1-1-2006

Three-dimensional profilometry based on shift estimation of projected fringe patterns

Jose F. Chicharo

University of Wollongong, chicharo@uow.edu.au

Jiangtao Xi

University of Wollongong, jiangtao@uow.edu.au

Yingsong Hu

University of Wollongong, yingsong@uow.edu.au

Zongkai Yang

Huazhong University of Science and Technology

Enbang Li

University of Wollongong, enbang@uow.edu.au

Follow this and additional works at: <https://ro.uow.edu.au/engpapers>

Part of the [Engineering Commons](#)

<https://ro.uow.edu.au/engpapers/2751>

Recommended Citation

Chicharo, Jose F.; Xi, Jiangtao; Hu, Yingsong; Yang, Zongkai; and Li, Enbang: Three-dimensional profilometry based on shift estimation of projected fringe patterns 2006, 678-687.
<https://ro.uow.edu.au/engpapers/2751>

Three-dimensional profilometry based on shift estimation of projected fringe patterns

Yingsong Hu,^{1,2} Jiangtao Xi,¹ Enbang Li,¹ Joe Chicharo¹ and Zongkai

Yang²

¹*School of Electrical Computer and Telecommunications Engineering, University of*

Wollongong, NSW 2522, Australia

²*Department of Electronic and Information Engineering, Huazhong University of*

Science and Technology, Wuhan City, 430074, China

This paper presents a new approach to fringe pattern profilometry. In this paper, a generalized model for describing the relationship between the projected fringe pattern and deformed fringe pattern is derived, where the projected fringe pattern can be arbitrary rather than being limited to be sinusoidal as those for the conventional approaches. Based on this model, a new approach is proposed to reconstruct the three-dimensional object surface by estimating the shift between the projected and deformed fringe patterns. Additionally, theoretical analysis and computer simulation results are presented, both of which show the proposed approach can significantly improve the measurement accuracy, especially when the fringe patterns are distorted by unknown factors. © 2005 Optical Society of America

OCIS codes: 120.6650, 120.2650, 150.6910.

1. Introduction

Fringe pattern profilometry (FPP) is one of the most popular non-contact methods for measuring the three-dimensional surface of an object in recent years. With FPP, a Ronchi grating or sinusoidal grating is projected onto a three-dimensional diffuse surface, the height distribution of which deforms the projected fringe patterns and modulates them in phase domain. Hence by retrieving the phase difference between the original and deformed fringe patterns, three dimensional profilometry can be achieved. A number of fringe pattern analysis methods have been developed for

FPP, including Fourier Transform Profilometry(FTP),¹⁻⁴ Phase Shifting Profilometry (PSP),⁵⁻⁷ Spatial Phase Detection (SPD),⁸ Phase Locked Loop (PLL)⁹ and other analysis methods,^{10,11} all of which are based on an assumption that the projected fringe patterns are or can be filtered to be sinusoidal.

In recent years, because of the simplicity and controllability, digital projectors have been widely used to yield fringe patterns for implementing FPP.^{12,13} However, it is very difficult for projectors to produce pure sinusoidal fringe patterns due to the existence of geometrical distortion and colour distortion. Although digital filtering can be used to reduce the distortion, the resulting fringe patterns may still not be pure sinusoidal, as the digital filters are usually not ideal either. Additionally, when the deformed fringe pattern has an overlapped spectra, bandpass filtering will be unavailable if a precise measurement is required. Therefore, errors will arise if the measurement is still based on pure sinusoidal assumption. This problem motivates us to look for a new method to reconstruct the 3-D profile based on non-sinusoidal fringe patterns.

In this paper, we present a generalized analysis model which describes a general relationship between the projected signal and the deformed signal in fringe pattern profilometry (FPP) systems. The derived mathematical model does not require the fringe pattern pure sinusoidal or of some other particular structure. This also implies that theoretically, distorted sinusoidal fringe patterns are still useful for profilometry.

Based on the derived mathematical model, this paper also proposed a new algo-

rithm, referred to as Shift Estimation (SE) method, to reconstruct 3-D surfaces using fringe pattern profilometry (FPP) technique. Compared with traditional methods, our algorithm has neither the requirement for the structure of projected fringe patterns, nor the prior knowledge of the characteristics of projection systems. The correctness of the proposed analysis model and Shift Estimation (SE) method has been confirmed by simulation results, which are provided to demonstrate that compared with the conventional analysis methods, the measurement accuracy has been significantly improved by Shift Estimation (SE) method, especially when the expected sinusoidal fringe patterns are distorted by unknown factors.

This paper is organized as follows. In Section 2, a mathematical model for FPP systems is derived and briefly compared with traditional analysis model. In Section 3, Shift Estimation (SE) method is proposed to reconstruct the 3-D surface based on the model proposed in Section 2. In order to compare the measurement accuracy using Shift Estimation (SE) method with that using traditional methods, in Section 4, we take PSP as an example, to theoretically analyze the measurement errors caused by nonlinear distortions, and estimate the errors using Shift Estimation (SE) method. Analytical results show that Shift Estimation (SE) method provides much better performance than PSP in terms of reconstruction accuracy. In Section 5, we give our comparative simulation results to demonstrate the improvement of using Shift Estimation (SE) method and confirm the theoretical analysis. Section 6 concludes this paper.

Fig. 1. Schematic diagram of FPP system

2. Generalized Analysis Model

A schematic diagram of a typical FPP system is shown in Fig.1. For simplicity, we consider a cross section of the object surface for a given y coordinate. Hence, the intensity of fringe patterns captured by CCD camera and the height distribution function can be expressed as a function with single variable x . Thus we use $s(x)$ and $d(x)$ to denote the intensity of the projected and deformed fringe pattern respectively and use $h(x)$ to represent the height distribution of the object surface.

In order to establish a relationship between $s(x)$ and $d(x)$, we consider a beam of

light corresponding to a pixel of the fringe pattern, denoted as E_pCH in Fig.1. It is seen that the light beam is projected at point C and reflected back to the camera if a reference plane exists. When the reference plane is removed, the same beam will be projected onto point H and reflected to the camera via point D. Assuming that the object surface and the reference plane have the same reflective characteristics, $s(x)$ at the location C should exhibit the same intensity as $d(x)$ does at the location D, because they originate from the same point of the fringe pattern created by the projector. Hence we have

$$d(x_d) = s(x_c) \quad (1)$$

where x_c and x_d are the coordinate locations of point C and point D respectively. We use u to denote the distance from C to D, that is

$$u = x_d - x_c \quad (2)$$

From Eq.(1) and Eq.(2), we have

$$d(x_d) = s(x_d - u) \quad (3)$$

Obviously, u varies with the height of point H on the object surface.

Meanwhile, because triangles E_pHE_c and CHD are similar, we have

$$\frac{x_c - x_d}{-h(x_h)} = \frac{d_0}{l_0 - h(x_h)} \quad (4)$$

where x_h is the x coordinate of point H, l_0 is the distance between the camera and the reference plane and d_0 is the distance between the camera and the projector.

As point H and point D are on the same reflected beam from point H to the camera, point has the same x coordinate as point D does in captured images, which implies $x_h = x_d$. So Eq.(4) can be rewritten as

$$\frac{x_c - x_d}{-h(x_d)} = \frac{d_0}{l_0 - h(x_d)} \quad (5)$$

As defined in Eq.(2), Eq.(5) can be expressed as

$$\frac{-u}{-h(x_d)} = \frac{d_0}{l_0 - h(x_d)} \quad (6)$$

As the height distribution $h(x)$ is a function of x_d , u should also be a function of x_d .

Then we have

$$u(x_d) = \frac{d_0 h(x_d)}{l_0 - h(x_d)} \quad (7)$$

Eq.(7) also can be written as

$$h(x_d) = \frac{l_0 u(x_d)}{d_0 + u(x_d)} \quad (8)$$

Therefore, Eq.(3) can be expressed as

$$d(x_d) = s(x_d - u(x_d)) \quad (9)$$

where $u(x_d)$ is given by Eq.(7).

As Eq.(8) and Eq.(9) should apply to arbitrary x_d , hence by letting $x_d = x$, we can simplify the mathematical expressions and derive a general model as follows:

$$d(x) = s(x - u(x)) \quad (10)$$

$$h(x) = \frac{l_0 u(x)}{d_0 + u(x)} \quad (11)$$

Eq.(10) reveals that the deformed signal $d(x)$ is a shifted version of $s(x)$, and the shift function $u(x)$ can be used to determine the object height distribution by Eq.(11).

It is interesting to note that by letting $s(x)$ be a cosinusoidal signal in Eq.(10), we can easily derive the conventional phase-modulation model. i.e. the conventional model is a special case of our proposed model. Clearly if we use sinusoidal or cosinusoidal fringe patterns, the projected signal can be expressed as:

$$s(x) = A \cos(2\pi f_0 x) \quad (12)$$

where A is the amplitude of intensity, and the deformed signal is

$$\begin{aligned} d(x) &= s(x - u(x)) \\ &= A \cos(2\pi f_0(x - u(x))) \\ &= A \cos(2\pi f_0 x - 2\pi f_0 u(x)) \\ &= A \cos(2\pi f_0 x + \phi(x)) \end{aligned} \quad (13)$$

where $\phi(x) = -2\pi f_0 u(x)$.

As expected, for the case where a sinusoidal fringe pattern is used, the deformed signal $d(x)$ is a phase-modulated signal. Hence, conventional analysis methods attempt to demodulate the deformed signals and retrieve the phase map $\phi(x)$ firstly. Then, by using the relationship between the phase shift function, $\phi(x)$, and the height distribution $h(x)$, the shapes of objects can be reconstructed. Actually, we can easily derive the relationship between $\phi(x)$ and $h(x)$ by our proposed model. Note in

Eq.(13), $\phi(x) = -2\pi f_0 u(x)$ .i.e. $u(x) = -\frac{\phi(x)}{2\pi f_0}$. By substituting this equation into Eq.(11), we can derive a well-known equation:

$$h(x) = \frac{l_0 \phi(x)}{\phi(x) - 2\pi f_0 d_0} \quad (14)$$

An equivalent form of Eq.(14) is

$$\phi(x) = \frac{2\pi f_0 d_0 h(x)}{h(x) - l_0} \quad (15)$$

As classical formulas, Eq.(14) and Eq.(15) appear in most of the articles on fringe pattern profilometry (for example^{2,4,14}). It can be seen that our proposed model has identical form with the conventional one when a pure sinusoidal fringe pattern is used for profilometry. However, in practical cases, especially when we are using a digital projector to generate fringe patterns, it is fairly difficult for us to obtain pure sinusoidal fringe patterns. Meanwhile, Eq.(10) and Eq.(11) reveal that the shift signal $u(x)$ contains all the 3-D information of object surface. Hence in theory we should be able to obtain the profile by projecting any signal. In next section, we present a Shift Estimation (SE) method to retrieve the shift function $u(x)$, so that the height distribution of three dimensional surfaces can be accordingly obtained.

3. Shift Estimation Method

Because the value of shift function $u(x)$ determines the height distribution $h(x)$, the height can be obtained if we have $u(x)$. Hence, we should track the values of shift function $u(x)$ for each point x by using $s(x)$ and $d(x)$. For this purpose, we use square

error defined below as an objective function with respect to $\hat{u}(x)$ that denotes the estimation of $u(x)$ at point x :

$$e^2(\hat{u}(x)) = [d(x) - s(x - \hat{u}(x))]^2 \quad (16)$$

In order to minimize the error e^2 , we use gradient-based method to obtain the estimation of $\hat{u}(x)$ in an iterative way:

$$\hat{u}_{m+1} = \hat{u}_m - \eta \frac{de^2}{d\hat{u}}|_{\hat{u}=\hat{u}_m} \quad (17)$$

where η is the learning rate. The gradient can be derived as:

$$\begin{aligned} \frac{de^2}{d\hat{u}}|_{\hat{u}=\hat{u}_m} &= 2e \frac{de}{d\hat{u}}|_{\hat{u}=\hat{u}_m} = -2e \frac{ds}{d\hat{u}}|_{\hat{u}=\hat{u}_m} \\ &= -2e \frac{s(x - (\hat{u}_m + 1)) - s(x - (\hat{u}_m - 1))}{(\hat{u}_m + 1) - (\hat{u}_m - 1)} \\ &= -e[s(x - \hat{u}_m - 1) - s(x - \hat{u}_m + 1)] \\ &= -[d(x) - s(x - \hat{u}_m)] \times \\ &\quad [s(x - \hat{u}_m - 1) - s(x - \hat{u}_m + 1)] \end{aligned} \quad (18)$$

Substituting Eq.(18) into Eq.(17), we can have an iterative equation to calculate the estimation of the value of shift function $u(x)$ at each point x .

$$\begin{aligned} \hat{u}_{m+1}(x) &= \hat{u}_m(x) + \eta[d(x) - s(x - \hat{u}_m(x))] \\ &\quad \times [s(x - \hat{u}_m(x) - 1) - s(x - \hat{u}_m(x) + 1)] \end{aligned} \quad (19)$$

For each point x , the iteration continues until convergence. In other words, if $|\hat{u}_{m+1} - \hat{u}_m|$ is less than a given lower bound, we can obtain an estimation of the value of

$u(x)$ at point x , $\hat{u}(x) = \hat{u}_m(x)$. Considering the continuity of the profiles, we can use the converged value $\hat{u}(x)$ as the initial value for the next point $x + 1$. i.e. let $\hat{u}_1(x + 1) = \hat{u}(x)$ and then continue doing the iteration for the next point $x + 1$, so that faster convergence can be achieved.

4. Error Analysis and Comparison

In order to demonstrate the advantages of proposed Shife Estimation (SE) method, we firstly study the error associated with the PSP approach due to nonlinear distortion of the projected fringe pattern, and then compare the results with those of using the proposed approach.

We only consider the effect of the second order harmonic, which is reasonable as the second order harmonic has the highest power than other higher harmonic components after the signals have been filtered by bandpass filters. We assume that in the ideal situation, the captured fringe pattern on the object surface is a pure sinusoid, which is given by

$$g_n(x) = a(x) + b(x) \cos(2\pi f_0 x + \phi(x) + 2\pi n/N)$$

$$\text{for } n=0,1,2,\dots,N-1 \quad (20)$$

where N is the number of phase shifting steps, $g_n(x)$ is the intensity of the image at point x , $a(x)$ is a slowly varying function representing the background illumination,

$b(x)$ is a slowly varying function representing the contrast between bright and dark fringes captured by the CCD, f_0 is the spatial carrier fringe frequency, and $\phi(x)$ is the phase shift caused by the object surface and the angle of projection. As mentioned in Section 2, the relationship between the height distributions and the shifted phase $\phi(x)$ is expressed by Eq.(15). Further, because the projector always locates far enough from the object, we have $l_0 \gg h(x)$. Therefore $\phi(x)$ can be expressed approximately as:

$$\phi(x) \approx \frac{2\pi f_0 d_0 h(x)}{-l_0} \quad (21)$$

then we have

$$h(x) \approx -\frac{l_0}{2\pi f_0 d_0} \phi(x) \quad (22)$$

For PSP method, the phase map $\phi(x)$ can be obtained by following formula:

$$\tan(2\pi f_0 x + \phi(x)) = -\frac{\sum_{n=0}^{N-1} g_n(x) \sin(2\pi n/N)}{\sum_{n=0}^{N-1} g_n(x) \cos(2\pi n/N)} \quad (23)$$

However, in practice there is always non-linear distortion with the projected fringe pattern, which can be described as harmonic components. For simplicity we only consider the second order harmonic distortion. Hence the actual fringe pattern projected on the surface of the object can be expressed as follows:

$$\begin{aligned} \tilde{g}_n(x) = & a(x) + b(x) \cos(2\pi f_0 x + \phi(x) \\ & + 2\pi n/N) + c(x) \cos(4\pi f_0 x + 2\phi(x) \\ & + 4\pi n/N) \end{aligned}$$

for $n=0,1,2,\dots,N-1$ (24)

Then if we still use Eq.(23) to calculate $\phi(x)$, errors will be introduced. Let we consider the situation when $N = 3$, and denote the error of $\phi(x)$ as δ . We have:

$$\tan(2\pi f_0 x + \phi(x) + \delta) = -\frac{\sum_{n=0}^{N-1} \tilde{g}_n(x) \sin(2\pi n/N)}{\sum_{n=0}^{N-1} \tilde{g}_n(x) \cos(2\pi n/N)} \quad (25)$$

Considering Eq.(24) and Eq.(25) together when $N = 3$, we have

$$\tan(2\pi f_0 x + \phi(x) + \delta) = \frac{b \sin(2\pi f_0 x + \phi(x)) - c \sin(4\pi f_0 x + 2\phi(x))}{b \cos(2\pi f_0 x + \phi(x)) + c \cos(4\pi f_0 x + 2\phi(x))} \quad (26)$$

Let θ denote $2\pi f_0 x + \phi(x)$, then we have

$$\tan(\theta + \delta) = \frac{b \sin(\theta) - c \sin(2\theta)}{b \cos(\theta) + c \cos(2\theta)} \quad (27)$$

then δ is also a function of θ and can be expressed as $\delta(\theta)$.

$\hat{\phi}_0$, the phase map of distorted fringe pattern on reference plane can be derived by Eq.(25) when $\phi(x) \equiv 0$:

$$\tan(\hat{\phi}_0) = \frac{b \sin(2\pi f_0 x) - c \sin(2 \cdot 2\pi f_0 x)}{b \cos(2\pi f_0 x) + c \cos(2 \cdot 2\pi f_0 x)} \quad (28)$$

As $\theta = 2\pi f_0 x + \phi(x)$, the equation above can be expressed as

$$\hat{\phi}_0 = \arctan\left(\frac{b \sin(\theta - \phi) - c \sin(2(\theta - \phi))}{b \cos(\theta - \phi) + c \cos(2(\theta - \phi))}\right) \quad (29)$$

In PSP, we will retrieve $\hat{\phi}$, an estimation of ϕ , which is the phase shifted by object

profiles using following equation

$$\begin{aligned}
\hat{\phi} &= \arctan\left(\frac{b \sin(\theta) - c \sin(2\theta)}{b \cos(\theta) + c \cos(2\theta)}\right) - \hat{\phi}_0 \\
&= \arctan\left(\frac{b \sin(\theta) - c \sin(2\theta)}{b \cos(\theta) + c \cos(2\theta)}\right) \\
&\quad - \arctan\left(\frac{b \sin(\theta - \phi) - c \sin(2(\theta - \phi))}{b \cos(\theta - \phi) + c \cos(2(\theta - \phi))}\right) \\
&= [\theta + \delta(\theta)] - [(\theta - \phi) + \delta(\theta - \phi)] \\
&= \phi + \delta(\theta) - \delta(\theta - \phi)
\end{aligned} \tag{30}$$

So, the phase measurement error ε can be expressed as

$$\varepsilon = \hat{\phi} - \phi = \delta(\theta) - \delta(\theta - \phi) \tag{31}$$

Hence, the maximum measurement error is:

$$\varepsilon_{max} = \delta_{max}(\theta) - \delta_{min}(\theta) \tag{32}$$

where $\delta_{max}(\theta)$ and $\delta_{min}(\theta)$ represent the maximum and minimum value of function respectively. Therefore, in order to calculate the phase errors, we have to derive $\delta(\theta)$ further from Eq.(27).

The left side of Eq.(27) is

$$\tan(\theta + \delta) = \frac{\sin(\theta) \cos(\delta) + \cos(\theta) \sin(\delta)}{\cos(\theta) \cos(\delta) - \sin(\theta) \sin(\delta)} \tag{33}$$

Substituting Eq.(33) into Eq.(27), we can have

$$\sin(\delta) = -\sqrt{p} \cdot \sin(3\theta + \delta) \tag{34}$$

where $p = \frac{c^2}{b^2}$, representing the power ratio of the second order harmonic to fundamental components.

Therefore the final expression of δ is

$$\delta = -\arctan\left(\frac{\sqrt{p}\sin(3\theta)}{1 + \sqrt{p}\cos(3\theta)}\right) \quad (35)$$

By setting $\frac{d\delta}{d\theta} = 0$, we can derive the maximum and minimum value of respectively.

That is

$$\delta_{max} = \arctan\left(\sqrt{\frac{p}{1-p}}\right) \quad (36)$$

and

$$\delta_{min} = -\arctan\left(\sqrt{\frac{p}{1-p}}\right) \quad (37)$$

Then Eq.(32) can be expressed as

$$\varepsilon_{max} = 2\arctan\left(\sqrt{\frac{p}{1-p}}\right) \quad (38)$$

By using Eq.(22) the maximum absolute error of measuring height distribution β is

$$\beta = \left| -\frac{l_0}{2\pi f_0 d_0} \cdot \varepsilon_{max} \right| = \frac{l_0}{2\pi f_0 d_0} \cdot \varepsilon_{max} \quad (39)$$

If we assume l_0 is 5 meter, d_0 is 2 meter, and the spatial period of fringe pattern is 0.1 meter, which implies $f_0 = 10/\text{meter}$, the height distribution errors are plotted as Fig.2. If we use this system to measure a profile whose maximum hight is 160mm when p equals 0.01, the relative height error will reach 4.98% respectively, which is not a negligible error for measurement.

Fig. 2. Height distribution error

On the other hand, by the mathematical model, Eq.(10), Eq.(11), and the Shift Estimation (SE) method Eq.(19), it can be seen that in the situation when the projected signal $s(x)$ and deformed signal $d(x)$ can be accurately obtained, the measurement accuracy of Shift Estimation (SE) method only depends on the learning rate η , which can be properly adjusted according to the different accuracy requirements. Therefore, in theory, measurement error of using Shift Estimation (SE) method can be controlled as small as possible, which is definitely better than the result by using PSP as analyzed above.

5. Simulation results

Simulations have been performed to verify effectiveness of our proposed Shift Estimation (SE) method. In our simulation, we use a paraboloid object surface whose maximum height is 160mm and the projected fringe pattern is generated from a cosinusoidal signal distorted by a nonlinear function given by:

$$s(g(x)) = 0.00156g^2(x) + g(x) + C \quad (40)$$

where C is a constant which can be ignored as it does not effect on reconstruction results, and $g(x) = A \cos(2\pi f_0 x)$ where f_0 is the spatial frequency of the fringe pattern, which is assumed to be 10/meter in our simulation. i.e. the spatial period of the fringe pattern is assumed to be 100mm. Assuming 8-bits quantification is used for CCD camera, A , the amplitude of the cosinusoidal signal $g(x)$ is assumed to be 128. Meanwhile, we assume l_0 and d_0 in Fig.1 equal to 5 meters and 2 meters respectively.

The spatial resolution of the captured image is assumed to be 1 pixel/mm.

Substituting $g(x) = A \cos(2\pi f_0 x)$ into Eq.(40) and discarding DC component, we have:

$$s(x) = 128 \cos(2\pi f_0 x) + 12.8 \cos(2\pi \cdot (2f_0)x) \quad (41)$$

Note for the projected fringe pattern given by Eq.(41), the second order harmonic only has -20db of power compared with the fundamental component. Corresponding to Eq.(41), the deformed fringe pattern can be expressed as:^{2,4}

$$\begin{aligned} d(x) &= 128 \cos(2\pi f_0 x + \phi(x)) \\ &+ 12.8 \cos(2\pi \cdot (2f_0)x + 2\phi(x)) \end{aligned} \quad (42)$$

where $\phi(x)$ is the phase shift caused by the object surface. It has been well-known that with conventional model, after demodulating $\phi(x)$, the surface height distribution can be calculated by the relationship between $\phi(x)$ and the height distribution $h(x)$.

Using the fringe pattern given by Eq.(41) and Eq.(42), we can reconstruct the object surface by conventional PSP method and Shift Estimation (SE) method respectively. The comparative results are shown in Fig.3. In Fig.3(a), The solid line is the measurement result by PSP method, the dash-dot line represents the reconstruction result by using Shift Estimation (SE) method, while the dashed line refers to the object surface, which is the true value of the height distribution. This figure shows nonlinear distortion introduces noticeable errors when PSP method is used, even though the nonlinear distortion is so slight that coefficient of square item in Eq.(40) is only 0.00156 and

Fig. 3. Reconstruction results and errors

the second order harmonic only has -20db of power compared with fundamental component. Moreover, the reconstructed surface by using PSP method is jagged and not smooth. Comparatively, by our proposed model and Shift Estimation (SE) method, we can obtain a much better reconstruction result, which is almost identical to the true values and the reconstructed surface is much smoother than using PSP method. Fig.3(b) shows the absolute errors of reconstructed height distribution. The solid line and dashed line represent the reconstruction error by using PSP method and Shift Estimation (SE) method respectively. In our simulation, the average absolute error and the standard error of the height distribution reconstructed by PSP are 2.7270mm and 3.5268mm. Comparatively by Shift Estimation (SE) method, the average absolute error and the standard error can be reduced to be only 0.0643mm and 0.2055mm respectively. We can see that measurement accuracy is significantly improved by the generalized model and Shift Estimation (SE) method.

6. Conclusion

In this paper, we have presented a generalized model for fringe pattern profilometry (FPP), which describes a general relationship between the projected signal and the deformed signal. Based on the generalized model, we present a new method, called Shift Estimation (SE) method for profilometry. With the generalized model and Shift Estimation (SE) method, the constraint of using sinusoidal signals has been completely removed. In other words, even if the original signal is distorted by some

unknown factors, we can still obtain an accurate reconstruction result without any prior knowledge of the characteristics of the profilometry system. The correctness and effectiveness of our proposed model and shift estimation (SE) method have been confirmed by our simulation results.

References

1. R. Green, J. Walker, and D. Robinson, “Investigation of the Fourier-transform Method of Fringe Pattern Analysis,” *Optics and Lasers in Engineering* **8**, 29–44 (1988).
2. X. Su and W. Chen, “Fourier transform profilometry: a review,” *Optics and Lasers in Engineering* **35**, 263–284 (2001).
3. J. Yi and S. Huang, “Modified Fourier transform profilometry for the measurement of 3-D steep shapes,” *Optics and Lasers in Engineering* **27**, 493–505 (1997).
4. X. Su, W. Chen, Q. Zhang, and Y. Chao, “Dynamic 3-D shape measurement method based on FTP,” *Optics and Lasers in Engineering* **36**, 49–64 (2001).
5. V. Srinivasan, H. Liu, and M. Halioua, “Automated phase-measuring profilometry of 3-D diffuse objects,” *Applied Optics* **23**, 3105–3108 (1984).
6. X. Su, W. Zhou, von Bally G, and D. Vukicevic, “Automated phase-measuring profilometry using defocused projection of Ronchi grating,” *Optics Communication* **94**, 561–573 (1992).
7. H. Su, J. Li, and X. Su, “Phase algorithm without the influence of carrier fre-

- quency,” *Optics Engineering* **36**, 1799–1805 (1997).
8. S. Toyooka and Y. Iwaasa, “Automatic profilometry of 3-D diffuse objects by spatial phase detection,” *Applied Optics* **25**, 1630–1633 (1986).
  9. R. Rodriguez-Vera and M. Servin, “Phase locked loop profilometry,” *Optics and Lasers Technology* **26**, 393–398 (1994).
  10. A. Moore and F. Mendoza-Santoyo, “Phase demodulation in the space domain without a fringe carrier,” *Optics and Lasers in Engineering* **23**, 319–330 (1995).
  11. J. Villa, M. Servin, and L. Castillo, “Profilometry for the measurement of 3-D object shapes based on regularized filters,” *Optics Communication* **161**, 13–18 (1999).
  12. L. Kinell, “Multichannel method for absolute shape measurement using projected fringes,” *Optics and Lasers in Engineering* **41**, 57–71 (2004).
  13. P. Huang, Q. Ho, F. Jin, and F. Chiang, “Colour-enhanced digital fringe projection technique for high-speed 3D surface contouring,” *Opt Eng* **38**, 1065–1071 (1999).
  14. F. Berryman, P. Pynsent, and J. Cubillo, “A theoretical comparison of three fringe analysis method for determining the three dimensional shape of an object in the present of noise,” *Optics and Lasers in Engineering* **39**, 35–50 (2003).