3$) have been zoomed in, and are shown in Fig. i(b), Fig. i(d), Fig. i(f) ($i=1,2,3$), respectively.

From the simulations results shown in Figs. 1-3, the error propagation and noise enhancement in the existing TWSP method can be clearly observed (by comparing the red curves in the Fig. i(b) Fig. i(d), Fig. i(f) ($i=1,2,3$)). We can also see that the proposed method achieves significant performance improvement thanks to the avoidance of the calculation of the equivalent phase maps. Next, we analyze the impact of the noise quantitatively. For this, 200 points in the phase map (from 400

to599) are selected, and the root mean squared errors (RMSE) are compared and shown in Table 1, where the RMSE is defined as:

$$S = \sqrt{[(x_1 - x_{ideal,1})^2 + (x_2 - x_{ideal,2})^2 + \dots + (x_n - x_{ideal,n})^2] / n} \quad (10)$$

where S is the RMSE, $x_{ideal,n}$ is the ideal wrapped phase of the point n , x_n is the wrapped phase of the point n , n is the point number.

Table 1: The root mean square errors (RMSE) of the two methods

x	The ideal wrapped phase of $\lambda = 1008$ (Rad)	The wrapped phase of λ_1 'in the proposed method (Rad)	The wrapped phase of λ_{123} in the existing method (Rad)
RMSE with SNR=10 dB	0	0.2421	0.5626
RMSE with SNR=15 dB	0	0.1246	0.3169
RMSE with SNR=20 dB	0	0.0642	0.1760

From the above table, it can be seen that these RMSEs of the proposed method are smaller than those of the existing method, and the difference becomes larger with the decrease of the SNR, i.e., compared to the existing method, the proposed method is more prominent when the measurement noise is larger.

Next, experimental results are provided to verify the proposed method. In the experiment, the fringe patterns are projected onto a plaster hand model, and the captured images are shown in Fig. 4.

Fig. 4. Projecting fringe patterns with different wavelengths. (a)The existing TWSP method. (b) The proposed TWSP method.

For the existing TWSP method in [19], the three original wavelengths are used to calculate the equivalent wavelength λ_{12} , λ_{23} and λ_{123} based on Eq. (1), yielding $\lambda_{12}=126$, $\lambda_{23}=144$ and $\lambda_{123}=1008$. The wrapped phase maps from the three original and equivalent wavelength of the existing TWSP method are shown in Fig.5(a). For the proposed TWSP method, the wavelength λ_1' in the proposed method is equal to the equivalent wavelength λ_{123} used in the existing TWSP method. The wavelength λ_2' in the proposed method is equal to the equivalent wavelength λ_{23} used in the existing TWSP method. Only λ_3' is the same for the two methods. The wrapped phase maps from the proposed TWSP method are shown in Fig. 5(b).

Fig. 5. The acquisition of the wrapped phase from the two methods. (a) The acquisition of the wrapped phase from the three original and equivalent wavelength of the existing TWSPSP method. (b) The acquisition of the wrapped phases from the proposed TWSPSP method

From the comparison of Fig.5(a) and Fig.5(b), the existing TWSPSP method needs to calculate twice equivalent wavelengths, while the equivalent wavelengths are not required to be calculated in the proposed method. So the existing TWSPSP is more complex and time consuming than the proposed method.

Now let us consider a row of pixels in the fringe images on the row 512. In the experiments, Fig. 6(a) shows the wrapped phase of image 3 (with wavelength λ_3) from the existing TWSPSP method, and Fig. 6(b) gives the wrapped phase of λ_3' from the proposed TWSPSP method. Because the two pattern images come from the same object and the wavelength is same, they are almost the same. The wrapped phase ϕ_{23} is computed from patterns 1 and 2, and it is shown in Fig. 7(a); while

the wrapped phase of λ_2 from the proposed TWPSP method is shown in Fig. 7(b).

The wrapped phase ϕ_{123} from the existing TWPSP method is shown in Fig. 8(a);

while the wrapped phase ϕ_1 from the proposed method is shown in Fig. 8(b).

Fig. 6. (a)The wrapped phase of λ_3 from the existing TWPSP method

(b)The wrapped phase of λ_3' from the proposed TWPSP method.

Fig. 7. (a) The wrapped phase of λ_{23} from the existing TWPSP method.

(b) The wrapped phase of λ_2' from the proposed TWPSP method.

Fig. 8. (a) The wrapped phase of λ_{123} from the existing TWSP method. (b) The wrapped phase of λ_1' from the proposed TWSP method

In the experiment, ϕ_{23} is calculated from ϕ_2 and ϕ_3 , so the noise on the latter will be transferred to the former. By comparing Fig. 7(a) and Fig. 7(b), it is seen that ϕ_2 from the proposed method is better than ϕ_{23} . From the comparison of Fig. 8(a) and Fig. 8(b), ϕ_1 from the proposed method is much better than the ϕ_{123} from the existing TWSP, although measurements with longer wavelength contain more noise normally.

With Eq. (9), the unwrapped phase ϕ_3 can be obtained, and the unwrapped phase ϕ_3 from the existing TWSP method is shown in Fig. 9(a), while the unwrapped phase ϕ_3 from the proposed TWSP method is shown in Fig. 9(b). It is seen that there are a lot of points are incorrectly unwrapped in Fig. 9(a). In contrast, almost all the points are unwrapped correctly in Fig. 9(b) except the region without the object.

Fig. 9. (a) The unwrapped phase of the existing TWSP method with λ_3 . (b) The unwrapped phase of the proposed TWSP method with λ_3' .

The results in the above figures clearly show that the noise contained in the unwrapped phases of the proposed method is much less than that of the existing method. In order to quantitatively analyze the noise, 20 points (70-89) are selected, and the RMSEs of the methods are shown in Table 2, which again demonstrate the advantage of the proposed method.

Table 2: RMSE of the two methods

ID	The expected unwrapped phase value (Rad)	The unwrapped phase from the existing method (Rad)	The unwrapped phase from the proposed method (Rad)
RMSE	0	33.0999	2.9645

In order to test the performance of the proposed method, four different methods are applied for the measurement of the same object in the same environment,

including the proposed method, the existing TWPSP method, the Gray Code method, and the Multi Wavelength method. The object is a bright hand model placed on a dark chair.

Fig. 10. Experiment results of the different methods. (a) The 3D reconstruction result from the GC (Gray code) PSP method. (b) The 3D reconstruction result from the MWPSP method. (c) The 3D reconstruction result using the existing TWPSP method. (d) The 3D reconstruction result using the proposed TWPSP method.

The 3D reconstruction result from the GCPSP method and MWPSP method are shown in Fig. 10 (a) and Fig. 10(b) respectively. For GCPSP method, the dark chair

region is lost, and only bright region remains. For the MWPSP method, the dark region and the bright region are all retrieved clearly. The 3D reconstruction result using the existing TWPSP method is shown in Fig. 10 (c). There are a lot of noise caused by the incorrectly phase value. The 3D reconstruction result using the proposed TWPSP method is shown in Fig. 10 (d). The 3D reconstruction result is much better , and the dark region is well reconstructed with small noise.

To further examine the performance of the proposed method, we conduct another experiment with two color dolls put on a dark chair. The reconstruction results with different methods are shown in Fig. 11.

(a)

(b)

(c)

(d)

Fig. 11. Experiment results of the different methods. (a) The 3D reconstruction result from the GCPSP method. (b) The 3D reconstruction result from the MWSP method. (c) The 3D reconstruction result using the existing TWSP method. (d) The 3D reconstruction result using the proposed TWSP method.

From the figure, we can find that quality of the reconstructed dark doll is different. The dark doll is well reconstructed in the MWSP method and the proposed TWSP method. However, the dark doll is lost in the GCPSP method, and the reconstructed dark doll contains a lot of noise in the existing TWSP method.

From the above two experiments, we can find that the MWSP method the TWSP method have similar performance, but the proposed method is faster than the MWSP method because the proposed one does not need to calculate the equivalent wavelengths and the equivalent phases. The comparisons among the four methods are summarized in Table 3.

Table 3. The comparison between the four methods

	The proposed TWSP method	The existing TWSP method	The MWSP method	The Gray Code method
Number of images captured	9	9	18	9
Precision	High	Middle	High	Low
Processing speed(in second)	12	27	45	15

Robustness	High	Low	High	Low
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4. Conclusions

A new TWSPSP method is presented in this paper. In the proposed method, three fringe patterns with proper wavelengths are projected on the object, and the wrapped phase can be obtained with the three proper wavelengths directly, i.e., the calculations of the equivalent wavelengths and their corresponding phase maps are not needed. This leads to the following advantages of the proposed method compared to the existing methods:

- 1) Low noise: In the proposed method, the wrapped phase can be obtained with the three proper wavelengths directly. Therefore, the noise enhancement problem is alleviated, thereby leading to better performance. It is found that the greater the noise in the environment, the more performance improvement the proposed method can achieve.
- 2) High speed: The proposed method does not need to calculate the equivalent phase maps, so the speed of the proposed method is higher compared with other methods. Moreover, through experiments, we also found that the reconstruction performance of the proposed method is close to that of the MWSPSP method.

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