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Special Class of Positive Definite Functions for Formulating Adaptive Micro/Nano Manipulator Control

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Abstract—This paper presents a special class of positive definite functions for the formulation of adaptive control strategies, specifically in the research of an effective control algorithm for piezoelectric actuation systems in micro/nano manipulation. To deal with the control problems of unknown system parameters, nonlinear hysteresis effects, and disturbances in the piezoelectric actuation systems, an adaptive control methodology is proposed. Using the saturation function derived from a positive definite function to formulate the control methodology, the closed-loop system stability can be guaranteed. Furthermore, the control methodology is proposed to track a desired motion trajectory in position, velocity, and acceleration. In this paper, a special positive definite function is introduced and a control formulation as well as a stability analysis is detailed. Implementation of the control methodology is practical and requires only a knowledge of the initial estimate of the system parameters. In this study, control experiments conducted using the proposed control approach on a piezoelectric actuation system has demonstrated a promising ability in tracking a specified motion trajectory. With this control capability in the presence of unknown system parameters, hysteresis, and external disturbance, the adaptive methodology is very attractive in the field of micro/nano manipulation in which high performance control applications could be realised.

I. INTRODUCTION

Piezoelectric actuation systems play a significant role in micro/nano manipulation for many research frontiers such as biomedical engineering, micro manufacturing and assembly, nano technology, nano robotics, and micro surgery. To achieve high-precision motion tasks, piezoelectric actuation systems have been identified as the most effective means due to their high stiffness, fast response, compact size, and physically unlimited motion resolution. In recent years, advancements in piezoelectric actuator (PEA) designs, sensing devices such as laser interferometer encoders, capacitive sensors, strain gauges, and linear variable displacement transducers, and developments of flexure-based mechanisms [1]–[3] have further boosted the importance of the micro/nano manipulation systems in these steadily growing technological areas.

In the control of the micro/nano manipulator systems, there exists in the PEAs a highly nonlinear hysteretic relationship

between the input (applied) voltage and the output displacement. This prevents the PEA from providing the desired high-precision motion resolution and accuracy. Research has been conducted in this area to model and compensate for the hysteresis effect. Some examples include a modelling of physical hysteresis [4], a dynamic model of hysteresis for a bi-morph beam [5], a comprehensive voltage-input electromechanical model [6], a differential model of hysteresis and its identification [7], and a charge steering model that bypasses hysteretic problem coupled with a comprehensive model of mechanical dynamics of the PEA [8]. In general, the modelling is done for a particular shape of hysteresis and the compensation is usually confined to feed-forward of a fixed pattern of input voltages to the PEA systems during positioning. As hysteresis effect is commonly considered as a nonlinear frequency-/rate-dependent [9] [10] or charge-dependent [6] phenomenon, it is therefore difficult to achieve a good positioning result for a PEA operating at uncertain rate and/or magnitude. Furthermore, there is no means to guarantee the positioning performance with the open-loop approaches.

On the other hand, appropriate control strategies can be formulated to take the nonlinear hysteresis effect into account to achieve high-precision positioning of the PEA systems. Recent examples include a combination of a feed-forward model in feedback control with an input shaper [11], an adaptive back-stepping approach [12], a PID-based control with iterative learning plus a disturbance observer [13], a model-based open-loop control [14], and a nonlinear observer-based variable structure control [15]. These control strategies are mostly formulated for specific applications or mechanisms and are usually implemented to track only reference positions.

In this paper, several ideas are gathered for the control of micro/nano manipulators, particularly for the piezoelectric actuation systems. First, a special class of positive definite functions is identified for the purpose of control formulation. Second, an adaptive control methodology is formulated for the PEA systems. Third, a control objective of tracking a

Fig. 1. Schematic model of a piezoelectric actuator (PEA)

desired motion trajectory in position, velocity, and acceleration is proposed for the closed-loop control of the PEA systems.

The proposed adaptive methodology is an enhancement to the original adaptive scheme proposed by [16]. As commented in [17], this original scheme ensures only convergence to zero of the velocity tracking error but does not ensure asymptotic zeroing of the positioning error. For this reason, the saturation function derived from a special class of positive definite functions is employed in the control formulation to resolve the shortcoming in the original scheme.

In this approach, an adaptive scheme is introduced to adjust the control signal to accommodate the unknown system parameters, nonlinear hysteresis effect, and external disturbance to the control system. The stability of the control methodology is proven theoretically and the controller is able to steer the PEA system to closely track a desired motion trajectory in position, velocity, and acceleration. Implementation of the control system is practical and requires only the estimated system parameters.

Control experiments conducted in a PEA system have validated the feasibility of the proposed adaptive control approach. Furthermore, a promising tracking performance has also been demonstrated. With the ability to handle unknown system parameters, hysteresis, and disturbance, the proposed control methodology is very attractive in the high performance PEA control applications, through which high-precision micro/nano-manipulation systems could be realised.

This paper is organised as follows. The model of a piezoelectric actuator is introduced in Section II. Section III describes a special positive definite function for the purpose of control formulation. The proposed adaptive control methodology is formulated in Section IV followed by the stability analysis in Section V. The experimental study is detailed in Section VI and the results are presented and discussed in Section VII. Finally, conclusions are drawn in Section VIII.

II. MODEL OF PIEZOELECTRIC ACTUATOR

An electromechanical model of a PEA is given in [6], [8]. This mathematical model can be divided into three stages of transformation from electrical to mechanical energy, and vice versa. The schematic model, as shown in Fig. 1, illustrates the transformation, which consists of voltage-charge stage, piezo stage, and force-displacement stage. Note that the model in Fig. 1 is formulated for a voltage-controlled amplifier. The

dynamic equation from the electrical input to the output motion stage can be described by the following set of equations:

$$v_{in} = v_h + v_z, \quad (1)$$

$$v_h = H(q), \quad (2)$$

$$q = C v_z + q_z, \quad (3)$$

$$q_z = T_{em} x, \quad (4)$$

$$f_z = T_{em} v_z, \quad (5)$$

$$m_z \ddot{x} + b_z \dot{x} + k_z x = f_z - f_{ext}, \quad (6)$$

where v_{in} represents the applied (input) voltage, v_h is the voltage due to hysteresis, v_z is the voltage related to mechanical side of the actuator, q is the total charge in the ceramic, H is the hysteresis effect, C is the linear capacitance connected in parallel with the electromechanical transformer having a ratio of T_{em} , q_z is the piezo charge related to the actuator output displacement x , and f_z is the transduced force from the electrical domain. The variables m_z , b_z , and k_z are the mass, damping, and stiffness, respectively, of the force-displacement stage, and f_{ext} is the force imposed by the external mechanical load. In PEA, hysteresis causes a highly nonlinear input/output relationship between the applied voltage and displacement.

For control purposes, (1) and (5) are substituted into (6) to yield

$$m_z \ddot{x} + b_z \dot{x} + k_z x = T_{em} (v_{in} - v_h) - f_{ext}, \quad (7)$$

and the PEA model is obtained by re-arranging (7),

$$m \ddot{x} + b \dot{x} + k x + v_h + f_e = v_{in}, \quad (8)$$

where $m = m_z / T_{em}$, $b = b_z / T_{em}$, $k = k_z / T_{em}$, and $f_e = f_{ext} / T_{em}$.

III. SPECIAL POSITIVE DEFINITE FUNCTION

A class of positive definite functions plays an important role in formulating a control strategy. A special function of this class which is twice continuously differentiable gives rise to a saturation function and a strictly positive function in its first and second derivatives, respectively. With these characteristics, the saturation function can be used to derive the control methodology and stability of the closed-loop system can be guaranteed. For example, in a control formulation, a special function, namely ρ , can be written as

$$\rho(e_p) = \sqrt{\varepsilon^2 + e_p^2} - |\varepsilon|, \quad (9)$$

where ρ is positive definite expressed as a function of a position tracking error e_p , which is bounded, and ε is an arbitrary constant with its absolute value chosen to be $|\varepsilon| > 0$. The saturation function $s(e_p)$, which is the first derivative, is expressed as

$$s(e_p) = \frac{d\rho(e_p)}{de_p} = \frac{e_p}{\sqrt{\varepsilon^2 + e_p^2}}, \quad (10)$$

and the second derivative $\varrho(e_p)$ is obtained as

$$\varrho(e_p) = \frac{ds(e_p)}{de_p} = \frac{\varepsilon^2}{\sqrt{(\varepsilon^2 + e_p^2)^3}}. \quad (11)$$

$$\dot{s}(e_p) = \varrho(e_p)\dot{e}_p = \frac{\varepsilon^2 \dot{e}_p}{\sqrt{(\varepsilon^2 + e_p^2)^3}}, \quad (12)$$

The saturation function given in (10) will be employed in the following section to formulate an adaptive control methodology. The special positive function described by (9) and its properties given in (10) and (12) will be used in the stability analysis of the proposed control system.

The structure of the proposed adaptive control methodology for the PEA system is summarised in Fig. 2.

$$w = (b + k_v) \dot{e}_p^2 + \alpha (k + k_p) e_p s(e_p) - \alpha m \dot{s}(e_p) \dot{e}_p, \quad (28)$$

and $\rho(e_p)$ is given in (9). The term u_1 in (27) can be rewritten as

$$u_1 = \frac{1}{4} [\dot{e}_p + 2\alpha s(e_p)] m [\dot{e}_p + 2\alpha s(e_p)] + \frac{1}{4} m \dot{e}_p^2 - \alpha^2 m s^2(e_p) + \alpha (b + k_v) \rho(e_p) + \frac{1}{2} (k + k_p) e_p^2. \quad (29)$$

Using (10) to replace the term $s^2(e_p)$ in (29),

$$u_1 = \frac{1}{4} m [\dot{e}_p + 2\alpha s(e_p)]^2 + \frac{1}{4} m \dot{e}_p^2 + \left[\frac{k + k_p}{2} - \frac{\alpha^2 m}{\varepsilon^2 + e_p^2} \right] e_p^2 + \alpha (b + k_v) \rho(e_p). \quad (30)$$

As $1/\varepsilon^2 \geq 1/(\varepsilon^2 + e_p^2)$, u_1 will be positive definite if the control gains, k_p and k_v , in (30) are chosen as

$$\begin{aligned} k_p &> \frac{2\alpha^2 m}{\varepsilon^2} - k, \\ k_v &> -b. \end{aligned} \quad (31)$$

Using (10) and (12) to replace the terms $s(e_p)$ and $\dot{s}(e_p)$, respectively, in (28),

$$w = \frac{\alpha (k + k_p)}{\sqrt{\varepsilon^2 + e_p^2}} e_p^2 + \left[b + k_v - \frac{\alpha m \varepsilon^2}{\sqrt{(\varepsilon^2 + e_p^2)^3}} \right] \dot{e}_p^2. \quad (32)$$

To ensure a positive definite w , the control gains in (32) must be selected such that

$$\begin{aligned} k_p &> -k, \\ k_v &> \frac{\alpha m}{\varepsilon} - b. \end{aligned} \quad (33)$$

It is possible to relate the left-hand side of (25) to a positive definite function u_2 given as

$$u_2 = \frac{1}{2} \Delta \varphi^T K \Delta \varphi. \quad (34)$$

Differentiating u_2 with respect to time yields

$$\frac{du_2}{dt} = \Delta \varphi^T K \Delta \dot{\varphi}. \quad (35)$$

Due to the fact that the parameters φ of system (14) are time-invariant, i.e. $\dot{\varphi} = 0$,

$$\Delta \dot{\varphi} = \dot{\varphi} - \varphi = \dot{\varphi}, \quad (36)$$

where $\dot{\varphi}$ is given in (16). Substituting (16) into (35),

$$\begin{aligned} \frac{du_2}{dt} &= \Delta \varphi^T K [-K^{-1} x_d y], \\ &= -[x_d^T \Delta \varphi]^T y, \\ &= -\Delta v y, \end{aligned} \quad (37)$$

where the scalar Δv is defined in (21).

Theorem: For the PEA system described by (8), the control law (19) with the adaptive scheme (16) assures the convergence of motion trajectory tracking with $e_p(t) \rightarrow 0$ and $\dot{e}_p(t) \rightarrow 0$ as $t \rightarrow \infty$ under the conditions of (31) and (33).

Fig. 3. Block diagram of PEA experimental system

Proof: In the closed-loop system formed by the system (8) and the control law (19) with the adaptive scheme (16), the functions, u_1 and w , from (27) and (28), respectively, are positive definite in all possible values of $e_p(t)$ and $\dot{e}_p(t)$ under the conditions of (31) and (33). Furthermore, the function u_2 is also chosen to be positive definite in (34). A continuous and non-negative Lyapunov function u can therefore be proposed as

$$u = u_1 + u_2. \quad (38)$$

The time derivative of u can be obtained by combining (26) and (37) as

$$\frac{d}{dt}(u_1 + u_2) = -w, \quad (39)$$

which shows that $u \rightarrow 0$ and implies $e_p(t) \rightarrow 0$ and $\dot{e}_p(t) \rightarrow 0$ as $t \rightarrow \infty$. Both system stability and tracking convergence are guaranteed by the control law (19) with adaptive scheme (16) driving the system (8) following the desired motion trajectory closely in terms of $x_d(t)$ and $\dot{x}_d(t)$. ■

VI. EXPERIMENTAL STUDY

In the process of developing a micro/nano manipulation system using the PEAs, a single-axis PEA is set up for an experimental study of the proposed control strategy. The experimental system, as shown in Fig. 3, consists of a PEA assembly, an amplifier module, a position signal processing unit, and a control PC installed with a digital-to-analogue (D/A) and an analog-to-digital (A/D) boards. The PEA employed is a PI (Physik Instrumente) multilayer PZT stacked ceramic translator, model P-843.30, capable of expansion of up to $45 \mu m$ corresponding to a range of operating voltage up to $100 V$. The PEA is preloaded $300 N$ by an internal spring and is incorporated with an high-resolution strain gauge sensor for position feedback. The amplifier module is a PI model E-505.00 with a fixed output gain of 10 providing voltage ranges from -20 to $+120 V$. The position signal processing unit is housed in a PI servo controller, model E-509-S3. A standard desktop computer is employed as the control PC. It is equipped with a Pentium 4 $2.8 GHz$ processor running on an operating system capable of hard real-time control. The D/A and A/D boards installed in the control PC are of 16-bit resolution, and they are used to generate the control signal and to read the analog position, respectively. In the control experiments, the sampling frequency of the control-loop is set at $2.5 kHz$.

The control experiments serve not only to validate the theoretical formulation of the control algorithm but also to examine the effectiveness of the proposed scheme in a physical

Fig. 4. Desired motion trajectory

PEA system. In the experimental study, the closed-loop system is required to follow a desired motion trajectory, which is shown in Fig. 4 for position, velocity, and acceleration. The desired motion trajectory is formed by segments of quintic polynomials [18] for the implementation and analysis of the tracking and steady-state performances of the system.

For the PEA system described in (8), the proposed adaptive control methodology presented in Section IV is implemented in the control PC. In following the desired motion trajectory, the tracking ability of the control system can be closely examined when it is subjected to uncertain parameters including unknown disturbance and hysteresis effects.

Notice that the proposed control law (19) consists of two parts; a conventional PD feedback control and an adaptive component $\hat{v}_{in}$. To study the effectiveness of the adaptive control strategy, a PD feedback control is implemented for comparison by omitting the adaptive component in (19), i.e.

$$v_{in} = -k_p e_p - k_v \dot{e}_p + \dot{f}_e. \quad (40)$$

In the control experiment, the initial estimate $\hat{\varphi}(0)$ for (16) is chosen to be zero. The control gains, k_p and k_v , in (19) are tuned to the following values:

$$k_p = 4 \times 10^6 \text{ (V/m)} \quad \text{and} \quad k_v = 1 \times 10^3 \text{ (Vs/m)}. \quad (41)$$

It is assumed that no external force is applied to the system and the f_e terms in (19) and (40) are ignored in the control experiment. Furthermore, the diagonal constant matrix $\mathbf{K}$ in (16) is selected as

$$\mathbf{K}^{-1} = 2 \times 10^6 \text{ diag}\{1, 1, 1, 1\}, \quad (42)$$

where the units of $\mathbf{K}^{-1}$ are (Vs^4/m^3) , (Vs^2/m^3) , (V/m^3) , and (V/m) , respectively. The saturation function $s(e_p)$ in (17) is implemented as given in (10) and the arbitrary constant ε is chosen as

$$\varepsilon = 1 \times 10^{-6} \text{ (m)}. \quad (43)$$

Furthermore, the positive scalar α in (17) is set at

$$\alpha = 1 \times 10^{-6} \text{ (m/s)}. \quad (44)$$

For comparison, the PD control in (40) is implemented using the same control gains as given in (41).

Fig. 5. Adaptive control: Actual position and estimated velocity

Fig. 6. Adaptive control: Control input and position tracking error

VII. RESULTS AND DISCUSSION

The experimental results of the adaptive control methodology are shown in Fig. 5 and Fig. 6. Given the desired motion trajectory as shown in Fig. 4, the PEA was commanded to travel within a range of $30 \mu\text{m}$ with a maximum velocity and an acceleration reaching 1.1 mm/s and 0.07 m/s^2 , respectively. The resulting PEA position and the estimated velocity are shown in Fig. 5. Despite system uncertainties and zero initial estimate for the adaptive scheme (16), the control law (19) was shown to be stable with the control input to the PEA as shown in Fig. 6. Furthermore, from the position tracking error, as presented in Fig. 6, the closed-loop system was shown to track the desired motion trajectory with position tracking error within $0.48 \mu\text{m}$ during motion and less than $0.03 \mu\text{m}$ at steady-state, which was almost at the noise level of the closed-loop system.

In comparison, the control results of the PD control, as shown in Fig. 7, indicates that the PD control does not provide enough actuating voltage to the PEA. The PD control resulted in relatively large position tracking error, which was more than $10 \mu\text{m}$. On the other hand, this implies that the adaptive component of the proposed control approach is effective and plays a significant role in the tracking performance.

Notice that the resulting position tracking error of the

Fig. 7. PD control: Control input and position tracking error

adaptive control methodology is much lower than the chosen value of ε in (43) for (10). For this fact, the saturation function $s(e_p)$ in (17) has little effect on the experimental study. Nevertheless, the use of the saturation function that is derived from the special class of positive definite functions described in Section III allows for the proof of stability of the proposed control system.

In summary, the proposed adaptive control methodology was shown to be stable and capable of following the desired motion trajectory under the unknown system parameters, nonlinear hysteresis effect, and disturbance. Good control results were achieved even with the initial estimate $\hat{\varphi}(0)$ set to zero for the adaptive scheme (16) in the experimental study. However, some effort was needed to tune the system for the desirable control performance.

VIII. CONCLUSIONS

Several ideas have been proposed in this paper. First, a special class of positive finite functions has been identified for the control formulation. Second, an enhanced adaptive control methodology has been formulated. Third, control objective of tracking a desired motion trajectory in the control methodology has been proposed for the closed-loop control of the piezoelectric actuation systems.

Using the saturation function derived from the special function in formulating the enhanced adaptive control methodology, the stability of the closed-loop system is guaranteed. Furthermore, the stability analysis has shown that both the position and velocity tracking errors converged to zero in tracking a desired motion trajectory.

Implementation of the proposed control methodology requires only the estimated system parameters for the initial conditions of the adaptive scheme. In the experimental study, a promising tracking ability has been demonstrated, and in comparison, the conventional PD scheme performed poorly without the adaptive compensation.

Being capable of motion tracking under unknown system parameters, nonlinear hysteresis effect, and disturbance, the enhanced adaptive control methodology is very attractive in the

field of micro/nano-manipulation in which high performance PEA control applications could be realised.

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REFERENCES

- [1] S. S. Ku, U. Pinsopon, S. Cetinkunt, and S. Nakajima, "Design, fabrication, and real-time neural network control of a three-degrees-of-freedom nanopositioner," *IEEE/ASME Transactions on Mechatronics*, vol. 5, no. 3, pp. 273–280, September 2000.
- [2] K. Spanner and S. Vorndran, "Advances in piezo-nanopositioning technology," in *Proceedings of 2003 IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, vol. 2, Kobe, Japan, 20–24 July 2003, pp. 1338–1343.
- [3] B. J. Yi, G. B. Chung, H. Y. Na, W. K. Kim, and I. H. Suh, "Design and experiment of a 3-DOF parallel micromechanism utilizing flexure hinges," *IEEE Transactions on Robotics and Automation*, vol. 19, no. 4, pp. 604–612, August 2003.
- [4] Y. I. Somov, "Modelling physical hysteresis and control of a fine piezo-drive," in *Proceedings of 2003 International Conference Physics and Control*, vol. 4, St. Petersburg, Russia, 20–22 August 2003, pp. 1189–1194.
- [5] T. S. Low and W. Guo, "Modeling of a three-layer piezoelectric bimorph beam with hysteresis," *Journal of Microelectromechanical Systems*, vol. 4, no. 4, pp. 230–237, December 1995.
- [6] M. Goldfarb and N. Celanovic, "Modeling piezoelectric stack actuators for control of micromanipulation," *IEEE Control Systems Magazine*, vol. 17, no. 3, pp. 69–79, June 1997.
- [7] R. Banning, W. L. D. Koning, H. J. M. T. A. Adriaens, and R. K. Koops, "State-space analysis and identification for a class of hysteretic systems," *Automatica*, vol. 37, no. 12, pp. 1883–1892, December 2001.
- [8] H. J. M. T. A. Adriaens, W. L. D. Koning, and R. Banning, "Modeling piezoelectric actuators," *IEEE/ASME Transactions on Mechatronics*, vol. 5, no. 4, pp. 331–341, December 2000.
- [9] D. Song and C. J. Li, "Modeling of piezo actuator's nonlinear and frequency dependent dynamics," *Mechatronics*, vol. 9, no. 4, pp. 391–410, June 1999.
- [10] X. Dang and Y. Tan, "Neural networks dynamic hysteresis model for piezoceramic actuator based on hysteresis operator of first-order differential equation," *Physica B: Condensed Matter*, vol. 365, no. 1–4, pp. 173–184, 1 August 2005.
- [11] T. Chang and X. Sun, "Analysis and control of monolithic piezoelectric nano-actuator," *IEEE Transactions on Control Systems Technology*, vol. 9, no. 1, pp. 69–75, January 2001.
- [12] H. J. Shieh, F. J. Lin, P. K. Huang, and L. T. Teng, "Adaptive tracking control solely using displacement feedback for a piezo-positioning mechanism," *IEEE Proceedings of Control Theory and Applications*, vol. 151, no. 5, pp. 653–660, 23 September 2004.
- [13] Y. C. Huang and C. H. Cheng, "Robust tracking control of a novel piezodriven monolithic flexure-hinge stage," in *Proceedings of 2004 IEEE International Conference on Control Applications*, vol. 2, Taipei, Taiwan, 2–4 September 2004, pp. 977–982.
- [14] G. Schitter and A. Stemmer, "Identification and open-loop tracking control of a piezoelectric tube scanner for high-speed scanning-probe microscopy," *IEEE Transactions on Control Systems Technology*, vol. 12, no. 3, pp. 449–454, May 2004.
- [15] C. L. Hwang, Y. M. Chen, and C. Jan, "Trajectory tracking of large-displacement piezoelectric actuators using a nonlinear observer-based variable structure control," *IEEE Transactions on Control Systems Technology*, vol. 13, no. 1, pp. 56–66, January 2005.
- [16] J. J. E. Slotine and W. Li, "Adaptive manipulator control: A case study," *IEEE Transactions on Automatic Control*, vol. 33, no. 11, pp. 995–1003, November 1988.
- [17] M. W. Spong, R. Ortega, and R. Kelly, "Comments on 'adaptive manipulator control: a case study'," *IEEE Transactions on Automatic Control*, vol. 35, no. 6, pp. 761–762, June 1990.
- [18] J. J. Craig, *Introduction to Robotics: Mechanics and Control*. Reading, MA: Addison-Wesley, 1989.